Cork Open Research Archive
Cartlann Taighde Oscailte Chorcaí

UCC Library and UCC researchers have made this item openly available. Please let us know how this has helped you. Thanks!

Title	Decoherence-assisted energy transfer and quantum criticalities
Author(s)	Giorgi, Gian Luca; Busch, Thomas
Publication date	2012
Original citation	Giorgi, G. L. and Busch, T. (2012) 'Decoherence-assisted energy transfer and quantum criticalities', Physical Review A, 86(5), 052112. (4pp). doi:10.1103/PhysRevA.86.052112
Type of publication	Article (peer-reviewed)
Link to publisher's version	https://journals.aps.org/pra/abstract/10.1103/PhysRevA.86.052112 http://dx.doi.org/10.1103/PhysRevA.86.052112 Access to the full text of the published version may require a subscription.
Rights	**© 2012, American Physical Society**
Item downloaded from	http://hdl.handle.net/10468/4494

Downloaded on 2021-11-28T23:04:48Z

University College Cork, Ireland
Coláiste na hOllscoile Corcaigh

Decoherence-assisted energy transfer and quantum criticalities

Gian Luca Giorgi[1] and Thomas Busch[1,2]
[1]*Department of Physics, University College Cork, Cork, Republic of Ireland*
[2]*Quantum Systems Unit, Okinawa Institute of Science and Technology, Okinawa 904-0411, Japan*
(Received 16 August 2012; published 14 November 2012)

We study the dynamics of a two-level quantum system interacting with an external environment that takes the form of an *XY* spin chain in the presence of an external magnetic field. While the presence of the bath itself can enhance the transition probability from the lower level to the upper level of the system, we show that this noise-assisted phenomenon is sensitive to a change of the quantum phase of the environment. The derivative of the transition probability displays a maximum in correspondence with the critical value of the applied field in the case of both isotropic and anisotropic chains.

DOI: 10.1103/PhysRevA.86.052112 PACS number(s): 03.65.Yz, 05.60.Gg, 82.20.Rp

I. INTRODUCTION

The unavoidable interaction of any realistic quantum system with its environment is responsible for decoherence and dissipation, which are usually considered detrimental effects for the implementation of quantum information and quantum communication tasks [1,2]. However, recent experimental studies have shown that some photosynthetic complexes exhibit long excitonic coherence despite the interaction with their own environment [3–5]. Furthermore, the possibility of decoherence-assisted quantum transport was recently discovered in a series of theoretical Hamiltonian models. Therefore, fully understanding the role played by the noise in assisting quantum transport phenomena has assumed a crucial importance and has become a rich subject of investigation (see, for instance, Refs. [6–14]).

While a big part of the theoretical investigations in this field deals with quantum systems in contact with bosonic environments, coherent transport induced by structured spin baths has for the first time been taken into consideration in Ref. [15]. There, it was shown that the transition probability from the lower to the upper level of a two-level system can be increased even for finite temperatures when modeling the environment as a series of spins 1/2 arranged in a star configuration. The authors also proved that the use of two separated baths, correlated with each other, can help to improve the transport compared to the use of a single environment.

At the same time, it is known that decoherence is sensitive to quantum critical changes in the bath and that the correlation length of the environment can influence the rate of loss of quantumness in a system [16–21]. A possible justification for this behavior can be found in the monogamous nature of entanglement [22].

Starting from the results of Ref. [15], here we take a step forward by considering the case of a spin bath that can experience a quantum phase transition and studying how a critical change in the environment will influence the noise-induced properties of a two-level system (TLS) coupled to the bath itself. The goal of our work is to find a direct link between the phase transition of the bath and the internal transition probability of the TLS.

We approach the problem by first considering the case of a TLS coupled to a homogeneous *XX* chain in the presence of a transverse field. If the system is coupled to the total magnetization of the bath, the problem is exactly solvable. We show that the transition probability inside the TLS is maximally sensitive to a change in the value of the applied field near its critical point and justify the result by noting that the bath renormalizes the system Hamiltonian parameters. The same qualitative behavior can be observed in the case of an inhomogeneous *XY* chain, where the analytical solution is no longer available, and where the simple mechanism of renormalization cannot be applied any more.

