Improved Implementation of Chua's Chaotic Oscillator Using Current Feedback Op Amp

A. S. Elwakil and M. P. Kennedy

***Abstract*—An improved implementation of Chua's chaotic oscillator is proposed. The new realization combines attractive features of the current feedback op amp (CFOA) operating in both voltage and current modes to construct the active three-segment voltage-controlled nonlinear resistor. Several enhancements are achieved: The component count is reduced and the chaotic spectrum is extended to higher frequencies. In addition, a buffered and isolated voltage output directly representing a state variable is made available. Based on a linearized model of Chua's circuit, the useful tuning range of the major bifurcation parameter (G) and the expected frequency of oscillation, are estimated.**

***Index Terms*—Chaos, chaos generators, chaotic oscillators.**

I. Introduction

Chua's chaotic oscillator has served as the primary reference for studying and generating chaos in electronic systems [1]–[3]. The circuit exhibits rich dynamics that demonstrate most well-known routes to chaos and has therefore been studied extensively [4]–[7]. The architecture of this oscillator is based on coupling a tank resonator (parallel L-C_2 section in Fig. 1(a)) to an active three-segment voltage-controlled nonlinear resistor (Chua's diode) through a filter section (R-C_1 section in Fig. 1(a)). A realization of the nonlinear resistor by connecting two voltage-controlled negative impedance converters in parallel has been accepted as a standard [8]. This realization uses two conventional voltage op amps (VOA's), operating in both their linear and nonlinear regions of operation, and six resistors. Since the nonlinear resistor is the only active element in Chua's circuit, it imposes the limit on the operating frequency due to the op amp's nonideal phase characteristics. Aiming for high-frequency chaotic signals, improved designs of Chua's diode should be introduced. Recently, proposed chaos based communication systems have used one of the two op amp outputs as the carrier signal. It is thus of great advantage to have this output buffered and isolated from the other circuit components. Moreover, if this output can directly represent one of the system state variables, subsequent analysis is simplified.

In this work, the attractive combined voltage-current capabilities of a CFOA are used to synthesize Chua's diode, resulting in a design that requires two fewer resistors than the standard of [8]. In addition, a buffered and isolated voltage output that directly represents a state variable is made available while the operating frequency is extended. We also investigate the possibility of providing a chaotic current output signal which might be useful in some applications. Based on a linearized model of Chua's circuit, we attempt to estimate the useful range for tuning the main bifurcation parameter $(G = 1/R)$ and the expected frequency of oscillation.

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II. The Current Feedback Op Amp

The four-terminal CFOA is a versatile analog building block that is recognized for its excellent performance in high-speed and high-slew–rate analog signal processing [9]. It is a cascade of a second-generation current conveyor (CCII) [10] and a voltage buffer. The CFOA describing matrix is given by

$$\begin{bmatrix} V_- \\ I_+ \\ I_C \\ V_O \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} I_- \\ V_+ \\ V_C \\ I_O \end{bmatrix} \tag{1}$$

where the inverting and noninverting inputs $(-, +)$ are characterized by a very low and a very high input impedance, respectively, while current and voltage outputs are delivered to the C and O terminals respectively.

By connecting an open circuit across the C terminal, the CFOA works as a VOA. Doing the same at the O terminal, the CFOA works as a CCII. The frequency and dynamic range limitations of a CFOA are inherited from its core CCII [11]. A comprehensive study of the CFOA's behavior can be found in [12]and [13]; a large number of application circuits were introduced in [14]. An excellent review of modern complementary bipolar high-speed op amps, including CFOA's, is given in [15].

