Implementation Issues for Optimized Hard Decision Energy Detector-Based Cooperative Spectrum Sensing

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ABSTRACT
Recent studies in cooperative energy detection have focused on the optimization of the threshold value and fusion center voting rule in an effort to minimize the sensing error probability. However, such studies operate under the assumption that the signal to noise ratio is equal at every node, which is rarely the case in practice.

In this paper, generalized formulas for the optimal threshold value and optimal fusion center voting rule are derived for hard decision energy detector-based spectrum sensing networks where the signal to noise ratio is distinct at each node. It is shown that the implementation of this solution requires more data to be transmitted than the optimal soft decision scheme, which is known to have superior performance.

Categories and Subject Descriptors
C.2.1 [**Computer-communication networks**]: Network architecture and design—*distributed networks, wireless communication*; G.1.6 [**Numerical analysis**]: Optimization; G.3 [**Probability and statistics**]: Statistical computing

General Terms
Algorithms, Performance, Theory

Keywords
Cognitive radio, spectrum sensing, cooperative networks, energy detector, hard decision, optimization

1. INTRODUCTION
Recent years have seen increasing utilization of the electromagnetic spectrum for a variety of military, civilian and commercial applications. For each new application, frequency resources are allocated based on the intended geographic range of the service, the number of channels proposed and the required bandwidth per channel. However, such resources are limited due to the variation of propagation characteristics across the electromagnetic spectrum and so, with each new application, the amount of usable spectrum decreases.

Recent studies by the Federal Communications Commission and National Telecommunications and Information Administration indicate that the majority of usable frequencies have been allocated in the United States [3, 9]. A similar study by the European Regulators Group found that several EU member countries lacked the resources for additional 2G/3G mobile networks [2]. Such bottlenecks pose serious risks to future commercial competition and represent a barrier to efficient spectrum regulation.

However, the problem is not intractable: studies have shown that spectrum usage in allocated bands can vary significantly depending on time and/or location [7]. By exploiting this variation, it is possible to recover bands which have been allocated but are unused for certain time periods or in certain locations. The process by which this is achieved is known as spectrum sensing. However, if a channel were inaccurately determined to be free, then there is a high risk that the service on that channel could be interfered with. This is an unacceptable situation for the service owner, who may have paid a license fee to use the channel. Thus, there is a requirement for spectrum sensing to be very reliable.

One method of spectrum sensing involves the use of energy detectors, which are less expensive to produce, but also less accurate than other available technologies such as matched filtering and cyclostationary feature detection [1]. However, when grouped together, a significant increase in performance can be achieved through cooperation, making networks of low cost energy detectors an ideal solution for reliable spectrum sensing [15].

Such cooperation requires that sensor nodes be able to communicate with each other. There are several methods by which this can be achieved [15, 6, 8]; however, available spectrum is scarce (recall that the nodes themselves are attempting to identify unoccupied bands to begin with), so it is necessary that communication between nodes is limited. One particular scheme, known as hard decision fusion, minimizes the amount of transmitted data by requiring that each node transmit a binary decision about band occupancy. The performance of this scheme depends partially on environmental factors beyond the control of the designer, such as the power of the signal (if one is present) and background noise, but also on arbitrary parameters at both the local node and network levels. Recent studies have focused on the optimization of these parameters so that sensing reliability is maximized [15, 5]. However, such analyses have relied on simplifying assumptions (e.g. that the signal to noise ratio at each node is equal) which are not generally applicable. In this paper, a generalized optimal solution is derived for hard decision sensor networks and it is shown that the implementation of this solution requires significantly more data to be transmitted than its soft decision equivalent.

2. SYSTEM MODEL

2.1 Signal model

In a network of cooperating energy detector-based spectrum sensor nodes, for a given channel, the received signal is typically represented as:

$$r_i(t) = \begin{cases} n_i(t) & H_0 \\ s_i(t) + n_i(t) & H_1, \end{cases} \quad (1)$$

where $r_i(t)$ represents the received signal at the i^{th} node, $n_i(t)$ represents the time-varying noise interference at the i^{th} node, $s_i(t)$ represents the transmitted signal at the i^{th} node and H_0 and H_1 are the null and alternative hypotheses, respectively.

