Pakrashi V., Basu, B., O'Connor A.J., 2005. Identification of open cracks using wavelet analysis. *IISC Structural Engineering Convention*. Bangalore, India

Identification of Open Cracks using Wavelet Analysis

V. Pakrashi, B. Basu and A. O' Connor

ABSTRACT

Damage detection in flexural members by wavelet analysis involves certain important factors such as the choice of wavelet function, the effects of windowing and the effects of masking due to the presence of noise during measurement. A numerical study has been performed in this paper addressing these issues for a beam element with an open crack. The first natural modeshape of a beam with an open crack has been simulated. Gaussian white noise has been synthetically introduced to the simulated modeshape and the onset of masking has been studied. A wavelet based method of damage detection can be useful in the identification of damaged bridge structures and is applicable under the presence of measurement noise as well.

Keywords: Open Crack, Wavelets, Signal to Noise Ratio, Windowing, Masking

INTRODUCTION

A major focus in the field of structural health monitoring and damage detection lies in consistent and efficient detection, localization and quantification of damage. Identification of damage through changes in natural frequencies and modeshapes of a beam with an open crack is a popular method. An open crack is often modeled as a rotational spring at the damage location, the stiffness of it being dependent on the crack depth ratio. Narkis [1] analysed one such model representing an open crack in beams and provided a closed form characteristic equation to calculate the cracked natural frequencies. Similar models have been used by Masoud et al [2], Dado [3], Hadjileontiadis et al [4] and Loutridis et al [5] as well. However, the changes are often quite small and they get affected when measurements are contaminated with noise.

A wavelet analysis based approach to analyse modeshapes is seen to provide a better and more robust methodology for identification of damage. On analysis of damaged modeshapes, a sharp change in wavelet coefficients is observed near the damage location and a local maximum of the coefficients is usually formed at the location of damage. The magnitude of this maximum value is related to the extent of damage. The detection of damage is based on the principle of detection of singularities of a function or any of its derivatives by wavelet analysis. The aspect of singularity detection through wavelets has been discussed in details by Mallat [6]. Gentile and Messina [7] carried out a study focussing mainly on Gaussian wavelets modelling the damage as an equivalent sub beam having a modified Young's modulus to cater for the sudden change at the damage location. Loutridis et al [5] used the same basis function to identify damage in a cracked cantilever beam using a rotational spring damage model. Chen [8] and Okafor and Dutta [9] have

considered similar problems concentrating on one wavelet basis function. Melhem and Kim [10] have analysed response of real structures and have shown the effectiveness of using wavelet transform over traditional Fourier transform in identification of damage in concrete structures. Kim and Melhem [11] showed the versatile uses of the method by applying it on damages related to mechanical gear and roller damage. Spatial response data from beam structures have been successfully analysed by wavelets to detect damage by Wang and Deng [12]. Hou et al [13] detected accumulated damage occurrence successfully in a multiple breakable spring model structure and validated it with the real accelerations of the San Fernando earthquake. Advantages of wavelet analysis over the usual eigenvalue analysis for a simply supported beam with non-propagating open crack were shown by Liew and Wang [14]. Moyo and Brownjohn [15] used wavelet analysis on a bridge structure to improve structural health monitoring. It is seen that although the effectiveness of wavelet analysis in damage detection is well studied, most of the works deal with a single basis function. Studies on comparative performance of the wavelet basis functions, effects of windowing and the presence of noise are still not very well probed into.
This paper considers a simply supported Euler Bernoulli beam element with an open crack. A rotational spring model has been considered to represent the system. The fundamental modeshape data have been simulated and analysed using two different non-Gaussian wavelet basis functions. Effects of windowing and presence of noise have been studied. It is seen that a proper choice of wavelet basis function with appropriate windowing of signal can lead to efficient and robust damage detection process.