II. MODEL

Let us consider a TLS described by the Hamiltonian

$$H_S = \frac{\Delta}{2}\sigma_z + T\sigma_x. \tag{1}$$

Because of the detuning Δ, between the states $|\uparrow\rangle$ and $|\downarrow\rangle$, a state initially prepared in $|\downarrow\rangle$ has a maximum probability of transition to $|\uparrow\rangle$ of $P_{\rm tr} = T^2/(T^2 + \Delta^2/4) \leqslant 1$, with perfect transition for $\Delta = 0$. As discussed by Sinayskiy *et al.* in Ref. [15], under certain circumstances, the presence of a single spin bath or two different, correlated baths can help increase the maximum transition probability for finite values of Δ.

Here our aim is to analyze the effect an internally structured environment can have on a decoherence-assisted transition process. We will consider a spin environment and study its influence as a function of an internal parameter which drives the system through a quantum critical point. For this, we model the bath as an *XY* chain in the presence of a transverse field, which can be described as

$$H_{XY} = -J\sum_{l=1}^{N}\left[\left(\frac{1+\gamma}{2}\right)\sigma_l^x\sigma_{l+1}^x + \left(\frac{1-\gamma}{2}\right)\sigma_l^y\sigma_{l+1}^y\right] - hM_z, \tag{2}$$

where the total magnetization along z is $M_z = \sum_{l=1}^{N}\sigma_l^z$ and where the boundary conditions are periodic $\sigma_{N+1}^\alpha = \sigma_1^\alpha$. From now on we will take $J = 1$ and use it as a scale of energy. In the thermodynamic limit the Hamiltonian H_{XY} possesses a critical point which can be observed when considering the average magnetization per spin, $m_z = M_z/N$ [23],

$$\lim_{N\to\infty} m_z = \frac{1}{\pi}\int_0^\pi \frac{\tanh[\beta\Lambda(h)/2]}{\Lambda(h)}(h - \cos\phi)\,d\phi, \tag{3}$$

 ©2012 American Physical Society

where β is the inverse temperature and $\Lambda(h) = [(h - \cos\phi)^2 + \gamma^2 \sin^2\phi]^{1/2}$. For finite temperatures, the derivative of m_z with respect to h shows a maximum for $h < h_c$, which turns into a divergence at $h_c = 1$ for $\beta \to \infty$.

A. Isotropic case

In the following, we will first pay special attention to the case of an isotropic chain, characterized by $\gamma = 0$, and indicate the Hamiltonian operator as H_{XX}. For the sake of simplicity and without any loss of generality, we will only consider non-negative fields ($h \geqslant 0$) and note that M_z is a conserved quantity since $[H_{XX}, \sum_l \sigma_l^z] = 0$. For $h > h_c = 1$, the ground state is completely ordered along the direction z and no spin-spin correlations are present. Lowering h below the critical value h_c, the ground state starts acquiring magnons (flipped spins) and for $h = 0$ the number of magnons is exactly equal to $N/2$. In a pictorial representation, exploiting the Jordan-Wigner mapping between spins and fermions, the transverse field h plays the role of a chemical potential and determines the value of the Fermi level: all the levels with energy less than h will belong to the Fermi sea and will be filled, while all the levels with energy greater than h will be left empty. This mechanism implies that for finite-size systems, moving the field from zero to one, $N/2$ transition points between different symmetry sectors of the Hamiltonian are crossed. In the thermodynamic limit, this leads to an infinite-order quantum phase transition, named Berezinsky-Kosterlitz-Thouless, which takes place without spontaneous symmetry breaking.

We assume that the interaction between the bath and the system is given by a coupling between the upper level $|\uparrow\rangle$ and the total magnetization of the chain [15]:

$$H_I = -\Gamma|\uparrow\rangle\langle\uparrow| \otimes m_z. \tag{4}$$

Since m_z is a constant of motion, one can see that $[H_{XX}, H_I] = 0$ and therefore the degrees of freedom of the environment can be eliminated. As a consequence, an exact solution for the dynamics of the system can be obtained. Notice that we are not dealing with a purely dephasing interaction, which would be characterized by $[H_S, H_I] = 0$, but that our model is suitable to describe a fully dissipative dynamical evolution.