2.2 Energy detection

In hard decision energy detection, in order to determine which hypothesis is true, i.e. whether the channel is occupied or unoccupied, the binary hypothesis test is applied. Thus, at each energy detector node, a test statistic is computed from discrete samples of the channel under investigation:

$$Y_i = \sum_{n=1}^{M_i} \mid r_i[n] \mid^2, \quad (2)$$

where Y_i is the test statistic at the i^{th} node in the network (i.e. the band energy assuming a 1Ω reference resistor), M_i is the number of samples at the i^{th} node and $r_i[n] = r_i(nT_s)$, where T_s is the sample period.

The test statistic is then compared to a threshold, and a decision is made according to a predefined rule. For hard decision energy detection, the rule is given by:

$$D_i = \begin{cases} H_0 & Y_i \leq \lambda_i \\ H_1 & Y_i > \lambda_i, \end{cases} \quad (3)$$

where D_i is the decision at node i and λ_i is the threshold at node i.

From (2), the distribution of the energy of the received signal at node i will be:

$$Y_i \sim \begin{cases} \chi^2_{2u_i} & H_0 \\ \chi^2_{2u_i}(2\gamma_i) & H_1, \end{cases} \quad (4)$$

where $\chi^2_{2u_i}$ and $\chi^2_{2u_i}(2\gamma_i)$ are the central and noncentral chi square distributions, respectively, u_i is the time-bandwidth product at the i^{th} node and γ_i is the noncentrality parameter at the i^{th} node [13].

The time-bandwidth product and noncentrality parameter at the i^{th} node are defined as $u_i = \frac{M_i}{2}$ and $\gamma_i = SNR_i$, respectively, where SNR_i is the signal to noise ratio at the i^{th} node [13;, 10, p. 45-47]. It is assumed that the number of samples is equal at each node; thus, $M_i = M$ and $u_i = u$, where M is the number of samples at every node and u is the common time-bandwidth product.

If the number of samples is large then, invoking the central limit theorem, the test statistic becomes approximately normally distributed:

$$Y_i \sim \begin{cases} \mathcal{N}(M\sigma_i^2, 2M\sigma_i^4) & H_0 \\ \mathcal{N}(M\sigma_i^2(1+\gamma_i), 2M\sigma_i^4(1+\gamma_i)^2) & H_1, \end{cases} \quad (5)$$

where σ_i^2 is the power of the noise signal $n_i(t)$ (assuming a 1Ω reference resistor) at the i^{th} node [12].

Thus, the decision probabilities at the i^{th} node are defined as:

$$P_{f_i} = Q\left(\frac{\lambda_i - M\sigma_i^2}{\sqrt{2M\sigma_i^4}}\right), \quad (6)$$

$$P_{a_i} = 1 - P_{f_i}, \quad (7)$$

$$P_{d_i} = Q\left(\frac{\lambda_i - M\sigma_i^2(1+\gamma_i)}{\sqrt{2M\sigma_i^4(1+\gamma_i)^2}}\right), \quad (8)$$

$$P_{m_i} = 1 - P_{d_i}, \quad (9)$$

where P_{f_i}, P_{a_i}, P_{d_i} and P_{m_i} are the probabilities of false alarm, acquisition (i.e. detecting an available unused channel), detection and missed detection at the i^{th} node, respectively, and $Q(\cdot)$ is the standard Gaussian complementary cumulative distribution function [12].

2.3 Cooperative networks

After each detector has made a decision about channel occupancy, the results are transmitted across a control channel to a designated master node or a fixed control center, called the fusion center, where a voting rule is applied to reach an overall decision. Generally, the k-out-of-N rule is used [15]:

$$D_{fc} = \sum_{i=1}^{N} g(D_i) \begin{cases} < k & H_0 \\ \geq k & H_1, \end{cases} \quad (10)$$

where D_{fc} is the decision at the fusion center, N is the total number of nodes in the network, k is the voting rule, $g(H_0) = 0$ and $g(H_1) = 1$.