III. Circuitry and PSpice Simulation

Our novel realization of Chua's circuit is shown in Fig. 1(a). The methodology of [8] in which two voltage-controlled negative impedance converters (VNIC's) are connected in parallel is followed. According to the analysis in [8], one of the VNIC's operates only in its linear region. Hence, this NIC can be simply realized using one CCII and one grounded resistor, instead of one VOA and three resistors. A CFOA is thus connected as a CCII with resistor R_4 to operate as a linear NIC in Fig. 1(a). With this connection, it turns out that the buffered output terminal (V_O) of this CFOA is available for direct access and is isolated from all other components. Recall from (1) that V_O is equal to V_C which is by connection forced to V_+. However, V_+ is just the voltage across C_1. Hence, V_O directly represents a state variable of the system. Resistors R_1, R_2, R_3 and the associated CFOA operate as a nonlinear VNIC, as required by [8]. In this case, the CFOA works as a VOA under the condition that the C terminal is an open circuit or, equivalently, the current I in Fig. 1(a) is zero. A PSpice simulation of a double scroll attractor is shown in Fig. 1(b) with $L = 180\ \mu$H, $C_2 = 1$ nF, $R = 1.6$ KΩ, $C_1 = 100$ pF, $R_1 = R_2 = 22$ KΩ, $R_3 = 3.3$ KΩ, $R_4 = 2.2$ KΩ and using the commercial AD844 CFOA biased with ±9 V. The same values of R, R_1, R_2, R_3, and R_4 as in [8] are used. However, the inductor and capacitors are scaled down to extend the operating frequency, making use of the excellent performance of the CFOA at high frequencies. Instead of wasting the current output (I) of the first CFOA, we investigated the effect on the chaotic behavior of using it to drive a load resistor. It was found that the smaller the load (higher driving capability) the smaller the value of R_3 required to maintain chaos. The output current (I) and voltage (V_O) are shown in Fig. 2 with a 5-kΩ load and with $R_3 = 500\ \Omega$. Current saturation is obvious since the operation of the first CFOA is not confined to its linear region.

