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Is Einstein's equivalence principle valid for a quantum particle?

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Einstein's equivalence principle in classical physics is a rule stating that the effect of gravitation is locally equivalent to the acceleration of an observer. The principle determines the motion of test particles uniquely (modulo very broad general assumptions). We show that the same principle applied to a quantum particle described by a wave function on a Newtonian gravitational background determines its motion with a similar degree of uniqueness.

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In this paper we address one of the conceptual issues arising from the efforts to reconcile quantum theory with gravitation, the question of the status of the equivalence principle for quantum matter. In classical physics the Einstein equivalence principle is a rule making one-half of the universal interdependence of geometry and matter, namely the influence of geometry on matter, more specific. It states that the effect of gravitation is locally indistinguishable from the effects arising from the acceleration of the observer [1]. Put differently, gravitational effects may be locally ''transformed away'' by an appropriate choice of the reference system. This is the principle used by Einstein himself; some authors call it ''strong equivalence principle'' [2]. We adopt this version of the equivalence principle in this paper as we believe that it hits the heart of the matter, other ''equivalence principles'' (cf. below) being more accidental or secondary.

Within Einstein's gravitation theory one shows that the above equivalence principle implies (modulo some natural assumptions on the general nature of the equations of motion) that all test particles placed in this spacetime move along geodesics. This fact is often expressed in one of two ways: (1) that the motion of the particle is mass independent or (2) that the inertial mass of the particle is equal to its gravitational mass. These two statements are sometimes used interchangeably as the ''weak equivalence principle'' in the literature [2]. This use of terminology is rather confusing, as the two statements are logically independent. They happen to coincide in the context of classical general relativity, but may diverge in another setting. This is, as we shall see below, what happens in the quantum case. The quantum dynamics of a test particle following from the Einstein (''strong'') equivalence principle is uniquely determined by one mass parameter. Thus the dynamics is mass dependent [(1) not true], but there is no room for independent inertial and gravitational masses [in other words: the masses are equal, (2) true].

We turn now to this case. In the literature various opinions on the status of the equivalence principle in the quantum world are expressed [3], and various, sometimes rather far removed from the original geometrical notion of equivalence, ideas are proposed [4] (but see also the final discussion). We think, however, that the extension of the Einstein equivalence principle in the form stated above to the quantum case experiences no logical difficulty, at least in the setting in which it has often been considered. We feel, therefore, that it may be of interest to see the simplicity of its action in this setting. The setting referred to consists of a structureless particle described by a wave function on a gravitational background of the nonrelativistic spacetime (we use this term reluctantly: it is deeply rooted in physicists' jargon, but misleading; see below). This setting has been adopted by several authors addressing the issue of covariance or equivalence [4–6]. Within the path-integral formalism conclusions similar to ours were reached earlier for the Feynamn propagator of a structureless particle by De Bièvre [6]. Our derivation, however, needs less assumptions, refers directly to the wave function, and has the advantage of great simplicity, both conceptual and technical (see also the closing discussion).

The reason for choosing the nonrelativistic rather than Einsteinian spacetime is that we want to avoid the complications arising from creation and annihilation of particles and their quantum field-theoretical description, which has to replace (''first-quantized'') quantum mechanics in this case (there existing no consistent relativistic quantum mechanics). The adopted setting is, however, nontrivial enough and, in fact, contrary to the customary name, possesses a geometrical structure (Newton-Cartan) interpretable in physical terms as a relativity theory, but with Galilean rather than Lorentzian local inertial observers [7].

We can now state the main claim of this note. Consider a quantum particle described by a wave function ψ in a geometrical background with the Newton-Cartan structure. Assume that the probability density of the particle is a scalar field. Then the Einstein equivalence principle determines the motion of the particle. This motion is not mass independent, but the inertial and gravitational masses are necessarily equal

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(which is what one observes in experiment, see Ref. [8]; we shall return to the experimental aspect of the equivalence principle in the concluding discussion). The choice of the mass parameter is the only freedom of the equation. The equation itself, when written in an appropriate coordinate system, is nothing else but the usual Schrödinger equation with the Newtonian potential term. We move now on to the details.