The process of making decisions at each node can be viewed as a series of N independent Bernoulli trials, where the overall decision D_{fc} is the sum of the outcomes of the trials. Thus, the fusion center decision probabilities are Poisson-binomially distributed (also known as Poisson's binomial distribution):

$$Q_f = \sum_{l=k}^{N} \sum_{A\in(F_l|H_0)} \prod_{x\in A} P_{f_x} \prod_{y\in A^C} P_{a_y}, \tag{11}$$

$$Q_a = 1 - Q_f, \tag{12}$$

$$Q_d = \sum_{l=k}^{N} \sum_{A\in(F_l|H_1)} \prod_{x\in A} P_{d_x} \prod_{y\in A^C} P_{m_y}, \tag{13}$$

$$Q_m = 1 - Q_d, \tag{14}$$

where Q_f, Q_a, Q_d and Q_m are the overall probabilities of false alarm, acquisition, detection and missed detection, respectively, F_l is the k-subset of size l of the power set[1] of all possible decision outcomes under a given hypothesis, and A^C is the complement of the set A [14].

3. OPTIMIZATION

In a network of cooperating nodes, the sensing error probability, G, is defined as the probability of either a false alarm or a missed detection event:

$$G = Q_f + Q_m. \tag{15}$$

Thus, G is a function of the number of cooperating nodes, the decision probabilities at each cooperating node and of the fusion center voting rule k; the decision probabilities are themselves functions of the SNR, noise power, number of samples and threshold at each node. However, the number of cooperating nodes, SNR and noise power cannot be specified by the designer and, since it is known that the number of samples required to achieve a given sensing error probability is a function of the SNR [12], the only variables which can be adjusted by the designer are the local node decision thresholds $\lambda_1, \lambda_2, ..., \lambda_N$ and the fusion center voting rule k.

3.1 Optimal local node decision thresholds

In order to minimize G by adjusting the threshold at the i^{th} node, it is necessary to solve:

$$\left.\frac{\delta G}{\delta \lambda_i}\right|_{\lambda_i=\lambda_{i_{opt}}} = 0, \tag{16}$$

where $\lambda_{i_{opt}}$ is the optimum threshold value at node i that minimizes G.

Differentiating (15) with respect to λ_i, it can be shown (see Appendix A for proof) that:

$$\frac{\delta G}{\delta \lambda_i} = \frac{\delta P_{m_i}}{\delta \lambda_i} \sum_{A\in(F_k|H_1)} \prod_{x\in A, x\neq i} P_{d_x} \prod_{y\in A^C, y\neq i} P_{a_y} - \frac{\delta P_{a_i}}{\delta \lambda_i} \sum_{A\in(F_k|H_0)} \prod_{x\in A, x\neq i} P_{f_x} \prod_{y\in A^C, y\neq i} P_{a_y}. \tag{17}$$

[1]The power set of a set S is the set of all subsets of S, including the empty set and S itself.

Now, combining (16) and (17) and simplifying:

$$\left.\frac{\frac{\delta P_{a_i}}{\delta \lambda_i}}{\frac{\delta P_{m_i}}{\delta \lambda_i}}\right|_{\lambda_i=\lambda_{i_{opt}}} = \frac{\sum_{A\in(F_k|H_1), i\in A} \prod_{x\in A, x\neq i} P_{d_x} \prod_{y\in A^C, y\neq i} P_{m_y}}{\sum_{A\in(F_k|H_0), i\in A} \prod_{x\in A, x\neq i} P_{f_x} \prod_{y\in A^C, y\neq i} P_{a_y}}. \tag{18}$$

Equation (18) can be solved to find the optimal decision threshold for each node, i.e. $\lambda_{1_{opt}}, \lambda_{2_{opt}}, ..., \lambda_{N_{opt}}$. As, to the best of the authors' knowledge, (18) cannot be simplified further, $\lambda_{i_{opt}}$ can be found at each node using numerical methods. It can then be verified numerically that the solution is a minimum by showing that the Hessian matrix of G is positive definite.