Assuming that at $t = 0$ the system and the bath are in a product state $[\rho(0) = \rho_S(0) \otimes \rho_B(0)]$, the density matrix of the system will evolve according to

$$\rho_S(t) = \mathrm{Tr}_B[e^{-i(H_S+H_I)t}\rho_S(0) \otimes \rho_B(0)e^{i(H_S+H_I)t}], \tag{5}$$

where we have now eliminated H_{XX} by exploiting $[H_{XX}, H_I] = 0$ and the invariance of the trace under cyclic permutations. Even if formally eliminated from the dynamics, H_{XX} is present through the initial state of the bath $\rho_B(0)$, which will be assumed to be in a thermal distribution $\rho_B(0) = e^{-\beta H_{XX}}/Z$ at the inverse temperature β, where the partition function is $Z = \mathrm{Tr}[e^{-\beta H_{XX}}]$.

Both H_{XX} and H_I can be written in a diagonal form by using the aforementioned Jordan-Wigner transformation, which maps spins into spinless fermions [24]. Labeling the eigenstates of the bath $|\lambda\rangle$ and the corresponding eigenvalues λ, the matrix elements $\langle i|\rho_S|j\rangle = \rho_S^{ij}$ will evolve in time according to

$$\rho_S^{ij}(t) = \frac{1}{Z}\sum_\lambda e^{-\beta\lambda}\langle i|e^{-iH_S^{(\lambda)}t}\rho_S(0)e^{iH_S^{(\lambda)}t}|j\rangle, \tag{6}$$

with $H_S^{(\lambda)} = H_S - \Gamma|\uparrow\rangle\langle\uparrow|\langle\lambda|m_z|\lambda\rangle = H_S + \Gamma(2n_\lambda/N - 1)|\uparrow\rangle\langle\uparrow|$, where n_λ is the fermionic occupation number of $|\lambda\rangle$.

Let us start analyzing the zero-temperature scenario, where the bath is in its ground state $|G\rangle$, which is characterized by its fermionic occupation number n_G, which in turn depends on h. In this case, the sum over all the Hamiltonian eigenstates in Eq. (6) reduces to only one term, $\rho_S^{ij}(t) = \langle i|e^{-iH_S^{(G)}t}\rho_S(0)e^{iH_S^{(G)}t}|j\rangle$, and the transition probability between $|\downarrow\rangle$ and $|\uparrow\rangle$ is given by $P_{\mathrm{tr}} = |\langle\downarrow|e^{iH_S^{(G)}t}|\uparrow\rangle|^2$. This allows renormalization of the diagonal elements of the system Hamiltonian, and the effective gap between $|\uparrow\rangle$ and $|\downarrow\rangle$ becomes

$$\tilde{\Delta} = \Delta - \Gamma(1 - 2n_G/N). \tag{7}$$

For $h = 0$, where $n_G = N/2$, the presence of the bath does not affect P_{tr}, but for increasing field h the fermionic occupation number starts decreasing until we have $n_G = 0$ at $h = h_c = 1$. The value of n_G/N in the thermodynamic limit for $\gamma \to 0$ and $\beta \to \infty$ can be obtained from Eq. (3). If $\Gamma \geqslant \Delta$, it is therefore possible to achieve perfect energy transfer by choosing an optimal h between zero and one such that $\tilde{\Delta} = 0$. For the case of $\Gamma < \Delta$, which is the experimentally more realistic scenario, a maximum enhancement will be reached for $h \geqslant h_c$, which corresponds to the zero-temperature case described in Ref. [15].

