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Routing quantum information in spin chains

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Two different models are presented that allow for efficiently performing routing of a quantum state. Both cases involve an XX spin chain working as a data bus and additional spins that play the role of sender and receivers, one of which is selected to be the target of the quantum state transmission protocol via a coherent quantum coupling mechanism making use of local and/or global magnetic fields. Quantum routing is achieved in the first of the models considered by weakly coupling the sender and the receiver to the data bus. On the other hand, in the second model, local magnetic fields acting on additional spins located between the sender and receiver and the data bus allow us to perform high-fidelity routing.

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I. INTRODUCTION

The development of quantum technologies relies on the ability to establish correlations between distant parties [1]. Whereas, photons are ideal carriers of quantum information in free space [2] since they interact weakly with the external environment, solid-state devices are probably more suitable for quantum communication within a computer.

In particular, spin chains with nearest-neighbor interaction offer a wide range of solutions for quantum state transfer (QST) protocols [3,4]. Apart from their simple theoretical description, they can be efficiently implemented in arrays of trapped ions [5–7] or by using cold atoms in optical lattices [8–10] where single spin addressing has recently been reported [11].

Since, in QST protocols, the initial state is usually confined to a small region of space, its transmission through a long unmodulated chain will inevitably involve all of the modes of the chain itself. As a consequence, state reconstruction in a different spatial location will be affected by the detrimental dispersion the spin wave packet is subjected to. Various proposals have been made to overcome this drawback. In Ref. [12], the authors suggested using engineered spin-spin coupling and found a way to obtain perfect QST independent of the chain length. However, such an implementation would require a high degree of control of the internal structure of the system, which is not desirable from the experimental point of view. Alternative methods are based on the use of trapped topological fields [13], on the extension of the encoding to more than one site [14], and on the use of strong dynamically switched on interactions between the sender and the receiver with the bus [15].

One of the more explored solutions consists of weakly coupling the sender and the receiver to the bulk chain. Roughly speaking, the resulting QST takes place in two distinct regimes: For very weak coupling, the bulk chain behaves merely like an information bus without being appreciably populated, and the probability amplitude of finding the excitation undergoes an effective Rabi oscillation between the sender and the receiver [16–20]; whereas, for nonperturbative end-point couplings, the relevant modes taking part in the quantum state dynamics reside mainly in the linear zone of the spectrum, thus, minimizing the effect of dispersion so that QST occurs in the so-called ballistic regime [21–23].

A step beyond QST is represented by the possibility of routing information from one sender to many possible receivers with minimal control of the system; that is, without modifying any of the spin-spin coupling parameters of the Hamiltonian. Achieving this goal would clearly increase the degree of connectivity of a spin bus by allowing the possibility to couple the quantum node of a spin network to many receivers.

Despite the large number of papers on QST involving one sender and one receiver, there are relatively few papers on quantum routing. Actually, a setup admitting QST from a sender to a single receiver may not be trivially extended to implement a routing scheme: By way of example, in Ref. [24], it is explicitly demonstrated that perfect quantum state routing is forbidden unless experimentally demanding operations or severe Hamiltonian engineering is performed. Even by relaxing the request of perfect QST, the problem still remains nontrivial, especially in the huge class of QST protocols based on mirror symmetry where a pivotal role is played by matrices being both persymmetric and centrosymmetric [23].

It is the aim of this paper to discuss the dynamical behavior of two coupling schemes that explicitly allow for an efficient routing to be performed.

Previous proposals in this direction were formulated by Zueco *et al.* in Ref. [25] and Bose *et al.* in Ref. [26]. In the former reference, the authors considered an XY chain

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in the presence of an external magnetic field harmonically oscillating in time and two possible receivers; whereas, in the latter, by exploiting the Aharonov-Bohm effect, high-fidelity three-party communication has been shown to be achievable. Routing between distant nodes in quantum networks has been proposed in Refs. [27,28] where perfect QST is investigated in a dual-channel quantum directional coupler and in a passive quantum network, respectively, and in Refs. [29,30] in the presence of local control of the network nodes. A scheme for routing entanglement has been also proposed in coupled two-impurity channel Kondo systems [31].

Here, instead, we propose two different quantum router protocols, which can be performed in XX spin chains and in which the local energies of the receiver do not need any control or manipulation during the whole process.

The paper is organized as follows. In Sec. II, we introduce the first routing scheme where the sender and receivers interact weakly with a spin ring and efficient QST is enabled by coupling resonantly the sender with a chosen receiver by means of a suitably chosen magnetic field. In Sec. III, the sender and receivers are not directly coupled to the spin bus but rather via effective "barrier qubits," on which strong magnetic fields act as knobs for the QST. In this latter scheme, a uniformly coupled spin chain is considered, thus, avoiding the need for bond control. Finally, in Sec. IV, conclusions are drawn, and future perspectives are discussed.

