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Title	Low-frequency fluctuations in a semiconductor laser with phase conjugate feedback
Authors	O'Brien, D;Huyet, Guillaume;McInerney, John G.
Publication date	2001
Original Citation	O'Brien, D., Huyet, G. and McInerney, J. G. (2001) 'Low-frequency fluctuations in a semiconductor laser with phase conjugate feedback', Physical Review A, 64(2), 025802 (4pp). doi:10.1103/PhysRevA.64.025802
Type of publication	Article (peer-reviewed)
Link to publisher's version	https://journals.aps.org/pra/abstract/10.1103/PhysRevA.64.025802 - 10.1103/PhysRevA.64.025802
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Download date	2026-11-01 22:28:05
Item downloaded from	https://hdl.handle.net/10468/4561



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Low-frequency fluctuations in a semiconductor laser with phase conjugate feedback

D. O'Brien, G. Huyet, and J. G. McInerney
Physics Department, National University of Ireland, University College, Cork, Ireland
(Received 30 January 2001; published 9 July 2001)

We analyze the dynamics of a semiconductor laser with phase conjugate optical feedback, using numerical simulations based on rate equations for the complex amplitude of the electric field and the carrier density. From this analysis we observe the presence of low-frequency fluctuations which are similar to those observed in a semiconductor laser with conventional optical feedback. The similarities and differences between phase conjugate and conventional optical feedback are discussed, and a mechanism for the appearance of low-frequency fluctuations in a semiconductor laser with phase conjugate feedback is suggested.

DOI: 10.1103/PhysRevA.64.025802 PACS number(s): 42.65.Sf, 05.45.−a

Semiconductor lasers with external optical feedback have been studied in order to enhance laser properties such as the spectral linewidth and stability, and have also proved useful in understanding time-delayed dynamical systems. Several different regimes have been identified, corresponding to various feedback levels for conventional optical feedback. Many of the same characteristics found in the case of conventional optical feedback have also been observed in lasers with phase conjugate feedback, such as enhancement of the laser linewidth, and frequency locking [1]. The phase conjugate feedback system displays a richer dynamical behavior than conventional optical feedback [2], and is superior to conventional feedback with regard to linewidth reduction, as the system displays both frequency and phase locking [1]. Low-frequency fluctuations very similar to those observed in conventional feedback were also recently observed experimentally in semiconductor lasers with phase conjugate feedback [3].

Low-frequency fluctuations (LFF) in semiconductor lasers with conventional optical feedback have been the subject of many numerical studies within the framework of the time-delayed Lang-Kobayashi rate equations [4]. It is now well accepted that low-frequency fluctuations are the result of a chaotic itinerancy between destabilized external cavity modes [5]. As a result, fast dynamics, associated with the destabilized external cavity modes, occur together with the slow dynamics.

Here we present numerical results obtained for a semiconductor laser with phase conjugate optical feedback. We observe both low- and high-frequency dynamics similar to those observed with conventional optical feedback. The low-frequency fluctuations correspond to power dropouts, while the high-frequency fluctuations are associated with fast pulses in the power. Due to the absence of conventional external cavity modes in phase conjugate feedback, current models do not predict this behavior. We also perform a linear stability analysis on the system, and discuss the nature of the solutions with respect to LFF.

The theoretical model used for describing a semiconductor laser with phase conjugate optical feedback is a modified version of the well-known Lang-Kobayashi (LK)-equation [2,4] for feedback in semiconductor lasers, and may be expressed in terms of the complex amplitude of the electric field $E(t)$, and the carrier density $N(t)$ in dimensionless variables,

$$\dot{E} = \kappa(1+i\alpha)(N-1)E + \gamma E^*(t-\tau),$$

$$\dot{N} = -\frac{1}{\tau_s}(N - J + |E|^2 N), \tag{1}$$

where κ is the field decay rate, τ_s is the electron lifetime, α is the linewidth enhancement factor, J is the pumping parameter, γ represents the feedback level, and τ is the external cavity round trip time.

