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A new latching control technology for improving wave energy conversion

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Abstract
Extracting wave energy from seas has been proven to be very difficult although various technologies have been developed since 1970s. Among the proposed technologies, only few of them have been actually progressed to the advanced stages such as sea trials or pre-commercial sea trial and engineering. One critical question may be how we can design an efficient wave energy converter or how the efficiency of a wave energy converter can be improved using optimal and control technologies, because higher energy conversion efficiency for a wave energy converter is always pursued and it mainly decides the cost of the wave energy production.

In this first part of the investigation, some conventional optimal and control technologies for improving wave energy conversion are examined in a form of more physical meanings, rather than the purely complex mathematical expressions, in which it is hoped to clarify some confusions in the development and the terminologies of the technologies and to help to understand the physics behind the optimal and control technologies. As a result of the understanding of the physics and the principles of the optima, a new latching technology is proposed, in which the latching duration is simply calculated from the wave period, rather than based on the future information/prediction, hence the technology could remove one of the technical barriers in implementing this control technology. From the examples given in the context, this new latching control technology can achieve a phase optimum in regular waves, and hence significantly improve wave energy conversion. Further development on this latching control technologies can be found in the second part of the investigation.

Keywords: wave energy converter, power take-off, WEC control, WEC optimum, latching control, interaction of wave and structure, WAMIT.
1 INTRODUCTION

Wave energy is a type of well-concentrated and predictable renewable energy, and its resources are huge (IEA’s estimation of the total wave energy is up to 80,000TWh a year [1], compared to the worldwide electricity production 17,400 TWh in the year of 2004). Extracting wave energy from seas may significantly contribute to the green target of sustainable development around the world. In the past three decades, it has been shown that the wave energy conversions are practically difficult, although the principles of wave energy conversion have been proven, and different technologies can be used for extracting wave energy from seas. One question is how expensive we can convert wave energy into useful energy, and this question must be answered by researchers and developers before any commercial wave energy farm is built.

Although difficulties, wave energy conversions have been seen some successful stories. It is reported a few hundred navigational buoys (oscillating water columns) have been deployed in the remote and harsh areas where the access for frequently changing batteries is not viable (see Chozas [2] and Falcao [3]). Also, the earlier breakthroughs in wave energy conversion in 1970-1980s on some prototype devices once convinced people that massive wave energy production would soon become a reality. For example, 2GW wave power plant producing power at a rate of 1.3p/kWh has been described in Whittaker et al.[4]. So far, more than 1000 patents of wave energy conversion techniques have been granted in Europe, Japan, and North America (McCormick [5]), but only few technologies have actually progressed to large practical devices which could produce useful energy, and some of them even achieve full-scale or pre-commercial sea-trial stages (see [6, 7]).

In the path to make wave energy production comparative to other conventional or renewable energy resources, researchers and developers have made their efforts to improve the wave energy conversion efficiencies, either by designing an efficient wave energy converter, such as the famous Salter Duck; or employing the advanced control technologies to improve wave energy capture capacity, such as the full reactive/phase control; or the less effective but yet more practical latching/de-latching control (see Salter et al.[8]); or both. In this research, the popular optimal and control technologies for improving wave energy conversions are discussed, with an emphasis on how practically these control technologies can be used for improving wave energy conversion.

Optimal and control technologies for improving wave energy conversion have been proposed and developed since 1970s (see Falnes [9]) and the developed optimal theories have shown if an ideal power take-off (PTO) can provide the required performance, the wave energy capture by the device can be made to or close to the theoretical maximum. However, such as ideal PTO can not be achievable in practice due to the very requirements for the control optima and due to the mechanical limitation of the PTO device and some other issues. As a result of the difficulties in realising the full optimal/control strategy, more
Principally, optimisation of wave energy devices can be obtained either by an optimum phase and/or an optimum amplitude (Falnes [9]). The full optimal/control technologies have satisfied both phase and amplitude optima completely, in which the PTO system is assumed as an ideal control device, and can perform as an additional inertia or a spring as required so to fully counteract the intrinsic reactance of the wave energy converter, that is, the mass and spring terms cancels each other in the mass-spring-dashpot system of the wave energy conversion. Under the assumptions of a full control, if the wave energy converter is resonant with the wave excitation, the phase optimum is fulfilled, and if the PTO damping is optimised, the amplitude optimum can be fulfilled. As a result of the full optima, the device could extract wave energy to (or close to) the theoretical maximum. Terminologically, the full optimal control (hereafter ‘full optimum’) can be also called the full complex-conjugate control (Nebel [19]) or the full reactive/phase control (Salter et al [8]). Examples given by Falnes [20] have shown in the full optimal control, a part of the extracted energy must be effectively fed back into the waves through the PTO. By extracting wave energy from and releasing partial energy back to waves, the control system can significantly improve wave energy production. This implies that the full control requires the PTO must have both very high energy conversion efficiencies in extracting energy from and feeding some energy back to waves. This has been proven to be too difficult for a practical PTO if it is not impossible.