II. QUANTUM ROUTER VIA WEAK BONDS

Let us consider N spins embedded in an XX chain in the presence of a transverse field plus $n+1$ spins (one sender and n receivers) locally connected to the chain. A pictorial view of this model is given in Fig. 1. The total Hamiltonian, describing the chain, the sender and receivers, and their coupling, respectively, reads $H = H_C + H_I + H_{CI}$, where

$$H_C = -J\sum_{l=1}^{N}\left(\sigma_l^x\sigma_{l+1}^x + \sigma_l^y\sigma_{l+1}^y\right) - h\sum_{l=1}^{N}\sigma_l^z,$$

$$H_I = -h_S\sigma_S^z - \sum_{i=1}^{n} h_{R_i}\sigma_{R_i}^z,$$

$$H_{CI} = -\frac{g}{2}\left(\sigma_{l_S}^x\sigma_S^x + \sigma_{l_S}^y\sigma_S^y\right) - \frac{g}{2}\sum_{i=1}^{n}\left(\sigma_{l_{R_i}}^x\sigma_{R_i}^x + \sigma_{l_{R_i}}^y\sigma_{R_i}^y\right).$$

We have labeled the chain sites with $l = 1,2,\ldots,N$, whereas, S stands for the sender, and $\vec{R} \equiv \{R_1, R_2, \ldots, R_n\}$ identifies the location of the n receivers. The chain site l_S is coupled

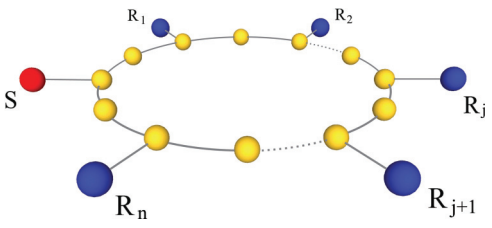


FIG. 1. (Color online) Schematic of the quantum router. The sender and the receivers are coupled to a common chain that acts as a quantum data bus.

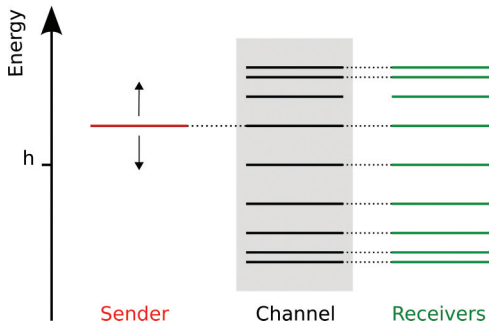


FIG. 2. (Color online) Energy-level scheme. Each of the receivers is in resonance with one of the (pairs of) levels of the discrete band. By locally tuning the sender energy, it is possible to select the desired receiver. The spectral separation determines a bound for the maximum number of receivers.

to the sender, whereas, the ith receiver is coupled to the site l_{R_i}. Boundary conditions are imposed by assuming $\sigma_{N+1}^{\alpha} = \sigma_1^{\alpha}$ $(\alpha = x,y,z)$. In the single excitation subspace, which will be used henceforth, an exact mapping can be performed by relating spin operators to fermion annihilation and creation operators. The mapping consists of $\sigma_i^- \to c_i^\dagger$, $\sigma_i^+ \to c_i$, and $\sigma_i^z \to 1 - 2c_i^\dagger c_i$. By applying the Fourier transform to the chain operators, we obtain

$$H = \sum_k \epsilon_k c_k^\dagger c_k - h_S(1 - 2c_S^\dagger c_S) - \sum_{i=1}^{n} h_{R_i}(1 - 2c_{R_i}^\dagger c_{R_i}) - \frac{g}{\sqrt{N}}\sum_k\left[c_k^\dagger\left(e^{ikl_S}c_S + \sum_{i=1}^{n} e^{ikl_{R_i}}c_{R_i}\right) + \text{H.c.}\right], \quad (1)$$

where $k = 2\pi q/(Na)$, a being the lattice constant and q being an integer number, $\epsilon_k = -2h - 4J\cos(ka)$, and

$$c_k = \frac{1}{\sqrt{N}}\sum_{l=1}^{N} c_l e^{ikl}. \quad (2)$$

Without loss of generality, we will assume a and $4J$, respectively, as the units of length and energy ($\hbar$ is, as usual, the unit of action).