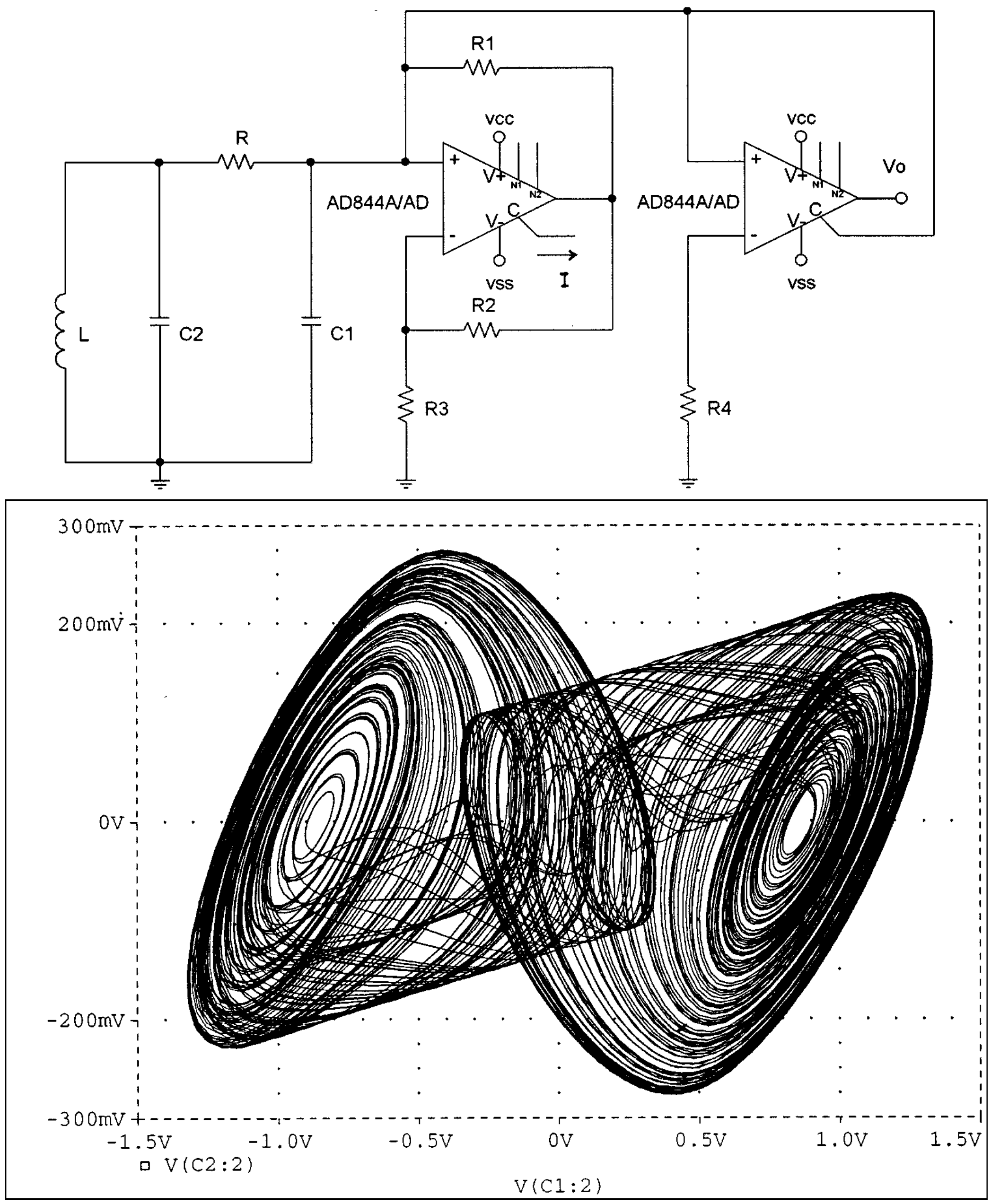

Fig. 1(a). The improved Chua's circuit realization. (b). PSpice simulation of the $\boldsymbol{V}_{C1} - \boldsymbol{V}_{C2}$ phase space trajectory.

IV. Estimation of the Tuning Range and Generated Frequency

The dynamic behavior of Chua's circuit can be adjusted by varying any of its elements. However, using resistor $R\ (1/G)$ as a bifurcation parameter is generally adopted. Thus estimating the possible tuning range of R is useful. Furthermore, estimation of the expected operating frequency of Chua's circuit for specific parameters is also useful. Chua's circuit equations are given by

$$\begin{aligned} C_1\dot{V}_{C1} &= G(V_{C2} - V_{C1}) - f(V_{C1}) \\ C_2\dot{V}_{C2} &= G(V_{C1} - V_{C2}) + I_L \\ L\dot{I}_L &= V_{C2} \end{aligned} \tag{2a}$$

and

$$f(V_{C1}) = m_1 V_{C1} + \frac{1}{2}(m_0 - m_1)[|V_{C1} + B_p| - |V_{C1} - B_p|] \tag{2b}$$

where m_1 and m_0 are the slopes of the outer and inner segments of the three-segment piecewise-linear characteristic of Chua's diode and $\pm B_p$ are the two break points.

A linearized Chua's circuit model has a characteristic equation given by

$$s^3 + a_2 s^2 + a_1 s + a_0 = 0 \tag{3}$$

where

$$a_2 = \frac{G}{C_2} + \frac{G + m_i}{C_1}, \quad a_1 = \frac{1}{LC_2} + \frac{G + m_i}{C_1 C_2} \text{ and } a_0 = \frac{G + m_i}{LC_1 C_2}.$$

