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A reflection on the use of 'electric flux' in introductory physics instruction

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The treatment of electric fields and flux in most textbooks on introductory physics, at least in the English language, differs radically from the nomenclature recommended by all relevant standards organizations and authorities. The general approach also conflicts with the usual methodology adopted in electrical engineering texts. The origin of the discrepancy is outlined and it is suggested that resolving the conflict would be beneficial to the teaching and learning of this and related topics.

I. INTRODUCTION – STANDARDISATION OF UNITS AND NOMENCLATURE

Prior to the 1960s learning introductory physics was complicated by the multiplicity of different systems of units used (cgs, MKS, British Imperial, British Engineering, e.s.u., e.m.u., Gaussian, Practical Electrical Units, etc.) not to mention the wide variety of symbols and nomenclature involved for the same physical quantity in textbooks and scientific publications in those days.

These difficulties were largely resolved by two important developments. Firstly, the formal adoption by the 11th Conférence Générale des Poids et Mesures (CGPM) in 1960 of the Système International d'Unités (SI).[1] At the same time a move toward further standardization was under way in the shape of the development of internationally agreed standards for symbols and nomenclature agreed by intergovernmental treaty organizations, national and international standards authorities and non-governmental subject agencies such as CGPM, Bureau International des Poids et Mesures (BIPM), the US National Institute of Standards and Technology (NIST), the International Union of Pure and Applied Physics (IUPAP), and the International Union of Pure and Applied Chemistry (IUPAC). The adoption and implementation, sometimes reluctantly, by the physics community of these developments had profound and beneficial effect on physics education.

In the years that followed, most physicists, chemists, engineers and others readily adopted the recommendations for symbols and nomenclature agreed by these authorities. This standardization in the form of internationally agreed conventions has greatly enhanced commercial, technical and scientific communication. In education such standardization applied to high school science curricula and textbooks is of considerable assistance to students who go on to study physics and/or chemistry at a university. At university level, the adoption in introductory physics textbooks of the same internationally agreed symbols and nomenclature makes it easier for students to proceed to intermediate and advanced level courses in areas such as physics, chemistry and engineering.

The agreed standards for symbols, units and nomenclature are laid down in documents published by BIPM (*SI Brochure*),[2] IUPAP (*Red Book*)[3] and IUPAC (*Green Book*).[4]

II. THE ISSUE OF ELECTRIC FLUX

There remains, however, one very obvious exception where agreed nomenclature is not universally embraced. The quantity electric flux is defined by IUPAP and IUPAC such that the total electric flux through a surface S is defined as

$$\Psi_E = \iint \boldsymbol{D} \,\boldsymbol{.} d\boldsymbol{S} \tag{1}$$

where $\boldsymbol{D}$ is the electric displacement at each point on the surface.

By contrast, the great majority of textbooks aimed at students taking introductory physics courses in university or at the upper level in high school use a different definition for the quantity 'electric flux', namely

$$\Psi'_E = \iint \boldsymbol{E} \,\boldsymbol{.} d\boldsymbol{S} \tag{2}$$

where $\boldsymbol{E}$ is the electric field strength (intensity) at each point on the surface.

In a survey of over 150 introductory physics textbooks in one university library,[5] only two were found to be in conformity with the IUPAP/IUPAC definition. Some thirty or so textbooks on intermediate or advanced electromagnetism in the same library divided evenly between the two approaches while almost all electrical engineering texts defined electric flux as in definition (1) above. An explanation of how this disparity arose historically is discussed in Section III below.

The incompatibility between different treatments is most noticeable in the way in which Gauss' law is presented. In the traditional approach and, crucially as we have seen, in most current electromagnetism/electromagnetics textbooks aimed at students of engineering, the total electric flux through a surface S is given by definition (1) and Gauss' law is written as

$$\oiint \boldsymbol{D} \,\boldsymbol{.} d\boldsymbol{S} = Q \tag{3}$$

where Q is the net electric charge within the (closed) surface. This approach is consistent with the definition of electric flux as defined by IUPAP and IUPAC. While official documents from BIPM[2] and NIST[6] on nomenclature generally avoid the use of the term 'electric flux', these documents define $\boldsymbol{D}$ as the *electric flux density*, thus indicating conformity with the IUPAP/IUPAC norm.