DAMAGE MODELLING

A simply supported Euler Bernoulli beam is considered with an open crack. The beam with an open crack is modelled as two uncracked beams connected through a rotational spring at the location of crack assuming that the effects of crack are applicable in the immediate neighbourhood of the damage location. The free vibration equation for both beams on either side of the crack can be written as

$$EI\frac{\partial^4 y}{\partial x^4} + mA\frac{\partial^2 y}{\partial t^2} = 0 \qquad (1)$$

By separation of variables in Equation 1 and solving the characteristic equation, a general solution of the modeshapes on each side of the crack is found as a combination of Sine, Cosine, Sine hyperbolic and Cosine hyperbolic. Both displacement and moment at the two supports of the beam are zero. Continuity in displacement, moment and shear are assumed at the location of crack. However, a discontinuity is introduced at the crack location for slope. The slope condition at the crack location is modelled as

$$\Phi_R'(a) - \Phi_L'(a) = \theta \quad \Phi''$$ (2)

Here, the term θ is the non-dimensional crack section flexibility dependent on the crack depth ratio. As per Narkis [1], the function is considered to be a polynomial of the crack depth ratio as

$$\theta = 6\pi\delta^{2}(h/L)(0.5-0.33\ 0\delta+0.022\ ^{2}\delta-3.412^{3}\delta+\ 3.1\ 6^{4}\delta \qquad (3)$$

where δ (=a/h) is the crack depth ratio. The boundary conditions are substituted in the general modeshape equation and a system of eight linear equations is found. The natural frequency of the cracked beam may be found by setting the determinant of the matrix derived from the system of equations to zero, expanding it and solving for the roots numerically. An explicit form of the polynomial thus formed has been given by Narkis [1]. Here however, the determinant was expanded and the roots were found using Brent's method in MATLAB. The discontinuity in modeshape because of the presence of the crack can be successfully detected by performing a wavelet analysis on it. Simulation of the first modeshape is used since it is convenient to measure the fundamental modeshape for real structures.

WAVELET ANALYSIS

In a square integrable function space, a wavelet is considered to be a zero average function [6]. Hence,

$$\int_{-\infty}^{+\infty} \psi(x)\,dx = 0 \qquad (4)$$

A wavelet family of functions may be obtained by considering

$$\psi_{b,s}(x) = \frac{1}{\sqrt{s}}\psi\left(\frac{x-b}{s}\right) \qquad (5)$$

The continuous wavelet transform of a function f(x) in the same square integrable space can be represented as

$$W_f(b,s) = \int_{-\infty}^{+\infty} f(x)\frac{1}{\sqrt{s}}\psi^{*}\left(\frac{x-b}{s}\right) \qquad (6)$$

The Calderon-Grossman-Morlet theorem requires a weak admissibility condition to ensure the completeness of the wavelet transform and to maintain energy balance. Mathematically, it is represented as

$$\int_{0}^{+\infty} \frac{|\hat{\psi}(\omega)|^{2}}{\omega}\,d\omega < +\infty \qquad (7)$$

The identification of a discontinuity in a function or any of its derivatives can be linked with the number of vanishing moments of the wavelet basis function chosen for analysis. A wavelet has m number of vanishing moments if

$$\int_{-\infty}^{+\infty} x^k \psi(x)\, dx = 0 \qquad k=0,1,2,\ldots m-1 \tag{8}$$

For a wavelet with no more than m number of vanishing moments, it can be shown that for very small values of s in the domain of interest, the continuous wavelet transform of a function f(x) can be related to the m^{th} derivative of the signal [6]. For any wavelet $\psi(x)$ with m vanishing moments, there exists a fast decaying function $\theta(x)$ satisfying

$$\psi(x) = (-1)^m \frac{d^m \theta(x)}{dx^m} \tag{9}$$

Under this condition, the relationship between the continuous wavelet transform of f(x) and its m^{th} derivative can be expressed as

$$\lim_{s\to 0} \frac{Wf(b,s)}{s^{m+1/2}} = K \frac{d^m f(x)}{dx^m} \tag{10}$$

where

$$\int_{-\infty}^{+\infty} \theta(x)\, dx = K \neq 0 \tag{11}$$

Hence it is possible for a wavelet to detect singularities in a signal or its derivatives through the incorporation of a proper choice of basis function.