We start by giving a brief account of the Newton-Cartan geometry. We shall not discuss the underlying axioms and the logical structure of this geometry, referring the reader to the existing literature [7], but rather we summarize the resulting structure in simple terms. The Newton-Cartan geometry is defined on a four-dimensional differential manifold. This manifold is equipped with an absolute time t defining the foliation of the spacetime by simultaneity hypersurfaces, a positive-definite metric on each of these hypersurfaces, and a covariant derivative (affine connection) compatible with these structures. However, as a metric on a hypersurface is a *form*, there is no unique way of embedding it in the four-dimensional manifold without additional structures. This is how a gauge freedom in the choice of a four-dimensional metric arises. Nevertheless, both the metric properties on the hypersurfaces of constant time as well as the compatible connection are unique (gauge independent). With respect to the thus defined connection the leaves of constant time are flat. The non-flatness of the geometry reflects only the way in which the leaves fit together to form the four-dimensional spacetime, and is encoded in one single scalar field ϕ. This field, however, is again non-uniquely determined by the connection, being subject to a gauge freedom. All this sounds rather more complicated than for a Lorentzian manifold of Einstein's theory of gravity, but now a great simplification comes. In the Newton-Cartan geometry there exists a class of privileged global coordinate systems, the so-called Galilean coordinates. One of the coordinates in each of these systems is always the time coordinate t up to a translation by a constant. The other will be denoted by x^i $(i=1,2,3)$. The space part of the coordinate basis is a Cartesian system with respect to the metric: $(\partial/\partial x^i)\cdot(\partial/\partial x^j)=\delta_{ij}$. Moreover, vectors $(\partial/\partial x^i)$ are parallel propagated by the connection, so the covariant derivative of $(\partial/\partial t)$ gives the only nontrivial characteristic of the connection, and must be expressible in terms of ϕ. In fact, with each choice of a Galilean system a natural gauge of the field ϕ is chosen by the formulas: $\nabla_\mu(\partial/\partial t)^\nu = \phi^\nu \nabla_\mu t$, where in the coordinate basis the vector field ϕ^ν is given by $\phi^0=0$, $\phi^i=(\partial\phi/\partial x^i)$. This fixes the choice of ϕ up to an addition of an arbitrary function of time. If (t,x^i) is a Galilean system, then (t',x'^i) is also a Galilean system if and only if it is related to (t,x^i) by a transformation of the form

$$t'=t+b, \quad \vec{x}'=R\vec{x}+\vec{a}(t), \tag{1}$$

where R is an orthogonal transformation and $\vec{a}(t)$ is an arbitrary time-dependent translation. Let us denote $(t,\vec{x})\equiv X$, $(t',\vec{x}')\equiv X'$ and let us write the transformation as $X'=rX$. The two fields ϕ and ϕ' correlated with the two systems are then related by the transformation $\phi'(X')=\phi(X) - \ddot{\vec{a}}(t)\cdot\vec{x}'+$ arbitrary function of time. We choose the function of time to be zero and write the transformation law in the form

$$\phi'(X)=\phi(r^{-1}X)-\ddot{\vec{a}}(t-b)\cdot\vec{x}. \tag{2}$$

The field ϕ is no longer a true scalar field, as with each of the two systems a different gauge has been fixed.

The Newton-Cartan spacetime is flat if and only if there exists a Galilean system in which ϕ is a function of time only (and may be chosen equal to zero). The same holds then in all those Galilean systems which are related by a transformation from the Galilean group $[\ddot{\vec{a}}(t)=0]$ to the first one (and, consequently, to each other). These special Galilean systems are called global inertial systems. If the spacetime is curved there are no global inertial systems, but, as is evident from the transformation law (2) and the form of the connection, for any chosen point of spacetime there exist Galilean systems in which connection vanishes in this point. The restrictions of these systems to a small neighborhood of this point are related to each other by Galilean transformations and are called local inertial systems. Both global (when they do exist) and local inertial systems have exactly the same physical interpretation in terms of special observers as in Einstein's theory.

The Newton-Cartan spacetime is the spacetime of the Newtonian world with gravitation. The field equation of the form: "the Ricci tensor of the connection$=0$" turns out to be identical in a Galilean system with the Laplace equation for ϕ, and the geodesic equation has in this system the form of the Newton's second law for a particle in the gravitational potential field ϕ. In parallel with the Einstein theory of gravity the geodesic law of motion of classical test particles may be obtained by the application of the Einstein equivalence principle. And here again the two minor equivalence principles are true in the classical case.

The existence of the global Galilean systems simplifies greatly the investigation of the covariance of equations. To see whether an equation has a geometrical, independent of the choice of coordinates, meaning, it is sufficient to check whether it has the same form in all Galilean systems. If it has, writing its coordinate independent form may pose some technical difficulties, but is possible. In what follows we use the Galilean systems only.