For systems at finite temperatures we need to consider all terms of the sum in Eq. (6), which, for any finite N, can be evaluated analytically. For a finite inverse temperature $\beta = 40$, we show P_{tr} in Fig. 1 as a function of h for chains of 8, 12, and 16 spins and compare it to its value in the

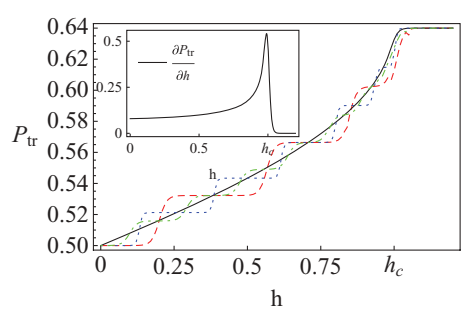

FIG. 1. (Color online) P_{tr} vs h for $N = 8$ sites [red (dashed) line], $N = 12$ sites [blue (dotted) line], $N = 16$ sites [green (dot-dashed) line], and $N = \infty$ sites [black (solid) line], in the isotropic case ($\gamma = 0$). The system parameters are $\Delta = 2T = 0.2$ and $\Gamma = 5 \times 10^{-2}$, while the inverse bath temperature is $\beta = 40$. In the case of 8, 12, and 16 sites, P_{tr} has been calculated using the analytical expression given in Eq. (6), while the thermodynamic limit has been calculated within a mean-field approximation (see text for details). In the absence of the bath, one would have $P_{\mathrm{tr}} = 0.5$. Inset: the derivative of P_{tr} with respect to h in the thermodynamic limit. For finite-size chains, the derivatives would exhibit $N/2$ peaks, the last of them falling around h_c, in correspondence with the degeneracy points of the Hamiltonian.

thermodynamic limit. The latter is calculated by introducing a mean-field approximation, obtained by replacing H_I with $H_I^{\rm mf} = -\Gamma|\uparrow\rangle\langle\uparrow|\langle m_z\rangle$, where the average magnetization per spin is given by Eq. (3) in the limit of vanishing γ. By comparing the different curves, it is evident that the curves for finite-size chains are rapidly converging toward the mean-field approximation, which can then be used to estimate $P_{\rm tr}$ in the limit of long chains. The finite-size solutions show clear signatures of the $N/2$ transition points in the ground state between the different symmetry sectors of the Hamiltonian and exhibit horizontal plateaus between them. Taking the thermodynamic limit, where these transition points become infinitely dense, we are left with a critical change for $P_{\rm tr}$ at $h \approx h_c$. The existence of a critical point in $P_{\rm tr}$ is shown in the inset of Fig. 1, where its derivative with respect to the external field is plotted. It shows a clear peak around h_c, which is due to the behavior of m_z.

In the zero-temperature case, the behavior of $P_{\rm tr}$ around h_c is a direct consequence of the commutativity between the bath and the interaction Hamiltonians. Indeed, the only effect of the bath is to replace the initial, unperturbed, system Hamiltonian H_S with the effective two-level Hamiltonian $H_S^{(G)}$, which has acquired the critical properties of the transverse magnetization. However, in the finite-temperature case, this approach does not hold any more. Nevertheless, the results of Fig. 1 show that for moderate temperatures, which can be expected in realistic scenarios, a connection between $P_{\rm tr}$ and the critical change in m_z still holds (further decreasing β would lead the bath out of its quantum domain and wash out the effect on $P_{\rm tr}$). We will discuss this point in more detail below.

B. Anisotropic case

Releasing the isotropy assumption by choosing $\gamma \neq 0$ in Eq. (2), an analytical solution for $\rho_S^{ij}(t)$ analogous to that in Eq. (6) cannot be obtained. Instead, the general expression $P_{\rm tr} = \max_t \langle\uparrow|{\rm Tr}_B[e^{-iH_{\rm tot}t}|\downarrow\rangle\langle\downarrow| \otimes \rho_B(0)e^{iH_{\rm tot}t}]|\uparrow\rangle$, where $H_{\rm tot} = H_S + H_{XY} + H_I$, must be evaluated. In the case of short chains, this can be done exactly by diagonalizing $H_{\rm tot}$ and explicitly performing the trace over the environment. In Fig. 2 we show $P_{\rm tr}$ for chains of 6 and 10 spins at finite temperature and compare the results to the mean-field approximation, performed, as before, by replacing m_z with its average value in H_I. One can see that the mean-field solution is a good approximation to the exact, finite-size solution even for short chains, and therefore becomes a suitable candidate to describe the thermodynamic limit. Furthermore, this behavior shows that the influence of a quantum phase transition in the bath on the transition dynamics between the two internal system states is not limited to a special choice of Hamiltonian or to possible special commutation properties, but lies in nature of the interaction.