RESULTS

Damage and modeshape data are simulated for a 1m long square beam whose cross sectional area, depth and the moment of inertia are taken as 0.001 m^2, 0.01 m and 8.33×10^{-10} m^4 respectively. The Young's modulus and the density are assumed to be 190×10^9 N/m^2 and 7900 kg/m^3. The crack is located at 0.4m from the left of the support and the crack ratio value is δ= 0.35. Wavelet analysis is performed with Haar and Coif4 basis function on the simulated fundamental modeshape data multiplied by Bartlett and Hanning window functions respectively of width equal to that of the signal to find the presence, locate and estimate the severity of damage. A proper choice of windowing for the wavelet bases suppresses edge oscillations and improves the damage detection scheme. It is seen in Figure 1 that even a simple wavelet function like Haar can successfully detect damage when considering open cracks since it has one vanishing moment being related to the first derivative of the modeshape, where a discontinuity is present. A jump (not an extremum) in the wavelet coefficients at the point of damage is observed. Hence, a bright cone near the damaged zone is not formed. A closed form solution to the jump of the wavelet coefficients at damage location for Haar basis is provided in Appendix 2.

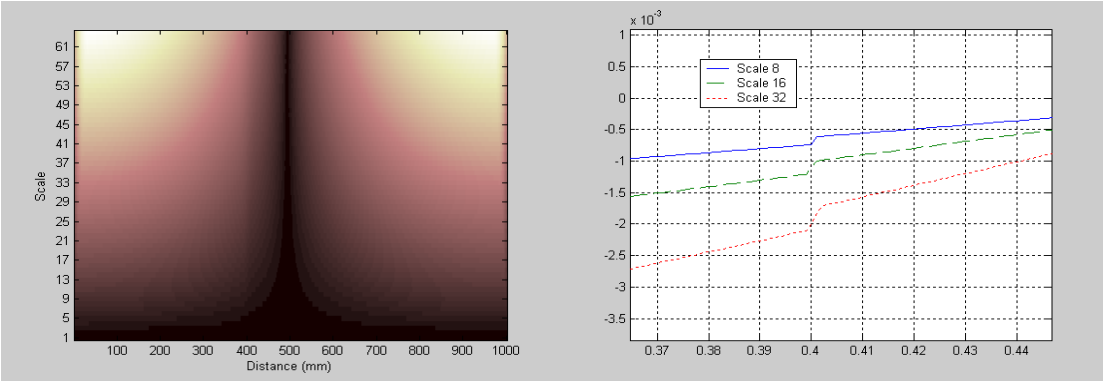

FIG. 1. Modeshape analysis with Haar basis function

Coiflets have higher number of vanishing moments and Coif4 (8 vanishing moments), in Figure 2 is seen to perform very efficiently. The damage is related to maxima of the wavelet coefficients in each scale and a definite bright cone is formed near the location of the damage.

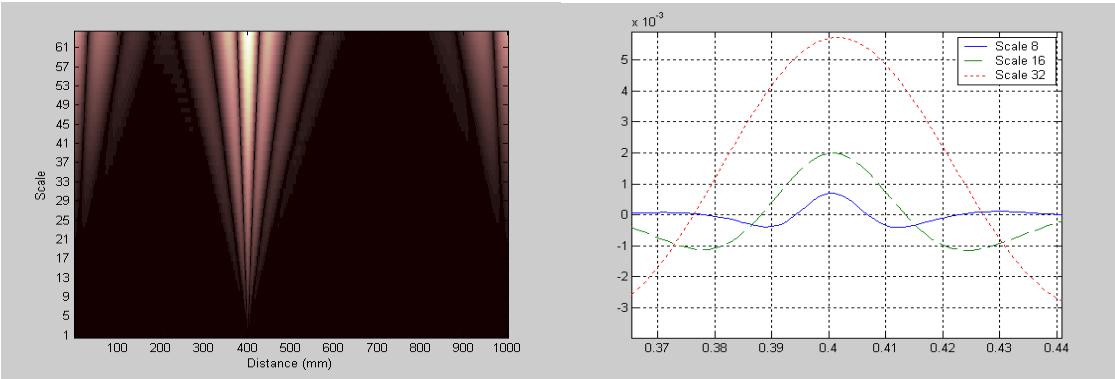

FIG. 2. Modeshape analysis with Coif4 basis function

Next, Gaussian white noise is introduced to the modeshape signal and the noisy modeshape is analyzed by the Coif4 basis function. The phenomenon of masking of damage is pronounced in finer (2,3 and 4) scales while the coarser (8,16 and 32) scales are comparatively more resistant to masking for the same Signal to Noise Ratio (SNR), as demonstrated by Figure 3.