The goal of a QST protocol is to act over an initial state encoded in the spin at the sender site with both the set of the receivers and the channel aligned in state $|\phi_{in}\rangle = (\alpha|0\rangle + \beta|1\rangle)_S|0\rangle_R^{\otimes n}|0\rangle_C^{\otimes N}$, and, by exploiting the dynamical evolution for a definite transfer time, transform it into $|\phi_{out}\rangle = |0\rangle_S(\alpha|0\rangle + \beta|1\rangle)_{R_j}|0\rangle_{\bar{R}_j}^{\otimes n-1}|0\rangle_C^{\otimes N}$, where $\bar{R}_j$ is the register of the $n-1$ receivers complementary to j. Since $|0\rangle_S|0\rangle_R^{\otimes n}|0\rangle_C^{\otimes N}$ is an eigenstate of H, it will be enough for our purpose to study the conditions under which $|1\rangle_S|0\rangle_R^{\otimes n}|0\rangle_C^{\otimes N}$ evolves into $|0\rangle_S|1\rangle_{R_j}|0\rangle_{\bar{R}_j}^{\otimes n-1}|0\rangle_C^{\otimes N}$ or, in the language of fermion excitation introduced before, we want to know if there exists a time t^* such that $c_S^\dagger(t^*)|0\rangle \simeq c_{R_j}^\dagger|0\rangle$. In order to get a full characterization of the QST, one should evaluate a fidelity averaged over all of the possible initial states (that is, over all the possible combinations of α and β such that $|\alpha|^2 + |\beta|^2 = 1$). It has been shown in Ref. [3] that this average fidelity only depends on the transition amplitude f_{R_jS} of an excitation from the sender to the jth receiver, through the relation

$\bar{F} = \frac{1}{2} + \frac{|f_{R_j s}|}{3} + \frac{|f_{R_j s}|^2}{6}$. As a result, for a generic R_j, the average fidelity is a monotonous function of the transition probability $F_{R_j}(t) = |f_{R_j S}|^2 \equiv |\langle 0|c_{R_j} c_s^\dagger(t)|0\rangle|^2$, and $\bar{F}$ reaches unity only for $F_{R_j}(t) = 1$. We can, therefore, consider the behavior of F_{R_j} itself and, as we want to route the information to many receivers, the protocol should be able to guarantee the highest possible value for this probability independent of the selected receiver's location.

The dynamical problem is completely specified by the following set of coupled equations for the $N + n + 1$ variables:

$$\dot{c}_S^\dagger = -i\Omega_S c_S^\dagger + i\frac{g}{\sqrt{N}}\sum_k c_k^\dagger, \tag{3}$$

$$\dot{c}_{R_i}^\dagger = -i\Omega_{R_i} c_{R_i}^\dagger + i\frac{g}{\sqrt{N}}\sum_k e^{-ikl_{R_i}} c_k^\dagger, \tag{4}$$

$$\dot{c}_k^\dagger = -i\epsilon_k c_k^\dagger + i\frac{g}{\sqrt{N}}\left(c_S^\dagger + \sum_{i=1}^{n} c_{R_i}^\dagger e^{ikl_{R_i}}\right), \tag{5}$$

where we have used the notation $c_j^\dagger = c_j^\dagger(t=0)$, $\Omega_j = -2h_j$ $(j = k, S, R_i)$ and have assumed that $l_S = 0$. As discussed in Ref. [18], in the weak-coupling limit, a solution can be worked out in the Laplace space and then can be brought back to the time domain.

In the following, we will describe how to obtain an efficient routing within the model described so far.

A. Chain-receivers resonance

In the scheme we are proposing, we exploit the resonance between the local energy at the receiver site and one of the modes of the chain in order to achieve the transfer. An efficient routing protocol, then, requires that we are able to resolve the different levels of the energy spectrum. To this aim, we must consider a finite-size system with a number of sites N limited by the minimal relevant energy separation that one is able to resolve.

To better illustrate our idea, we start by considering the ideal case of a channel where all the energy levels are well separated and resolved. The sender and the receivers are coupled to different sites of the channel by a hopping term whose strength we assume weak with respect to the intrachannel one. The local energy of every receiver can be made resonant with a different mode of the channel. In our specific case, since the channel levels are twofold degenerate, with the exception of the $k = 0$ and $k = \pi/2$ modes, the number of receivers can be, at most, $N/2 + 1$. In the presence of a small hopping constant between receiver R_j and chain site j, the degeneracy is resolved, and the resonant states are split into two new levels separated by an energy amount δ. The weak-coupling condition holds when the splitting is smaller than the original energy separation in the chain.

