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<th>Bistability in an injection locked two color laser with dual injection</th>
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</tr>
<tr>
<td>Publication date</td>
<td>2011</td>
</tr>
<tr>
<td>Type of publication</td>
<td>Article (peer-reviewed)</td>
</tr>
</tbody>
</table>
[http://dx.doi.org/10.1063/1.3605584](http://dx.doi.org/10.1063/1.3605584)  
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Optical injection of laser diodes gives rise to a range of nonlinear phenomena including chaos and multistability and has been studied extensively for the single mode laser.\(^1\) Recently, dual-mode lasers, a family that includes vertical cavity surface emitting lasers (VCSELs), ring lasers, photonic crystal lasers, and two color Fabry-Pérot (FP) lasers, subjected to a single optical injection have been studied due to potential applications in all-optical signal processing.\(^2\)–\(^6\) An additional injected mode can also add functionality to multimode FP lasers.\(^7\)–\(^9\) However, the understanding of these dual injected FP lasers is complicated by the effects of the uninjected modes.\(^10\)

In this letter, we examine both theoretically and experimentally a two color FP laser subject to dual optical injection. Our model allows us to identify a bistable region between two locked states, which is born from a swallow-tail bifurcation. We then demonstrate that this bistability can be used as an all-optical memory element.

We consider a two color FP laser of length 545 \(\mu\)m with an InAlGaAs/InP active region. Mode selection is achieved by perturbing the FP mode spectrum using slotted regions etched into the ridge waveguide. The exact positions of these slots are determined by relating the refractive index profile in real space to the threshold gain in wavenumber space and then solving the corresponding inverse problem.\(^10\)

The optical spectrum of the free running two-color laser is shown in Fig. 1, where the spacing between the short wavelength mode \(\nu_2\) and long wavelength mode \(\nu_1\) is 480 GHz corresponding to a frequency separation between six fundamental FP modes. The large frequency spacing results in weak coupling between the modes and therefore stable simultaneous lasing of both modes is observed, which is in contrast to both the VCSEL and ring laser. In the following, we choose the pump current such that both lasing modes have equal optical intensity in the absence of optical injection.

Using two independent tunable master lasers, we now study the response of the two-mode laser to simultaneous optical injection into both modes. We have previously shown that a four dimensional rate equation model accurately describes the dynamics of a two color laser subject to optical injection into one of the two modes.\(^6\) In the dual injection case, this model generalizes to a five-dimensional system for the normalized complex fields, \(E_{1/2}\), and the normalized excess carrier density, \(n\), whose dynamic evolution is given by

\[
\begin{align*}
\dot{E}_1 &= \left[\frac{1}{2} (1 + i\chi)(g_1(2n + 1) - 1) - i\Delta v_1\right] E_1 + K_1, \\
\dot{E}_2 &= \left[\frac{1}{2} (1 + i\chi)(g_2(2n + 1) - 1) - i\Delta v_2\right] E_2 + K_2, \\
\dot{N} &= P - n - (2n + 1) \sum_m g_m |E_m|^2, \\
&= \frac{1 + \epsilon \sum_n |\beta_{mn}|^2}{1 + \epsilon \sum_n |\beta_{mn}|^2} = g_m
\end{align*}
\]

where the nonlinear modal gain is

\[g_m = \frac{1 + \epsilon \sum_n |\beta_{mn}|^2}{1 + \epsilon \sum_n |\beta_{mn}|^2} \quad (2),\]

In this system, the bifurcation parameters are the normalized injection strengths, \(K_1\) and \(K_2\), and the normalized angular frequency detunings, \(\Delta v_1\) and \(\Delta v_2\). Other values used are chosen to be in agreement with experimental data with

\[\begin{align*}
K_1 &= \frac{1}{2}, \\
K_2 &= 0.001, \\
\Delta v_1 &= 2\pi \times 1.34 \text{ GHz}, \\
\Delta v_2 &= 2\pi \times 1.32 \text{ GHz}, \\
\end{align*}\]

and the normalized excess carrier density, \(n\), whose dynamic evolution is given by

\[
\dot{\chi} = \frac{1}{2} (1 + i\chi)(g_1(2n + 1) - 1) - i\Delta v_1
\]

where the nonlinear modal gain is

\[g_m = \frac{1 + \epsilon \sum_n |\beta_{mn}|^2}{1 + \epsilon \sum_n |\beta_{mn}|^2} \quad (2),\]