To develop more realistic optimum and control technologies, different technologies are proposed, and the most popular control technology would be the latching control technologies [10, 12, 16, 17, 21-23]). Among the latching control technologies, different control strategies have been proposed on how the latching control can help to reach a sub-optimal condition. For instance, the phase control by latching has been achieved by implementing different control strategies. Babarit et al [21] have compared three different latching control strategies, and concluded all three technologies can help to improve wave energy conversion significantly. However, in implementing these latching control technologies, some future information must be predicted or forecasted (the method of Falcao et al [12] is an exception), and they are often named as the ‘predictive method’. The requirement of the future information applies a challenge to practical applications.

In this research, the optimal methods are first examined and studied in a manner that the complex control theory is replaced with the method of more evident physical meaning and implementation, and based on the understanding and the principle of the phase optimum, a new latching control method is proposed, for which the latching duration is simply calculated based on the wave period (further development of the technology can be found in the second part of the research [24]). To illustrate the optimal and control technologies, a generic point absorber, which is close to some practical point
absorbers, is used for the investigation. Hence the numerical results can be sensible and realistic in terms of the hydrodynamic parameters and energy conversion. It has been shown that the generic wave energy converter with optimal or control technologies could extract more energy from waves, and among them, the full optimum technology could improve to extract energy close to the theoretical maximum.

2 A SIMPLE WAVE ENERGY CONVERTER

In studying wave energy conversion and control technologies for how to improve power production, a simple wave energy converter is used as an example through all the applications in this research. The wave energy converter is a generic point absorber of a cylinder with a radius \( R = 3.0 \text{m} \) and a draft \( D = 1.5 \text{m} \). The point absorber is a single body device with a reference to a fixed point, for example, the seabed. To improve wave energy production by the point absorber, a PTO with a control is applied (see Figure 1). The idealized PTO is capable of providing the required inertia, damping and spring effects, and it will be shown that how the PTO can possibly maximize wave energy production.

![Figure 1 A point absorber with a full PTO referencing to the seabed](image)

The same cylinder point absorber has been analysed in Sheng et al [25], and the panels for WAMIT analysis are for the wetted surfaces, shown in Figure 2.

For wave energy conversion, the single motion mode, heave, is considered as the motion for power take-off. And we will show the improvements of wave energy conversion by the inclusions of inertia and spring effects from the power take-off system.

To illustrate the hydrodynamic performances of the cylinder in waves, Figure 3 to Figure 5 show the responses of the heave motion for the cylinder freely floating in waves. Figure 3 shows the added mass...
and the hydrodynamic damping coefficient. It can be seen that the cylinder has a minimum added-mass at \( \omega = 2.18 \text{ rad/s} \), and its added-mass approaches a constant value at large frequencies; while the hydrodynamic damping has a maximum value at \( \omega = 1.47 \text{ rad/s} \), and becomes zero when the frequencies become large.

![Figure 2](image1.png)

**Figure 2** Panels on the wet body of the wave energy converter

![Figure 3](image2.png)

**Figure 3** Added mass and hydrodynamic damping coefficient for the cylinder WEC

Figure 4 shows the excitation response on the cylinder. It can be seen that the excitation force is a mono function with wave frequencies: the force becomes smaller when the frequency becomes larger.

Figure 5 shows the heave motion response (RAO). The heave motion has a resonance at \( \omega_0 = 1.8 \text{ rad/s} \) (that is, the resonance period \( T_0 = 3.49s \)). The maximum response of the heave motion is 2.12, which is a
reasonable resonance response, because the cylinder has a shallow draft which tends to have a large hydrodynamic damping coefficient. As a result of this, the viscous damping in this case is not important and will be ignored.

Figure 4 Excitation response of the cylinder WEC

Figure 5 Heave RAO (response amplitude operator), showing the resonance at the frequency $\omega_0=1.8$ (rad/s)
3 THEORY FOR WAVE ENERGY CONVERSION

3.1 Maximum wave energy conversion

The theoretical maximal wave energy capture width by a wave energy converter has been studied by several scholars respectively (see Falcao [3] and Babarit et al. [26]). It is noted that the theoretical maximum wave energy capture width may only depend on the types of the wave energy converters. For example, an axi-symmetrical point absorber which has a desired heave motion for wave energy conversion will have a maximum wave energy capture width as

$$W_0 = \frac{\lambda}{2\pi}$$

(1)

where $W_0$ is the wave energy capture width, and $\lambda$ the wave length.

It has been also shown that if the converter has a desired pitch motion about one symmetrical axis, it has a same maximum wave energy capture width as given in eq. (1) (see Falnes [27]). However, if the surge motion of the device is used for wave energy conversion, its maximum wave energy capture width can be

$$W_0 = \frac{\lambda}{\pi}$$

(2)

The theoretical maxima for wave energy conversion means that for a practical wave energy converter, its wave energy capture width will be never larger than the theoretical maximum, regardless of how the wave energy converter is designed and what control strategy is used.

