**Low loss photonic nanocavity via dark magnetic dipole resonant mode near metal**

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**Supporting Information**

1. ***Hz solution of TE mode in the dielectric-semiconductor-dielectric-metal 4-layered planar structure***

To complete the solution of the simplest TE mode in the 4 layered structure of planar geometry as given in the main texts, the Hz components are given below:

$$\left\{\begin{matrix}\begin{matrix}H\_{z}\left(z\right)=B\_{t}\frac{β}{ωμ\_{0}}e^{iβx}e^{-k\_{d}(z-a)} \\H\_{z}\left(z\right)=A\frac{β}{ωμ\_{0}}e^{iβx}\cos(\left(k\_{c}z-θ\right)) \\H\_{z}\left(z\right)=B\_{b1}\frac{β}{ωμ\_{0}}e^{iβx}e^{k\_{d}\left(z+a\right)}+B\_{b2}\frac{β}{ωμ\_{0}}e^{iβx}e^{-k\_{d}(z+a)}\end{matrix}\\H\_{z}\left(z\right)=C\frac{β}{ωμ\_{0}}e^{iβx}e^{k\_{m}(z+a+h)}\end{matrix}\right. \begin{matrix}for z>a\\for |z|<a\\for -\left(a+h\right)<z<-a\\for z<-(a+h)\end{matrix}$$

1. ***Plasmonic mode at a single dielectric-metal interface***

Starting from the central equation of electromagnetic wave theory:

 $∇^{2}E-\frac{ε}{c^{2}}\frac{∂^{2}E}{∂t^{2}}=0$

Assuming the **E** has a single harmonic time dependence **E**(**r**,t)=**E**(**r**)e*-iωt*, we can then obtain Helmholtz equation:

$ ∇^{2}E+\frac{ω^{2}ε}{c^{2}}E=0$ (**S1**)

For a single dielectric-metal interface, there are only two media and one interface. We choose x as the propagation direction of the plasmonic wave and z the direction perpendicular to the infinitely large planes. The solution to Eq. (S1) can be found from ref. [[1](#_ENREF_17)]. The boundary conditions allow only TM mode to propagate at the interface of infinitely large planes, with ${k\_{d}}/{k\_{m}=-{ε\_{d}}/{ε\_{m}}}$, and $k\_{i}^{2}=β^{2}-\frac{ω^{2}ε\_{i}}{c^{2}}$ (*i=d,m* and *ki* > 0). Combining these three equations, we can obtain the well known surface plasmon dispersion relation at a single interface:

$ ω=βc\sqrt{\frac{ε\_{m}+ε\_{d}}{ε\_{m}ε\_{d}}}$ (**S2**)

The analytical solution to Eq. (S2), assuming $ε\_{m}=1-\frac{ω\_{p}^{2}}{ω^{2}}$ is:

$$ω^{2}=\frac{ω\_{p}^{2}}{2}\left\{1+\frac{β^{2}c^{2}}{ε\_{d}ω\_{p}^{2}}\left(1+ε\_{d}\right)-\sqrt{1+\frac{β^{4}c^{4}}{ε\_{d}^{2}ω\_{p}^{4}}\left(1+ε\_{d}\right)^{2}+\frac{2β^{2}c^{2}}{ε\_{d}ω\_{p}^{2}}\left(ε\_{d}-1\right)}\right\}$$

The above formula is used to calculate the plasmonic dispersion curves at a single metal-dielectric interface used in Fig. 1c. For real transition metals (Ag, Au, Cu) with *ω* < *ωp*, the permittivity is better described by $ε\_{m}=ε\_{\infty }-\frac{ω\_{p}^{2}}{ω^{2}+iγω}$ [1], where γ is a damping constant. Simulation results presented in Fig. 1d, Fig. 2 and Fig. 3 are obtained using realistic material parameters.

