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Trapping and cooling particles using a moving atom diode and an atomic mirror

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We propose a theoretical scheme for atomic cooling, i.e., the compression of both velocity and position distribution of particles in motion. This is achieved by collisions of the particles with a combination of a moving atomic mirror and a moving atom diode. An atom diode is a unidirectional barrier, i.e., an optical device through which an atom can pass in one direction only. We show that the efficiency of the scheme depends on the trajectory of the diode and the mirror. We examine both the classical and quantum mechanical descriptions of the scheme, along with the numerical simulations to show the efficiency in each case.

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I. INTRODUCTION

One standard cooling technique for neutral atoms is using magneto-optical traps [1]. Evaporative cooling of bosons is used for achieving condensates [2] and ultracold, spin-polarized Fermi gases are usually cooled to temperatures below the Fermi temperature through sympathetic cooling [3].

Recently another method has been introduced, called single-photon cooling [4–6], which allows one to cool atoms and molecules which cannot be handled in a standard way. The method is based on an atom diode or one-way barrier [7,8]. An atom diode is a device which allows the atom to pass through it only in one direction, whereas the atom is reflected if coming from the opposite direction. Such a device has been studied theoretically [5,9–12] and also experimentally implemented as a realization of a Maxwell demon [13,14].

A way of changing or reducing the velocity of particles (which does not necessarily correspond to cooling) is letting particles collide with a moving mirror. An early example is the production of an ultracold beam of neutrons colliding with a moving Ni surface [15]. Atomic mirrors can be built using reflection by an evanescent light field [16,17]. Moving such mirrors for cold atom waves has been also implemented with a time-modulated, blue-detuned evanescent light wave propagating along the surface of a glass prism [18–20]. More recently, the diffraction of a Bose-Einstein condensate on a vibrating mirror potential created by a blue-detuned evanescent light field was studied [21] and the reflection of an atomic cloud from an optical barrier of a blue-detuned beam was used to study first-order and second-order catastrophes in the cloud density [22]. Even Rb atoms which fall on a magnetic mirror have been examined [23] and Rb atoms have even been stopped using a moving magnetic mirror [24]. Furthermore solid atomic mirrors have been used for focusing neutral atomic and molecular beams [25–27]. Si crystals on a spinning rotor have been used as solid atomic mirrors to slow down beams of helium atoms [28,29].

A stream of particles can be slowed by collision with a moving mirror traveling in the same direction as the particles. One limitation of standard settings at present is that for a fixed mirror velocity only pulses of particles with a specific and well-defined initial velocity are stopped. In [30], it was shown that by designing a particular trajectory for the mirror it is even possible to stop a pulse in which the initial velocities are broadly distributed or possibly unknown. But slowing an ensemble of atoms solely with one mirror of course does not result in phase-space compression. In order to achieve this, we introduce a required irreversible step.

In this work we develop a scheme to cool (i.e., compress in phase space) a traveling cloud of particles. This is done by combining the idea of a moving mirror with an irreversible atom diode also in motion.

In the next section, we present and investigate our cooling method, first in an idealized classical setting, i.e., assuming a point particle with classical motion. In Sec. III, we discuss a quantum-mechanical implementation of our cooling scheme. The paper ends with a conclusion.

II. COOLING CLASSICAL PARTICLES WITH DIODE AND MIRROR

First we shall investigate a classical scheme for achieving our goals before moving on to a full quantum treatment of the problem. We assume classical point particles and restrict the scenario to a one-dimension motion. The setting consists of two main objects: a moving atomic mirror potential and an atom diode. The particles move freely between the collisions with these two objects. Let us start by reviewing properties of a single moving atomic mirror potential.

A. Elastic collision stopping a single particle with moving mirror

A collision between a number of bodies is called elastic if there is no loss of mechanical energy during the collision. With this in mind consider the collision of a particle (moving with velocity $v_f$) with a moving mirror (with velocity $v_m$). The velocity of the particle after the elastic collision is given by

$$v_f = 2v_m - v_i. \tag{1}$$

It is immediately apparent that if we let $v_m = \frac{v_i}{2}$ the particle is stopped instantly by the collision. We can see that, in particular, if a particle has trajectory $x(t) = v_i t$, the trajectory of the mirror is $x_m(t)$, and the collision occurs at time $t_c$, then we
from one direction (here from the left to the right) which passes the diode and the mirror with trajectories (c) \( \alpha_m > \alpha_d \) with \( \alpha_d > 0 \). This trap trajectory has been displaced by a constant distance. With these trajectories a slight compression in velocity has been achieved.