Some further points can be made as follows. First of all, for more wave energy conversion, more than one motion modes for wave energy conversion are desirable, due to the aforementioned limits for each motion mode for energy conversion. For example, if the heave and pitch motions can be both used for wave energy conversion, then the maximum wave energy capture width could be given by Eq. (2), instead of Eq. (1). However, a practical reason for using multi-motion modes for wave energy extraction, although difficult, is that the different motion modes may have different resonance frequencies/periods, so that the multi resonances of the device can widen the bandwidth of the device for wave energy extraction from seas.

Secondly, the maximum wave energy capture width for the motion about its symmetrical axis is limited by eq. (1), while for the surge motion as the mode for wave energy conversion, the limit is doubled (see eq.(2)). For the surge motion, we can actually take it as an extreme case: the surge motion can be regarded as a pitching motion about an axis at infinity above the device. As it is well known that
when the pitching axis is located away from the symmetrical axis, the device has a higher energy conversion capture width (thus the conversion efficiency). This is the case in the development of the Salter’s Duck (Chapter 2, Cruz [28]). For a conventional pitching wave energy converter, its pitching axis is always between the symmetrical axis and the case of the surge motion (the pitching axis is in infinity). So it can be deduced that the theoretical maximum of the wave energy capture width for a pitching device will be between $\frac{\lambda}{2\pi}$ and $\frac{\lambda}{\pi}$, very much depending on the position of the pitching axis.

### 3.2 Frequency-domain analysis

In the numerical simulation, WAMIT is, which is based on the conventional boundary element method and potential flow, employed (the relevant potential theory can be found in Refs. [20, 29, 30]). In frequency domain, the dynamic equation for the floating body has a form (following the expression given by Falnes [20]) as,

$$i\omega(m + a_{33}) + b_{33} + \frac{c_{33}}{i\omega}u_3 = f_3$$

(3)

where $m$ is the mass of the device, $a_{33}$, $b_{33}$, and $c_{33}$ are the added mass, hydrodynamic damping coefficient and the stiffness coefficient, respectively; $u_3$ the complex velocity amplitude; $f_3$ the complex excitation force amplitude for heave motion and $\omega$ the wave circular frequency.

The corresponding solution of the frequency-domain equation is

$$u_3 = \frac{f_3}{i\omega(m + a_{33}) + b_{33} + \frac{c_{33}}{i\omega}}$$

(4)

and the corresponding complex amplitude of the heave motion is

$$\xi_3 = \frac{f_3}{-\omega^2(m + a_{33}) + i\omega b_{33} + c_{33}}$$

(5)

with a resonance frequency

$$\omega_0 = \sqrt{\frac{c_{33}}{m + a_{33}}}$$

(6)
When a full linear power take-off is applied in further analysis, the PTO force can be expressed by following Babarit et al [31],

\[ F_{PTO}(t) = -\left[ a_{PTO} \ddot{X}_3(t) + b_{PTO} \dot{X}_3(t) + c_{PTO} X_3(t) \right] \]

(7)

where \( X_3(t) \) is the time-dependent heave motion, \( a_{PTO}, b_{PTO} \) and \( c_{PTO} \) are the linear coefficients of the PTO additional mass, PTO damping and PTO spring coefficient.

Ideally, the PTO can be used as a device for energy extraction as well as a control device. To maximise the wave energy conversion, the PTO device is needed to be controlled so that the power take-off devices would have abilities to control the device as well as to convert energy from the moving body. Mathematically, it can be understood that the power take-off system will apply an external force on the oscillating body, and its mathematical expression of the force can be made as the linear terms of the acceleration, velocity and position, given by eq. (7).

In frequency-domain, the PTO force is given in a form

\[ f_{PTO} = -\left( i \omega a_{PTO} + b_{PTO} + \frac{c_{PTO}}{i \omega} \right) u_3 \]

(8)

where \( f_{PTO} \) is the complex PTO force amplitude.

When the power take-off system is applied to the floating body, the dynamic equation of the floating point absorber becomes,

\[ i \omega (m + a_{33} + a_{PTO}) + (b_{33} + b_{PTO}) + \frac{c_{33} + c_{PTO}}{i \omega} u_3 = f_3 \]

(9)

its corresponding solution is

\[ u_3 = \frac{f_3}{i \omega (m + a_{33} + a_{PTO}) + (b_{33} + b_{PTO}) + \frac{c_{33} + c_{PTO}}{i \omega}} \]

(10)

The corresponding motion complex amplitude in this case is given as

\[ \xi_3 = \frac{f_3}{-\omega^2 (m + a_{33} + a_{PTO}) + i \omega (b_{33} + b_{PTO}) + (c_{33} + c_{PTO})} \]

(11)

The average power extraction by the power take-off system is calculated as
\[
\bar{P} = \overline{F_{PTO}(t) \cdot U_3(t)} = \frac{1}{2} \text{Re} \left( f_{PTO} \cdot u_3^* \right)
\]  

(12)

where * denotes the conjugate, overbar the average value over time and \(U_3(t)\) is the time-dependent velocity.

