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Damage detection in beams with an open crack using S transform |
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Application of S Transform in Structural Health Monitoring

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Abstract
The successful detection of change in a data or in any of its derivatives in the presence of noise is a critical component of structural health monitoring and damage detection. This sudden change can be brought about by a sudden change in the strain or the stress field of the structural system under consideration. Two very typical examples of such sudden changes are the sudden change in stiffness of a vibrating single degree of freedom system in time and the local perturbation of stress and strain fields of a beamlike structure in space due to the presence of an open crack. New methods and analysis techniques have become popular in the field of structural health monitoring to detect and characterise such changes. Time–frequency techniques, like wavelet analysis are being more widely used in this regard in the recent times for the detection of presence, location and the calibration of the extent of these changes. This paper presents the application of S transform for the successful detection and calibration of damage in time and in space in the presence of additive Gaussian white noise. The performance of S transform based detection is compared with wavelet based and statistics based methodologies. The application and use of S transform in the field of structural health monitoring is observed to be extremely promising.

Résumé
La détection réussie d'un changement de données ou de ses dérivés en présence de bruit est un composant critique de la surveillance de la santé et de la détection structurale de dommages. Ce changement soudain peut être dans la contrainte ou les champs de déformation du système structural à l'étude. Deux exemples typiques sont le changement soudain de la rigidité d'un seul degré de vibration du système libre en temps et la perturbation locale des champs d'effort et de contrainte d'un faisceau due à la présence d'une fente ouverte. Les nouvelles méthodes et techniques d'analyse sont devenues populaires dans le domaine de la surveillance de la santé structurale pour détecter et caractériser de tels changements. La technique temps–fréquence, comme l'analyse en ondelette est employée couramment récemment pour la détection de la présence, la localisation et le calibrage de l'amplitude de ces derniers changements. Ce document présente l'application de la transformée S pour la détection et le calibrage réussis des dommages dans le temps et dans l'espace en présence du bruit blanc gaussien additif. La performance de la détection basée sur la transformée S est comparée aux méthodologies basées sur les statistiques et les ondelettes. L'application et l'utilisation de la transformée S dans le domaine de la surveillance de la santé structurale semblent être extrêmement prometteuses.

Keywords
S Transform, Kurtosis, Pseudofractal, Damage, Wavelet, Noise

1 Introduction
This paper presents an application of S transform in the field of structural health monitoring [1]. The detection of the presence, the location and the extent of damage in a
structure through non-destructive techniques has become popular in recent times using time-frequency analyses [2-4] and statistical measure based analyses [5-7]. Although a significant number of applications of various methods have been validated through experiments [8–10], a good amount of literature uses numerically simulated data to test the efficiency of a proposed detection method [11]. A successful application of any new method in the field of structural health monitoring thus requires the method to identify various types of damage in the space and the time domain. A way to test such a new method is to apply it on simulated data and observe its efficiency. Noise stress tests and comparison with other existing detection techniques on the same simulated data is important in this regard.

This paper proposes the use of S transform for structural health monitoring and illustrates this application on two systems in time and space domains respectively. The first problem assumes a sudden loss of stiffness in a single degree of freedom system where the objective is to detect the instant of time where this loss takes place. Additionally, it is also investigated whether such detection can take place in the presence of high noise. The second problem considers an open crack in a simply supported beam. Here, the objective is to identify the location of damage from the simulated first natural modeshape. The calibration of the extent of damage is compared with three other existing non-destructive damage detection and calibration methods. The application of S transform is observed to be very promising. The first example can be related to a sudden loss of stiffness of a structural system in time, typically brought about by the failure of structural components like braces or cables under a wide range of excitation forces including earthquake and wind. The second example is the considerably widely known problem of an open crack in a damaged beam which can be related to the local damage within a bridge. The successful application of S transform on the chosen problems significantly raises hope for the technique to be successfully applied on the abovementioned systems in real life.

2 S Transform

The S transform of a signal $h(t)$, $S(\tau, f)$ is given as [1]

$$S(\tau, f) = \frac{|f|}{\sqrt{2\pi}} \int_{-\infty}^{\infty} h(t) e^{-\frac{(t-\tau)^2}{2}} e^{-i2\pi f t} dt$$ (1)

where $\tau$ is the instant of time, $f$ is the instantaneous frequency commonly referred to as voice and $t$ is the time variable required for integration. This transform is different from a wavelet transform as the admissibility condition is not satisfied [1]. It is a local spectrum over time and averaging the local spectrum does give the Fourier transform of the original signal. Since the S transform captures the time-frequency characteristics of a signal locally, it can be expected that a sudden frequency change in time or a singularity is space leading to significant local deviation of a signal or any of its derivative will be captured in the transformed coefficients in the form of an extremum.

3 Problem Description

The application of S transform on structural health monitoring is illustrated through two different problems, details of which are provided below.

