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# Semi-active Frequency Tracking Algorithm for Control of Flapwise Vibrations in Wind Turbine Blades

John Arrigan\*, Vikram Pakrashi\*, Biswajit Basu\*, Satish Nagarajaiah\*\*

\*Department of Civil, Structural and

Environmental Engineering,

Trinity College Dublin

email: arriganj@tcd.ie

pakrashv@tcd.ie

basub@tcd.ie

\*\*Department of Civil and Env. Eng. and Mech.

Eng. and Mat. Sc.,

Rice University, Houston,

TX, United States

email: nagaraja@rice.edu

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**Abstract**—The increased size and flexibility of modern multi-Megawatt wind turbines has lead to the dynamic behaviour of these structures becoming an important design consideration. The aim of this paper is to study the potential use of Semi-Active Tuned Mass Dampers (STMDs) in reducing vibrations in the flapwise direction. The model developed in this study considers only the structural dynamics of the turbine and includes the coupling between the blades and tower. The semi-active algorithm employs a frequency tracking technique based on the Short Time Fourier Transform (STFT). This allows real-time tuning of the dampers to the dominant frequencies in the system. Numerical simulations have been carried out to study the effectiveness of the STMDs for steady and turbulent wind loading.

**Keywords** – Wind turbine blades, Semi-active control, Short Time Fourier Transform, STMDs

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## I INTRODUCTION

Over the last decade, wind turbine technology has taken huge leaps forward to become a leading alternative energy resource to traditional fossil fuels. Turbines with outputs as large as 5MW are being constructed with tower heights and rotor diameters of over 80m and 120m respectively. With the increased size of the blades comes increased flexibility making it important to understand their dynamic behaviour. Ahlstrom carried out research into the effect of increased flexibility in turbine blades and found that it can lead to a significant drop in the power output of the turbine (1). Significant research has been carried out into the area of blade design and their failure characteristics (2, 3). However, it is only over the last few years that research has started to focus on the dynamic behaviour of the turbine blades and the interaction that occurs between the blades and the tower. Two main types of vibration occur in wind turbine blades, flapwise and edgewise. Flapwise vibrations are vibrations occurring out of the plane of rotation of the blades while edgewise vibrations occur in the plane of rotation. Ronold and Larsen (4) studied the failure of a wind turbine blade in flapwise bending during normal operating conditions of the turbine

while Murtagh et al. (5) studied the flapwise motion of wind turbine blades and included their dynamic interaction with the tower.

Efforts to mitigate the increased vibration problems that are occurring in wind turbine blades have thus far concentrated on the actual design of the blades themselves. The possibility of using dampers in the blades to control their dynamic behaviour has not yet been investigated to a large extent.

Vibration mitigating devices have been used in engineering systems for many decades; Tuned Mass Dampers (TMDs) being one of the first types. TMDs consist of a mass connected to the primary structure through the use of springs and dashpots. Tuning the TMD to the natural frequency of the structure can result in significant reduction in vibration through the out of phase motion of the TMD to the primary system. Many different types of TMDs exist but they all fall into the three general categories of dampers: passive devices, active devices and semi-active devices. Passive TMDs require no external energy and are very popular due to their low cost and simplicity. However once the damper is tuned its properties cannot be altered. Active TMDs make use of an external power source which allows their stiffness to be adjusted rapidly to account for any variation in the dynamic behaviour of the structure.

Their adaptability is a big advantage over passive devices as they are effective over a much larger frequency bandwidth. However, the need for a large power source often makes their cost infeasible. Semi-active devices however are a compromise between the two; their properties can be adjusted in real-time over a relatively large frequency bandwidth with only a small amount of power required.

Passive TMDs have been used widely throughout civil engineering applications, particularly in tall buildings subjected to wind or earthquake loadings. Over the last few decades extensive research has been carried out into the use of passive TMDs and their suitability for vibration control (6, 7). However, due to the non-linearity of nearly all engineering dynamical systems research has more recently focused on Semi-Active TMDs (STMDs) due to their 'active' capabilities without the need for an external power supply. Pinkaew and Fujino (8) looked at the use of STMDs for vibration mitigation in structures excited by harmonic loads, while Nagarajaiah and Sonmez (9) applied Short Time Fourier Transform (STFT) techniques to track the dominant frequencies of non-linear structures. This allowed real-time tuning of the STMD resulting in a more effective reduction in response.