Under the assumption that the SNR and noise power are equal at every node, (18) reduces to a more concise statement of the solution presented in [15].

3.2 Optimal fusion center voting rule

As discussed in Section 2.3, the voting rule is chosen at the fusion center after the decisions have been transmitted from the local nodes. Thus, in order to find the optimal voting rule, it is necessary to solve:

$$\left.\frac{\delta}{\delta k}(G(\lambda_{1_{opt}}, \lambda_{2_{opt}}, ..., \lambda_{N_{opt}}))\right|_{k=k_{opt}} = 0. \tag{19}$$

The second derivative test can be applied to verify that the solution is a minimum.

Noting that k is an integer:

$$\begin{aligned}\frac{\delta G}{\delta k} &= \frac{G(k+1) - G(k)}{(k+1) - k} \\ &= \sum_{A\in(F_k|H_1)} \prod_{x\in A} P_{d_x} \prod_{y\in A^C} P_{m_y} \\ &\quad - \sum_{A\in(F_k|H_0)} \prod_{x\in A} P_{f_x} \prod_{y\in A^C} P_{a_y}.\end{aligned} \tag{20}$$

Combining (19) and (20) and simplifying:

$$\sum_{A\in(F_{k_{opt}}|H_1)} \prod_{x\in A} P_{d_x}(\lambda_{x_{opt}}) \prod_{y\in A^C} P_{m_y}(\lambda_{y_{opt}}) = \sum_{A\in(F_{k_{opt}}|H_0)} \prod_{x\in A} P_{f_x}(\lambda_{x_{opt}}) \prod_{y\in A^C} P_{a_y}(\lambda_{y_{opt}}). \tag{21}$$

The solution to (21) is the optimal fusion center voting rule, k_{opt}. Since, to the best of the authors' knowledge, (21) cannot be simplified further, k_{opt} must be computed using numerical methods.

Again, under the assumption that the SNR and noise power are equal at every node, (21) reduces to the solution presented in [15].

4. IMPLEMENTATION

4.1 Data transmission requirements

From (18), it can be seen that the optimal threshold at the i^{th} node is a function of the decision probabilities and, by extension, the SNR and noise power at every other node.

Table 1: Data transmission requirements.

Fusion method	$\overline{SNR}$ at each node	Bits required
Soft	distinct	$2Nb$
Hard	distinct	$2Nb + N$
Soft	equal	$Nb + b$
Hard	equal	$N + b$

Therefore, to compute the optimal threshold at a given node, it is necessary to have knowledge of the values of the SNR and noise power at every other node. To reduce bandwidth requirements, the local SNR can be normalized with respect to the local noise power [6]. As it cannot be assumed that every node will be able to communicate with every other node, data must be relayed via the fusion center. Thus, if b bits are used to represent the normalized SNR, $\overline{SNR}$, at each node, then Nb bits must be relayed. Additionally, as the local decisions are binary (see (3)), each node will transmit one decision bit to the fusion center. Thus, the total number of bits to be transmitted before a decision is reached is $2Nb + N$.

In soft decision fusion, each node transmits its test statistic and normalized SNR value to the fusion center [6]. If the normalized SNR is again represented using b bits, and a further b bits are used to represent the test statistic, then the total number of bits to be transmitted before a decision is made is $2Nb$.

In certain circumstances, it may be known that the normalized SNR is equal at every node [15, 5]. In such cases, only one node need transmit the common normalized SNR value to the fusion center. Therefore, b bits must be transmitted to the fusion center and a further N bits for the decisions at each of the nodes. The equivalent soft decision scheme still requires that each node transmit its test statistic to the fusion center, although only one node is required to transmit the value of the common normalized SNR value. Thus, $Nb + b$ bits are required in total.