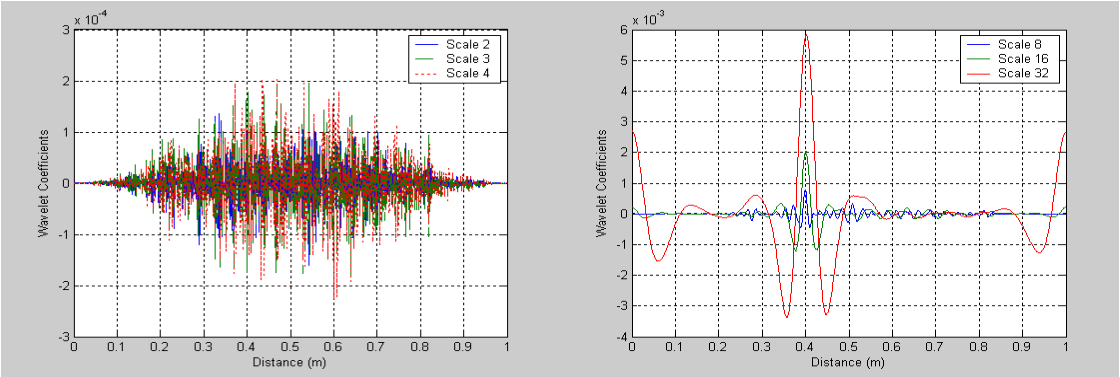

FIG. 3. Modeshape analysis with Coif4 for SNR 75 dB

CONCLUSIONS

A numerical study regarding the importance of the proper choice of basis functions and windows for efficient and robust damage detection by wavelet analysis of fundamental modeshape in the presence of noise in measured data for beam-like structures is presented. Simulations based only on the first modeshape have been used since it is convenient to measure the fundamental modeshape of real structures. The presence, location and extent of the damage are demonstrated to be found efficiently by wavelet analysis when the modeshape is appropriately windowed with respect to the basis function used. Coiflets are demonstrated to be extremely robust and versatile. Haar analysis coefficients show a jump at the location of the damage and can be used for the detection purpose when the presence of noise is very low. While the Bartlett window was found suitable for Haar, the Hanning window was more suitable for Coif4. Noise affects the coefficients at finer scales much more than the coarser ones and lower scales have early onset of masking even after windowing of the modeshape.

REFERENCES

1. Narkis. Y, *Identification of Crack Location in Vibrating Simply Supported Beams*, Journal of Sound and Vibration, 1994, 172(4), 549-558
2. Masoud S, Jarrah M.A and Maamory M.L, *Effect of Crack Depth on the Natural Frequency of a Prestressed Fixed-Fixed Beam*, Journal of Sound and Vibration, 1998, 214(2), 201-212
3. Dado M.H, *A Comprehensive Crack Identification Algorithm for Beams under Different End Conditions*, Applied Acoustics, 1997, Vol. 51, No. 4, 381-398
4. Hadjileontiadis L.J, Douka E and Trochidis A, *Fractal Dimension Analysis for Crack Identification in Beam Structures,* Mechanical Systems and Signal Processing, 2005, 19, 659-674
5. Loutridis S, Douka E and Trochidis A, *Crack Identification in Double Cracked Beams using Wavelet Analysis*, Journal of Sound and Vibration, 2004, 277, 1025-1039
6. Mallat S, 2001, *A Wavelet Tour on Signal Processing*, Academic Press, New York

