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An Educational Tool for RF LNA Analysis Based on an Implementation of the Smith Chart in Excel

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***Abstract*— This paper describes a simple tool for the analysis and design of single-stage Radio Frequency Low Noise Amplifiers based on standard Smith-Chart procedures that has been implemented in Excel. The tool allows the user to enter the transistor's s-parameters and details of the source and load networks and to interactively explore the effects of these quantities on design variables such as gain, noise figure and stability. It also facilitates the design of two-element matching networks to transform the source and load impedances to optimum values to achieve the desired gain and noise performance.**

***Keywords* – RF Education, LNA, Smith Chart, Matching, Excel**

I INTRODUCTION

Many undergraduate programmes in Electrical and Electronic Engineering include one or more courses on Radio Frequency (RF) circuit design such as the module EE4011 "RF IC Design" at University College Cork [1]. These RF courses usually include some discussion of Low Noise Amplifier (LNA) design and may illustrate the basic concepts relating to gain, noise, stability and input and output matching networks by means of the Smith Chart [2]. While a practicing RF design engineer will make use of specialised Computer Aided Design (CAD) software for detailed circuit analysis and optimization, the engineering student is also encouraged to perform a range of basic calculations manually using a standard scientific calculator, possibly in conjunction with a paper version of the Smith Chart. These manual calculations help to develop the student's understanding of the fundamental LNA design relationships and also provide a basis for potential examination questions. Even the simplest LNA gain formulas can be tedious to evaluate manually because of the complex numbers involved and therefore an Excel [3] spreadsheet was developed to allow EE4011 students to quickly verify the results of their hand calculations. It was subsequently determined that Excel had enough interactive features to implement a tool for single-stage LNA design incorporating the Smith Chart and two-element matching networks. This interactive tool is the subject of the remainder of this paper. This tool uses a very simplified version of the Smith Chart but other authors have used Excel to implement highly sophisticated versions [4].

II PORTABILITY AND INTERACTIVITY

The core requirement of the tool is that it can be used by students on a range of platforms – for example on a departmental PC network where they may not have administrative rights to install software, or on their personal laptops, where they may not have licenses for commercial/educational software that may be available on the departmental network. Additionally, in the case of EE4011, which is made available to postgraduate students in Ireland who are participants of a graduate school known as the "International Centre for Graduate Education In Micro- & Nano-Engineering" (ICGEE) [5], the tool should be easy to download from the ICGEE web site and available for use on college or personal PCs/laptops without installation privileges or licenses.. These portability requirements are readily met by Excel because of its widespread installed base.

From an interactivity perspective, it was desirable to include features that would provide instant feedback to the student such as instant updating of gain, noise or stability circles on a Smith Chart to allow the student to develop an intuitive grasp of these concepts in LNA design. Initially, it was thought that Excel might not be able to support such a level of interaction but, in recent years, there has been a growing use of Excel to develop interactive spreadsheets to illustrate basic scientific concepts – in some cases these type of spreadsheets

are known as “Excelets” [6] and these have provided the inspiration for the tool described here. Interactivity is achieved by using the Form Control features in Excel. Four Form Control features are used in particular: a “check box” is used for yes/no choices such whether to display a particular feature on the Smith Chart or not; the “option button” and “group box” Form Controls allow the user to select from multiple choices such how to determine the input and output reflection coefficients on the Smith Chart; and, finally, the “scroll bar” Form Control allows the user to vary quantities (reflection coefficients in this case) smoothly between upper and lower limits and observe the effect of these on the LNA performance.

III THE SMITH CHART IN EXCEL

The Smith Chart [2] is a plot of reflection coefficients on a complex plane and the circles and arcs on the Smith Chart correspond to the reflection coefficients of standard series or parallel connected R-C or R-L networks. Consider a simple network consisting of a resistance, R, in series with an inductance, L. At any frequency, the impedance, Z, of this network and the associated reflection coefficient are defined by equation (1).

$$Z = R + j\omega L \qquad \Gamma = \frac{Z - Z_0}{Z + Z_0} \tag{1}$$

where ω is the frequency in radians/s and Z_0 is the characteristic impedance of the system, typically 50Ω. In equation (1), if R is kept constant and either ω or L swept from 0 to ∞, then the resultant values of Γ will trace out a semi-circle on the complex plane. If ω and L are kept constant and R varied from 0 to ∞, the resultant values of Γ will trace out an arc on the complex plane. Similarly, a series connected R-C circuit will produce its own characteristic set of semi-circles and arcs on the complex plane. Plotting these circles and arcs on the complex plane for a range of resistance and reactance values leads to the familiar Smith Chart. In most cases, for stable circuits, only reflection coefficients with a magnitude of 1 or less are of practical interest and thus the region of interest in the Smith Chart is contained within the unit circle surrounding the origin of the complex plane and this unit circle serves as a natural boundary for any plots.