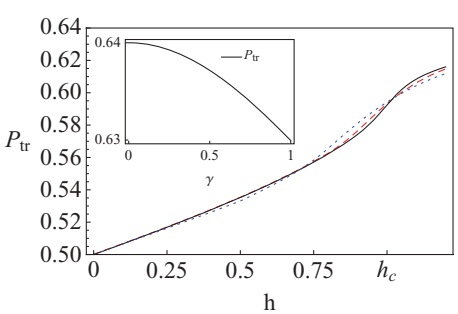

FIG. 2. (Color online) $P_{\rm tr}$ vs h for $N = 6$ sites [blue (dotted) line], $N = 10$ sites [red (dashed) line], and $N = \infty$ sites [black (solid) line], in the presence of an anisotropic bath ($\gamma = 0.8$). The system parameters are $\Delta = 2T = 0.2$ and $\Gamma = 5 \times 10^{-2}$, while the inverse bath temperature is $\beta = 40$. Here, the finite-size lines have been calculated by explicitly solving the Hamiltonian evolution, while the mean-field approximation has been used in the thermodynamic limit. Inset: $P_{\rm tr}$ as a function of γ for strong fields ($h = 2$) and for $\beta = 40$. The transition probability is a monotonically decreasing function of the anisotropy.

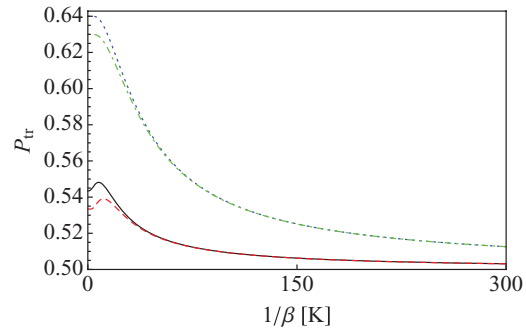

FIG. 3. (Color online) $P_{\rm tr}$ vs $1/\beta$, in the thermodynamic limit, for $\gamma = 0$ and $h = 2$ [blue (dotted) line], $\gamma = 1$ and $h = 2$ [green (dotdashed) line], $\gamma = 0$ and $h = 0.5$ [black (solid) line], and $\gamma = 1$ and $h = 0.5$ [red (dashed) line]. Here, we have taken $J/\hbar = 2{\rm ps}^{-1}$ and the inverse temperature is measured in Kelvin degrees.

C. Temperature effects

Let us finally discuss the interplay between temperature effects and anisotropy for both small ($h = 0.5$) and large fields ($h = 2$). In Fig. 3 we show $P_{\rm tr}$ as a function of β for the isotropic case $\gamma = 0$ and for the Ising case $\gamma = 1$, by assuming $J/\hbar \simeq 2{\rm ps}^{-1}$ (see Ref. [15]) to give a realistic estimation for the inverse temperature. Two remarkable results emerge: (i) The bath can have a positive effect on $P_{\rm tr}$ for a large range of temperatures and (ii) γ is in fact important only in the very low temperature regime. It is important to stress that (ii) not only holds for strong fields, where the $x - y$ component of the interaction is dominated by the external field, but also in the symmetry-broken region, where the in-plane and the transverse terms have comparable strength.

III. CONCLUSIONS

In conclusion, we have considered the problem of a TLS immersed in a spin chain that plays the role of the environment. We have shown that the transition probability from the lower level to the upper level of the TLS is sensitive to the critical properties of the bath. Describing the bath as an isotropic XX chain and the interaction with the system through its transverse magnetization, the problem becomes exactly solvable, since the bath Hamiltonian commutes with the interaction term. In the zero-temperature case, the bath renormalizes the system Hamiltonian, making it possible to improve the transition

probability. Even if the mechanism is more complicated, the same qualitative enhancement can be observed in the finite-temperature regime. Finally, the decoherence-assisted process identified in our work has a more general range of validity as we have shown that the same qualitative effect can be observed by using an anisotropic chain, where the commutativity property between bath and interaction Hamiltonian does not apply any more.