In this work, we show that the efficiency depends strongly on the trajectories of diode and mirror. By considering different trajectories, we show that significant phase-space compression can be achieved. Motivated by Sec. IIA, we first consider a square-root scheme where the trajectories of diode (d) and mirror (m) are

\[
x_d(t) = v_d \sqrt{t}, \quad x_m(t) = \alpha_m \sqrt{t},
\]

with \( \alpha_m > \alpha_d \); see also Fig. 1(c).

Alternatively, we consider a linear scheme where the trajectories of diode (d) and mirror (m) are

\[
x_d(t) = v_d t, \quad x_m(t) = v_m t,
\]

with \( v_m > v_d \); see also Fig. 1(d). As it will turn out later that the linear scheme is more advantageous than the square-root scheme, we derive some general formulas and properties for the linear scheme first.

C. Properties of the linear scheme

In the linear case, there is an explicit formula for the velocity of the classical particle after the \( n \)th collisions, namely

\[
v_n = \begin{cases} 
(n(v_d - v_m) + v_0, & n \text{ even,} \\
(n - 1)(v_m - v_d) + 2v_m - v_0, & n \text{ odd.}
\end{cases}
\]

where even \( n \) corresponds to the velocity after a diode collision and odd \( n \) corresponds to the velocity after a mirror collision. We can also write down an expression for the corresponding time \( t_n \) for which the \( n \)th collision happens

\[
t_n = \frac{x_i}{v_m - v_0} \left( \prod_{k \geq 2}^{n-1} \frac{v_k - v_d}{v_k - v_m} \prod_{j \text{ odd}}^{n-1} \frac{v_l - v_m}{v_l - v_d} \right).
\]

We can use Eq. (5) to calculate the maximum number of collisions (if there is no further time restriction): after the last collision \( (n = n_{\text{max}}) \), we have \( v_d \leq v_{n_{\text{max}}} \leq v_m \). From this, it
From Eq. (5), it also follows immediately that after a collision: parameters for linear scheme (green dots): a function of time; each symbol indicates the velocity of the particle where

\[ a_m = v_m \sqrt{T}, \quad \alpha_d = v_d \sqrt{T}. \]

follows that

\[ v_d \leq v_0 - n_{\text{max}} \Delta v_{\text{md}} \leq v_m, \]

\[ \frac{v_0 - v_m}{\Delta v_{\text{md}}} \leq n_{\text{max}} \leq \frac{v_0 - v_d}{\Delta v_{\text{md}}}, \]

\[ r - 1 \leq n_{\text{max}} \leq r, \]

where \( \Delta v_{\text{md}} = v_m - v_d > 0 \) and \( r = \frac{v_0 - v_m}{\Delta v_{\text{md}}} \). For an even \( n \), with \( 1 < n < n_{\text{max}} \), it follows therefore that \( n \leq r \) and therefore

\[ v_0 - n \Delta v_{\text{md}} \geq (n - 2) \Delta v_{\text{md}} + 2v_m - v_0, \]

\[ v_n \geq v_{n-1}. \]

We want to recall that \( v_n \) is an algebraic value here, not the absolute value of the velocity. In the case of \( n \) odd (after a collision with the mirror), \( v_n \) and \( v_{n-2} \) are almost always negative; therefore, from the statement \( v_n - v_{n-2} > 0 \) it follows that almost always \( |v_n| < |v_{n-2}| \).

**D. Comparison of the square-root and linear schemes for a single particle**

Let \( v_m = d/T \) where \( d \) is the final position of the mirror and \( T \) is the total time; \( v_m \) is also the velocity of the mirror in the linear scheme. For comparison, we chose \( \alpha_{d/m} = v_{d/m} \sqrt{T} \) in the square-root scheme in such a way that the initial and final position of diode and mirror is the same in both schemes.

In Fig. 2, the velocity of the particle \( v_n \) after a collision is shown versus time, for the square-root scheme as well as for the linear scheme. We see the velocity of the particle in the trap tend towards \( v_d \leq v_p(t) \leq v_m \) for larger \( t \); furthermore the particle is localized \( x_{d}(t) \leq x_p(t) \leq x_{m}(t) \). We see this behavior in the linear case and in the case of the square root; however, we do not see the same level of velocity reduction in Fig. 2 in the square-root case as in the linear case: the reducing of the velocity occurs in the linear trap on a much shorter time scale than that of the square-root trap (it takes much longer to achieve the same reduction in velocity for the square-root trap).