7. Gentile A and Messina A, *On the Continuous Wavelet Transforms Applied to Discrete Vibrational Data for Detecting Open Cracks in Damaged Beams*, International Journal of Solids and Structures, 2003, 40, 295-315
8. Chang C.C and Chen L.W, *Vibration Damage Detection of a Timoshenko Beam by Spatial Wavelet Based Approach,* Applied Acoustics, 2003, 64, 1217-1240
9. Okafor A.C and Dutta A, *Structural Damage Detection in Beams by Wavelet Transforms*, Smart Materials and Structures, 2000, 9, 906-917
10. Melhem H and Kim H, *Damage Detection in Concrete by Fourier and Wavelet analyses*, ASCE Journal of Engineering Mechanics, 2003, May, 571-577
11. Melhem H and Kim H, *Damage Detection of Structures by Wavelet Analysis*, Engineering Structures, 2004,26, 347-362
12. Wang Q and Deng X, *Damage Detection with Spatial Wavelets,* International Journal of Solids and Structures, 1999, 36, 3443-3468
13. Hou Z, Nouri M and Amand R. St, *Wavelet Based Approach for Structural Damage Detection*, ASCE Journal of Engineering Mechanics, 2000, July, 677-683
14. Liew K.M and Wang Q, *Application of Wavelet Theory for Crack Identification in Structures*, ASCE Journal of Engineering Mechanics, 1998, February, 152-157
15. Moyo P and Brownjohn J.M.W, *Detection of Anomalous Structural Behaviour using Wavelet Analysis,* Mechanical Systems and Signal Processing, 2002, 16(2-3), 429-445

APPENDIX I. NOTATION

The following symbols were used in this paper:

L = Length of the beam
a= Location of damage from the left hand support of the beam
c= Crack depth
h= Overall depth of the beam
E= Young's modulus
I= Moment of inertia
A= Cross sectional area
ρ= Density of the material of the beam
y(x,t)= Displacement of the beam from its static equilibrium position
x = Distance from the left hand support along the length of the beam
t= Time
ω= Natural frequency of the cracked beam
$\Phi(.)$= Modeshape
θ= Non-dimensional crack section flexibility
δ= Crack depth ratio
f(.)= Function in square integrable space
b= Translation parameter
s= Scale
$\psi(.)$= Wavelet basis function
R= Right hand side of the damage
L= Left hand side of the damage

APPENDIX II. WAVELET COEFFICIENTS FOR HAAR BASIS

A closed form representation of the difference of wavelet coefficients immediately to the left and right of the damage location using Haar basis function for the first fundamental modeshape of the cracked beam without any presence of noise is provided in this section. This jump of the wavelet coefficients is given as

$$\Delta_{Diff} = s^{-1/2}\lambda^{-1}(-2C_{1L}\mathrm{Cos}(\lambda a) + C_{1L} + C_{1L}\mathrm{Cos}(\lambda a + \frac{\lambda s}{2}) + 2C_{2L}\mathrm{Sin}(\lambda a) - C_{2L}\mathrm{Sin}(\lambda a + \frac{\lambda s}{2}) + 2C_{3L}\mathrm{Cosh}(\lambda a) - C_{3L} - C_{3L}\mathrm{Cosh}(\lambda a + \frac{\lambda s}{2}) +$$
$$2C_{4L}\mathrm{Sinh}(\lambda a) - C_{4L}\mathrm{Sinh}(\lambda a + \frac{\lambda s}{2}) + 2C_{1R}\mathrm{Cos}(\lambda a) + C_{1R} - C_{1R}\mathrm{Cos}(\lambda a + \frac{\lambda s}{2}) + 2C_{2R}\mathrm{Sin}(\lambda a) + C_{2R}\mathrm{Sin}(\lambda a + \frac{\lambda s}{2}) - 2C_{3R}\mathrm{Cosh}(\lambda a) +$$
$$C_{3R} + C_{3R}\mathrm{Cosh}(\lambda a + \frac{\lambda s}{2}) - 2C_{4R}\mathrm{Sinh}(\lambda a) - C_{4R}\mathrm{Sinh}(\lambda a + \frac{\lambda s}{2}) \qquad 0 \le a < s/2$$