3.1 Sudden Loss of Stiffness

The first system is a linear single degree of freedom system comprising of a mass $(m)$, a dashpot $(c)$ and a number of springs connected in parallel $(k_1$ to $k_n)$ (Figure 1). A sudden loss
of stiffness can occur if one or some of the springs break at a certain instant of time. The response of the system would change due to its abrupt change in natural frequency.

\[ F(t) \]

\[ k \]

\[ c \]

\[ k \]

\[ k \]

\[ m \]

**Figure 1. Sudden Loss of stiffness in a linear SDOF system**

The system is acted on by an arbitrary time varying force. For this paper, a sinusoidal force has been considered. Additive Gaussian white noise corrupting the output signal is considered as well.

### 3.2 Local Damage

The second system considered in this paper is an example of a local damage in a simply supported Euler Bernoulli beam in the form of an open crack (Figure 2). The beam is of length \( L \) and the crack is located at a distance ‘\( a \)’ from the left hand support of the beam. The beam is modelled as two uncracked beams connected through a rotational spring at the location of the crack. The crack depth is taken as ‘\( c \)’ and the overall depth of the beam is ‘\( h \)’. The crack depth ratio (CDR) is defined as \( c/h \) and is a measure of the damage extent.

**Figure 2. Open crack in a simply supported beam**

The damage is modelled as a rotational spring connected by two sub-beams on each side respectively. Although continuity in displacement, moment and shear at the location of crack exists, a slope discontinuity is present at the location of the crack location of the crack dependent on the CDR. Details of the model can be found in Narkis [12].

### 4 Detection Capability of S Transform

The responses of the two mechanical systems illustrated in the previous sections are simulated. The displacement of the SDOF and the first natural modeshape of the damaged beam are assumed to be the outputs from the systems respectively. A sudden loss of 50% of the stiffness is considered at the 25th second from the start in the case of the SDOF system while a damage is considered at a location of 0.4m from the left support for a 1m beam where CDR is equal to 0.35. Figure 3 illustrates these output signals and the respective S transform under ideal condition. No noise is considered.

It is observed that S transform can successfully detect the presence of a sudden stiffness change in time and a local damage in space under ideal conditions. Thus, it can be a
promising method for damage detection. Additionally, the method is observed to be quite flexible since the nature of the damage is quite different in the two cases considered. The SDOF system elicits a time domain response due to a sudden frequency change and the sudden jump is present in the signal itself. On the other hand, the discontinuity of the derivative of the modeshape acts as a detector in the second example. Here, even under ideal conditions it is impossible to visually identify the location of the damage.

Once the application of S transform is observed under ideal conditions, its performance under noise required to be found.

5 Performance against Noise

The performance against S transform against noise is shown for the problem of sudden loss of stiffness. The example of space domain is not spelled out due to restriction of the length of the paper. The sudden stiffness loss occurs at about 32 seconds from the start.

Additive Gaussian white noise is considered here. It is observed that S transform can successfully isolate the event even when it is not apparent due to the presence of high noise.

Figure 3. Successful detection of damage in time(a-b) and space domains(c-d)

Figure 4. Noisy displacement (a) and successful detection (b) using S Transform
6 Comparison with other Techniques

The application of S transform is compared with three other existing damage detection techniques namely wavelet transform [3], Kurtosis Crack Detector (KCD) [6] and Pseudofractal Crack Detector (PFCD) [7]. The comparison is made in terms of the comparative change of the calibration values for an evolving crack located 0.4m from the left support of the beam. Of these techniques, the wavelet transform is a time–frequency method and would be closest to the class of transform S transform belongs in. The other two methods are essentially a measure of local deviation of a signal from Gaussianity. Details regarding the methods can be obtained from the listed references. Each calibration curve has been normalized to its absolute maximum value of calibration within the range of CDR considered (0.05 to 0.35 here).

It is observed that S transform creates a monotonously decreasing calibration curve for increasing CDR. The PFCD method is the least sensitive in terms of damage calibration while wavelet transform being the most sensitive. A Coif4 wavelet basis function at scale 22 has been used in this paper. The sensitivity of calibration is comparable for KCD and S Transform. However, a major advantage of S Transform is that the local spectra is obtained for a range of frequencies and each coefficient vector of the transform for a certain frequency over the length of the entire time would show a different sensitivity. Additionally, KCD and PFCD both are dependent on certain empirical values that are used to slice the signal and their performances can be severely affected if the value is not optimized [6–7]. Preprocessing techniques, like windowing may improve the calibration efficiency of S Transform based damage calibration as it has been found to be effective for wavelet transform based calibration before [3].

![Figure 5. Comparison of methods for damage extent calibration](image)

7 Conclusions

This paper illustrates the application of S Transform on a structural health monitoring problem. The proposed application has been successfully validated on simulated signals related to two different damaged systems, the first being a sudden stiffness deterioration of a linear SDOF system under harmonic excitation while the second being a local open crack in a simply supported damaged beam. The method has been observed to be effective even in the
presence of noise. The performance of S Transform for damage calibration has been compared with three other existing damage detection techniques.

References