These equations assume a single-laser-mode operation. We have neglected the reinjection of light after multiple reflections in the external cavity, and so we are analyzing a weak feedback regime. If multiple reflections were included, we would expect to see competing effects of conventional LFF occurring at twice the external cavity round trip time. The finite response time of the phase conjugate mirror was not included in this description, as the main objective of this work is to identify the main features of the dynamics.

If we neglect fluctuations in the amplitude of the electric field and in the carrier density, Eqs. (1) reduce to a single-phase equation, which reads

$$\dot{\varphi} = \gamma\sqrt{1+\alpha^2}\,\sin[\varphi(t) + \varphi(t-\tau) + \arctan(\alpha)]. \tag{2}$$

The steady-state solutions of this equation, obtained for $\dot{\varphi} = 0$, correspond to $\varphi(t) + \varphi(t+\tau) + \arctan\alpha = 2\varphi(t) + \arctan\alpha = 2\pi n$, where n is an integer. It is therefore useful to introduce the mode parameter m defined as

$$m = \frac{\varphi(t) + \varphi(t-\tau) + \arctan\alpha}{2\pi}, \tag{3}$$

which is an integer for the steady-state solutions. A similar variable, called the mode number, was introduced in Ref. [6] to describe the dynamics of a semiconductor laser with conventional feedback, so it is an integer when the laser operates on a single external cavity mode. It is, however, worth noting that conventional external cavity modes do not exist for phase conjugate optical feedback.

Numerical solutions of Eqs. (1), integrated with a fourth-order Runge-Kutta algorithm, are depicted in Figs. 1 and 2. The parameter values used were $\kappa = 1$, $\alpha = 5$, $\tau_s = 200$, $J = 1$, $\tau = 1000$, and $\gamma = 5$, although the observed features of the dynamics occur over a wide parameter range.

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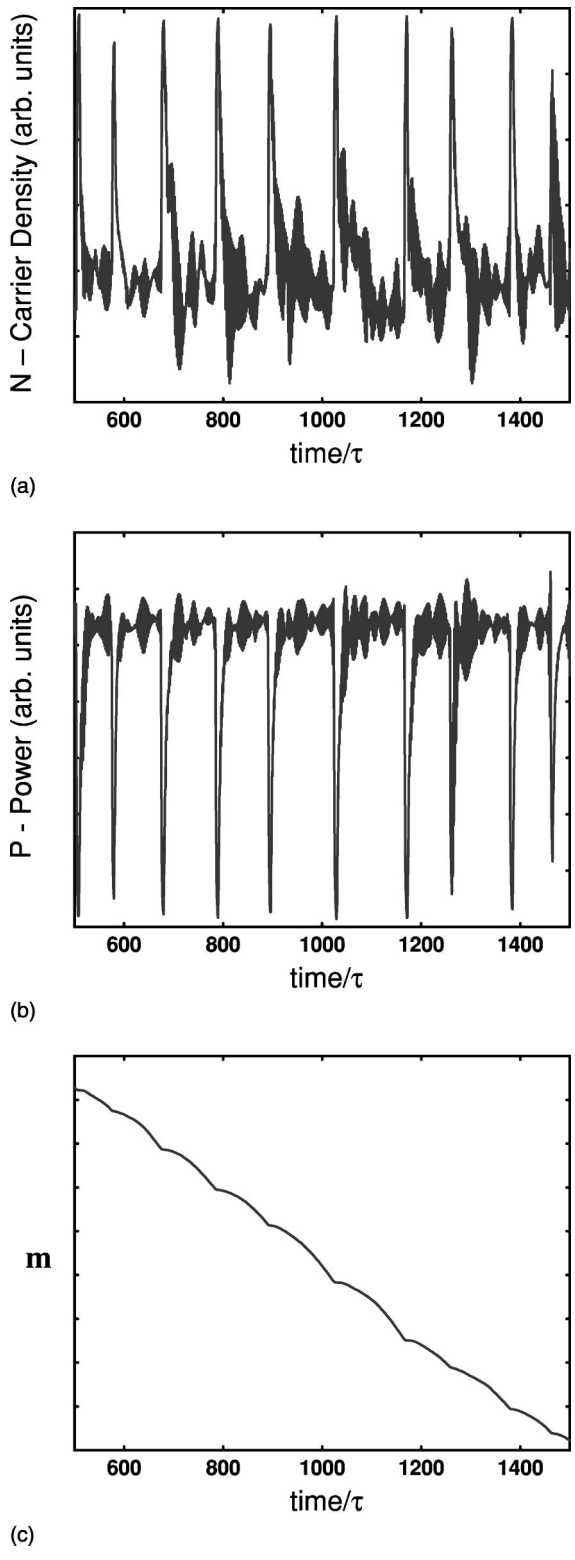