We believe that our results can elucidate the interplay between quantum coherence and correlations and can help understand the basic mechanisms underlying energy transport in biologic systems.

ACKNOWLEDGMENTS

This work was supported by Science Foundation of Ireland under Project No. 10/IN.1/I2979. We would like to thank Tony Apollaro, Mauro Paternostro, Gabriele De Chiara, and Ferdinando de Pasquale for valuable discussions and comments.

[1] M. A. Nielsen and I. L. Chuang, *Quantum Computation and Quantum Information* (Cambridge University Press, Cambridge, 2000).
[2] H.-P. Breuer and F. Petruccione, *The Theory of Open Quantum Systems* (Oxford University Press, New York, 2002).
[3] G. S. Engel, T. R. Calhoun, E. L. Read, T.-K. Ahn, T. Mančal, Y.-C. Cheng, R. E. Blankenship, and G. R. Fleming, Nature (London) **446**, 782 (2007).
[4] G. Panitchayangkoon, D. Hayes, K. A. Fransted, J. R. Caram, E. Harel, J. Wen, R. E. Blankenship, and G. S. Engel, Proc. Natl. Acad. Sci. **107**, 12766 (2010).
[5] E. Collini, C. Y. Wong, K. E. Wilk, P. M. G. Curmi, P. Brumer, and G. D. Scholes, Nature (London) **463**, 644 (2010).
[6] M. Plenio and S. Huelga, New J. Phys. **10**, 113019 (2008).
[7] A. Olaya-Castro, C. F. Lee, F. F. Olsen, and N. F. Johnson, Phys. Rev. B **78**, 085115 (2008).
[8] M. Mohseni, P. Rebentrost, S. Lloyd, and A. Aspuru-Guzik, J. Chem. Phys. **129**, 174106 (2008).
[9] P. Rebentrost, M. Mohseni, I. Kassal, S. Lloyd, and A. Aspuru-Guzik, New J. Phys. **11**, 033003 (2009).
[10] F. Caruso, A. W. Chin, A. Datta, S. F. Huelga, and M. B. Plenio, J. Chem. Phys. **131**, 105106 (2009); Phys. Rev. A **81**, 062346 (2010).
[11] M. Sarovar, A. Ishizaki, G. R. Fleming, and K. B. Whaley, Nat. Phys. **6**, 462 (2010).
[12] F. L. Semião, K. Furuya, and G. J. Milburn, New J. Phys. **12**, 083033 (2010).
[13] L. Campos Venuti and P. Zanardi, Phys. Rev. B **84**, 134206 (2011).
[14] I. Kassal and A. Aspuru-Guzik, New J. Phys. **14**, 053041 (2012).
[15] I. Sinayskiy, A. Marais, F. Petruccione, and A. Ekert, Phys. Rev. Lett. **108**, 020602 (2012).
[16] S. Paganelli, F. de Pasquale, and S. M. Giampaolo, Phys. Rev. A **66**, 052317 (2002).
[17] H. T. Quan, Z. Song, X. F. Liu, P. Zanardi, and C. P. Sun, Phys. Rev. Lett. **96**, 140604 (2006).
[18] D. Rossini, T. Calarco, V. Giovannetti, S. Montangero, and R. Fazio, Phys. Rev. A **75**, 032333 (2007); J. Phys. A **40**, 8033 (2007).
[19] D. Rossini, P. Facchi, R. Fazio, G. Florio, D. A. Lidar, S. Pascazio, F. Plastina, and P. Zanardi, Phys. Rev. A **77**, 052112 (2008).
[20] G. L. Giorgi, S. Paganelli, and F. Galve, Phys. Rev. A **81**, 052118 (2010).
[21] B. Damski, H. T. Quan, and W. H. Zurek, Phys. Rev. A **83**, 062104 (2011).
[22] C. M. Dawson, A. P. Hines, R. H. McKenzie, and G. J. Milburn, Phys. Rev. A **71**, 052321 (2005).
[23] E. Barouch, B. M. McCoy, and M. Dresden, Phys. Rev. A **2**, 1075 (1970).
[24] E. Lieb, T. Schultz, and D. Mattis, Ann. Phys. (NY) **16**, 407 (1961).