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## Appendix A. List of symbols

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$T$	periods in the planning horizon
$d_t$	random variable
$\zeta_t$	value of random variable $d_t$
$\tilde{d}_t$	the expected value of random variable $d_t$
$g(\cdot)$	probability density function
$F_t$	realised demand set at the beginning of period $t$
$I_t$	inventory level at the end of period $t$
$I_0$	initial inventory level at the beginning of the planning horizon
$Q_t$	ordering quantity placed at the beginning of period $t$
$u(\cdot)$	ordering cost
$K$	fixed ordering cost
$c$	proportional ordering cost
$h$	proportional holding cost
$b$	proportional penalty cost
$f_t(I_{t-1}, F_t; Q_t)$	immediate cost of period $t$ with opening inventory level $I_{t-1}$ , realised demand set $F_t$ , and order quantity $Q_t$
$C_t(I_{t-1}, F_t)$	the expected total cost of an optimal policy over period $t, \dots, T$ with opening inventory level $I_{t-1}$ and realised demand set $F_t$
$S_t$	order-up-to-level of period $t$
$\delta_t$	binary variable
$\bar{C}_1(I_0)$	expected total cost over period $1, \dots, T$ under $(R, S)$ policy with initial inventory level $I_0$
$d_{jt}$	a random variable denotes the demand over period $j, \dots, t$ , i.e. $d_{jt} = d_j + \dots + d_t$
$\zeta_{jt}$	value of random variable $d_{jt}$
$\tilde{d}_{jt}$	expected value of the convolution $\tilde{d}_j + \dots + \tilde{d}_t$
$\omega$	a random variable
$x$	a scalar value

$L(x, \omega)$	first order loss function
$\hat{L}(x, \omega)$	complementary of first order loss function
$P_{jt}$	a binary variable which is set to one if the most recent replenishment up to period $t$ was issued in period $j$ , where $j \leq t$ — if no replenishment occurs before or at period $t$ , then we let $P_{1t} = 1$ , this allows us to properly account for demand variance from the beginning of the planning horizon
$\Omega$	support of $d_{jt}$
$W$	number of regions in a partition of $\Omega$
$i$	region index ranging in $1, \dots, W$
$\Omega_i$	the $i^{th}$ subregion of $\Omega$
$p_i$	$Pr(d_{jt} \in \Omega_i)$
$E[d_{jt} \Omega_i]$	conditional expectation of $d_{jt}$ in $\Omega_i$
$\tilde{H}_t$	the upper bound to the true value of $\sum_{j=1}^t \hat{L}(S_j, d_{jt})P_{jt}$
$\tilde{B}_t$	the upper bound to the true value of $\sum_{j=1}^t L(S_j, d_{jt})P_{jt}$
$e_W^{jt}$	approximation error
$\sigma_{jt}$	the standard deviation of $d_{jt}$
$Z$	a standard normal random variable
$\mathcal{N}(\mu, \sigma^2)$	a normal random variable with mean $\mu$ and variance $\sigma^2$
$H$	forecast horizon
$D_{s,t}$	demand forecasts made in period $s$ for period $t$
$\epsilon_{s,t}$	the forecast update made at the end of period $s$ for period $t$
$\epsilon_s$	forecast update vector generated at the end of period $s$

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Table A.1: A list of symbols

## Appendix B. Expected demands of the computational study

This section presents the expected demands for various demand patterns.

## Appendix C. Optiamlity gap% for different correlation coefficients

time period	LCY1	LCY2	SIN1	SIN2	EMP1	EMP2	EMP3	EMP4	STA
1	15	3	15	12	3	2	6	9	10
2	16	6	4	7	8	12	7	3	10
3	15	7	4	7	13	14	4	11	10
4	14	11	10	10	22	25	6	11	10
5	11	14	18	13	12	20	8	26	10
6	7	15	4	7	8	13	16	27	10
7	6	16	4	7	11	10	6	11	10
8	3	15	10	12	5	16	24	11	10

Table B.2: Expected demands for various demand patterns

Settings		Mean	Median	IQR	Maximal	Minimal
demand patterns	LCY1	0.38	0.08	0.08	3.02	0.02
	LCY2	0.09	0.09	0.03	0.17	0.04
	SIN1	0.13	0.07	0.05	0.84	0.01
	SIN2	0.05	0.05	0.05	0.10	0.01
	STA	0.07	0.09	0.07	0.18	0.01
	EMP1	0.25	0.08	0.07	1.37	0.00
	EMP2	0.23	0.09	0.20	1.03	0.02
	EMP3	0.07	0.05	0.04	0.21	0.03
fc	150	0.24	0.08	0.07	3.02	0.01
	300	0.08	0.07	0.06	0.34	0.00
uc	0	0.23	0.08	0.03	3.02	0.01
	1	0.09	0.07	0.07	0.69	0.00
pc	5	0.07	0.08	0.06	0.17	0.02
	10	0.15	0.07	0.07	0.97	0.00
	20	0.26	0.08	0.05	3.02	0.01
cv	0.1	0.07	0.07	0.06	0.42	0.00
	0.2	0.25	0.08	0.07	3.02	0.01
rho	-0.75	0.07	0.08	0.05	0.17	0.00
	-0.25	0.10	0.07	0.06	1.03	0.01
	0.25	0.29	0.07	0.09	3.02	0.01
	0.75	0.16	0.08	0.11	0.84	0.01
Overall		0.16	0.07	0.07	3.02	0.01

Table C.3: Optimality gaps % for different pivot parameters