The aim of this paper is to study the effectiveness of STMDs in the vibration control of wind turbine blades. Investigation into the natural frequencies of rotating blades is first considered for different rotational speeds. Two techniques have been employed for comparison. The first considers the natural frequencies of a rotating Bernoulli-Euler cantilever beam using the Frobenius method. This is then compared to the frequencies obtained from an eigenvalue analysis of the turbine model developed in this paper which includes blade tower interaction.

The model developed considers purely the structural dynamics of the turbine including the blade-tower interaction. Flapwise vibration only has been considered. The model consists of three rotating cantilever beams (representing the turbine blades) connected at their root to a large mass (which models the nacelle) allowing the inclusion of blade-tower interaction. The masses, lengths etc. were chosen to replicate those of a real wind turbine to accurately capture the dynamic interaction between the blades and tower. An STMD was connected to each blade tip and to the nacelle. This gave the completed model including STMDs a total of 8 Degrees of Freedom (DOF). Steady and turbulent wind loading was applied to the model acting in the flapwise direction.

## II ANALYSIS

### a) Determination of Blade Natural Frequencies using Frobenius Method

The governing differential equation for a rotating Euler Bernoulli beam under flapwise vibration is

$$\mathbf{r}A \frac{\partial^2 w}{\partial t^2} + \frac{\partial^2}{\partial x^2} \left( EI \frac{\partial^2 W}{\partial t^2} \right) - \frac{\partial}{\partial x} \left( T \frac{\partial w}{\partial x} \right) = f(x,t) \quad (1)$$

where  $\mathbf{r}$  is the density of the beam,  $A$  is the cross sectional area,  $w$  is the relative displacement of a point with respect to its static deflected position,  $E$  is the Young's modulus of the material of the beam,  $I$  is the moment of inertia of the beam,  $T$  is the centrifugal tension force on the beam and  $f$  is the applied force per unit length. The cross sectional area,  $A$ , and bending rigidity,  $EI$ , are taken as constant along the length of the beam,  $x$ . Both  $w$  and  $f$  are dependent on the location on the beam with respect to the origin,  $x$ , and time,  $t$ . The centrifugal tension  $T$  is expressed as

$$T(x) = \int_x^L \mathbf{r}A\Omega^2(r+x)dx \quad (2)$$

where  $L$  is the length of the beam,  $r$  is the radius of the rigid hub to which the flexible beam is attached and  $\Omega$  is the rotational speed of the beam, assumed to be constant. The effect of gravity on the rotation of the beam is assumed negligible compared to the centrifugal effect.

Setting  $f(x,t) = 0$  in equation 1 and substituting in some non-dimensional parameters, the modeshape equation is obtained in a dimensionless form as

$$D^4 W(X) - 0.5\mathbf{n}(1+2\mathbf{r}_0)D^3 W(X) + \mathbf{n}\mathbf{r}_0 D[XDW(X)] + 0.5\mathbf{u}D(X^2 DW(X) - \mathbf{m}W(X)) = 0 \quad (3)$$

where  $D = \frac{d}{dX}$ ,  $X = \frac{x}{L}$ ,  $W(X,t) = \frac{w(x,t)}{L}$ ,  $\mathbf{r}_0 = \frac{r}{L}$

Employing the Frobenius method of series solution of differential equations as in ref (10) and considering ideal clamped-free boundary conditions for a cantilever, the natural frequency equation is obtained to be

$$D^2 F(1,2)D^3 F(1,3) - D^3 F(1,2)D^2 F(1,3) = 0 \quad (4)$$

Where  $F(X,c) = \sum a_{n+1}(c)X^{c+n}$

Choosing

$$a_1(c) = 1, \quad a_2(c) = 0, \quad a_3(c) = \frac{0.5\mathbf{n}(1+2\mathbf{r}_0)}{(c+2)(c+1)} \quad \text{and}$$

$$a_4(c) = \frac{-\mathbf{n}\mathbf{r}_0 c}{(c+3)(c+2)(c+1)},$$

the recurrence relation is obtained as

$$(c+n+4)(c+n+3)(c+n+2)(c+n+1)a_{n+5}(c) - 0.5\mathbf{n}(1+2\mathbf{r}_0)(c+n+2)(c+n+1)a_{n+3}(c) + \mathbf{n}\mathbf{r}_0(c+n+1)^2 a_{n+2}(c) + [0.5\mathbf{n}(c+n)(c+n+1) - \mathbf{m}]a_{n+1}(c) = 0 \quad (5)$$

and the normalised modeshape equation can be derived as

$$\Phi_n(X) = \frac{[D^2F(1,3)F(X,2) - D^2F(1,2)F(X,3)]}{[D^2F(1,3)F(1,2) - D^2F(1,2)F(1,3)]} \quad (6)$$

The results obtained using the Frobenius technique are discussed later in the paper.