If we consider again the linear case in Fig. 2, then we will also see all the general properties of Eq. (9): the upper branch (corresponds to \( n \) even, i.e., velocities after diode collisions) is decreasing with increasing time (which corresponds to increasing number of collisions), the lower branch (corresponds to \( n \) odd, i.e., velocities after mirror collisions) is increasing with increasing time (which corresponds to increasing number of collisions), and the upper branch is always above the lower branch.

The ratio between final particle velocity after the last collision with the mirror and initial particle position \( x_0 \): (a) velocity \( v_f \) after the last collision with the mirror; (b) velocity \( v_f \) after the last collision with the diode. Linear scheme (green, lower planes) and square-root scheme (red, higher planes); other parameters are the same as in Fig. 2.

**E. Compression in classical phase space**

We now discuss the more general case where we have a cloud of noninteracting particles characterized by some probability density \( \rho(t,x,v) \). In particular, we look at a Gaussian initial distribution given by

\[ \rho(0,x,v) = \frac{1}{2\pi \Delta v \Delta x} e^{-\left(\frac{x^2}{\Delta x^2} + \frac{v^2}{\Delta v^2}\right)}. \]
We calculate the final probability distribution at time \( t = T \), \( \rho(T, x, v) \) for the linear and square-root schemes, and compare the ability in each case to cool the cloud. In Fig. 4 this comparison between the initial and final velocity distributions (\( \rho(t, v) = \int dx \rho(t, x, v) \) for \( t = 0, T \)) is shown and we see that both schemes achieve a reduction in velocity. The linear scheme however achieves a greater reduction in velocity than the square root one similar to the single-particle case shown in Fig. 2 and Fig. 3. It is interesting that the final velocity distribution is independent of the initial average velocity \( v_0 \) for the linear scheme. The dots in Fig. 4 correspond to the final velocities after the mirror collision (diode collision) which is achieved if we consider a single particle in the diode-mirror system with \( v_0 \) and \( x_0 \) being the average velocity and position of the ensemble. We find that the positions of the peaks correspond approximately to these velocities. To underline the compression in phase space, the initial and final distribution \( \rho(0, x, v) \) [\( \rho(T, x, v) \)] is shown in Fig. 5 for the linear scheme.

For clarification, both distributions are shown scaled such that their maximum is one. Linear scheme, \( v_0 = 10v_m \); other parameters as in Fig. 4.

FIG. 4. Classical setting: comparison between velocity distribution using the linear and square-root schemes: initial velocity distribution for both schemes (shifted, black, lowest broad distribution) and final velocity distribution for the square-root scheme, \( v_0 = 10v_m \) (red, thick, solid line) and \( v_0 = 15v_m \) (red, thick, dashed line), and for the linear scheme, \( v_0 = 10v_m \) (green, thin, solid line) and \( v_0 = 15v_m \) (green, thin, dashed line). The dots above the plots correspond to a single-particle simulation with initial velocity \( v_0 \) and initial position \( x_0 \); other parameters: \( x_0 = -0.8d_1 \), \( \Delta x = 0.1d_1 \), \( \Delta v = 5v_m \); other parameters are the same as in Fig. 2.

We have shown that the efficiency depends strongly on the trajectories of atom diode and atomic mirror. It turns out that the linear scheme is much more efficient that the square-root scheme in the classical setting. Therefore, we will consider now solely the linear scheme in a quantum setting.

III. QUANTUM CATCHER

Inspired by the preliminary and promising classical results, we would like to consider if such a similar cooling is possible using a quantum mechanical treatment. We again consider a single quantum particle moving in one dimension. We want the quantum diode-mirror system (which we call the quantum catcher) to operate similar to the classical case; we expect however differences as there will be quantum effects and the dependence on mass in the Schrödinger equation.

A. Implementing a quantum atom diode and mirror

While the reflection mirror can be realized for example in experiments by an optical potential, the implementation of an atom diode is less straightforward. A theoretical proposal for such a diode is found for example in [12] and a similar one (see Fig. 6) we use throughout the remaining paper.

We assume a three-level atom where the three levels are represented by \( |1\rangle \), \( |2\rangle \), and \( |3\rangle \); see Fig. 6; the states \( |1\rangle \) and \( |2\rangle \)

FIG. 5. Classical setting: (a) shifted initial distribution \( \rho(0, x, v) \) and (b) final distribution \( \rho(T, x, v) \). Both distributions are scaled such that their maximum is one. Linear scheme, \( v_0 = 10v_m \); other parameters as in Fig. 4.