Unfortunately, Excel does not have an inbuilt library for drawing circles or arcs and so every circle and arc has to be decomposed into a set of X-Y datapoints which can be plotted as an X-Y graph using the scattergram option. A circle is described by its centre point and radius on the complex plane while an arc is considered to be a circle segment traced out by sweeping the radial line of the circle through a range of angles with respect to the positive x-direction of the graph. Figure 1 shows the Smith Chart template constructed in Excel and used by this tool. This shows the circles representing the reflection coefficients of series R-C or R-L networks with normalized resistances of 0, 1/3, 1 and 3 (the normalized resistance is defined as R/Z_0). It also shows the arcs representing the reflection coefficients of series R-C or R-L networks with normalized reactances of 0, +/- ½, +/-1 and +/- 2. Although this is a very simplified implementation of Smith Chart, it is sufficient for the present purpose.

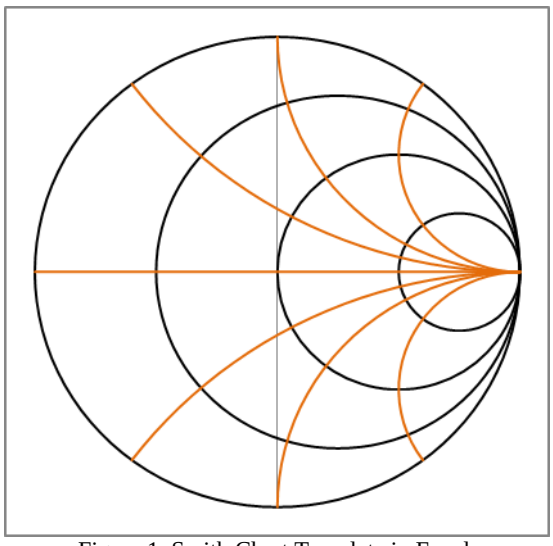


Figure 1: Smith Chart Template in Excel.

IV GAIN AND NOISE CALCULATIONS

A typical one-transistor LNA configuration is shown in Figure 2. The transistor is described by s-parameters and noise parameters at the operating frequency and system impedance Z_0. The source impedance is Z_S (with corresponding reflection coefficient, Γ_S) and the load impedance is Z_L (with corresponding reflection coefficient, Γ_L).

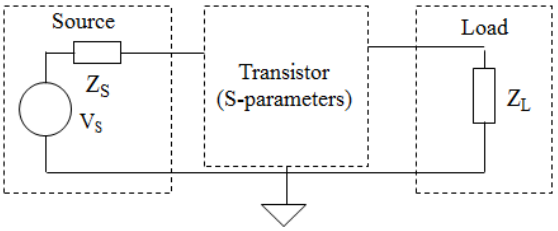


Figure 2: One-Transistor LNA Configuration.

Many different formulas can be developed to illustrate the power gain from input to output for this one-transistor LNA [7, 8] with one of the most commonly specified gains being the Transducer Gain, G_T, given by equation (2). It is seen that G_T depends on the s-parameters of the transistor (with $\Delta = s_{11}s_{22} - s_{12}s_{21}$) and the source and load reflection coefficients.

$$G_T = \frac{\left(1-|\Gamma_S|^2\right)|s_{21}|^2\left(1-|\Gamma_L|^2\right)}{|1-s_{11}\Gamma_S - s_{22}\Gamma_L + \Delta\Gamma_S\Gamma_L|^2} \quad (2)$$

This expression for Transducer Gain is commonly simplified by assuming that the transistor is unilateral (i.e. s_{12}=0) giving an expression for Unilateral Transducer Gain, G_{TU}, as specified in equation (3).