Table 1 details the data transmission requirements of the optimized hard and soft decision fusion schemes. As can be seen, hard decision fusion requires more data to be transmitted unless it is known that $\overline{SNR}$ is equal at each node.

4.2 Performance

The IEEE 802.22 draft standard for cognitive wireless regional area networks specifies that sensing technologies must be able to ensure an overall probability of detection greater than or equal to 0.9 (i.e. a probability of missed detection less than or equal to 0.1) and an overall probability of false alarm less than or equal to 0.1 at $SNR = -21dB$ [11].

Using these specifications, and the optimized parameters resulting from (18) and (21), the performance of optimized hard decision fusion can be measured by the sample complexity, i.e. the number of samples required to achieve given probabilities of false alarm and missed detection, for a certain network size, as shown in Figure 1. As can be seen, optimized hard decision fusion requires a higher sample complexity than optimized soft decision fusion.

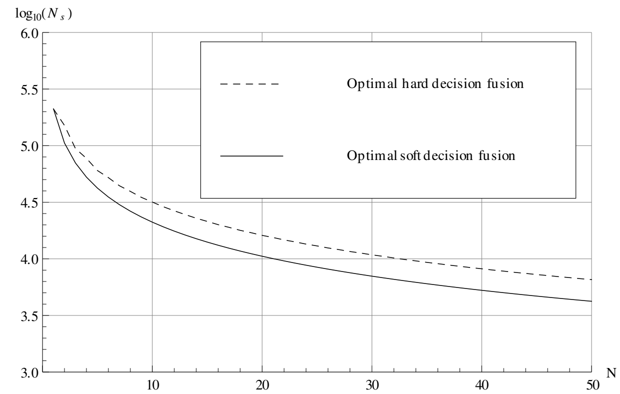

Figure 1: Log-linear plot of sample complexity against network size for IEEE 802.22 specifications.

5. CONCLUSION

Optimized energy detector-based cooperative spectrum sensing can significantly reduce the sensing error probability of a cognitive radio network. However, this reduction comes at the cost of increased data transmission, which is limited.

When the normalized SNR is distinct at each node, optimal hard decision fusion requires higher data transmission than optimal soft decision fusion, which has been shown to have a lower sample complexity.

If the normalized SNR is equal at every node, then optimal hard decision fusion becomes more practical as the data transmission requirements are reduced significantly. However, such scenarios are not common.

Based on these findings, it must be concluded that hard decision fusion, based on threshold and voting rule optimization, is, in general, unlikely to be of practical use. Further reduction of data transmission requirements may be achieved through the optimization of quantized soft decision algorithms.

6. ACKNOWLEDGMENTS

This work was funded by the Irish Research Council for Science, Education and Technology (IRCSET) through the Embark Initiative.

APPENDIX

A. FIRST DERIVATIVE OF G

To show that (17) is true, an alternative, discrete Fourier transform based formulation for the probability mass function of the Poisson binomial distribution may be used:

$$P(k; N, \mathbf{P}_N) = \sum_{n=0}^{N} \left(\frac{e^{\frac{-j2\pi nk}{N+1}}}{N+1} \prod_{x=1}^{N} \left(P_{f_x} e^{\frac{j2\pi n}{N+1}} + P_{a_x} \right) \right), \tag{22}$$

where $P(k; N, \mathbf{P}_N)$ is the probability mass function and $\mathbf{P}_N$ is the set of success probabilities at each node, in this case given by $\{P_{f_1}, P_{f_2}, ..., P_{f_N}\}$ [4].