FIG. 6. Quantum atom diode and atomic mirror scheme.
are (meta)stable and there is spontaneous emission from state \([3]\) to state \([2]\). We start with the mirror potential \(V_m(x)\) which acts on the atom independent of whether it’s in state \([1]\) or \([2]\). For implementation of the atom diode, we assume a coupling between levels \([1]\) and \([3]\) with a Rabi frequency \(\Omega_p(x)\). State \([3]\) decays quickly with decay constant \(\gamma\) to the stable state \([2]\). Finally, there is a state selective potential \(V_d(x)\) placed on the left-hand side of \(\Omega_p(x)\) and \(V_m(x)\), which affects the atom only if it is in state \([2]\). Assume the particle is now incident from the left in state \([1]\); it is then pumped to state \([3]\) where it decays to state \([2]\) in such a way that it is then located and therefore trapped between the two potentials \(V_d(x)\) and \(V_m(x)\).

The master equation for the three-level diode-mirror system described above (neglecting recoil) is

\[
\frac{\partial}{\partial t} \rho(t) = -\frac{i}{\hbar} [\hat{H}_3L, \rho(t)] - \frac{\gamma}{2} \rho(t)[3][3] + [3][3] \rho(t) \\
+ \gamma [2][3][3][2].
\]  

(11)

The Hamiltonian is

\[
\hat{H}_3L = -\frac{\hbar^2}{2m} \frac{\partial^2}{\partial x^2} + \\
+ \begin{pmatrix}
V_m(x,t) & 0 & \hbar \Omega_p(x,t)/2 \\
0 & V_d(x,t) + V_m(x,t) & 0 \\
\hbar \Omega_p(x,t)/2 & 0 & 0
\end{pmatrix}.
\]

(12)

The situation is quite different from the classical case because here the probability density depends on the mass \(m\) of the particle chosen.

At initial time \(t = 0\), we start in a pure state and the initial wave function of the particle is a Gaussian (not necessarily a minimum-uncertainty product one),

\[
\psi_0(x) = A \exp \left\{ -\frac{1}{1 + i\epsilon} \frac{m^2 \Delta v^2}{\hbar^2} (x - x_0)^2 \\
+ \frac{i m v_0}{\hbar} (x - x_0) \right\},
\]

(13)

where \(\epsilon = \sqrt{\frac{\Delta x^2 m^2 \Delta v^2}{\hbar^2}} - \frac{1}{2}\) and \(A\) is a normalization constant. Note that \(\epsilon \geq 0\) due to the Heisenberg uncertainty relation.

We use the quantum-trajectory approach \([32–35]\) to solve the above 1D master equation (11) numerically. In the quantum-jump approach, the master equation (11) is solved by averaging over “trajectories” with time intervals in which the wave function evolves with the conditional Hamiltonian interrupted by random jumps (decay events). In the dynamics before the first spontaneous photon emission, we assume that the quenching laser \(\Omega_p\) and the decay can be approximated by an effective complex potential \(-i V_L(x - x_c(t)) = -i \hbar \Omega_p(x - x_c(t))\). To be more explicit, before the jump we model our effective Hamiltonian by

\[
\hat{H}_A = -\frac{\hbar^2}{2m} \frac{\partial^2}{\partial x^2} + V_m[x - x_m(t)] - i V_L[x - x_c(t)]
\]

(14)

and after the jump we model our Hamiltonian by

\[
\hat{H}_B = -\frac{\hbar^2}{2m} \frac{\partial^2}{\partial x^2} + V_m[x - x_m(t)] + V_d[x - x_d(t)].
\]

(15)

where

\[
V_{d/m}(x) = V_{0,d/m} e^{\frac{-x^2}{2\sigma_{d/m}^2}}, \quad V_c(x) = V_{0,c} e^{\frac{-x^2}{2\sigma_c^2}}.
\]

(16)

This means that the atomic mirror and the reflecting potential of the atom diode are both implemented with Gaussian potentials \(V_{d/m}(x)\). To avoid having the diode, mirror, and imaginary potential all starting in the same point, we assume that all potentials are at rest until a given time \(t_{rest}\) and only then begin

\[
V_{d/m}(x) = V_{0,d/m} e^{\frac{-x^2}{2\sigma_{d/m}^2}}, \quad V_c(x) = V_{0,c} e^{\frac{-x^2}{2\sigma_c^2}}.
\]