In an analytical form, the average captured power can be given,

\[
\bar{P} = \frac{1}{2} \frac{b_{PTO} |f_3|^2}{(b_{33} + b_{PTO})^2 + \left[ \omega (m + a_{33} + a_{PTO}) - \frac{c_{33} + c_{PTO}}{\omega} \right]^2}  
\]

(13)

where \(|f_3|\) is the excitation force module for heave motion. If \(f_3\) is calculated for unit-amplitude wave, then \(\bar{P}\) given in Eq. (13) is the average power capture response (Sheng et al. [25]).

The wave energy capture width for the deep water waves is given by

\[
W = \frac{\bar{P}}{\frac{\rho g^2}{8\pi} T}  
\]

(14)

where the denominator is the wave energy flux of regular waves with a wave height \(H=2\text{m}\) (i.e., a unit wave amplitude) and a period, \(T\).

4 OPTIMUM ANALYSIS

For the PTO optimal and control technologies, different optimal strategies are studied in this section. The PTO considered here is linear with the acceleration, velocity or motion of heave, as given by eq.(7).

4.1 Optimized damper (PTO1)

The optimal damper for the PTO is considered as a pure damper with \(a_{PTO} = c_{PTO} = 0\). Since the PTO is controlled to produce maximum wave energy, maximizing the power conversion (13) yields an optimised damping coefficient as

\[
b_{PTO1} \left( b_{33}^2 + \left[ \omega (m + a_{33}) - \frac{c_{33}}{\omega} \right]^2 \right)^{1/2}  
\]

(15)
Obviously, the optimised damping coefficient is very frequency dependent. Figure 6 shows the optimised damping coefficient for the point absorber. It is interesting to note that the minimum optimised PTO damping coefficient is at $\omega=1.83$ rad/s, a slightly different from the heave motion resonance frequency $\omega_0=1.80$ rad/s.

Corresponding to the optimised damping coefficient, the heave response is shown in Figure 7. It can be seen that the heave response no longer achieves unit when the frequency is small (i.e., in the long period waves). The reason for this is the much increasing PTO damping at low frequencies. When the frequency is small, from eq. (15), the optimized PTO damping will be dominated by

$$b_{PTO} \approx \frac{C_{33}}{\omega}$$

(16)

The corresponding heave RAO is given by

$$\xi_3 \approx \frac{f_3}{(i+1)C_{33}} = \frac{1}{2} (1-i) \frac{f_3}{C_{33}}$$

(17)

In the free floating situation, $\frac{f_3}{C_{33}}$ is unit when the frequency is small (from eq.(5)). Hence from eq.(17), the heave response at very low frequency will be 0.707, which is also confirmed in Figure 7, and the response has a lagging phase of 45º to the wave excitation.

![Figure 6 Comparison of hydrodynamic damping coefficient and optimized PTO damping coefficient](image-url)
The corresponding average wave energy extraction by following Falnes ([20], P.51) as

\[ P_1 = \frac{1}{4} \frac{|f_3|^2}{b_{33} + b_{33}^2 + \left[ \omega(m + a_{33}) - \frac{c_{33}}{\omega} \right]^2} \]

(18)

From the look of the equation of the captured average energy by the eq. (18), it seems to have a maximum value at the resonance frequency, because the term \( \omega(m + a_{33}) - \frac{c_{33}}{\omega} \) in the denominator simply disappears at the resonance frequency, and this conclusion has been used or claimed by some researchers in their papers and books. However, it needs to be pointed out that this is only true when the added mass, hydrodynamic damping coefficient and excitation response are all frequency-independent, because only under those conditions, an analytical derivation could prove that the captured energy calculated by eq. (18) has a maximum value at the resonance frequency. Since these hydrodynamic parameters are all frequency-dependent (and this is also true for most practical wave energy converters), the maximal value given by the eq. (18) becomes much more complicated. In fact, there is no analytical formula for the maximum energy conversion. However, it will be safe to say that at the resonance frequency, the wave energy conversion has reached its phase optimum, that is, the motion velocity of the device is in phase with the wave excitation.

To illustrate that the maximum wave energy conversion may not happen at the resonance frequency, the numerical analysis for the cylinder WEC is used. As shown in Figure 8, the wave energy conversion turns out that there are two maxima at \( \omega=1.04 \) rad/s and \( \omega=1.72 \) rad/s, respectively. Both maximums
happen at different frequencies from the resonance frequency of the heave motion, but the second maximum occurs at a frequency very close to the resonance frequency (1.72 rad/s compared to 1.80 rad/s). The first maximum of the wave energy conversion happens at a much lower frequency and it is due to the fact that the optimized PTO damping increases significantly when the frequency reduces (see Figure 6), hence the corresponding velocity may be modest.