FIG. 1. Low-frequency dynamics filtered at the round-trip time for power (P), carrier density (N), and mode parameter (m), vs time/τ.

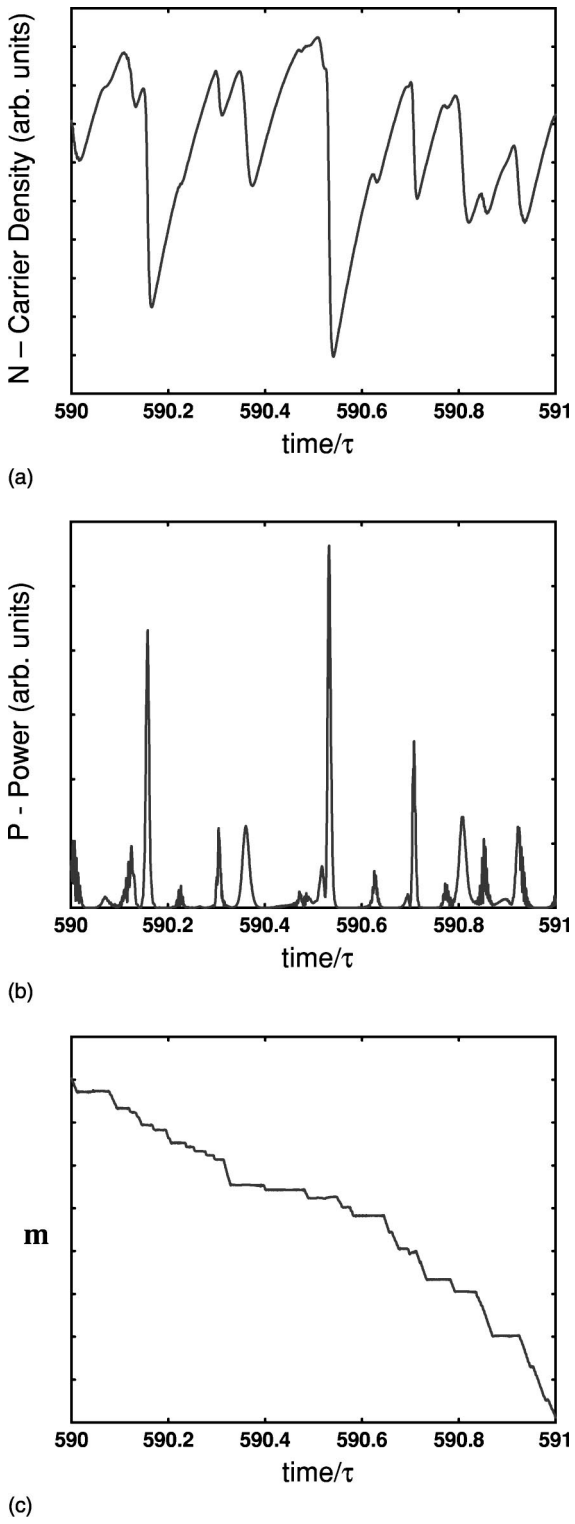


FIG. 2. High-frequency dynamics: power (P), carrier density (N), and mode parameter (m), vs time/τ.