The flux model underlying definition (1) above can be found in the early (1856) work of James Clerk Maxwell.[7,8] As Longair[9] has pointed out, Maxwell invoked formal analogies between the mathematics of hydrodynamical systems and electromagnetic phenomena. He drew particular attention to the application of incompressible fluid flow concepts to Faradays of notion electric and magnetic 'lines of force', which correspond to streamlines in the hydrodynamic model. Maxwell, of course, went on to develop a complete self-consistent field theory of classical electromagnetism which no longer required such analogies and primitive models.

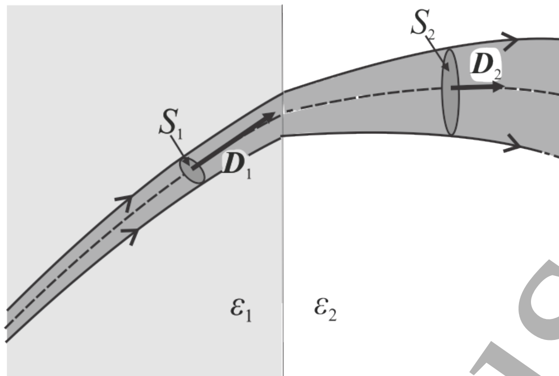

FIG. 1. A tube of electric flux crossing an interface between dielectric media.

One significant feature of the flux model underlying definition (1) lies in the fact that the flux density at a point in a medium due to a given system of charges is independent of the dielectric properties of the medium and that flux is continuous across interfaces between media (Fig. 1),[10] in other words $\iint \boldsymbol{D}_1 \,.\, d\boldsymbol{S}_1 = \iint \boldsymbol{D}_2 \,.\, d\boldsymbol{S}_2$.

On the other hand, the great majority of introductory physics textbooks in the English language adopt definition (2) for 'electric flux' and Gauss' law is usually presented in the form

$$\oiint \boldsymbol{E} \,.\, d\boldsymbol{S} = \frac{Q}{\epsilon_0} \tag{4}$$

where ϵ_0 is the permittivity of vacuum. From this viewpoint, $\boldsymbol{E}$ is the 'electric flux density', in clear conflict with the IUPAP/IUPAC compatible definition (1) of the term.

'Electric flux' as defined by (2) has quite distinct characteristics from the IUPAP/IUPAC definition and Maxwell's analogy. For example, referring to the tube of flux in Fig. 1, $\iint \boldsymbol{E}_1 \,.\, d\boldsymbol{S}_1$ is *not* equal to $\iint \boldsymbol{E}_2 \,.\, d\boldsymbol{S}_2$. Indeed, in this context, $\iint \boldsymbol{E} \,.\, d\boldsymbol{S}$ does not have the characteristics of a flux at all. Similarly, standard treatments of the electric field due to a uniformly polarized dielectric specimen[11] or of the field inside and outside a dielectric body placed in an external electric field[12] show that, while the $\boldsymbol{D}$-field lines are continuous across interfaces, the $\boldsymbol{E}$-field lines are not.[13]

The use of the permittivity of vacuum in Eq. (4) suggests an intention *ab initio* to only deal with electric fields in vacuum. For reasons outlined in Section III below, many introductory physics curricula do indeed take this approach. Note that, in this particular context, the nomenclature could be brought into more conformity with IUPAP/IUPAC standards by *defining* electric flux as $\iint(\epsilon_0 \boldsymbol{E}) \,.\, d\boldsymbol{S}$ instead of definition (2). Before proceeding to deal with electric fields in materials, however, this definition requires generalisation to definition (1) but, once so extended, it enables more simple treatments than is possible by maintaining definition (2). This is particularly useful for learners who go on to study intermediate level courses in electromagnetism, materials science, applied physics and engineering.

It should be pointed out that the situation tends to be more nuanced in textbooks written in languages other than English. In Continental European languages, for example, the tendency is to identify two distinct fluxes: that defined by (1) above may be translated as 'electric displacement flux' and the flux defined by (2) as 'electric field strength flux'. While not strictly conforming to IUPAP and IUPAC conventions, this approach is far less unsatisfactory than current English language practice.