The wave energy capture width in this case is shown in Figure 9. The energy capture width has a maximum at $\omega=1.77$ rad/s, with a maximum capture width $W_1=2.96$ m (theoretical maximum capture width $W_0=3.13$ m). Again, it does not happen at the resonance frequency, but very close (1.77 rad/s to 1.80 rad/s).

Figure 8  Wave energy conversion via an optimized PTO

Figure 9  Wave energy capture width against the theoretical maximum capture width (device width=6m)
4.2 Tuned constant damper (PTO2)

This may be the simplest optimum case, in which the PTO is acting as a pure constant damper which is tuned to produce maximum wave energy conversion at a specified frequency. It may be regarded as a special case of the optimized damper, but it is probably the most practical way to increase wave energy production because an active PTO control due to its frequency-dependency is impractical in many practical applications.

Generally, the tuned PTO damping is set to be equal to the hydrodynamic damping at certain frequency, $\omega_1$.

$$b_{PTO2} = b_{33}(\omega_1)$$

(19)

Frequently, the tuned PTO damping as the hydrodynamic damping at the resonance frequency $\omega_1=1.8$ rad/s, i.e., $b_{pto2}=21.839$ kN/(m/s), or at the frequency $\omega_1=1.72$ rad/s ($b_{pto2}=22.883$ kN/(m/s)) when the maximum wave energy conversion occurs can be chosen.

When the constant damping coefficient is decided, the average wave energy conversion is given as

$$\bar{P}_2 = \frac{1}{2} \frac{b_{PTO2} f_3^2}{(b_{33} + b_{PTO2})^2 + \left[ \omega (m + a_{33}) - \frac{c_{33}}{\omega} \right]^2}$$

(20)

Figure 10 and Figure 11 show the heave motion responses and the wave energy capture widths for different damping levels. From Figure 10, it can be expected that a slightly larger damping coefficient creates a slightly smaller response at the resonance frequency, but the difference in the wave energy capture width at the resonance frequency is very small because the difference between the two damping levels are relatively small. However, a slightly larger damping ($b_{pto2}=22.883$ kN/m/s) creates a better energy capture width in both higher and lower frequencies, as shown in Figure 12. If the damping level is further increased to $b_{pto2}=30.0$ kN/m/s, it can be seen the obvious improvement of energy capture in higher and lower frequencies at the cost of a small reduction at the resonance frequency. Further increase in the PTO damping level to $b_{pto2}=50$ kN/m/s, a better wave energy conversion can be seen in the higher and low frequencies, but a larger reduction around the resonance frequency can be seen (see Figure 14).

Comparatively, if the PTO damping level is reduced to $b_{pto2}=15$ kN/m/s (an underdamping case), the energy capture width is reduced for all frequencies (see Figure 15). Obviously, the underdamping in PTO is not favorable.
Figure 10  Heave responses for different constant damping coefficients

Figure 11  Wave energy capture widths with different damping levels
Figure 12  Enlarged wave energy capture widths with different damping levels

Figure 13  Large PTO damping coefficient increases the band of wave energy capture
4.3 Full PTO control (PTO3)

For the full PTO optimum, it is supposed that the PTO has a full optimum and control capability so that the phase and amplitude optima can be fulfilled completely. For achieving this, the PTO has an ability in adjusting the coefficients of the PTO inertia, damping and spring terms in such a manner that a maximum average energy extraction can be achieved for the wave energy converter. In realisation, the PTO device needs to be an idealised device in both efficiently taking energy out from the oscillating body
and feeding energy back to the oscillating device, thus back to wave, in an alternative way through the mass and spring control. And at same time, the PTO must behave as a damper to convert part of the energy into useful energy.

From eq. (13), it can be easily seen that a significant average energy extraction from waves can be achieved by the phase optimum, if the PTO can produce an appropriate an inertia or spring stiffness or both to ensure

\[ \omega (m + a_{33} + a_{PTO}) - \frac{c_{33} + c_{PTO}}{\omega} = 0 \]

(21)

Under the condition, the velocity of the device is calculated as

\[ u_3 = \frac{f_3}{b_{33} + b_{PTO}} \]

(22)

Obviously, the wave excitation force and the body motion velocity are in phase. Hence the condition (21) which requires the system has counteracted the reactance of the device.