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\[\begin{align*}
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\Delta v_2 &= 2\pi \times 1.32 \text{ GHz}, \\
\end{align*}\]
the phase-amplitude coupling, $\alpha = 2.6$, and the normalised pump current, $P = 0.4$. The parameter $T = \tau_p / \tau_{ph} = 800$ is the ratio between carrier lifetime, $\tau_c$, and the photon lifetime $\tau_{ph} = 1.02 \text{ ps}$. The modal gain includes the linear modal gain, $g_m^0 = 1$, and the nonlinear gain taking into account the cross and self saturation, which are determined by $c_{ph} m_0$. We choose $e = 0.01$ and $\beta_1 = \beta_2 = 2/3$, and $\beta_1 = \beta_2 = 1$ to be consistent with the stability of the free running two-color laser.\(^6\)

We first consider the case of equal negative detuning frequencies $\Delta \nu_1 = \Delta \nu_2 < 0$. Using the numerical continuation software AUTO (Ref. 11), we find that the dynamical system (1) has a stable fixed point for large injection strengths $K_1$ and $K_2$. This region is bounded from below by a saddle-node (SN) bifurcation as shown in the right hand panel of Fig. 1. The SN line is almost circular in the $K_1$-$K_2$ plane and manifests itself experimentally as an unlocking transition from a two-mode locked state with decreasing injection strengths of the master lasers. The system (1) also possesses a stable limit cycle solution for small injection strengths which is destroyed via a subcritical torus bifurcation with increasing injection strengths (TR line in Fig. 1). Similar to the SN line, the TR line is again almost circular in the $K_1$-$K_2$ plane, and both lines are concentric around the origin. For sufficiently negative detuning $\Delta \nu_1 = \Delta \nu_2 < -12 \text{ GHz}$, the radius of the torus line is larger than the radius of the SN line, which opens up a region of bistability between the two bifurcation lines, where a stable limit cycle and a stable fixed point solution coexist. Experimentally, the torus bifurcation is identified by a transition from an unlocked to a locked state with increasing injection strength. The experimental data in the right hand panel of Fig. 1 accurately confirm the predicted shape of the bistability region.

In the limit of $K_1 \rightarrow 0$, the limit cycle solution corresponds to a two-mode equilibrium solution of the four-dimensional single mode injection model. In this case, the bistability in Fig. 1 reduces to the previously reported bistability between single mode locked and two mode equilibrium state.\(^6\) However, this bistability involves partly unlocked states and the switching times are limited to values $> 2 \text{ ns}$.

In order to obtain faster switching, it is therefore desirable to use a bistability between states in which both modes are locked. This can be achieved by using unequal detunings, $\Delta \nu_1 \neq \Delta \nu_2$. Figure 2 shows the calculated SN bifurcation in the $K_1$-$K_2$ plane for various $\Delta \nu_1$ and fixed $\Delta \nu_2$. The SN line is deformed from its circular form with equal detuning, and at a swallow-tail bifurcation, a pair of cusps are born around $\Delta \nu_1 = -12 \text{ GHz}$. The cusps, which indicate the presence of hysteresis,\(^12\) form two corners of a triangular shaped region with two stable equilibria which we label by $F_1^2$ and $F_2^2$. We note that this bistable region is a unique feature of optical injection in both modes. In contrast to the bistability in Fig. 1, it does not extend to the single mode injection limit and is a consequence of the SN bifurcation only, without involving the torus bifurcation.

Let us now fix $K_1$ at a value which intersects the bistable region. Increasing $K_2$ from zero, the laser is initially locked to the $F_1^2$ state and remains in this state until the SN curve is reached. At this point, the stable $F_1^2$ disappears and the laser is locked to the $F_2^2$ state. Likewise, now decreasing $K_2$, the laser remains locked in the $F_2^2$ state until the lower part of the triangular region is reached and $F_2^2$ collides with an unstable fixed point. Then, the system returns to the $F_2^2$ state.

This predicted hysteresis behavior is confirmed experimentally in the inset of Fig. 3, where a typical hysteresis loop for the intensity of mode 1 is shown for varying $K_2$.

\[ P_1 / P_1 (\text{dB}) = \begin{cases} 0.023 & \text{I} \\ 0.028 & \text{II} \\ 0.033 & \text{III} \end{cases} \]

\[ P_2 / P_2 (\text{dB}) = \begin{cases} 0.024 & \text{I} \\ 0.028 & \text{II} \\ 0.032 & \text{III} \end{cases} \]

FIG. 1. (Color online) The SN bifurcation for $\Delta \nu_1 = -18 \text{ GHz}$ and various values of $\Delta \nu_1$ in the $K_1$-$K_2$ plane. For $\Delta \nu_1 > -12 \text{ GHz}$, a bistable region appears as shown by the shaded region with two cusp bifurcations in the corners. The inset shows the bifurcation diagram for $\Delta \nu_1 = -4 \text{ GHz}$, including two Bogdanov-Takens points (BT) and the associated Hopf (HB) and homoclinic (P) bifurcation lines. The black shaded region, which is bounded by the Hopf bifurcation line and the horizontal saddle node bifurcation line, indicates the presence of a stable limit cycle. In the black and gray striped region within the triangle bounded by saddle node bifurcation lines, a stable limit cycle and a stable fixed point coexist.