### b) Dynamic Model Formulation

The dynamic model was formulated using the Lagrangian formulation expressed in equation 9 below

$$\frac{d}{dt} \left( \frac{dT}{dq_i} \right) - \frac{dT}{dq_i} + \frac{dV}{dq_i} = Q_i \quad (7)$$

where:  $T$  = kinetic energy of the system,  $V$  = potential energy of the system,  $q_i$  = displacement of degree of freedom  $i$  and  $Q_i$  = generalized loading for degree of freedom  $i$ .

Each blade was modelled as a cantilever beam with uniformly distributed parameters. They were assumed to be vibrating in their first mode with a quadratic modeshape. The blades were attached at their root to a large mass representing the nacelle of the turbine. This allowed for the inclusion of the blade-tower interaction. Four STMDs were attached to the model, one at each blade tip and one at the nacelle to mitigate the vibration in the flapwise direction. These were modelled as mass-spring-dashpot systems whose tuning was controlled by the semi-active algorithm outlined later in this paper. A schematic of the model is shown in figure 1.

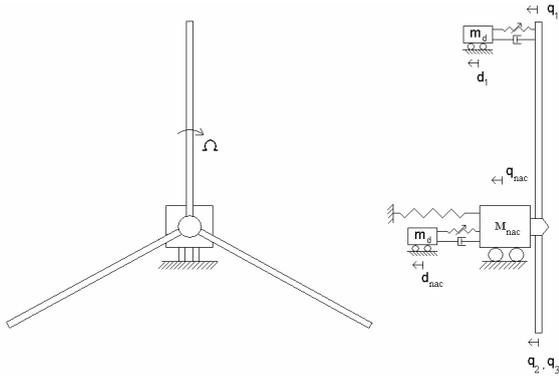


Figure 1: Dynamic Model

The degrees of freedom (dof) marked  $q_1$ ,  $q_2$ ,  $q_3$  and  $q_{nac}$  represent the motion of the blades and nacelle and the STMD displacements are labelled as  $d_i$ , where  $i$  corresponds to the relevant dof. For simplicity just two STMDs are shown in the diagram. One attached to the nacelle and the other attached to the blade in the upright vertical position. The final model with STMDs attached consisted of a total of 8 dof expressed in the standard form as in equation 8 below.

$$[M]\{\ddot{q}\} + [C]\{\dot{q}\} + [K]\{q\} = \{Q\} \quad (8)$$

where  $[M]$ ,  $[C]$  and  $[K]$  are the mass, damping and stiffness matrices of the system respectively.  $\ddot{q}$ ,  $\dot{q}$  and  $q$  are the acceleration, velocity and displacement vectors and  $Q$  is the loading. Centrifugal stiffening was added to the model as per the formula developed by Hansen (11, p.182). Mass proportional structural damping was included.

### c) Loading

Two simple load cases were studied. The first loading scenario looked at a steady wind load that varied linearly with height. Equation 9 shows the resulting expression for the loading on blade 1. The loads on blades 2 and 3 are shifted by angles of  $2p/3$  and  $4p/3$  respectively.

$$Q_1 = \left( \frac{v_{nac}^2 A}{3} + \frac{v_{nac+L}^2 A}{10} \right) + \left( \frac{v_{nac} v_{nac+L} A}{2} \right) \cos(\Omega t) \quad (9)$$

where:  $v_{nac}$  = wind speed at nacelle height,  $v_{nac+L}$  = wind speed at the maximum blade tip height, i.e. when blade is in upright vertical position.  $A$  = Area of blade, taken as 1 to normalize the load, with  $\Omega$  as before equal to the rotational speed of the blade. The loading on the nacelle was assumed to be zero so that any motion of the nacelle was due to coupling with the blades.