Roughly speaking, the dispersion ϵ_k can be divided into a parabolic region at the bottom and at the top of the energy band and a linear region in the middle of the band. In the parabolic region, the energy separation is on the order of $\Delta_p \simeq \pi^2/(2N^2)$, whereas, in the linear region of the band, $\Delta_l \simeq 2\pi/N$. Therefore, weak-coupling conditions are fulfilled whenever $\delta \ll \Delta_p$. In this way, every receiver is coupled to the channel only via its resonant mode, whereas, transitions via the other modes can be neglected. The energies of the channel and the receivers are fixed, whereas, the sender can tune its energy. The sender selects the receiver R_j to send the state to by tuning its energy Ω_S to Ω_{R_j} (see Fig. 2). In this way, the system behaves as an effective model in which only the sender, the receiver, and the resonant modes of the channel are involved in the dynamics. As pointed out in Ref. [18], for a channel with an odd number of sites, destructive interference occurs, and the excitation only oscillates between sender and channel, without arriving at the receiver. So, we will restrict ourselves to consider the case of an even number of sites N. Moreover, in order to achieve efficient state transfer, the receiver has to be coupled to a site with an even position label.

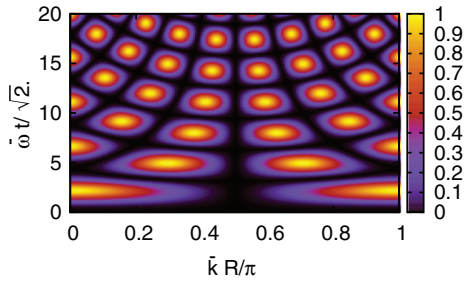


FIG. 3. (Color online) Transition probability as a function of time (scaled with $\bar{\omega}$) and $\bar{k}R$. Integer values of $\bar{k}R/\pi$ guarantee high quality transfer after shorter times [yellow (light) spots in the density plot].

Following the calculation of Ref. [18], a weak-coupling expansion in g can be performed to solve the system of Eqs. (3)–(5). When $\Omega = \Omega_S = \Omega_{R_j}$ is chosen to be resonant with two modes $\pm\bar{k}$ of the channel, because of the interaction, these four degenerate levels are split into $\Omega \pm \delta_\pm$, where

$$\delta_\pm \simeq \frac{\bar{\omega}}{\sqrt{2}}\sqrt{1 \pm \cos\,\bar{k}R_j}, \tag{6}$$

and where $\bar{\omega} = 2g/\sqrt{N}$. The transition probability for the receiver R_j, then, has the form

$$F_{R_j}(t) \simeq \tfrac{1}{4}(\cos\,\delta_+ t - \cos\,\delta_- t)^2. \tag{7}$$

As mentioned before, high-fidelity QST is achieved if, for a certain time, F_{R_j} approaches 1. Since it depends on the product $\bar{k}R_j$, then, it is clear that, for every position around the chain, there is an optimal energy in the band spectrum. In Fig. 3, the transition probability (7) is plotted. It appears evident that the choice $\bar{k}R_j = \pi s$, with s as an integer, is always optimal since the time F_{R_j} takes to reach its maximum is shorter.

As shown in Fig. 3, the maximum can also be achieved for different choices of $\bar{k}R_j$ but only after longer times. This means that decoherence effects, caused by the unavoidable presence of some external environment, are more likely to come out. As a consequence of this environmental intrusion, the quality of the routing protocol can be seriously affected.

For $\bar{k} = \pm\pi/2$, which corresponds to the linear part of the dispersion, Eq. (7) reduces to

$$F_{\rm lin}(t) \simeq \sin^4\left(\frac{gt}{\sqrt{N}}\right). \tag{8}$$

This case corresponds to the most efficient configuration since the energy separation with the closest levels is the highest.

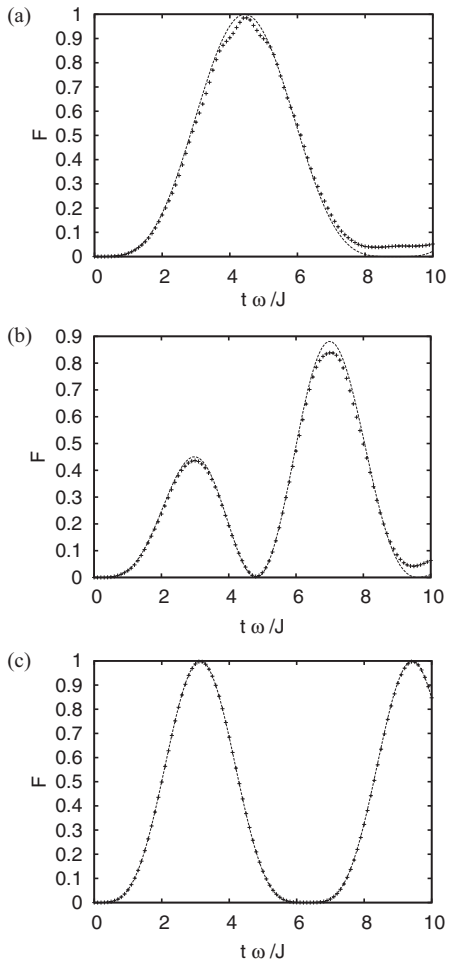


FIG. 4. Comparison of the exact transition probability F (dashed line) with the analytic result obtained by applying the weak-coupling approximation (continuous line) for $N = 16$ and $g = 0.01$. (a) $\bar{k} = 0$ and $R = 12$; (b) $\bar{k} = 7\pi/4$ and $R = 10$; (c) $\bar{k} = \pi/2$ and $R = 4$.