$$\Delta_{Diff} = s^{-1/2}\lambda^{-1}(-2C_{1L}\mathrm{Cos}(\lambda a) + C_{1L}\mathrm{Cos}(\lambda a - \frac{\lambda s}{2}) + C_{1L}\mathrm{Cos}(\lambda a + \frac{\lambda s}{2}) + 2C_{2L}\mathrm{Sin}(\lambda a) - C_{2L}\mathrm{Sin}(\lambda a + \frac{\lambda s}{2}) - C_{2L}\mathrm{Sin}(\lambda a - \frac{\lambda s}{2}) + 2C_{3L}\mathrm{Cosh}(\lambda a)$$
$$-C_{3L}\mathrm{Cosh}(\lambda a - \frac{\lambda s}{2}) - C_{3L}\mathrm{Cosh}(\lambda a + \frac{\lambda s}{2}) + 2C_{4L}\mathrm{Sinh}(\lambda a) - C_{4L}\mathrm{Sinh}(\lambda a + \frac{\lambda s}{2}) - C_{4L}\mathrm{Sinh}(\lambda a - \frac{\lambda s}{2}) + 2C_{1R}\mathrm{Cos}(\lambda a) - C_{1R}\mathrm{Cos}(\lambda a - \frac{\lambda s}{2}) -$$
$$C_{1R}\mathrm{Cos}(\lambda a + \frac{\lambda s}{2}) - 2C_{2R}\mathrm{Sin}(\lambda a) + C_{2R}\mathrm{Sin}(\lambda a + \frac{\lambda s}{2}) + C_{2R}\mathrm{Sin}(\lambda a - \frac{\lambda s}{2}) - 2C_{3R}\mathrm{Cosh}(\lambda a) + C_{3R}\mathrm{Cosh}(\lambda a - \frac{\lambda s}{2}) + C_{3R}\mathrm{Cosh}(\lambda a + \frac{\lambda s}{2}) -$$
$$2C_{4R}\mathrm{Sinh}(\lambda a) + C_{4R}\mathrm{Sinh}(\lambda a + \frac{\lambda s}{2}) + C_{4R}\mathrm{Sinh}(\lambda a - \frac{\lambda s}{2}) \qquad s/2 \le a < L - s/2$$

$$\Delta_{Diff} = s^{-1/2}\lambda^{-1}(-2C_{1L}\mathrm{Cos}(\lambda a) + C_{1L}\mathrm{Cos}(\lambda a - \frac{\lambda s}{2}) + C_{1L}\mathrm{Cos}(\lambda L) + 2C_{2L}\mathrm{Sin}(\lambda a) - C_{2L}\mathrm{Sin}(\lambda L) - C_{2L}\mathrm{Sin}(\lambda a - \frac{\lambda s}{2}) + 2C_{3L}\mathrm{Cosh}(\lambda a)$$
$$-C_{3L}\mathrm{Cosh}(\lambda a - \frac{\lambda s}{2}) - C_{3L}\mathrm{Cosh}(\lambda L) + 2C_{4L}\mathrm{Sinh}(\lambda a) - C_{4L}\mathrm{Sinh}(\lambda L) - C_{4L}\mathrm{Sinh}(\lambda a - \frac{\lambda s}{2}) + 2C_{1R}\mathrm{Cos}(\lambda a) - C_{1R}\mathrm{Cos}(\lambda a - \frac{\lambda s}{2}) -$$
$$C_{1R}\mathrm{Cos}(\lambda L) - 2C_{2R}\mathrm{Sin}(\lambda a) + C_{2R}\mathrm{Sin}(\lambda L) + C_{2R}\mathrm{Sin}(\lambda a - \frac{\lambda s}{2}) - 2C_{3R}\mathrm{Cosh}(\lambda a) + C_{3R}\mathrm{Cosh}(\lambda a - \frac{\lambda s}{2}) + C_{3R}\mathrm{Cosh}(\lambda L) - 2C_{4R}\mathrm{Sinh}(\lambda a)$$
$$+C_{4R}\mathrm{Sinh}(\lambda L) + C_{4R}\mathrm{Sinh}(\lambda a - \frac{\lambda s}{2}) \quad L - s/2 \le a < L$$