

**UCC Library and UCC researchers have made this item openly available.
Please [let us know](#) how this has helped you. Thanks!**

Title	Boolean rings are definitely commutative!
Author(s)	MacHale, Desmond
Publication date	2015
Original citation	MacHale, D. (2015) 'Boolean Rings are Definitely Commutative!', Bulletin of the Irish Mathematical Society, 76, pp. 77-78.
Type of publication	Article (peer-reviewed)
Link to publisher's version	http://banach.ucd.ie/bull76/index.php Access to the full text of the published version may require a subscription.
Rights	© 2015 Irish Mathematical Society
Item downloaded from	http://hdl.handle.net/10468/9603

Downloaded on 2021-09-27T09:11:29Z



UCC

University College Cork, Ireland
Coláiste na hOllscoile Corcaigh

Boolean Rings are Definitely Commutative!

DESMOND MACHALE

ABSTRACT. A ring $\{R, +, \cdot\}$ is called Boolean if $r^2 = r$ for all $r \in R$. We present four proofs that a Boolean ring is commutative.

A ring $\{R, +, \cdot\}$ is called Boolean if $r^2 = r$ for all $r \in R$. In this bicentenary year of Boole's birth we present four proofs that a Boolean ring is commutative. Our first proof is the standard one found in many textbooks.

Proof 1. For all $r \in R$ we have $r = r^2 = (-r)^2 = -r$, so $r + r = 0$. Next, for all x and y in R , $x + y = (x + y)^2 = x^2 + xy + yx + y^2$, so by cancellation in the group $\{R, +\}$, we have $xy + yx = 0 = xy + xy$, by the above. Again by cancellation we have $xy = yx$, as required. \square

Proof 2. As in Proof 1, $xy + yx = 0$, for all x and y in R . Since for all $r \in R$, $0 \cdot r = 0 = r \cdot 0$ we have $(xy + yx)x = x(xy + yx)$ or $xyx + y \cdot x^2 = x^2 \cdot y + xyx$. Cancelling xyx and remembering that $x^2 = x$, we get $xy = yx$, as required. \square

Proof 3. Since for all r , $r^2 = r$ it follows that if $r^2 = 0$ then $r = 0$. Now for all x and y in R we have $(xy - xyx)^2 = xyxy + xyxxyx - xyxyx - xyxxy = xyxy + xyxxyx - xyxyx - xyxxy = 0$. So $xy - xyx = 0$ and $xy = xyx$. Then $(yx - xyx)^2 = yxyx + xyxxyx - yxxyx - xyxyx = yxyx + xyxxyx - yxxyx - xyxyx = 0$. So $yx - xyx = 0$ and $yx = xyx$. Thus $xy = yx$ as required. \square

Proof 4. For $a, b \in R$ if $ab = 0$, then $ba = (ba)^2 = b(ab)a = 0$. Now, $0 = xy - xy = xy - x^2y = x(y - xy)$, so $0 = (y - xy)x = yx - xyx$. Also, $0 = yx - yx = yx - yx^2 = (y - yx)x$, so $0 = x(y - yx) = xy - xyx$. Thus $xy = yx$ for all x and y in R . \square

We note it is immediate in all four proofs that $xy = yx = xyx = yxy$, for all x and y .

2010 *Mathematics Subject Classification.* 19E50.

Key words and phrases. Boolean Rings.

Received on 8-6-2015.

Desmond MacHale received his Ph.D. from the University of Keele and is Emeritus Professor of Mathematics at University College Cork where he taught for nearly forty years. His research interests include commutativity in groups and rings, automorphisms of groups, Euclidean geometry, number theory, and mathematical humour.

SCHOOL OF MATHEMATICAL SCIENCES, UNIVERSITY COLLEGE CORK.
E-mail address: `d.machale@ucc.ie`

Copyright of Bulletin of the Irish Mathematical Society is the property of IMS Bulletin and its content may not be copied or emailed to multiple sites or posted to a listserv without the copyright holder's express written permission. However, users may print, download, or email articles for individual use.