$$G_{TU} = \frac{1-|\Gamma_S|^2}{|1-s_{11}\Gamma_S|^2}|s_{21}|^2\frac{1-|\Gamma_L|^2}{|1-s_{22}\Gamma_L|^2} \quad (3)$$

To more clearly identify the factors that effect LNA gain, equation (3) is often rewritten in terms of three individual gain components as shown in equations (4) and (5).

$$G_{TU} = G_S G_O G_L \quad (4)$$

where

$$G_S = \frac{1-|\Gamma_S|^2}{|1-s_{11}\Gamma_S|^2} \quad G_O = |s_{21}|^2 \quad G_L = \frac{1-|\Gamma_L|^2}{|1-s_{22}\Gamma_L|^2} \quad (5)$$

In equation (5) G_S is commonly referred to as the source gain and includes the effect of the source reflection coefficient, G_L is load gain and includes the effect of the load reflection coefficient and G_0 is the intrinsic power gain of the transistor and only depends on the s_{21} parameter of the transistor.

G_{TU} reaches a maximum value when the source and load are conjugate matched to the transistor i.e. when $\Gamma_S=s_{11}^*$ (causing G_S to reach a maximum) and $\Gamma_L=s_{22}^*$ (causing G_L to reach a maximum) (* denotes a complex conjugate). In this case, the gain, G_{TUmax}, is given by equation (6).

$$G_{TU,\max} = \frac{1}{1-|s_{11}|^2}|s_{21}|^2\frac{1}{1-|s_{22}|^2} \quad (6)$$

From an educational perspective, design examples frequently work with the formulas for the Unilateral Transducer Gain (G_{TU}) and the Maximum Unilateral Unilateral Transducer Gain (G_{TUmax}) because they are simpler and more manageable for hand calculations than the more general formula for Transducer Gain (G_T) . However, the unilateral assumption is only an approximation and the error introduced by this approximation can be quantified through another quantity known as the Unilateral Figure of Merit, M, as given in equation (7).

$$M = \frac{|s_{11}||s_{12}||s_{21}||s_{22}|}{\left(1-|s_{11}|^2\right)\left(1-|s_{22}|^2\right)} \quad (7)$$

Other gain definitions are also commonly used such as operating gain (G_P) and available gain (G_A). Additionally, the stability of the amplifier can be assessed by calculating the Rollet Stability Factor, K. Finally, the noise performance of the amplifier can be determined [7, 8] using the noise parameters of the transistor and the source reflection coefficient as shown in equation (8).

$$F = F_{\min} + \frac{4R_N}{Z_0}\frac{|\Gamma_S - \Gamma_{opt}|^2}{\left(1-|\Gamma_S|^2\right)|1+\Gamma_{opt}|^2} \quad (8)$$

Here, F is the noise factor of the LNA and F_{min}, R_N and Γ_{OPT} are the noise parameters of the transistor. In particular, Γ_{OPT} is the special value of source reflection coefficient at which the LNA will achieve its best (lowest) noise factor, F_{min}. All of these quantities are calculated by the Excel spreadsheet and can thus be used by the student to verify the results of hand calculations.

V VISUALISING THE DESIGN TRADE-OFFS

One of the main uses of the Smith Chart in LNA design is to illustrate the trade-offs that occur as the source and load reflection coefficients (or impedances) vary. In particular, the gain depends on both source and load reflection coefficients while the noise figure depends on the source reflection coefficient. These design trade-offs are illustrated very effectively by means of circles on the Smith Chart as shown in Figure 3.

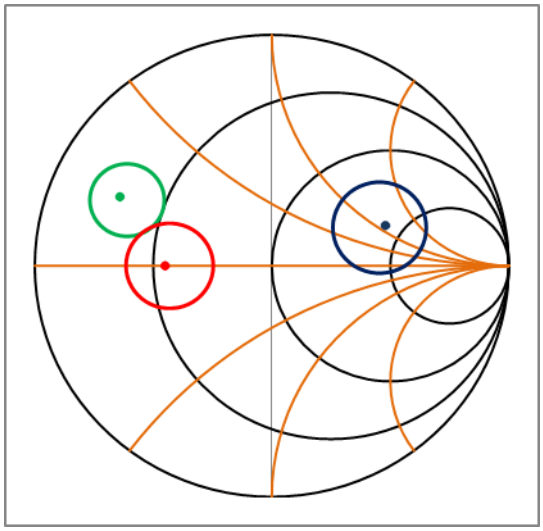


Figure 3: Gain and Noise Circles on the Smith Chart.