The second loading scenario considered the same load case as the first with an added random component modelling turbulent wind. This turbulent velocity component was generated at a height equal to that of the nacelle using a Kaimal spectrum (12) defined by equations 10, 11 and 12 below. Uniform turbulence was assumed for the blades.

$$\frac{f S_{vv}(H, f)}{v_*^2} = \frac{200c}{(1 + 50c)^5} \quad (10)$$

where:  $H$  = nacelle height,  $S_{vv}(H, f)$  is the PSDF (Power Spectral Density Function) of the fluctuating wind velocity as a function of the nacelle elevation and frequency,  $v_*$  is the friction velocity from equation 14, and  $c$  is known as the Monin coordinate which comes from equation 15.

$$\bar{v}(H) = \frac{1}{k} v_* \ln \frac{H}{z_0} \quad (11)$$

$$c = \frac{fH}{\bar{v}(H)} \quad (12)$$

where  $k$  is Von-Karman's constant (around 0.4 (13)),  $z_0 = 0.005$  (the roughness length), and  $\bar{v}(H)$  is the

mean wind speed. This results in a turbulence intensity of 0.115 in the generated spectrum.

#### d) STFT Based Tracking Algorithm

STFT is a commonly used method of identifying the time-frequency distribution of non-stationary signals. It allows local frequencies to be picked up in the response of the system that may only exist for a short period of time. These local frequencies can be missed by normal Fast Fourier Transform (FFT) techniques. The STFT algorithm splits up the signal into shorter time segments and an FFT is performed on each segment to identify the dominant frequencies present in the system during the time period considered. Combining the frequency spectra of each of these short time segments results in the time frequency distribution over the entire time history.

The STFT algorithm developed in this study allows the STMDs to be tuned in real-time to the dominant frequencies in the system. Before each time segment is Fourier analyzed it is multiplied by a Hanning window function centred on the current time emphasising the frequencies more recently present in the response. Once the weighted signal is obtained an FFT is performed and the frequency spectrum obtained. The dominant frequencies are then identified and the STMDs tuned to these frequencies. The algorithm is repeated every second allowing real time tuning of the STMDs. The semi-active algorithm is outlined in the flow chart shown in figure 2.

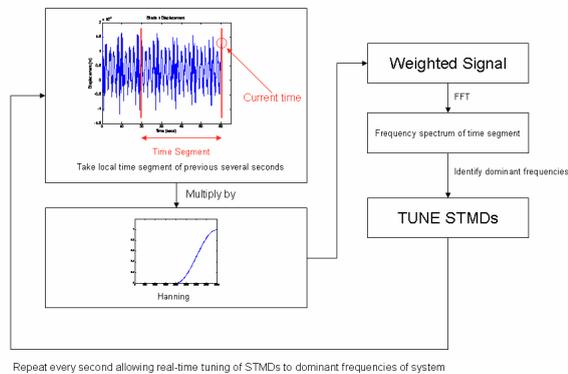


Figure 2: STFT Algorithm

### III RESULTS

#### a) Natural Frequency Estimation

The natural frequency was first calculated using the Frobenius method for a stationary Bernoulli Euler beam, i.e.  $O = 0$ . This value was then used in our dynamic model which added in the effect of centrifugal stiffening. A 14<sup>th</sup> term expansion was deemed sufficient for the Frobenius results. All natural frequencies calculated are for the first mode of vibration. The results are shown in Table 1. The 3 blade model results include blade tower coupling.

<b>O</b> (revs/min)	<b>Bernoulli- Euler Frobenius (Hz)</b>	<b>3-blade model (Hz)</b>
0	1.5588	1.5588
10	1.5703	1.5707
30	1.9274	1.9399
60	2.8010	2.7863

Table 1: Natural Frequency Results

As can be observed good agreement is seen between the 2 different models and the different methods used. It is important to note that the blade tower coupling in the 3 blade turbine model results in a different natural frequency for one of the blades compared to the other two. The closest corresponding one to the Frobenius result is included in the table. Omission of this coupling results in all 3 blades having the same natural frequency, close to those obtained for the Bernoulli Euler beam.

#### b) Dynamic Control-Steady Wind Load

The following section looks at the results of the STMD system for the steady wind loading described above in section 2c. Three different parametric variations were considered.

##### Vary O, rotational speed of the blades

The first parameter varied was the rotational speed of the blades,  $O$ . The variation considered the blades slowing down linearly over 180 seconds from 3.14 rads/s to 1.57 rads/s. Figure 3a shows the undamped and damped response of blade 1 with figure 3b showing the corresponding STMD behaviour by plotting the blade displacement, STMD displacement and STMD tuning all with respect to time.