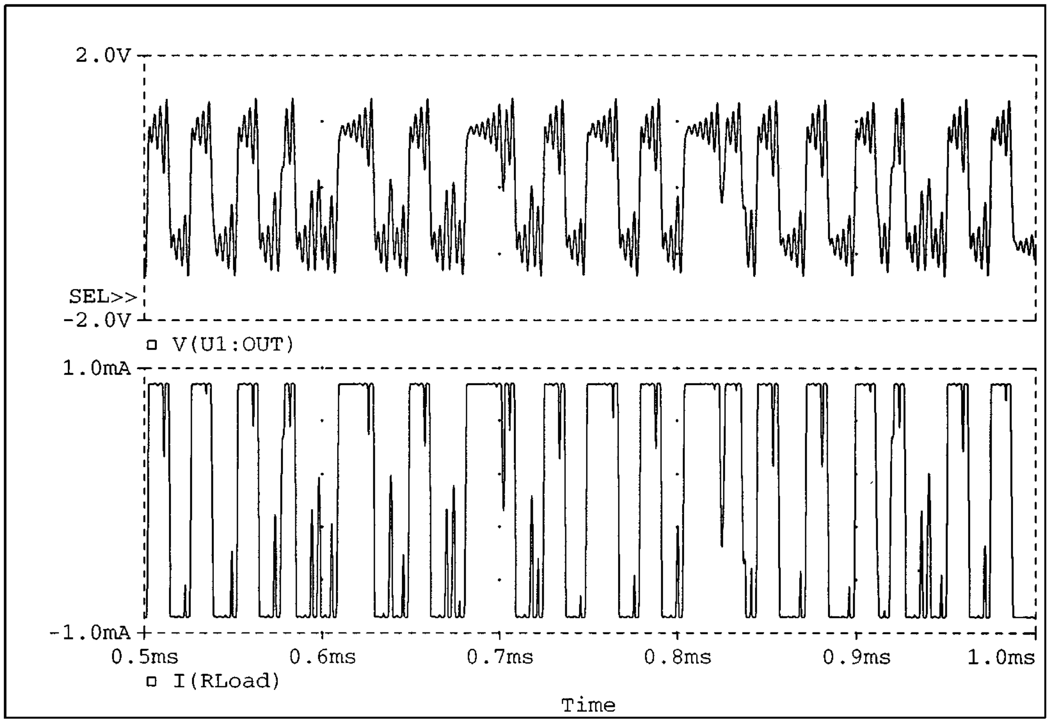

Fig. 2. Voltage and current outputs with a 5-KΩ load.

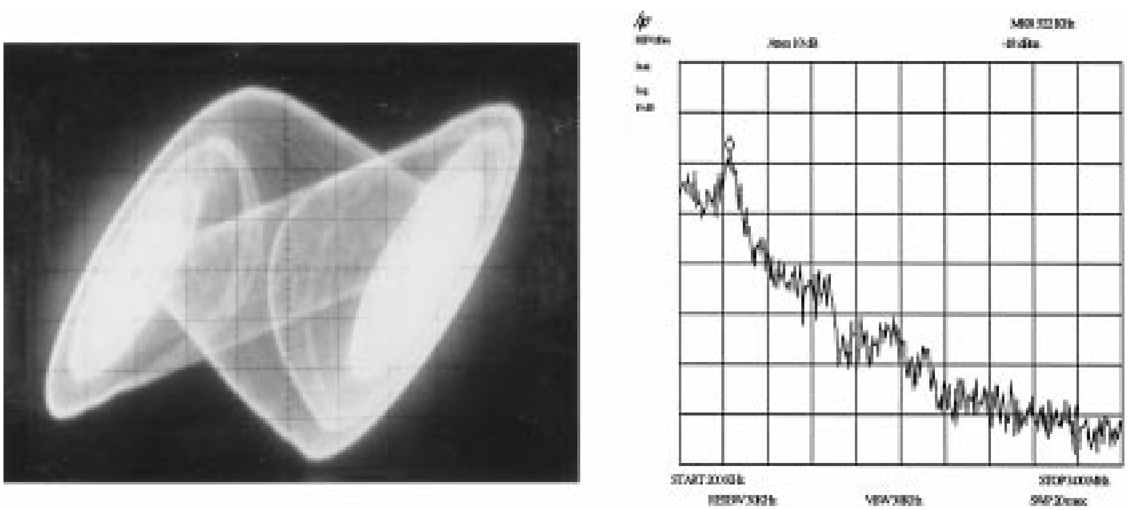

Fig. 3. (a) An experimental $\boldsymbol{V}_{C1} - \boldsymbol{V}_{C2}$ trajectory. Horizontal axis: $\boldsymbol{V}_{C1}$ (0.2 V/cm), Vertical axis: $\boldsymbol{V}_{C2}$ (0.2 V/cm). (b) Experimental measurement of the chaotic spectrum.

The condition for oscillation $(a_1 a_2 = a_0)$ and radian frequency of oscillation $(\omega_0 = \sqrt{a_1})$ are thus found to be

$$G = -\frac{C_1^2 + LC_2 m_i^2}{Lm_i(C_1 + C_2)} \tag{4a}$$

$$\omega_0 = \frac{1}{\sqrt{LC_2}} \sqrt{1 + \frac{GLm_i}{C_1}} \tag{4b}$$

with the constraint $|\frac{GLm_i}{C_1}| < 1$. Note that with no coupling $(G = 0)$, the oscillating frequency is that of the resonant tank, as expected.