Figure 3 shows the Smith Chart drawn in Excel for a sample transistor and illustrates design trade-offs by means of three circles. A load gain circle (blue circle on R.H.S. of graph) is drawn for Γ_L =1dB and indicates that to achieve a G_L of 1dB or higher, the load reflection coefficient should be placed on or inside this circle. The point s_{22}* is indicated as a (blue) dot inside this circle and corresponds to the maximum value of G_L. A source gain circle (green circle on upper L.H.S. of graph) is drawn for Γ_L =2.6dB and indicates that to achieve a G_S of 2.6dB or higher, the source reflection coefficient should be placed on or inside this circle. The point s_{11}* is indicated as a (green) dot inside this circle and corresponds to the maximum value of G_S. A noise circle (red circle on mid L.H.S. of graph) is drawn for F=3.1dB and indicates that to achieve a noise figure of 3.1dB or higher, the source reflection coefficient should be placed on or inside this circle. The point Γ_{OPT} is indicated as a (red) dot inside this circle and corresponds to the lowest noise figure, F_{MIN}. The noise and gain circles directly illustrate a typical trade-off in LNA design because it is very rare that the quantities s_{11}* and Γ_{OPT} have the same value and thus the source reflection coefficient that leads to maximum gain will nearly always be different from the source coefficient that leads to minimum noise figure. The source reflection coefficient will thus have to be chosen as a compromise between achieving highest gain and the lowest noise. For the example in Figure 3, it is seen that if a noise figure of less than 3.1dB is required, then Γ_L will have to be placed *inside* the noise circle shown – but this will be *outside* the source gain circle shown and therefore the source gain will be less than 2.6dB. It is thus apparent that for this transistor improving one target (gain or noise figure) will degrade the other and so a trade-off is needed. The user can interactively change the gain and noise figures for which the circles are drawn and thus determine the best compromise between the gain and noise performance. Additionally, the spreadsheet displays the numerical values for the centres and radii of these circles so that the results of hand calculations can be verified.

For maximum flexibility in setting the input and output reflection coefficients, the user can choose from four options for entering these values as illustrated in figure 4. The value of the source reflection coefficient can be set to s_{11}* (to maximise the source gain term), it can be set to the point on the chosen source gain circle closest to the origin of the Smith Chart (common for some matching strategies), it can be set to a value entered manually (in magnitude and phase form) or, finally, it can be set by using sliders. The slider Form Control is one of the best options for building interactivity into an Excel spreadsheet. Each time the slider is moved, the user can see the position of Γ_S moving on the Smith Chart (Γ_S is indicated by a crosshair on the Smith Chart) and the formulas for gain and noise figure are updated, thus providing instant feedback on the effect of Γ_S .

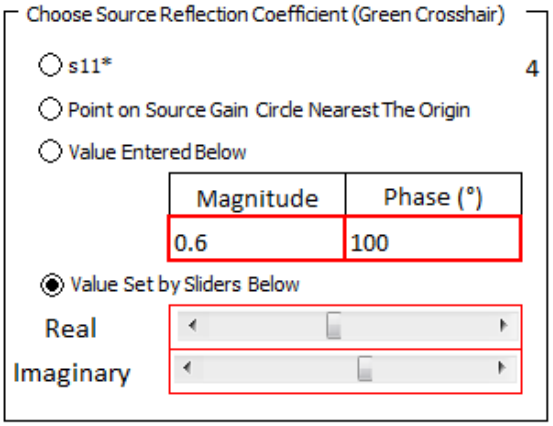


Figure 4: Available Options for Setting Γ_S.

A similar range of options are available to the user to set the output reflection coefficient, Γ_L, providing instant feedback on its effect as well.

VI MATCHING CIRCUIT DESIGN

Once values for Γ_S and Γ_L have been determined that meet the design requirements, it is necessary to design matching networks to transform the reflection coefficients of the actual source and loads (the input and outputs) to these desired values. A stand-alone LNA will usually be driven by a signal source with an impedance equal to the system impedance, Z_0, and will, in turn, drive at its output a load whose impedance will also equal the system impedance, Z_0. In this case, both source and load reflection coefficients (Γ_S, Γ_L) will be zero (corresponding to the centre of the Smith Chart) and will need to be transformed to the optimum values for gain and noise determined earlier. This transformation can be achieved by placing matching networks between the source and load and the transistor as shown in figure 5. These matching networks only use reactive components (capacitors and inductors) so as not to dissipate power or add extra noise [9].