To ensure (21), you can either change \( a_{PTO} \) or \( c_{PTO} \), or both. For PTO inertia control, an anti-mass can be used in such a way that the relevant hydrodynamic parameters, including added-mass, damping and the excitation, can remain unchanged (Vantorre et al.[32]). However, a more practical way may be the change of the overall mass of the device as a part of the PTO control, though the hydrodynamic coefficients may also be changed accordingly. For example, Wavebob [33] is reported to have a capacity of pumping the ballast water into and out of the device, so to change the reactance of the device for maximizing wave energy production. Increasing the spring effect by the PTO control may be useful if a higher resonance frequency is expected, because practically only a positive spring coefficient can be produced if a spring is used. The spring coefficient can be calculated as

\[ c_{PTO} = \omega^2 (m + a_{33}) - c_{33} \]

(23)

The required PTO spring coefficient is negative when the frequency is lower than the resonance frequency, shown in the eq. (23). As pointed out by Falcao [34], the negative spring effect may be produced by the hydraulic accumulators.

It must be noted the required PTO spring coefficient is increasing with the frequency squared. Increasing the frequency may require a significant increase in the PTO spring coefficient. Besides, how to
adjust different PTO spring coefficient based on the different frequency is another challenge to the device control system.

Under the condition (21), the corresponding average power is

\[ \bar{P} = \frac{1}{2} \frac{b_{\text{PTO}} |f_3|^2}{(b_{33} + b_{\text{PTO}})^2} \]  

(24)

Maximizing the average power in (24) yields a condition as

\[ b_{\text{PTO3}} = b_{33} \]  

(25)

\[ \bar{P}_3 = \frac{1}{8} \frac{|f_3|^2}{b_{33}} \]  

(26)

this is the maximal wave energy conversion by a wave energy device, which can be found in many references (see Falnes [20], and Evans et al [35]).

Under the full control condition, the complex motion amplitude of the heave motion is calculated as

\[ \xi_3 = \frac{1}{2 i \omega b_{33}} f_3 \]  

(27)

As a result of the active control on the PTO given by eqs. (21) and (25), the heave response can be very high in both high and low frequencies. Figure 16 shows a comparison of the heave RAOs of an optimised damper (PTO1) and the full control PTO (PTO3). The two RAOs are close only around the resonance frequency. In low frequencies, the period of the heave motion is long.

Figure 17 shows the comparison of the wave energy extractions via different PTOs. It can be seen that the PTO full control (PTO3) produces much higher power at higher and lower frequencies than that of the optimised damper (OPT1), but around the resonance frequency, both PTOs extract similar power from waves. Figure 18 shows the comparison of wave energy capture widths for the two PTOs. An interesting result is that the wave energy capture width with the PTO full control is very close to the theoretical maximum. This should be understood that when the full control is applied to the PTO, the wave energy device is always resonant with the wave, regardless of the wave frequencies, and at the resonance frequency, the wave energy capture is very close to the theoretical maximum if the optimised damping is used, even for different PTO control strategies (PTO1 and PTO3), which can be seen from Figure 9 and Figure 11.
Figure 16  Heave responses for optimized damper (PTO1) and full control PTO (PTO3)

Figure 17  Wave energy conversions for optimized damper (PTO1) and full control PTO (PTO3)
5 LATCHING CONTROL

5.1 Latching duration

For a latching control, it is very beneficial and well accepted that the latching happens at the instant when the device velocity becomes zero or very small. However, the best instant when the unlatching is applied can be different for different latching control strategies (see Babarit et al [21] and Hals et al [13]). So far, almost all the proposed latching control strategies need information into the future for at least a few seconds so that the required future information can be available for determining the instant of unlatching. For instance, if the latching control aligns the maximum velocity to the peak excitation, the short-term prediction of the peak excitation must be made for a few seconds to accurately predict the instant when the peak excitation occurs. There is one exception for the latching control technologies which is proposed by Falcao [12, 34], in which the unlatching happens at the instant when the PTO force exceeds a given threshold so that the future information is not required by the latching control technology.

As it is shown in the previous chapters, the significantly increased wave energy conversion can be extracted at a phase optimal condition, in which the motion of the wave energy converter is resonant with the wave excitation, so that the velocity of the WEC can be in phase with the wave excitation (22).

Based on the phase optimal condition, a new method for deciding the latching duration is proposed here. The new latching duration is only based on the wave (characteristic) period for regular waves for...
this part of the research (for the case in irregular waves it can be seen in the second part of the research [24]): the latching duration is calculated as

\[ t_{latch} = \frac{T_w - T_0}{2} \]  

(28)

where \( t_{latch} \) is the latching duration, \( T_0 \) the resonance period and \( T_w \) the wave period (it will be shown this proposal has essentially removed the requirement of the future information in [24]).

Based on the latching duration in Eq.(28), the unlatched time in a wave cycle can be calculated as

\[ t_{unlatched} = T_w - 2t_{latch} = T_0 \]  

(29)

It can be seen that the unlatched period by Eq. (29) in a wave cycle is completely coincident with the device resonance period. This implies that the latching control has essentially changed the wave excitation of a long period into the resonance period. When the device is in un-latched condition, its motion is resonant with wave excitation. As it has been illustrated previously, when in resonance, the device automatically satisfies the condition of phase optimum, this is why the latching control often called as latching for phase control.