1. ***Guided fundamental TE mode in the dielectric-semiconductor-dielectric planar structure***

For a multilayer (> 2 layers) system, we can solve Eq. (S1) in each domain and determine their coefficients using appropriate boundary conditions. For the simplest 3 layered system, we can choose the top and bottom domains the same materials, with the core using a different material. In photonic case, both the core and cladding materials are dielectric with *ε* > 0. In this case, when *εcore > εcladding*,guided modes are supported. Both TE and TM modes can be achieved. If we choose z = 0 at the middle of the core layer and let thickness of the core be 2a. For the lowest order TE mode, the electric and magnetic fields satisfy [2,[3](#_ENREF_23)]:

$\left\{\begin{matrix}E\_{y}\left(z\right)=B\_{t}e^{iβx}e^{-k\_{d}(z-a)}\\H\_{x}\left(z\right)=-iB\_{t}\frac{k\_{d}}{ωμ\_{0}}e^{iβx}e^{-k\_{d}(z-a)}\\H\_{z}\left(z\right)=B\_{t}\frac{β}{ωμ\_{0}}e^{iβx}e^{-k\_{d}(z-a)}\end{matrix} for z>a\right.$

$\left\{\begin{matrix}E\_{y}\left(z\right)=Ae^{iβx}cos⁡(k\_{c}z)\\H\_{x}\left(z\right)=-iA\frac{k\_{c}}{ωμ\_{0}}e^{iβx}sin⁡(k\_{c}z)\\H\_{z}\left(z\right)=A\frac{β}{ωμ\_{0}}e^{iβx}cos⁡(k\_{c}z)\end{matrix} for |z|<a\right.$

$$\left\{\begin{matrix}E\_{y}\left(z\right)=B\_{b}e^{iβx}e^{k\_{d}(z+a)}\\H\_{x}\left(z\right)=iB\_{b}\frac{k\_{d}}{ωμ\_{0}}e^{iβx}e^{k\_{d}(z+a)}\\H\_{z}\left(z\right)=B\_{b}\frac{β}{ωμ\_{0}}e^{iβx}e^{k\_{d}(z+a)}\end{matrix} for z<-a\right.$$

Continuity of *Ey* and *Hx* at the interface leads to the conditions that $B\_{b}=B\_{t}=Acos(k\_{c}a)$ and $tank\_{c}a=\frac{k\_{d}}{k\_{c}}$, with $\left\{\begin{matrix}k\_{c}^{2}=\frac{ω^{2}ε\_{c}}{c^{2}}-β^{2}\\k\_{d}^{2}=β^{2}-\frac{ω^{2}ε\_{d}}{c^{2}}\end{matrix}\right.$ .

1. ***Electric field distribution of TE mode in photonic 3 layered planar structure, TE mode in 4 layered planar structure and TM mode in 4 layered planar structure***



**Figure S1** (a-c) Normalized E field |E| distribution of TE mode in the dielectric-semiconductor-dielectric 3 layered structure, TE mode in the dielectric-semiconductor-dielectric-metal 4 layered structure and TM mode in the dielectric-semiconductor-dielectric-metal 4 layered structure, respectively. In this 2D simulation, d = 110 nm, h = 6 nm, εcore = 13, εd = 2.9 and wavelength is 820 nm. The metal is assumed lossless with plasma frequency at 2.27×1015 Hz.

1. ***Additional field distribution of TE01 on glass and TE01 and TM11 modes on Al2O3/Ag***



**Figure S2** (a, b) COMSOL simulations of normalized TE01 mode magnetic field Hy distribution on zy plane at x = 0, for semiconductor disk on glass and semiconductor disk on Al2O3/Ag (diameter of 200 nm for both cases). (c) Normalized TM11 mode electric field Ez distribution on zy plane at x = 0, for semiconductor disk (200 nm in diameter) on Al2O3/Ag. The dashed rectangles indicate the physical contours of the semiconductor disks.

1. ***Q of TE01 on Al2O3/Ag as a function of Al2O3 thickness***



**Figure S3** COMSOL simulated quality factor Q of TE01 mode on Al2O3//Ag vs. the thickness of

Al2O3. In this simulation, the diameter of the AlGaInP disk is fixed at 200 nm.

Reference:

1. S. A. Maier, *Plasmonics: Fundamentals and Applications* (Springer, 2007).
2. J. D. Jackson, *Classical Electrodynamics* (John Wiley&Sons, Inc., New York, 1999), 3rd edn.
3. D. K. Cheng, *Field and wave electromagnetics* (Addison – Wesley Publishing Company, Inc., 1983).