Moreover, the value of F is independent both of the receiver's position and of the time for reaching the maximum. Another important case is given by the resonance with the modes $\bar{k} = 0$ or $\bar{k} = \pi$ where the dispersion is quadratic. This case cannot be deduced as a limit of Eq. (7) because these two modes are not degenerate. Following the procedure of Ref. [18], one obtains

$$F_{\text{quad}}(t) \simeq \sin^4\left(\frac{gt}{\sqrt{2N}}\right). \tag{9}$$

In this case too, F does not depend on R_j, but here, the energy separation is smaller, and other levels could couple to the dynamics, making the perturbative approximation less accurate.

In Fig. 4, the exact numerical evaluation of the transition probability (calculated for a chain of $N = 16$ sites, assuming $g = 10^{-2}$) is compared with the analytical expressions given in Eqs. (7)–(9). The best efficiency is achieved for those receivers that are resonant with the linear and quadratic parts of the dispersion, whereas, F is reduced in the intermediate cases.

The need to be resonant with a mode which lies in one region or another of the spectrum of the chain, by itself, introduces an inhomogeneity among the receivers. Moreover, the energy separation in the quadratic part of the dispersion decreases very rapidly with the number of sites, posing a limit to the length of the channel. In principle, this problem could be overcome by decreasing the coupling g, but this would imply longer transmission times, and the decoherence effect would start to be relevant.

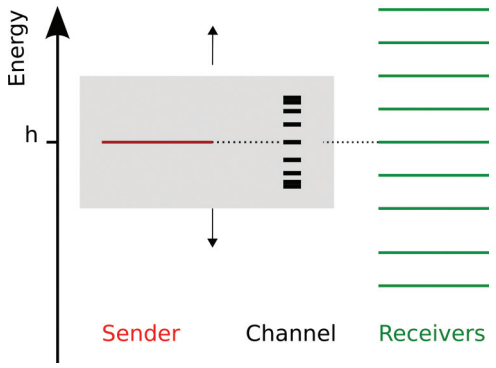


FIG. 5. (Color online) Modified scheme of levels. The receivers are well separated in energy ($\Delta_R > 2\pi/N$) by tuning the sender and by translating the whole chain spectrum through a uniform external magnetic field.

B. Equally spaced energies

In order to increase the possible number of receivers and to get an equivalent fidelity for each of them, a different energy configuration can be considered. As sketched in Fig. 5, let us assume the sender to always be resonant with the mode $\bar{k} = \pi/2$ in the linear dispersion region so that the dispersion of the channel is $\Omega_S - \cos\bar{k}$. Let us also assume that the energies of the receivers do not match the band levels but are separated by $\Delta_R > 2\pi/N$.

A receiver R_j is selected by tuning $\Omega_S = \Omega_{R_j}$ (and by changing, accordingly, the field on the chain). Because of the validity of the weak-coupling approximation, all of the other receivers are not involved in the process. Given that we are working in the linear dispersion region, the transition probability is given by Eq. (8).

This improves the previous scheme since the effects of the quadratic part of the band are now corrected, and the fidelity is the same for every receiver.

Since the typical energy separation is now the one in the middle of the band, for a fixed value of g, longer chains can be employed, and a larger number of receivers can be included. However, for this scheme to work, it is not sufficient to act only on the sender anymore, but a global control over the chain is necessary. This could be obtained by applying a global magnetic field that has the effect of translating the whole band spectrum by the desired amount. As in the former proposal, no control over the receivers is needed.

C. Off-resonance

Finally, we just mention a third possible scheme consisting of an almost continuous channel with a weak out-of-resonant coupling with the sender and the receivers [18,32]. In this scheme, the off-resonant continuous channel creates an effective coupling between sender and receiver tuned at the same

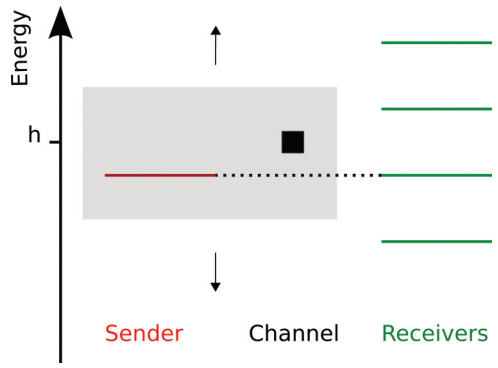


FIG. 6. (Color online) The sender and one of the receivers are in resonance, whereas, the continuous (as seen from outside) channel is detuned. The effective two-body frequency oscillation associated with this scheme is lower than the other schemes proposed in this section.