Using (22), the overall probability of false alarm can be written as:

$$\begin{aligned} Q_f &= \sum_{l=k}^{N} P(l; N, \mathbf{P}_N) \\ &= \sum_{n=0}^{N} \left(\left(\sum_{l=k}^{N} \frac{e^{\frac{-j2\pi nl}{N+1}}}{N+1} \right) \prod_{x=1}^{N} \left(P_{f_x} e^{\frac{j2\pi n}{N+1}} + P_{a_x} \right) \right). \quad (23) \end{aligned}$$

Differentiating Q_f with respect to the threshold at the i^{th} node, (23) becomes:

$$\begin{aligned} \frac{\delta Q_f}{\delta \lambda_i} =& \frac{1}{N+1} \frac{\delta P_{a_i}}{\delta \lambda_i} \sum_{n=0}^{N} \left(\left(\sum_{l=k}^{N} e^{\frac{-j2\pi nl}{N+1}} \right) \left(1 - e^{\frac{j2\pi n}{N+1}} \right) \right. \\ & \left. \times \prod_{x=1, x\neq i}^{N} \left(P_{f_x} e^{\frac{j2\pi n}{N+1}} + P_{a_x} \right) \right). \quad (24) \end{aligned}$$

Now, defining $f(l) = e^{\frac{-j2\pi nl}{N+1}}$, (24) becomes:

$$\begin{aligned} \frac{\delta Q_f}{\delta \lambda_i} =& \frac{1}{N+1} \frac{\delta P_{a_i}}{\delta \lambda_i} \sum_{n=0}^{N} \left(\left(\sum_{l=k}^{N} f(l) - f(l-1) \right) \right. \\ & \left. \times \prod_{x=1, x\neq i}^{N} \left(P_{f_x} e^{\frac{j2\pi n}{N+1}} + P_{a_x} \right) \right). \quad (25) \end{aligned}$$

This can be simplified by noting that:

$$\sum_{l=k}^{N} f(l) - f(l-1) = f(N) - f(k-1). \quad (26)$$

Thus, (25) becomes:

$$\begin{aligned} \frac{\delta Q_f}{\delta \lambda_i} =& \frac{1}{N+1} \frac{\delta P_{a_i}}{\delta \lambda_i} \sum_{n=0}^{N} ((f(N) - f(k-1)) \\ & \times \prod_{x=1, x\neq i}^{N} \left(P_{f_x} e^{\frac{j2\pi n}{N+1}} + P_{a_x} \right) \Bigg) \\ =& \frac{\delta P_{a_i}}{\delta \lambda_i} \sum_{n=0}^{N} \left(\frac{f(N)}{N+1} \prod_{x=1, x\neq i}^{N} \left(P_{f_x} e^{\frac{j2\pi n}{N+1}} + P_{a_x} \right) \right) \\ & - \frac{\delta P_{a_i}}{\delta \lambda_i} \sum_{n=0}^{N} \left(\frac{f(k-1)}{N+1} \prod_{x=1, x\neq i}^{N} \left(P_{f_x} e^{\frac{j2\pi n}{N+1}} + P_{a_x} \right) \right) \\ =& \frac{\delta P_{a_i}}{\delta \lambda_i} P(N; N-1, \mathbf{P}_{N-1}) \\ & - \frac{\delta P_{a_i}}{\delta \lambda_i} P(k-1; N-1, \mathbf{P}_{N-1}), \quad (27) \end{aligned}$$

where $\mathbf{P}_{N-1} = \mathbf{P}_N \setminus P_{f_i}$.

Logically, $P(N; N-1, \mathbf{P}_{N-1}) = 0$, so (27) reduces to:

$$\begin{aligned} \frac{\delta Q_f}{\delta \lambda_i} &= - \frac{\delta P_{a_i}}{\delta \lambda_i} P(k-1; N-1, \mathbf{P}_{N-1}) \\ &= - \frac{\delta P_{a_i}}{\delta \lambda_i} \sum_{A \in (F_k | H_0)} \prod_{x \in A, x\neq i} P_{f_x} \prod_{y \in A^C, y \neq i} P_{a_y}. \quad (28) \end{aligned}$$

A similar method can be used to show that:

$$\frac{\delta Q_m}{\delta \lambda_i} = \frac{\delta P_{m_i}}{\delta \lambda_i} \sum_{A \in (F_k | H_1)} \prod_{x \in A, x\neq i} P_{d_x} \prod_{y \in A^C, y \neq i} P_{a_y}. \quad (29)$$

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