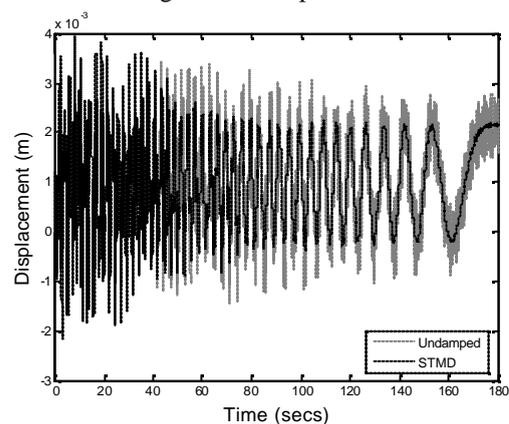


Figure 3a: Blade 1 response, vary  $O$

As can be seen in figure 3a a significant reduction is achieved in the response of the blade. The behaviour of the STMD in figure 3b clearly shows the semi-active behaviour kicking in at  $t = 41$  seconds and the tuning of the STMD changing with respect to time catering for the changing system properties.

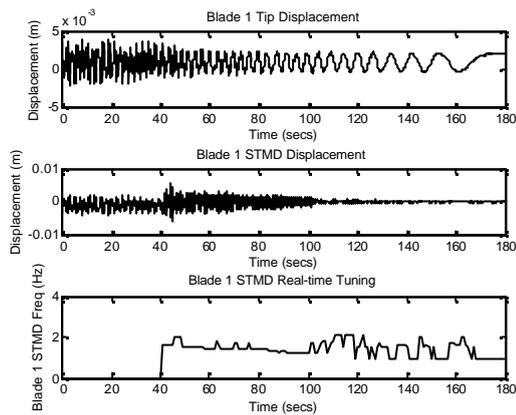


Figure 3b: Blade STMD behaviour, vary  $O$

A large reduction is also achieved in the nacelle response when the STMD kicks in at  $t = 41$  seconds as seen in figure 4.

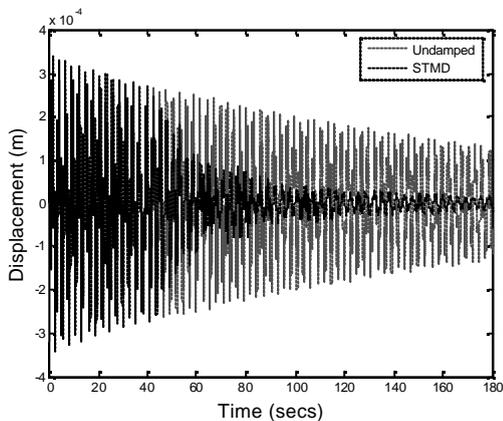


Figure 4: Nacelle Displacement, vary  $O$

Vary  $\omega_{b1}$ , the natural frequency of blade 1

The natural frequency of blade 1 was varied from 1.5588 Hz (9.79 rads/s) to 1.2398 Hz (7.79 rads/s) at  $t = 75$  seconds. This loss of blade stiffness simulates damage occurring in the blade. The other two blades were assumed to remain unchanged.

Figure 5a plots the displacement response of blade 1.

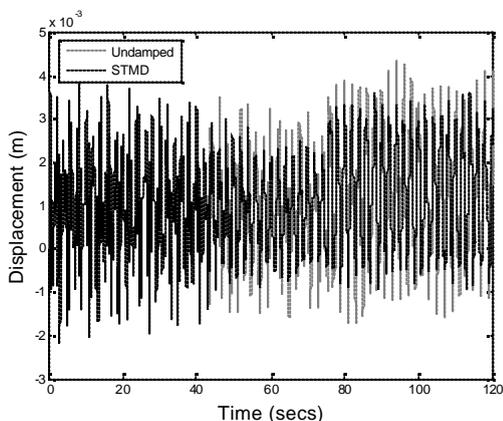


Figure 5a: Blade Displacement, vary  $\omega_{b1}$

As can be observed at  $t = 75$  seconds the behaviour of the blade changes due to the change in its natural frequency. The tuning of the STMD adapts for this as can be seen in figure 5b. This results in an effective reduction in the response of the blade before and after the change in natural frequency, as can be observed in figure 5a.