The definition of the latching duration in eq.(28) gives a similar latching duration as the Falnes’ proposal [9] in which the unla tching of the device occurs at the instant of \( T_0/4 \) ahead of the next ‘peak excitation’ in regular waves (it can be seen that the unlatching needs the future information of the instant of the next peak excitation). If the unlatching happens at the instant of \( T_0/4 \) ahead of the next peak excitation, and next latching would very likely occur at the instant of \( T_0/4 \) after the peak excitation. Thus a single period of the unlatching in this latching method is \( T_0/2 \). Hence in a wave cycle, two unlatching periods together would make a period of \( T_0 \), which is the same as the resonance period of the device heave motion.

### 5.2 Wave energy conversion with a latch control

From the frequency-domain analysis above, it is shown that the cylinder wave energy converter has a resonance period \( T_0=3.49s \) (\( \omega_0=1.80\text{rad/s} \)). Obviously, this device has a rather short resonance period when compared to those of the waves in energetic seas, which normally have a period between 7-12s (i.e., 0.5-0.9 rad/s given by Falcao [12]). To the energetic waves, this particular point absorber will be very inefficient because it will work far off its resonance in such a wave condition. Take an example of a regular waves of a period of \( T_w=6s \) (wave height \( H=1.0m \), which is a rather short period of sea waves, but it is still significantly larger than the device’s resonance period. For instance, when a linear PTO with
a damping level $b_{PTO}=50$ kN/(m/s) (this is a damping coefficient for the device to take maximal energy out from sea waves), the responses of the device to the wave are well off the optimal condition. Figure 19 shows a phase comparison of the velocity and the excitation. Obviously, the velocity and excitation is out of phase by about $\pi/2$, because the wave period is much longer than the resonance period of the heave motion of the point absorber, and the motion will be in phase with the wave excitation (i.e., the point absorber performs like a wave rider). That explains a phase of $\pi/2$ between the velocity and the wave excitation.

To improve wave energy extraction, the new latching control technology is applied to the device. The new latching control is implemented in a manner that the device is latched at the instant when its velocity becomes zero or very small, and after the given period by (28), the device is released (unlatched) so the device and PTO can convert the wave power into useful energy.

The dynamic equation for the latching controlled device can be expressed as,

$$\left[ M + A_{33}(\infty) \right] \ddot{X}_3(t) + \int_0^t K_{33}(t-\tau) \dot{X}_3(\tau) d\tau + b_{PTO} \times \dot{X}_3(t) + C_{33} X_3(t) = F_3(t) + F_{control}(t)$$

where $X_3$ is the heave motion, $A_{33}(\infty)$ the added mass for have at infinite frequency, $C_{33}$ the hydrostatic restoring coefficient for heave, $F_3$ the excitation force for heave, $F_{control}$ a force applied for latching control, $K_{33}$ the impulse function for the heave motion. These parameters can be assessed via WAMIT and its frequency to time domain analysis tool.

For the cylinder point absorber, the impulse function for the heave is shown in Figure 20.
Once the equation (30) is solved, then the wave energy extraction can be calculated as

\[
\overline{P} = \frac{1}{T} \int_0^T b_{PTO} \times V_3^2(t) \, dt
\]

where \(T\) is the total period for the simulation.

To illustrate the benefit of latching control for improving wave energy conversion, a simple example is given as follows.

Figure 21 and Figure 22 show the comparisons of the heave motions and velocity with no control and with a latching control \((b_{PTO}=50 \text{ kN/(m/s)})\). In this example, the wave period is 6s (wave height \(H=1.0\text{m}\)). The latching duration is simply taken as the half of the difference between the wave period and the resonance period \((T_0=3.49\text{s})\) (28). Obviously, the latching control has changed the motion phase greatly so that the velocity is pretty much in-phase with the excitation. It can be seen that, by latching control, the heave velocity amplitude has been significantly increased at about 3 times (Figure 21) while the motion amplitude increases at about 1.6 times (Figure 22). Figure 23 shows the comparison of the heave motion velocity (with latching control) and excitation. It is clear that the velocity is very much in phase with the excitation (compared to Figure 19). In this regard, the phase optimum is said to be achieved. As a result of the latching control, the average power conversion has increased from 6.38kW to 32.95kW, an increase by 416%. If the wave period is longer, the increase of the wave energy extraction with the latching control can be more significant.
Figure 21  Comparison of motions for latching control to no control in a regular wave of $H=1.0\text{m}$ and $T_w=6.0\text{s}$

Figure 22  Comparison of velocity for latching control to no control in a regular wave of $H=1.0\text{m}$ and $T_w=6.0\text{s}$

Figure 23  Velocity and excitation after latching in a regular wave of $H=1.0\text{m}$ and $T_w=6.0\text{s}$
6 RESULTS AND ANALYSIS

In this section, the wave energy extractions with different optimum/control strategies are investigated. The optimum/control strategies include:

1) **Optimised damper (PTO1)**
   The optimized damper is given by eq. (15), which is obviously frequency-dependent. If regular waves are considered, corresponding to the single wave frequency, the optimized damper is a constant. The corresponding power conversion is given by (18).