FIG. 2. (Color online) The SN bifurcation for $\Delta \nu_1 = -18 \text{ GHz}$ and various values of $\Delta \nu_1$ in the $K_1$-$K_2$ plane. For $\Delta \nu_1 > -12 \text{ GHz}$, a bistable region appears as shown by the shaded region with two cusp bifurcations in the corners. The inset shows the bifurcation diagram for $\Delta \nu_1 = -4 \text{ GHz}$, including two Bogdanov-Takens points (BT) and the associated Hopf (HB) and homoclinic (P) bifurcation lines. The black shaded region, which is bounded by the Hopf bifurcation line and the horizontal saddle node bifurcation line, indicates the presence of a stable limit cycle. In the black and gray striped region within the triangle bounded by saddle node bifurcation lines, a stable limit cycle and a stable fixed point coexist.

FIG. 3. (Color online) Partition of the $K_1$-$K_2$ plane for $\Delta \nu_1 = -8 \text{ GHz}$ and $\Delta \nu_2 = -18 \text{ GHz}$ into three regions: I contains no locked states; II comprises of one locked state and one unlocked state; and region III consists of only locked states. The inset shows hysteresis of the linear power in mode 1 at $K_1 = 0.01$ for up ($P_1$) and down ($P_1$) sweep in $K_2$ as indicated by the arrows in the main panel. The color of region III represents the contrast ratio $P_1$ to $P_1$ (in dB) between these up and down sweeps, where the dark shading indicates a large contrast of optical power in mode one.
From Fig. 2, the size of the bistable region can be increased by simply increasing the ratio $\Delta v_2: \Delta v_1$. However, as the detuning of $v_1$ becomes comparable to the relaxation oscillation ($\sim 5$ GHz), Bogdanov-Takens bifurcations appear along the SN bifurcation line, which result in the appearance of a Hopf bifurcation (HB) and a homoclinic bifurcation (P) as shown in the inset of Fig. 2. Between the HB and P bifurcation lines, we observe stable limit cycle dynamics. While these bifurcations are interesting from a dynamical point of view, they limit the region of bistability between locked states and do not contribute to improve switching. For the current letter, we therefore focus on the analysis of the dynamics in the region where only locked states exist.

To explore this bistability experimentally, we fix $\Delta v_1$, $\Delta v_2$, and $K_2$, and sweep $K_1$ up and down measuring the optical power in both modes. By monitoring the rf power spectrum, we are able to distinguish between locked and unlocked regions. This allows us to partition the $K_1$-$K_2$ plane into three regions as shown in Fig. 3. In region I no locked states exist, while in region II, bistability between locked and unlocked states is observed. Here, we focus on region III, where only locked states exist. The predicted triangular shape of the bistability region is confirmed experimentally by the shape of the colored region in Fig. 3. In this region, the contrast ratio of the power in mode 1 reaches values up to 8 dB.

We exploit this bistability to form an all-optical memory element which is triggered by modulating the optical injection in mode 2 while keeping the injection in mode 1 fixed at a constant value $K_1$. As shown in the upper panel of Fig. 4, the modulation consists of a 10 ns set pulse with 100 ps risetime and a 10 ns reset pulse with 5 ns risetime. This modulation is also indicated by the arrows in Fig. 3. In the middle and lower panels of Fig. 4, we observe that the laser is switched between two stable locked states with different modal intensities. The contrast ratio in $v_1$ is 8 dB and the fall time is $< 500$ ps. This is faster than in the single injection case, where the switching speed is limited by large transients of the order of several ns, and the emitted optical power is of the order several mWs. Switching times of $< 20$ ps have been reported; however, optical powers are limited to 100 s of $\mu$W.

In conclusion, we have studied a two-color FP laser subject to dual injection and demonstrated a bistability between locked states. We have explained the bistability as originating from a swallow-tail bifurcation. Although, we have focused our analysis on a two-color FP laser, we note that this bistability will be present in other dual injection systems.

This work was supported by Science Foundation Ireland. The authors thank Eblana Photonics for fabrication of lasers.

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**References**