We are now prepared to place a quantum particle in this Newtonian geometry. We assume that it is described by a wave function $\psi(X)$, such that the correlated "probability density" $\rho(X)\equiv\bar{\psi}\psi(X)$ is a scalar field: $\rho'(X')=\rho(X)$. We use this quantum-mechanical language, but the argument is purely field theoretic in spirit, and no *a priori* assumptions on the integration of probability need to be made. The scalar transformation law of ρ does not determine the transformation law of ψ, as it says nothing about the phase of ψ. Therefore, the problem to be solved is this: Can we ascribe in each Galilean system a phase to the ψ in such a way that a consistent transformation law of the form

$$\psi'(X)=e^{-i\theta(r,X)}\psi(r^{-1}X) \tag{3}$$

would hold and ψ would satisfy a form invariant equation in all those systems?

Following standard assumptions we restrict considerations to the class of linear equations of second order at most. Wishing to make use of the equivalence principle we first have to answer this question for ψ in flat space, with the restriction of coordinate systems to inertial ones. One could make use of the standard quantum-mechanical arguments, which would produce the free particle Schrödinger equation with the well known transformation law of ψ consistent in the quantum-mechanical sense, as a projective unitary representation of the Galilean group. However, we think that it is instructive to see how the same result follows by purely geometrical arguments, without any use of a Hilbert space. We sketch the argument. The coordinate transformations are now restricted to the Galilean group, $\vec{a}(t)=\vec{v}t+\vec{a}$ in Eq. (1). We assume that the equation for ψ has the following form (the same in all inertial systems):

$$[a\partial_t^2+b_i\partial_i\partial_t+c_{ik}\partial_i\partial_k+d\partial_t+f_i\partial_i+g]\psi=0,$$

where $a(X),\ldots,g(X)$ are the same functions in each coordinate system, and ψ transforms according to a law of the form (3) with θ to be determined. In the "unprimed" version of this equation we substitute $\psi(X)=e^{i\theta(r,X')}\psi'(X')$ in accordance with Eq. (3), express the differentiations in terms of the new variables X' and divide the resulting equation by the phase factor function $e^{i\theta(r,X')}$. In the resulting equation the ratios of the coefficient functions standing at the distinct differential operators must be equal to the ratios of $a(X'),\ldots,g(X')$. It is easy to see that the transformations of the coefficient functions at the second order operators remain unaffected by the phase function θ. Considering first spacetime translations $X'=X+Y$, $Y=(b,\vec{a})$, one finds, in particular, that the ratios of $a(X)$, $b_i(X)$, $c_{ik}(X)$ must be equal to those of $a(X+Y)$, $b_i(X+Y)$, $c_{ik}(X+Y)$ for all X and Y. It follows that rescaling the original equation by an appropriate factor function of X one can assume without loss of generality that a, b_i and c_{ik} are constant. Consider now general Galilean transformations. For the first two coefficients the covariance now demands that $a=\lambda a$, $R\vec{b}+2a\vec{v}=\lambda\vec{b}$ (where λ may be a function of the transformation). These conditions can be satisfied for all R and $\vec{v}$ only if $a=0$ and $\vec{b}=0$. The transformation of the third coefficient is now $R_{ij}R_{kl}c_{jl}=\lambda c_{ik}$. This may be satisfied only if $c_{ik}=c\delta_{ik}$. Rescaling the equation by an appropriate phase factor one can assume that $c\geqslant 0$. However, if $c=0$ the equation is at most of the first order, and then considerations similar to ours show that $d=0$ and $\vec{f}=0$. This case is trivial. Thus $c>0$ and $\lambda=1$. Now one writes the transformation of the equation in full. From the invariance of the ∂_t term one finds that d is a constant. The condition for $\vec{f}(X)$ takes the form

$$\vec{f}(X)=R^{-1}[\vec{f}(X')-d\vec{v}-2ic\vec{\partial}'\theta(r,X')].$$

Considering this condition for translations and rotations one finds that $\mathrm{Re}\,\vec{f}(X)$ is an invariant vector field, thus $\mathrm{Re}\,\vec{f}(X)=0$, and then $d=ik$, with real k. Applying $\vec{\partial}\times$ to the imaginary part of the condition one finds that also $\vec{\partial}\times\mathrm{Im}\,\vec{f}(X)$ is an invariant field, so $\mathrm{Im}\,\vec{f}(X)=\vec{\partial}\,h(X)$. At this point looking back to the equation we realize that what remains of this term may be gotten rid of by a redefinition of the phase of ψ [by $-(i/2c)h(X)$]. We can assume then that $\vec{f}(X)=0$ and find that $\theta(r,X)=-(k/2c)\vec{v}\cdot\vec{x}+\tilde{\theta}(r,t)$. Finally, the condition for $g(X)$ reads now