2) **Constant damper (PTO2)**
   Constant damper here is an optimized damper corresponding to the optimized damper at the resonance frequency (in this case, \( \omega_0 = 1.8 \text{ rad/s} \)). This damper is independent of the wave frequency. The converted power is calculated by (20).

3) **Full optimum/control (PTO3)**
   Full optimum/control is achieved in two conditions: the phase optimal condition (21) and optimized damping condition (25). Accordingly, the power conversion is calculated by eq.(26).

4) **Latching control (PTO4)**
   The latching control here is implemented via the proposed latching duration given by (28), and the converted power is calculated by (31).

![Figure 24 Power conversions with different optimal and control strategies](image)

The wave height is taken 2m (that is, wave amplitude is 1.0m). In the consideration, such a wave height is not possible in the very short waves, but this will not affect the analysis as follows.
Figure 24 shows the comparison of the wave energy conversion via different optima and control. For the constant damper (PTO2), the PTO damping is tuned for the maximum power conversion as the resonance period/frequency. It can be seen that the power conversion reaches its maximum at the resonance period, which is also coincident with the theoretical maximum for the device. But away from the resonance period, the capacity of wave power extraction decreases on both sides for longer and shorter waves.

For the optimized damper (PTO1), the damping coefficient is very frequency dependent (see Figure 6). Again, at the resonance period, the converted power is very close to the theoretical maximum, but the maximum converted power is not happened at the resonance period. It is interesting to see that the power conversion from long waves seems a constant, regardless of the wave period. However, the power conversion in short waves, the capacity of power conversion reduces rapidly, similar to the case of the constant damper (PTO2).

For the full optimum (PTO3), the power conversion is very close to the theoretical maximum, regardless of the wave periods. It can be said that the full optimum is the most effective control, but we must recognize the implementation of the full optimum is very difficult in reality. For a practical wave energy device taking energy out from the random waves, the changes in the added control mass or the spring coefficient are impractical to implement, which can be seen from the eq. (21).

For the latching control (PTO4), the energy conversion is very close to the theoretical maximum for the wave period up to 5 s (the resonance period is 3.49s). In long waves, latching control can help to extract more energy out up to wave period of 8s. If the wave period is further increased to 10s, the power capture is reduced, however, not too much.

From the comparison of the power capture capacities, it can be seen that the full control/optimum is the most effective control technology for improving wave energy conversion. Though less effective than the full control/optimum, latching control is indeed very effective in improving wave energy conversion. The optimized damper though better than the tuned constant damper is not very affective, but the rapid increase in damping coefficient on the wave frequency/period can be a very difficult factor in implementing the technology.

Practically, constant damper may be the easiest control technology to be implemented, hence it finds many applications in the practical wave energy converters, though the method is not so effective as other control/optimal technologies.
7 CONCLUSIONS

In this part of the investigation, some conventional optimal and control technologies have been examined. Our purpose is to provide a clarified summary and understanding in the technologies. Based on the understanding to the optimal conditions, we could propose a method to calculate the latching duration for a latching control technology (more details and application of the technology will be given the second part of the research [24]). Though it is simple, it is very effective to reach a phase optimal condition.

Generally, the optimal/control parameters are often frequency-dependent. For the regular waves of a single frequency, the full optimum can be implemented with the constant control parameters due to the single frequency. However, in real seas, the waves are irregular, changing frequencies and amplitudes from wave to wave, hence the full control system must be adjusted accordingly if an ideal PTO can be obtained. As a result of the wave to wave control, the practical application is very difficult. For achieving the phase optimum, latching control has been proposed for improving wave energy conversion, and for a more practical implementation of the latching control technology, in this research a new latching duration calculation method has been proposed so that the future information is no longer needed.

Based on the investigation in the research, the following conclusions can be made:

1) Maximum wave energy extraction may not happen at the resonance frequency/period, even with the full PTO optimum. In the context, it has been shown that there is no analytical expression, unless the added mass, the hydrodynamic damping and the excitation force are all frequency-independent, which is not true for most of practical wave energy converters. The numerical simulation further confirms the maximum wave energy extraction does not happen at the resonance frequency.

2) The full PTO optimum aims to cancel the reactance of the device, which means the PTO can also performs as an additional mass and/or spring. Therefore, the PTO needs feeding energy back into waves as well as extracting energy from waves both with very high efficiency. This can be hardly achieved for practical PTOs.

3) A new latching duration is proposed which is based on the understanding to the phase optimal condition. Principally, such a latching control can change the whole dynamic system into a system in resonance, thus the phase optimal condition can be attained.

4) Latching control can improve the wave energy conversion for the device significantly. Though it is not so effective as the full optimum technology, it is much better than the technologies in the damping optimisations.
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