energy (Fig. 6). The setup is similar to the previous one, but now $h = \Omega_S \pm \nu$ with $\nu > 1$. This condition ensures that the sender is not resonant with the channel. As for the receivers, the energy separation condition becomes $\Delta_R > 2$. This scheme allows for a high-fidelity transfer over longer distances since, at least, within the limits of validity of the weak-coupling approximation, the system undergoes an effective two-level oscillation between sender and receiver, whereas, the chain, which is never populated, acts as a mere connector. As a drawback, longer times are required to accomplish the protocol and, as in the chain-receiver resonance case, environmental decoherence effects are more likely to affect the quality of the protocol.

III. QUANTUM ROUTER VIA LOCAL FIELD BARRIER

In the model presented in Sec. II, we were supposed to be able to reduce the coupling between the chain and the sender-receiver sites. In this section, we propose an alternative configuration where the hopping is assumed to be equal between all the spins, and routing is performed by tuning the local magnetic field acting on the spin adjacent to the sender. As there is no need to operate on the sender and/or the receiver couplings, this may result in a simpler implementation depending on the experimental setup.

Let us consider a linear XX chain composed by N sites plus $n+1$ pairs of spins. Following the notation in Sec. II, we can write the total Hamiltonian as $H = H_C + \sum_X H_{I_X} + \sum_X H_{CI_X}$, where

$$\begin{aligned} H_{I_X} &= -J\left(\sigma^x_{X_A}\sigma^x_{X_B} + \sigma^y_{X_A}\sigma^y_{X_B}\right) - h_X\sigma^z_{X_B}, \\ H_{CI_X} &= -J\left(\sigma^x_{l_X}\sigma^x_{X_B} + \sigma^y_{l_X}\sigma^y_{X_B}\right), \end{aligned} \tag{10}$$

and an open boundary condition of H_C are assumed. Here $X_{[A,B]}$ stands for the spins composing the sender block $S_{[A,B]}$ and the receiver blocks $R_{k[A,B]}$.

Each block is composed of a pair of spins: The first one, labeled A, acts as the effective sender (receiver), and the second one, labeled B, is connected with the site l_X belonging to the linear chain.

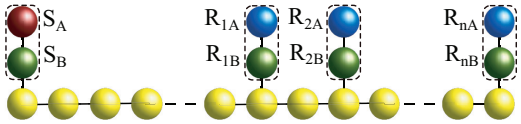


FIG. 7. (Color online) The model: A linear XX chain is used as a transmission channel between the sender S_A and one of the receivers R_{kA}.

We assume that magnetic fields with (different) intensities h_X act on the second site of each block. By controlling h_S, it is possible to confine the excitation on the sender or to perform QST from S_A to R_{kA} by choosing $h_S = h_{R_k}$. As specified in Ref. [33], for even chains, the optimal transfer time t^* is proportional to the square of the intensity of the magnetic field h_S and, for large enough h_S, it is also independent of the number of sites. Therefore, we will consider a configuration of the router in such a way that there are an even number of spins between the sender and each receiver, as depicted, e.g., in Fig. 7.

As in Sec. II, we assume the initial state to be prepared with all spins in the down state, $|\mathbf{0}\rangle = |0\rangle^{\otimes N+2(n+1)}$. Then, we prepare the sender site S_A in the state $|\psi_{in}\rangle = \alpha|0\rangle + \beta|1\rangle$ and let the complete system evolve according to Eq. (10). Because of the invariance of the subspace with a fixed number of flipped spins, the fidelity averaged over all possible initial states is, again, given by the expression $\bar{F} = \frac{1}{2} + \frac{|f_{RS}|}{3} + \frac{|f_{RS}|^2}{6}$ in Ref. [3], and the transition amplitude f_{RS} reads

$$f_{R_{jA},S_A}(t) = \sum_{k=1}^{N+2(n+1)} \langle \mathbf{R}_{jA}|\mathbf{a}_k\rangle\langle\mathbf{a}_k|\mathbf{S}_A\rangle e^{-i\lambda_k t}, \tag{11}$$

and λ_k, $|\mathbf{a}_k\rangle = \sum_{j=1}^{N+2(n+1)} a_{kj}|\mathbf{j}\rangle$, are, respectively, the eigenvalues and the corresponding eigenvectors of H written in the position basis $|\mathbf{j}\rangle = |0\cdots 01_j 0\cdots 0\rangle$ (with $\mathbf{j} = 1,\ldots,N,S_{[A,B]},R_{1[A,B]},\ldots,R_{n[A,B]}$) where the spin at the jth site has been flipped to the $|1\rangle$ state. In order to perform an efficient QST in the setting under scrutiny, it is necessary to achieve a modulus of the transmission amplitude between sites