FIG. 7. Probability distributions: initial distribution (shifted, black, solid lines), final distributions for the classical setting (red, thick line), quantum setting with \(v_0 = 10 v_{m}\) (green, thin line) and quantum setting with \(v_0 = 8 v_{m}\) (blue, dashed line); (a) velocity space, (b) velocity space zoomed in, and (c) position space. Common parameters: \(v_d = 0.9 v_{m}, v_0 = 10 v_{m}, \Delta v = 5 v_{m}, x_0 = -0.8 d, \Delta x = 0.1 d\). Additional parameters in the quantum setting: \(v_{0,d/m} = 5 \times 10^8 \hbar / T, V_{0,c} = 4 \times 10^7 \hbar / T, v_{c} = 0.98 v_{m}, \sigma_{c} = 0.0006 d, \sigma_{d} = 0.0001 d, \) and \(m = 1000 \hbar / d^2\).
moving linearly, i.e., their trajectory is

$$x_{d/m/c} = \begin{cases} v_{d/m/c} t_{\text{rest}}, & 0 \leq t \leq t_{\text{rest}}, \\ v_{d/m/c} t, & t > t_{\text{rest}}. \end{cases}$$

(17)

At final time the velocity-probability distribution is given by $\rho(T,v) = \langle v|\rho(T)|v\rangle$, and the position-probability distribution is given by $\rho(T,x) = \langle x|\rho(T)|x\rangle$.

### B. Results

In the following, we choose the parameters shown in the caption of Fig. 7. The classical results are independent of the particle mass (as only free motion and ideal, elastic collisions with ideal walls are considered). The quantum-mechanical result depends on the mass. First, we set $m = 1000T\hbar/d^2$ and later we will examine different mass values.

In Fig. 7 the initial and final velocity distribution are shown and there is a good qualitative correlation between the classical and quantum distributions. In Fig. 7(a) we see that both the quantum and classical distributions are much compressed compared to the original very broad distribution. As expected the particles are confined between the two walls of the catcher [see Fig. 7(c)]. Therefore, the position distribution is much narrower than the initial distribution, and together with the compression in velocity distribution gives us the cooling we desired. The quantum scheme even retains another interesting property of the classical system; we see in Fig. 7(b) that, similar to the classical version, the velocity at final time $T$ is almost independent of the initial velocity.

In Figs. 7(a) and 7(b) a difference between the two cases can be seen: the quantum distribution is significantly broader than the classical; further they are less smooth. This appears to be partly because of the quenching of the wave function when it has to transition from being in state $|1\rangle$ to state $|2\rangle$.

An interesting effect to note, however, is that the quantum system performs better than the classical. This effect appears to be due to the broadness of our potentials $V_{d/m}$; in the classical simulation we treat these walls as infinitely high, while in the quantum case they have the form of Eq. (16).

Heuristically this cooling scheme works through repeated collisions with the diode-mirror system and so the effect of the broad potential increases cooling as the particle is reflected far from the center of the potential. Therefore, in Fig. 8, we examine the effect of reducing $\sigma_{d/m}$. We see that for smaller $\sigma_{d/m}$ we get closer agreement between quantum and classical schemes. This is because for smaller values of $\sigma_{d/m}$ our quantum potentials behave more and more like the infinite potential barriers in the classical case. As there are so many collisions that take place in the diode-mirror system it is quite sensitive to tuning of the parameter $\sigma_{d/m}$, with broader potentials enabling better cooling in the trap.

To underline further the generality of this cooling method, we now consider different mass values in Fig. 9. We see that for all mass values examined, we get a similar compression of the velocity distribution. In all these cases, the position distribution is also located at the end between diode and mirror potential [similar to the case shown in Fig. 7(c)]. Therefore, for all mass values examined, we get a similar compression of the velocity and position distribution, i.e., cooling of the quantum particle.

All the results are presented using dimensionless variables to underline the broad applicability of the cooling method. Therefore, the results can correspond to several, different physical settings. For example, in the case of $^7\text{Li}$ assuming a $1/e^2$ beam waist of 1 $\mu$m the dimensionless parameters in Fig. 9 (red, dotted line) correspond to $\sigma_{d/m} = 0.5$ $\mu$m, $d = 500$ $\mu$m, $T \approx 13.8$ ms, and $v_0 \approx 0.36$ ms$^{-1}$.

### IV. CONCLUSION

In this paper we have presented a method for trapping and cooling particles using an atom diode-mirror system. We investigated different trajectories for the diode and the mirror. In particular, we found a strong dependence of the efficiency on the trajectory: through classical numerical simulations of linear and square-root trajectories we deduced the advantages of the linear scheme for cooling. We propose a way to implement the atom diode and mirror system quantum mechanically; we then applied it to the trapping and cooling of a quantum particle resulting in a quantum catcher. Through further numerical
simulations we demonstrated that we can achieve cooling also in this quantum setting. Especially, we examined several parameter settings to underline the broad applicability of this cooling method.

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