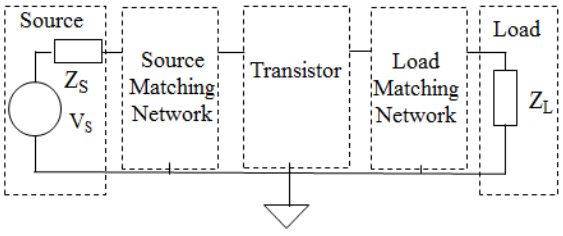


Figure 5: LNA with Source and Load Matching Networks.

The simplest form of matching networks are two-element networks consisting of a series connected component followed by a shunt (parallel)

connected component or a shunt (parallel) connected component followed by a series connected component [9]. In most cases (though not always) if one component is a capacitor (C) then the other component is an inductor (L) and vice versa. These simple two-element matching networks are often referred to as "L-networks" because of the arrangement of the passive components. Figure 6 illustrates a typical matching network placed between the signal source and the input of the transistor where it is seen that the arrangement of the inductor and capacitor looks like an "L" shape.

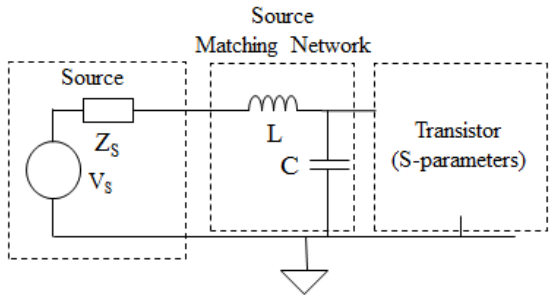


Figure 6: Example 2-element Source Matching Network.

The tool developed here allows the user to select one of four matching networks and then determines the component values needed to transform the source and load reflection coefficients (assumed to be zero corresponding to an impedance of Z_0) to the values determined earlier for the optimum design. Figure 7 illustrates the options available to the user for the case of the source matching network. A similar set of options is available for the load matching network.

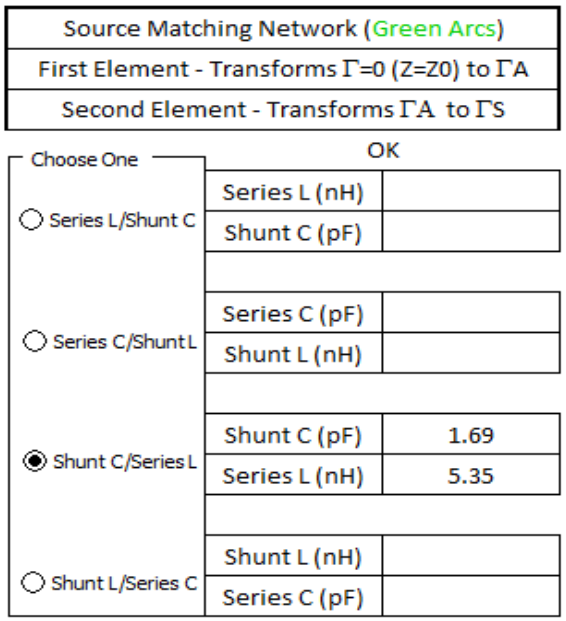


Figure 7: Available Options for Source Matching Network.

If the source and load network topologies chosen by the user are physically possible the tool will display "OK" and display the element values as illustrated in figure 7. If the chosen topology does not yield a physically valid solution (i.e. roots do not exist for the underlying quadratic equation [9]), the message "Chosen Option Not Possible" is displayed and the user can try another topology.

An example of a completed matching design is shown in figure 8. This shows the same source, load and noise circles as illustrated in figure 3 but also shows the matching network design. Two small cross-hairs (crosses) are also shown on the figure. One of these (blue) on the R.H.S. of the chart shows the value of Γ_L and the other one (green) on the L.H.S. of the chart shows the value of Γ_S chosen to meet the design requirements for gain and noise.