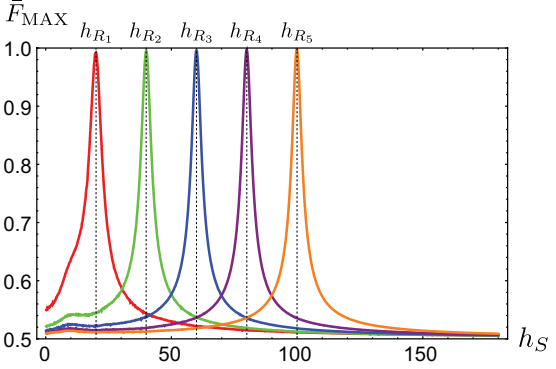


FIG. 8. (Color online) Maximum of the average fidelity in a fixed time interval $Jt < 5\times 10^4$ for a channel of $N = 30$ sites with $n = 5$ receiver blocks. By tuning the magnetic field h_S, respectively, to h_{R_k}, k=1, ...,5, we can perform a routing with high efficiency. The magnetic fields are ordered according to the position of the receivers $h_{R_{(k-1)}} < h_{R_k}$.

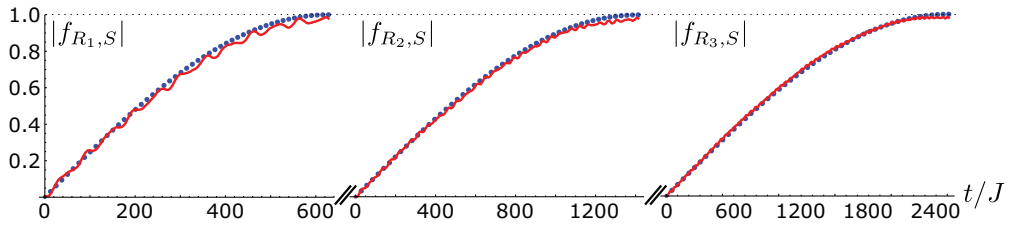


FIG. 9. (Color online) Comparison of the exact transition amplitude in Eq. (11) (red solid line) with the result of Eq. (12) (dotted blue line) obtained after a perturbative analysis for $h_X \gg J$ in a chain of $N = 20$ sites with $n = 3$ receiver blocks. (Time is given in units of $1/J$.)

S_A and R_{jA} as close as possible to 1 at a certain time t^*. The local field h_X exactly produces this result.

Indeed, the presence of h_X has two consequences: First, it causes the appearance of an eigenstate localized on the sites B of each block with energy much larger than that of the rest of the system; and, second, an effective weak coupling of the spin at sites A of each block to that at site l_X of the linear chain arises.

This can easily be seen by writing H_{CI_X} as

$$H_{CI_X} = -\frac{2J}{\omega_X^a - \omega_X^b}\left(\omega_X^a |\mathbf{I}_X\rangle\langle\psi_X^a| - \omega_X^b |\mathbf{I}_X\rangle\langle\psi_X^b| + \text{H.c.}\right),$$

where $\omega_X^{a,b}$ and $|\psi_X^{a,b}\rangle$ are the eigenvalues and eigenvectors of H_{I_X} after rescaling the ground-state energy,

$$\omega_X^a = -h_X + \sqrt{h_X^2 + 4J^2}; \qquad |\psi_X^a\rangle = \frac{\omega_X^b}{2J}|\mathbf{X}_A\rangle + |\mathbf{X}_B\rangle,$$

$$\omega_X^b = -h_X - \sqrt{h_X^2 + 4J^2}; \qquad |\psi_X^b\rangle = \frac{\omega_X^a}{2J}|\mathbf{X}_A\rangle + |\mathbf{X}_B\rangle.$$

In the limit $h_X \gg J$, the eigenstates $|\psi_X^a\rangle$ and $|\psi_X^b\rangle$ become $|\mathbf{X}_A\rangle$ and $|\mathbf{X}_B\rangle$, respectively, the scaling of their coupling to the chain's site behaves as

$$\frac{\omega_X^a}{\omega_X^a - \omega_X^b} \to \frac{J^2}{h_X^2} \quad \text{and} \quad \frac{\omega_X^b}{\omega_X^a - \omega_X^b} \to -\left(1 - \frac{J^2}{h_X^2}\right).$$

It follows that we can write

$$H_{CI_X} = -2J\left[\frac{J^2}{h_X^2}|\mathbf{I}_X\rangle\langle\mathbf{X}_A| + \left(1 - \frac{J^2}{h_X^2}\right)|\mathbf{I}_X\rangle\langle\mathbf{X}_B| + \text{H.c.}\right].$$

This implies that we can effectively consider the first spin of each block weakly coupled to the chain's spin with strength $\sim 1/h_X^2$; whereas, the second spin, still coupled with strength $\sim J$ to the chain, experiences the large magnetic field h_X of H_{I_X}, which freezes its dynamics.