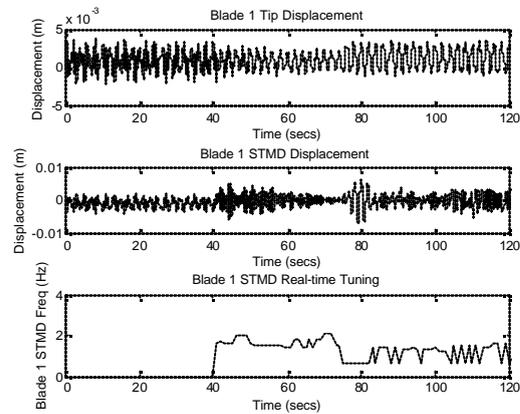


Figure 5b: Blade STMD behaviour, vary  $\omega_{b1}$

A similar reduction is achieved in the nacelle response both before and after the change in blade stiffness.

Vary  $\omega_{nac}$ , the natural frequency of the nacelle

Further simulations found that variation in the natural frequency of the nacelle could also be adequately catered for by the semi-active algorithm.

*c) Dynamic control-Turbulent Wind Load*

Response of the model to the turbulent wind load described in section 2c was also considered for the same three parametric variations. The algorithm again catered well for this load case. Figure 6 shows the response of blade 1 for varying  $O$  and figure 7 shows the response for a sudden lose in blade stiffness.

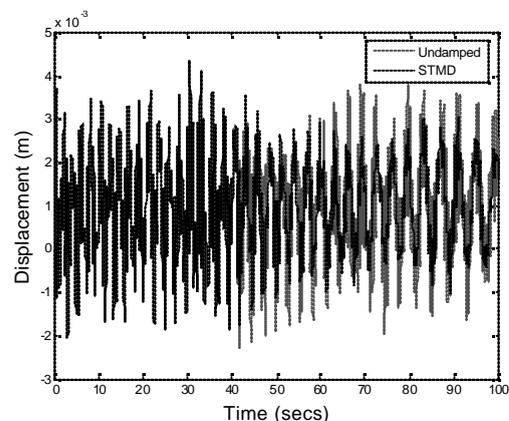


Figure 6: Blade 1 Response, vary  $O$ , turbulent load

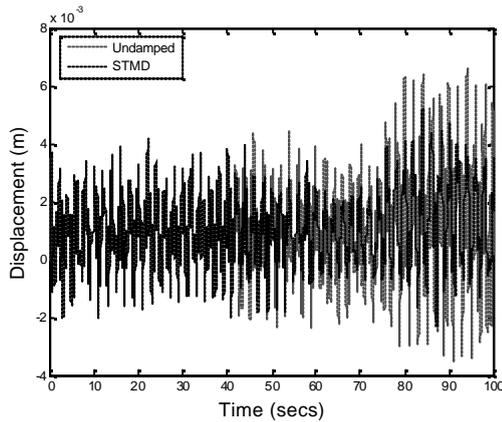


Figure 7: Blade 1 response, vary  $\omega_{b1}$ , turbulent load

Again a significant reduction was achieved in the response of the blades. Similarly effective results were seen for a sudden change in nacelle stiffness.

#### IV CONCLUSIONS

In this study, the authors investigated the use of an STFT based algorithm for the semi-active control of wind turbine blades in flapwise bending. The natural frequencies of the rotating blades for different rotational speeds,  $\Omega$ , were calculated by performing an eigenvalue analysis on the system. The results were compared to those obtained by applying the Frobenius method to a rotating Bernoulli Euler beam with the same stationary natural frequency. Good agreement was seen between the models and the methods used.

Four STMDs were added to the model, one at each blade tip and one at the nacelle to control the response the system. The displacement response of each dof was controlled in real time by inputting the previous 40 seconds of the response into the semi-active algorithm. This allowed a very fine sampling frequency of 0.025Hz thus ensuring no mistuning of the STMDs. The dampers were then repeatedly tuned every second to allow real-time tuning in the system. Numerical simulations were carried out to ascertain the effectiveness of the STMDs in mitigating flapwise vibrations when variations were considered in three of the system parameters; the rotational speed,  $\Omega$ , the natural frequency of blade 1,  $\omega_{b1}$ , and the natural frequency of the nacelle,  $\omega_{nac}$ . This allowed the simulations to model a variable speed wind turbine and take account of damage in the blades and nacelle which may occur during the life cycle of the turbine. Significant reduction was achieved by the semi-active algorithm for both steady and turbulent wind loading highlighting the viability of STMDs in controlling flapwise vibrations in wind turbines. A separate study by the authors into the edgewise vibration of the blades is also being carried out.

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