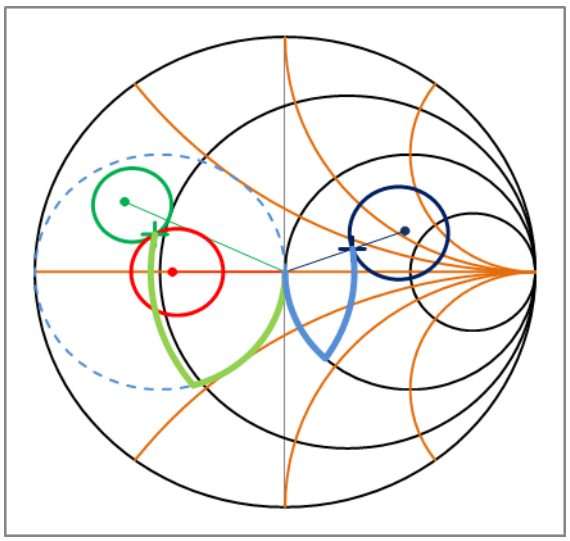


Figure 8: Matching Element Design on the Smith Chart.

Figure 8 also shows two approximate "V" shapes. These illustrate how the source and load reflection coefficients are transformed from the origin to either Γ_S or Γ_L. Each "V" is in fact made up of two arcs, each of which shows how the reflection coefficients are transformed by adding the individual shunt and series components. The spreadsheet also displays the impedance (resistance and reactance) and admittance (conductance and susceptance) at the nodes of the "V" shapes so that the user can check these against values determined using a paper version of the Smith Chart, thus allowing the manual procedure (as would be used in an exam) to be quickly verified. (Note there is nothing special about the "V" shapes – they just correspond to the matching transformations chosen for this example). Finally, figure 8 also includes an additional circle (dashed blue) on the L.H.S. of the Smith Chart. This is circle corresponds to the reflection coefficients that would be traced out by parallel connected R-L and R-C network where R is set to the system impedance, Z_0, and frequency is varied from 0 to ∞. This is the so-called "unit

conductance" circle and is used during the matching circuit design.

VII STABILITY

Another consideration when designing matching networks for LNAs is the issue of stability. Depending on the transistor s-parameters, the LNA may become unstable for certain source and load reflection coefficients and these must be avoided for stable operation. Stability can be determined by evaluating the Rollet Stability Factor [7,8] and regions of potential instability can be seen by plotting source and load stability circles on the Smith Chart. These circles and the associated calculations are also displayed by the spreadsheet as shown in figure 9.

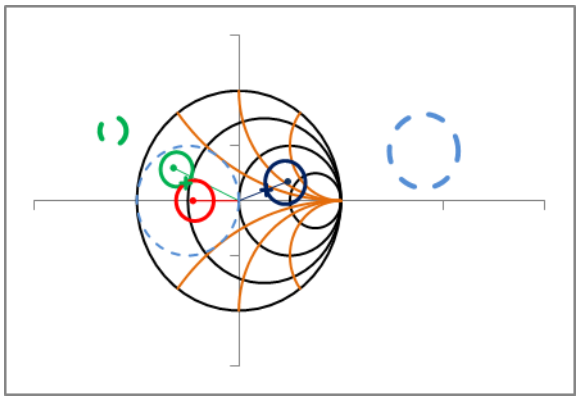


Figure 9: Stability Circles on the Smith Chart.

Figure 9 shows an expanded view of the complex plane with two extra circles in addition to the conventional Smith Chart. These are the source gain circle (dashed green on the L.H.S.) and load gain circle (dashed blue on the R.H.S.). In both cases, there circles do not encroach on the unit circle of the Smith Chart indicating that the LNA configuration is unconditionally stable for this particular example.

VIII SUMMARY

This paper has outlined an Excel-based educational tool that provides an insight into single-stage LNA design procedures. It allows the user to interactively study the impact of source and load reflection coefficients on gain, noise and stability performance and offers a range of choices for matching network design. The values of all quantities that would have to be determined when doing a design manually, in conjunction with a paper version of the Smith Chart, are shown so that the user can check his or her use of the manual procedures to verify that a full understanding of the procedure has been mastered. The paper also shows how the widely available spreadsheet program Excel contains a range of options for creating interactivity and is thus a convenient widely-available platform for demonstrating basic concepts in a range of educational fields, such as Radio Frequency LNA design in this instance.

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