Interestingly, there is no similar dispute concerning the definition of magnetic flux which all authors and authorities agree is defined as

$$\Psi_M = \iint \boldsymbol{B} \,.d\boldsymbol{S} \tag{5}$$

where $\boldsymbol{B}$ is the magnetic flux density or magnetic induction. Magnetic flux is the direct analogue of electric flux as defined by (1) and has exactly the same behavior at an interface between magnetic media as shown by electric flux between dielectric media.

III. MID-TWENTIETH CENTURY CURRICULUM REFORM

The 1950 also saw the start of major curriculum reform at both high school and university level, particularly in the US. These developments were driven by a number of different factors, including the experience of the part played by physicists during World War 2, the reaction to the launch of Sputnik in 1957 and the perceived needs of the Cold War. The setting up of the National Science Foundation (NSF) in 1950 created a vehicle for lobbying and supporting curriculum development initiatives and providing in-service training and materials for teachers.

Also significant were the reforms introduced in the United States by two important projects sponsored by the NSF which were directed at improving teaching in high school, namely, the Physical Science Study Committee (PSSC) program,[14–16] initiated during 1956 - 1960, and Project Physics (1962 - 1972).[17–19].

A significant influence on the reformed curriculum was a desire to include 20th century topics. For example, a PSSC report[20] in 1960 stated that

> the Committee believed that it would be to the benefit of physics teaching if the presentation of the subject matter were focused towards one goal, and that goal ought to be *the atomic picture of the universe*. (italics original)

The curriculum and courses developed by the PSSC were designed for use in upper level secondary school and entry level at university. The curriculum makes interesting reading at this remove. In particular, it introduced 'the student to electricity and through it to the physics of the atom'. This intermixing of atomic physics with electromagnetism was novel and radical and reflected the contemporary zeitgeist – indeed, in the early PSSC syllabus electromagnetism was entirely incorporated within 'Atomic Physics'.

University based physicists were heavily involved in these projects and this had some effect on curriculum development at College level. A more important influence on the physics curriculum in universities, however, was the available textbooks. Between 1944 and 1946 the first editions of the three volumes of Francis Weston Sears' *The Principles of Physics*[21] were published followed in 1949 by the very influential *College Physics* and *University Physics*[22] by Sears and Zemansky. In 1960, Wiley published *Physics for Students of Science and Engineering*[23] by Halliday and Resnick which partly supplanted Sears and Zemansky as the standard introductory physics text in US universities. These books effectively defined a new curriculum for introductory physics texts which emphasized basic principles and the unity of physics while dropping many traditional topics. The atomistic approach of the PSSC approach was considered, but rejected, by Halliday and Resnick in their hugely influential 1960 textbook.[23] In the Preface to the first edition they state that they considered 'the adoption of an atomic approach' but opted instead for an approach based on 'modifying the selection and treatment of topics within the traditional organization'. In practice this resulted in following the traditional treatment in the case of Mechanics and Thermal Physics but adopting a somewhat hybrid version of the PSSC approach when it came to Electromagnetism. The atomistic aspect of the approach to Electromagnetism can be seen in the way current is described in terms of electron motion, Coulomb's law is treated as applying exclusively to charges in vacuum and dielectric materials are introduced *ab initio* as comprising atomic dipoles.

This new canon of topics became widely accepted, particularly in the English-speaking world[24] and is currently adopted in the majority of English language textbooks on introductory physics.

IV. CONCLUSIONS

It is important that nomenclature should conform to internationally agreed conventions. The terms electric flux and electric flux density should be confined to the usage represented by definition (1) above (that is, electric flux measured in coulomb). Should it be desired to continue to consider the quantity represented by definition (2) as a 'flux', different terminology should be employed as in Continental European practice. Furthermore, the current tendency to use the term 'electric field' (instead of *electric field strength* $\boldsymbol{E}$) and 'magnetic field' (instead of *magnetic flux density* $\boldsymbol{B}$), both also in violation of agreed international terminology, should be discouraged.