Figure 1 shows the low-frequency dynamics in the power, the carrier density, and the mode parameter m. We see clear dropouts in the laser intensity, similar to those observed in conventional optical feedback, while the mode parameter shows a mean drift together with a slow modulation which corresponds to the power dropouts. The mean drift in the mode number is a result of the laser operating at a frequency lower than the solitary laser frequency.

In Fig. 2 we plot the fast dynamics, again looking at the carrier density, the power, and the mode parameter. The tem-

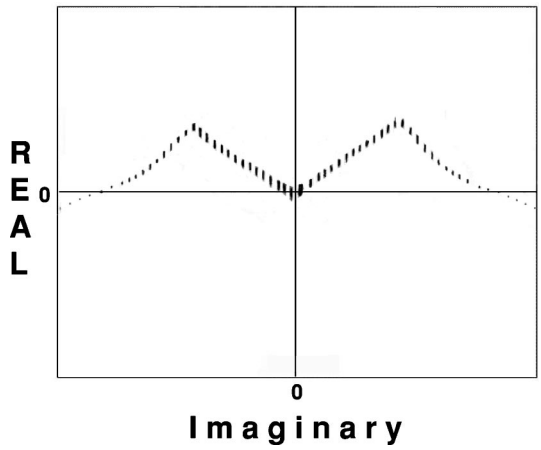


FIG. 3. Distribution of eigenvalue solutions to the characteristic equation in the complex plane, for parameter values corresponding to the LFF regime.

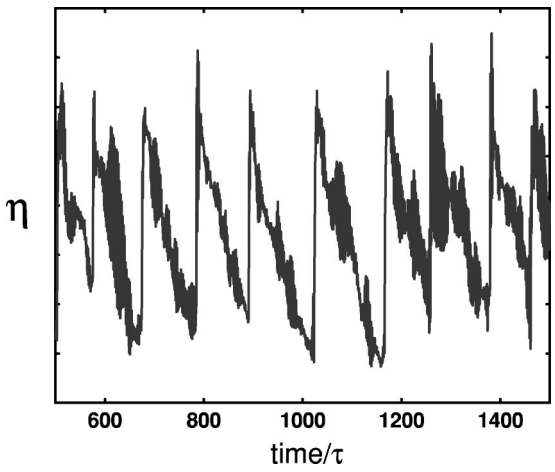


FIG. 4. Temporal evolution of the average instantaneous optical frequency η.

poral evolution of the mode parameter consists of periods of quasistationary behavior near the steady-state solutions, followed by fast jumps. These jumps correspond to changes of $m \pm 1$, with a general trend toward lower values of m.

This behavior is very similar to that observed with conventional optical feedback. The usual interpretation for low-frequency fluctuations observed in the LK equations involves chaotic itinerancy among external cavity modes [5,7]. In this picture, the solitary laser relaxation oscillations become undamped under the influence of optical feedback, generating limit cycles around the external cavity modes. Each limit cycle may become unstable for certain parameter values, leading to a chaotic attractor via a quasiperiodic, or period-doubling route, to chaos. These chaotic attractors may also merge to generate the LFF chaotic attractor. A simple picture of the route to LFF can be obtained by reducing the LK equations to a three-dimensional dynamical system [8].

In the case of phase conjugate feedback, there are no conventional external cavity modes. Equations (1) have only five steady-state solutions. The first is associated with the nonlasing operation, while two solutions are always unstable above the threshold, and two solutions are stable for certain levels of the feedback and pump parameters. An analogy between conventional and phase conjugate feedback can, however, be drawn in order to explain the low-frequency fluctuation observed in both sets of equations. From the general features of time delay differential equations, one can expect multiple resonances when a Hopf bifurcation occurs under the influence of delay. In the case of conventional feedback, this leads to the appearance of external cavity modes. For phase conjugate feedback, the Hopf bifurcation which destabilizes the continuous wave operation should present similar features. Here the complex solutions of characteristic equations having positive real parts will play the role of external cavity modes.