A widely used design set is $C_1 = 1/9, C_2 = 1, L = 1/7$ and $m_i = \begin{cases} -0.8 \text{ in the inner segment} \\ -0.5 \text{ in the outer segments} \end{cases}$. The corresponding range for G calculated from (4a) is: $0.6055 < G < 0.8172$. Choosing $G = 0.7$, the range for the frequency of oscillation from (4b) is $1.4 < \omega_o < 1.962$. Appropriate scaling can then be used to estimate actual values.

V. Experimental Results

The circuit of Fig. 1(a) was constructed taking $L = 120$ μH, $C_2 = 470$ pF, $R = 2$ kΩ pot., $C_1 = 56$ pF, $R_1 = R_2 = 22$ kΩ, $R_3 = 5$ kΩ pot. and $R_4 = 2.2$ kΩ. The observed $V_{C1} - V_{C2}$ trajectory is shown in Fig. 3(a). It was observed that a wider range for tuning can be obtained by varying R_3, rather than R. The chaotic frequency spectrum, shown in Fig. 3(b), is centred approximately around 522 kHz. It is worth noting that the parasitic capacitances at the CFOA C terminals have negligible effect, since this terminal is unused for one CFOA while for the other C_1 has a dominant swamping effect.

VI. Conclusion

An improved implementation of Chua's circuit has been introduced. The circuit provides a higher bandwidth of chaotic signal with buffered output. Functionality was demonstrated using a commercial CFOA. However, even higher operating frequencies can be achieved with recently reported CFOA designs, such as that in [16] which has a 3-dB bandwidth of 110 MHz. By adding a voltage buffer to the CCCII given in [17], [18], CFOA's that are suitable for telecommunication applications can be obtained. We note that another CFOA-based implementation of Chua's circuit has been reported recently [19]. Our design is superior in that it provides a buffered output voltage that directly represents a state variable, in addition to a current output signal. It should also be noted that other extended frequency chaotic oscillators using the CFOA have also been introduced recently [20], [21].

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DC-AC Power Inversion Using Σ-Δ Modulation

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***Abstract*—A feasibility study of Σ-Δ modulation as a control mechanism for low-frequency dc-ac power inverters with sinusoidal output voltage is presented. The sinusoidal dc-ac inverter, composed of a Σ-Δ 1-bit A/D converter and a half-bridge inverter, was built and tested. Measured Bode plots of the closed-loop response are given and the output voltage waveform observed at 50 Hz is shown.**

***Index Terms*—DC-AC inverters, Σ-Δ 1-bit A/D modulators.**

I. Introduction

Low-frequency dc-ac power inverters have been the focus of research for the last few decades [1]–[5]. These inverters have many applications, such as audio power amplifiers, uninterruptible power supplies, motor drives, and line conditioners. Pulse-width modulation (PWM) has been the workhorse technique in this field with a large selection of methods [1], [2]. Some of these methods are complex and sophisticated, ranging from vector PWM [3] and neural networks [4] to random PWM [5]. Other methods include sliding mode and multiple-feedback control schemes [8]–[11].

The purpose of this paper is to briefly investigate the feasibility of Σ-Δ modulation [6], [7] as a control strategy for switching-mode dc-ac low-frequency sinusoidal output voltage inverters. Advantages of this technique include simplicity of implementation, low-cost, low-switching noise, and high-quality sinusoidal output voltage.

II. Circuit Description

A buck circuit, from one viewpoint, is merely a high-power, one-bit digital-to-analog converter. By connecting the one-bit digital output of a Σ-Δ modulator to the one-bit digital input of a buck circuit, an inverter can be constructed.

A. Σ-Δ Modulator

The Σ-Δ modulator contains a summation amplifier, an integrator, two comparators, and a D flip-flop, as shown in Fig. 1. The circuit was constructed around three LF347 op amps and one 74C74 flip-flop, as depicted in Fig. 2. The summation and integration circuits were combined, using 10-kΩ resistors and a 0.01-μF capacitor, providing a reasonable approximation of an integrator with dc compensation. The forward-path comparator was made using a 5.1-V Zener diode to clamp

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