V. ACKNOWLEDGEMENTS

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[1] Bureau International des Poids et Mesures, *Comptes Rendus des Séances de la Onzième Conférence Générale des Poids et Measures*, (Paris: Gauthier-Villars & Cie 1960).

[2] Bureau International des Poids et Mesures, *The International System of Units (SI)*, 8th edition 2006, updated 2014, Table 4, ⟨`http://www.bipm.org/utils/common/pdf/si_brochure_8_en.pdf`⟩ BIPM.org, Retrieved 2018-02-22.

[3] E. R. Cohen and P. Giacomo, *Symbols, Units, Nomenclature and Fundamental Constants in Physics*, (Document IUPAP-25; SUNAMCO 87-1 1987); *Physica*, **146A**, 1–68 (1987).

[4] E. R. Cohen et al, *Quantities, Units and Symbols in Physical Chemistry*, IUPAC Green Book 3rd Edition, 2nd printing (IUPAC & RSC Publishing, Cambridge, 2008).

[5] Cambridge University Library ⟨`http://www.lib.cam.ac.uk/`⟩, Retrieved 2018-02-22.

[6] A. Thompson and B. N. Taylor *Guide for the Use of the International System of Units (SI)*, NIST Special Publication 811 2008 Edition, Table 4, ⟨`http://physics.nist.gov/cuu/pdf/sp811.pdf`⟩ NIST.gov, Retrieved 2018-02-22.

[7] J. C. Maxwell (1856). "Analogies in nature"; in *The scientific letters and papers of James Clerk Maxwell* (P. M. Harman, ed.), Vol. 1, pp. 376383 Cambridge, UK: Cambridge University Press (1990).

[8] J. C. Maxwell . "On Faraday's lines of force", Trans. Camb. Phil. Soc. **10**, 155188 (1856).

[9] M. S. Longair, " '...a paper ...I hold to be great guns': a commentary on Maxwell (1865) 'A dynamical theory of the electromagnetic field' ", Philos. Trans. R. Soc. **A373**, 20140473 (2015).

[10] The latter condition also necessitates that the component of $\boldsymbol{D}$ perpendicular to an interface is continuous across the interface.

[11] See, for example, D. J. Griffiths *Introduction to Electrodynamics, 3rd ed*, (Prentice Hall, Upper Saddle River NJ, 1999), Figure 14.10, p 169; O. D. Jefimenko *Electricity and Magnetism, 2nd ed*, (Electret Scientific Company, Star City WV, 1969), Fig. 8.16, p. 254.

[12] D. J. Griffiths, op cit, Figure 4.27, p. 187; O. D. Jefimenko op cit, Fig. 8.11, p. 244.

[13] This situation can be remedied, of course, by the introduction of 'surface charges' that are sources of electric field but not of electric displacement. From a macroscopic perspective such sources are 'fictitious' entities but can be understood as real in the context of a microscopic model.

[14] Physical Science Study Committee, "The Physical Science Study Committee A Planning Conference Report, Physics Today 28–9 (March 1957).

[15] Physical Science Study Committee, *Physics*, (D. C. Heath and Company, Boston, 1960).

[16] "PSSC: 50 Years Later", ⟨`http://www.compadre.org/portal/pssc/pssc.cfm?view=author`⟩ Compadre.org, Retrieved 2017-07-14.

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[19] G. Holton, "The Project Physics Course: Then and Now", Science & Education, **12**, 8, 779-786 (November 2003).

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[21] F. W. Sears, *Principles of Physics I, II and III*, (Addison-Wesley Press, Cambridge MA, 1946–7).

[22] F. Sears and M. Zemansky, *College Physics Parts 1 and 2*, (Addison-Wesley Press, Cambridge MA, 1948); *University Physics*, (Addison-Wesley Press, Cambridge MA, 1949).

[23] D. Halliday and R. Resnick, *Physics for Students of Science and Engineering*, (John Wiley and Sons, New York, 1960).

[24] There were a few notable exceptions to this approach; for example, Marshall, Pounder and Stewart's *Physics* (Macmillan of Canada 1967) which, while fully adopting SI and the IUPAP recommendations, followed the traditional historical sequencing even in electromagnetism