Let us consider the stability analysis of one of the steady-state solutions which are stable for low feedback levels. For very low feedback levels, the stability analysis provides three solutions, one negative real solution, and two complex conjugate solutions associated with the solitary laser relaxation oscillations. When the feedback level is increased, the relaxation oscillations become undamped as their real parts become positive. Other pairs of complex conjugate solutions may also appear, giving rise to multiple resonances. This generates a whole family of roots with a positive real part, with their imaginary parts separated by $2\pi/\tau$ [9], as shown in Fig. 3.

These roots are analogous to the external cavity modes for conventional feedback. A scenario leading to the appearance of LFF from these solutions is therefore possible.

An instability similar to the usual LFF instability can therefore be observed among the nonconventional external cavity modes for certain parameter ranges. To verify this conjecture, we compared the amplitude of the frequency fluctuations with the number of solutions of the characteristic equation having a positive real part.

Looking at the spectral evolution of the system, the average instantaneous frequency over the course of one round trip is

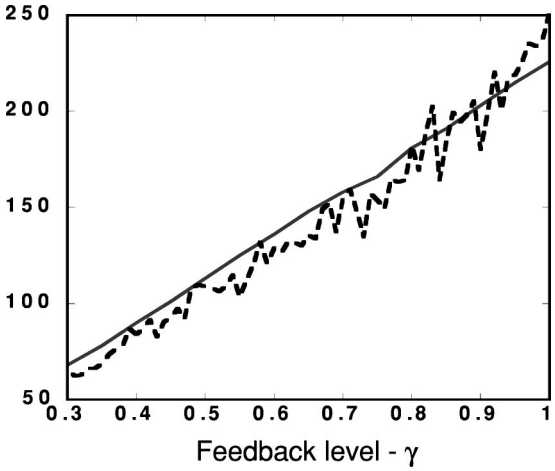


FIG. 5. Comparing the number of solutions to the characteristic equation with a positive real part (resonances in the power spectrum) (solid line) to the drift in the mean optical frequency η (dashed line).

$$\eta = \langle \dot{\varphi} \rangle = \frac{1}{\tau} \int_{t-\tau}^{t} \dot{\varphi}(t) dt = \frac{\varphi(t) - \varphi(t-\tau)}{\tau}. \qquad (4)$$

Figure 4 shows the evolution of η as a function of time. The system changes optical frequency drastically at a power dropout. After a power dropout the system is at the solitary laser frequency. There is then a steady drift away from the solitary laser frequency as the system recovers, and this drift continues until a critical point is reached. At this point, it collapses back to the solitary laser frequency.

In order to calculate the amplitude of the fluctuation of η (average instantaneous optical frequency), we compute the root-mean-square of η, and assume it is a triangular wave to calculate the peak-to-peak value. This method is more reliable than directly measuring the peak-to-peak value from the time trace. The result of this calculation is shown in Fig. 5 together with the number of roots of the characteristic equation with a positive real parts for different feedback levels. We note that there is very good agreement between the curves over a wide range of feedback levels. This agreement was also checked for different values of the electron lifetime.

These observations indicate that the underlying physics of low-frequency fluctuations is similar to that observed in the Lang-Kobayashi equations for conventional optical feedback. Although the two systems differ in many ways, the main features of LFF are observed in both systems. For instance, the existence of external cavity modes with conventional feedback is closely connected with the phase invariance of the LK equation. This phase invariance is not present in phase conjugate feedback, and therefore does not play a major role in the underlying mechanism for the observation of LFF's. This suggests that the mechanism responsible for generating LFF in a time-delay dynamical system is more general than previously seen in conventional feedback. The slow dynamics of the carrier dynamics and the time delay seem to be the two most important ingredient for these fluctuations.

This work was motivated by the experimental observations of Professor D. Kane, Macquarie University, Australia. We are grateful to her for her interesting and stimulating talks during her visit. We are also indebted to Dr. V. Voignier for useful advice in the course of this work. This work was supported by the Institute for Nonlinear Science at University College, Cork.

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