Editorial: Conceptual Climate Modelling

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Abstract

Modelling the climate is notoriously difficult and generally associated with high-dimensional general circulation models that may be quite unwieldy from the mathematical perspective. At the other end of the spectrum are seemingly simple conceptual models that focus on underlying mechanisms, such as the roles of different types of delayed feedback loops and/or switching phenomena for a specific climate phenomenon. This special issue is designed to highlight the usefulness of conceptual modelling in climate. It presents a number of conceptual climate models, discusses the mathematical techniques available for their analysis, and showcases how relevant insights can be gained from them, including informing more realistic modelling of the climate.

Climate models can be characterised in terms of their complexity: from high-resolution general circulation models, via intermediate models that enable longer timescale or large-ensemble studies, to conceptual models that focus on specific mechanisms. In this hierarchy, conceptual models are most amenable to mathematical analysis, especially with advanced techniques from dynamical systems. This makes it possible in many instances to study in detail how different constituent effects, such as different types of feedback loops, periodic or stochastic forcing of different kinds and/or switching phenomena, interact under different conditions to produce the overall behaviour under investigation.

The goal of this special issue is to provide a kaleidoscopic view of some recent developments in the related areas of conceptual modelling of climate phenomena, the development of associated analysis tools, and case studies of specific climate models. While they present only a snapshot, we hope that the contributions collected here will give a useful overview of the present state of the field with the secondary aim of stimulating future research.

As a starting point, Dijkstra provides an introduction to conceptual modelling in climate research, whose focus and leading example is the Atlantic Meridional Overturning Circulation (AMOC) [1]. This review shows that conceptual AMOC models are a necessary part of the modelling hierarchy, where they help ensure that key physical concepts can be identified

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and correctly interpreted in more detailed and complicated models. A number of contributions follow that provide new insights into the nature of the AMOC and its possible tipping. Bailie and Krauskopf study a box model due to Welander for the vertical mixing in the North Atlantic, which is a key part of the AMOC [2]. This model features weak and strong mixing regimes driven by density differences between the top layer and cold deep ocean. A thorough bifurcation analysis determines the observable behaviour as a function of freshwater influx and density threshold, which includes bistability and self-sustained oscillations between the two types of mixing with the deep ocean. Neff *et al.* introduce a box model that captures the deepwater convection of the AMOC at two distinct sites in the subpolar region of the North Atlantic Ocean [3]. They show by means of bifurcation analysis that the tipping of AMOC will involve a 'partially shutdown' state with deepwater convection only occurring at one site, which is in agreement with simulations of AMOC tipping in more sophisticated models. The dynamics of a non-smooth fold bifurcation that is typically found in ocean box models is analysed by Budd and Kuske [4]. They show how the non-smoothness effects the behaviour of tipping from one state to another in both cases of non-forced and periodically forced systems.

Mehling *et al.* consider the limitations to predicting the tipping of AMOC by studying the dynamics of an ocean box model that is coupled to a chaotic atmosphere [5]. The authors quantify predictability of the final state by analysing the resulting chaotic attractors, transients and saddles of the system. Moreover, they highlight that further work on defining a safe operating space is needed to avoid overshooting an AMOC tipping point. Results on the predictability and structural stability of general chaotic systems, which reflect important properties of atmospheric states, are presented by Maiocchi *et al.* [6]. Specifically, the authors calculate Lyapunov exponents and detect underlying unstable periodic orbits of the Lorenz 96 chaotic attractor, and they highlight vastly different instability properties.

Geogdzhayev *et al.* take the classic Lorenz model, now with a time-varying parameter, as an abstract model for addressing fundamental questions about how weather patterns respond to slow smooth changes [7]. With a Markov chain-based methodology, the authors show that the statistics of ensemble averages compare well to those of temporal averages. Atmospheric dynamics is also studied by Kobras *et al.* in the form of a minimal dynamical system derived from the classical Phillips two-level model [8]. They study the link between the structure of eddies and the position of the jet stream, and give new insights into the properties of the storm track in the mid-latitude atmosphere. Slyman *et al.* study a conceptual model for tropical cyclones and show that bifurcation-induced, rate-induced, and noise-induced tipping can each provide mechanisms for transitioning between storm and non-storm states [9].

An energy balance model with orbital forcing is studied by Widiasih *et al.* to investigate the dynamics of glacial cycles [10]. When a bifurcation parameter is tuned to paleoclimate records, the model exhibits behaviour similar to the Mid-Pleistocene Transition, and a bifurcation analysis reveals that the underlying mechanism relies on shifts between regions of frequency locking. Goodwin and Williams study another energy balance model, which incorporates explicit cloud representation for shortwave and longwave radiation [11]. Their analysis suggests that the hemispheric asymmetry in global warming, known as Arctic Amplification, may be due to relatively simple thermodynamic and radiative processes, as well as upwelling in the Southern Ocean. Consolo *et al.* introduce a three-compartment biomass-water-toxicity extension of the Klausmeier model to study vegetation patterns emerging on sloped terrains in the presence of toxic compounds, including a transition from a uniformly-vegetated area to desert via non-stationary vegetation stripes [12]. Their analysis reveals how the combined effect of mean annual rainfall, plant mortality and plant sensitivity to toxicity leads to different ecological scenarios.

A number of papers in this special issue focus on developing new modelling techniques and reduction methods for creating conceptual climate models. A brief review of reduction methods in climate dynamics by Hummel *et al.* provides the main mathematical ideas and benchmark examples across a broad range of available methods [13]. Chekroun and Liu employ a combination of centre-unstable manifold reduction and Galerkin-Koornwinder approximations to derive a simple delay differential equation model for the El Niño Southern Oscillation (ENSO) [14]. A bifurcation analysis reveals how tipping solution paths through a saddle-node bifurcation of periodic orbits provide a mechanism to reproduce ENSO-like behaviour. Chen and Zhang introduce a mathematical framework for deriving conceptual ENSO models from a spatially-extended stochastic dynamical system [15]. The resulting models display outputs for the atmosphere, ocean, and sea surface temperature components of the system with detailed spatiotemporal patterns that reproduce large-scale dynamical and non-Gaussian statistical features of eastern Pacific El Niño events and, furthermore, allow for a clearer understanding of the role of stochastic noise in ENSO dynamics. The role of stochastic noise is consider by Kypke and Ditlevsen in the context of the paleoclimate phenomenon of Dansgaard-Oeschger events [16]. They derive data-driven models and show that the stability of the interstadial state depends on whether the Itô or the Stratonovich interpretation of the stochastic integral is applied. This work highlights the need for physical constraints when constructing conceptual stochastic models from climate data. Misra and Jha formulate a nonlinear mathematical model to investigate the strategy of planting leafy trees during development activities, such as building roads and housing complexes, in order to mitigate the resulting elevations in atmospheric carbon dioxide (CO_2) levels [17]. The analysis reveals bifurcations in relation to the deforestation rate, as well as a critical proportion of cleared land that should be utilised for leafy trees in order to maintain the CO_2 level.

We hope that this collection of recent work will provide an overview of conceptual climate modelling in all its facets, and will stimulate future work in this important and rapidly developing area.

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