As in the case of Sec. II C, when h_S is close to the energy of receiver block h_{R_k}, there exists a pair of eigenenergies outside the spectrum of the chain whose corresponding eigenstates are localized (symmetrically and antisymmetrically) on the B parts of each block involved in the transfer [18,34]. Moreover, in this case, we have the emergence of another quasidegenerate pair of eigenvalues (inside the energy band of H_C but out of resonance with any of its eigenvalues), whose corresponding eigenvectors have a non-negligible superposition with states $|\mathbf{S}_A\rangle$ and $|\mathbf{R}_{kA}\rangle$ so that they give rise to an effective Rabi-like oscillation mechanism of the spin excitation between the sender and the selected receiver site of the router. As a result, both the transition amplitude and the average fidelity become very close to unity at half the Rabi period.

In Fig. 8, we report the maximum of the average fidelity over all initial states within a fixed time interval $Jt < 5 \times 10^4$ for a channel of $N = 30$ sites with $n = 5$ receiver blocks: It is clearly shown that, by properly tuning the magnetic field h_S, one can perform a QST with high efficiency towards each of the targeted receiving sites.

As shown in Ref. [33], the transition amplitude between the sender and a receiver, connected by an even number of sites, is well approximated by

$$f_{R_{jA},S_A}(t) \approx \sin\left(\frac{J^3 t}{h_j^2}\right), \tag{12}$$

as checked in Fig. 9 against the numerical solution for a chain of $N = 20$ sites with $n = 3$ possible receiving blocks.

IV. CONCLUSIONS

The implementation of many quantum information protocols requires the transfer of a quantum state from an input to different output locations, and quantum routing has to be implemented in order to build a large network. Depending on the physical system used for this purpose, the control over many interaction parameters may be unfeasible, and it is necessary to study efficient routing protocols that require minimal engineering and external manipulation.

In this paper, we have presented two different possible implementations of a router that allows quantum state transfer from a sender to a chosen receiver by means of a resonant coupling mechanism. In the first scheme, the key ingredient is the weak coupling between the sender and receivers to the spin bus, and three different configurations of local and global magnetic fields are considered: (a) Every receiver energy is resonant with a different mode of the channel, and QST occurs by tuning the sender energy; (b) the energies of the receivers do not match the band levels but are equally spaced, and the sender is always resonant with a mode of the channel so that QST can be performed by translating the whole band spectrum by the desired amount of energy through the application of a global magnetic field; (c) a weak, out-of-resonance coupling between the chain and the sender and the receivers. In this scheme, the off-resonant continuous channel creates an effective coupling between the sender and the receiver, provided they are tuned to the same energy.

Finally, for the case in which the couplings between adjacent qubits are constrained to be equal, we have proposed a second model for the quantum routing protocol in which a linear chain is used as a data bus, and the single sender and

receiver spins are substituted by sender and receiver blocks made of pairs of spins. One of these two spins is effectively involved in the communication, whereas, the second (the barrier spin, effectively working as a gateway), is acted upon by a local field which plays the role of a knob that permits the QST. As a consequence of the use of strong local magnetic fields, an effective weak coupling is established either between sender and receiver and spin bus (in the resonant case) or between the sender and the receiver (in the off-resonant case). Moreover, the presence of the barrier spin makes it not necessary to act directly on the sender qubit, which is, therefore, involved only in the state encoding step, which may result in an experimental simplification.

In the resonance regime, the information transfer is due to collective degrees of freedom (i.e., the single-particle excitations of the spin chain), and therefore, to obtain a good transmission performance, it is necessary to be able to set the energy levels in a precise way. This kind of control can be achieved in the context of atomic Mott insulators where it has been shown experimentally that different lattice potentials can be tailored with high accuracy [11]. On the other hand, by working in the off-resonance regime, the precise shaping of the medium's energy level is unnecessary, and naturally occurring systems as well as separated nitrogen vacancy centers in diamond would represent a feasible experimental implementation [35].

Finally, since our theoretical treatment exploits a model Hamiltonian, which received much experimental attention in the past few years, it is definitely worthwhile to investigate routing implementations based on it. These could be further developed and improved in various ways; in particular, by allowing for the possibility of multiple sending sites, connected at will to a selected set of receivers, in order to perform multiple quantum state transfer over a single data bus.

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