$$g(X)=g(X')-(k^2/4c)\vec{v}^2+k\partial_{t'}\tilde{\theta}(r,t').$$

Applying $\vec{\partial}$ to this condition and considering translations and rotations we find that $\vec{\partial}g$ vanishes, so $g(X)=g(t)\equiv G'(t)$. Here again we realize that redefining the phase of ψ [by $(i/k)G(t)$] we remove the remaining freedom of the phase in ψ and the g term from the equation. (If $k=0$ then g is invariant, so $g=\mathrm{const}$. We get the Helmholtz equation and scalar transformation law for ψ. This, being not a dynamic equation, we discard.) We finally get, with the standard notation of constants,

$$\left[i\hbar\,\partial_t+\frac{\hbar^2}{2m}\vec{\partial}^2\right]\psi=0,$$

$$\theta(r,X)=-\frac{m}{\hbar}\vec{v}\cdot\vec{x}+\frac{m}{2\hbar}\vec{v}^2t,$$

which, of course, is the standard free particle theory.

Einstein's equivalence principle implies now that if in the flat space we transform the Schrödinger equation to all arbitrary Galilean systems (noninertial), then we can identify all local modifications to the equation which can appear in an arbitrary Galilean system in curved spacetime. We assume the transformation law (3) and find that the transformed equation differs from the Schrödinger equation at most by additional terms on the left-hand side (lhs) of the form $i\vec{\chi}\cdot\vec{\partial}\psi+\Lambda\psi$, where $\vec{\chi}$ is real. In curved spacetime the Einstein equivalence principle gives then the equation

$$\left[i\hbar\partial_t+\frac{\hbar^2}{2m}\vec{\partial}^2+i\vec{\chi}(X)\cdot\vec{\partial}+\Lambda(X)\right]\psi(X)=0,$$

where $\vec{\chi}(X)$ and $\Lambda(X)$ are now fields characterizing geometry. We assume that these fields are determined locally by the geometry. Assuming the transformation law of the form (3) and demanding the covariance of the equation we find the condition

$$\vec{\chi}(X)=R^{-1}\left[\vec{\chi}'(X')-\hbar\dot{\vec{a}}(t)-\frac{\hbar^2}{m}\vec{\partial}'\theta(r,X')\right].$$

Applying $\vec{\partial}\times$ to this equation we find that $\vec{\partial}\times\vec{\chi}$ is a vector field, in particular it is a vector field with respect to rotations. But ϕ, the only characteristic of the geometry, is a scalar field with respect to rotations, so there is no local way in which a $\vec{\chi}$ giving rise to a nonzero vector field $\vec{\partial}\times\vec{\chi}$ can be formed with it. Hence $\vec{\chi}(X)=\vec{\partial}\xi(X)$. We observe now that

this longitudinal field may be absorbed into the phase of ψ (with the appropriate modification of Λ), so one can assume $\chi=0$. The transformation condition then simplifies to $\theta(r,X)=-(m/\hbar)\dot{\vec{a}}(t-b)\cdot\vec{x}+\tilde{\theta}(r,t)$. The covariance condition for the Λ term now takes the form

$$\Lambda(X)=\Lambda'(X')-m\ddot{\vec{a}}(t)\cdot\vec{x}'-(m/2)\dot{\vec{a}}^2(t)+\hbar\partial_{t'}\tilde{\theta}(r,t').$$

At this point let us look back once more to the flat space case and assume that X is an inertial system. Then $\Lambda(X)=0$ and we find that the additional terms in the operator acting on ψ' arising from the non-inertiality of the system X' are $m\ddot{\vec{a}}(t'-b)\cdot\vec{x}'+(m/2)\dot{\vec{a}}^2(t'-b)-\hbar\partial_{t'}\tilde{\theta}(r,t')$. We learn two things. First, the terms are real, so by the equivalence principle $\Lambda(X)$ is real in general. Second, a change of coordinates produces definite terms up to the linear order in $\vec{x}$. The equivalence principle then implies that in the general case it should be possible by a change of coordinates to eliminate in the neighborhood of a given point X_0 terms independent of, and linear in $\vec{x}-\vec{x}_0$. Put differently, it should be possible to transform away the value and the first derivative $\vec{\partial}$ of $\Lambda(X)$ at this point. Let us introduce $\tilde{\Lambda}(X)$ by $\Lambda(X)=-m\phi(X)+\tilde{\Lambda}(X)$. Using Eq. (2) we find that the covariance condition now takes the form

$$\tilde{\Lambda}(X)=\tilde{\Lambda}'(X')-\frac{m}{2}\dot{\vec{a}}^2(t)+\hbar\partial_{t'}\tilde{\theta}(r,t'),$$

which implies $\vec{\partial}'\tilde{\Lambda}'(X')=R\vec{\partial}\tilde{\Lambda}(X)$. It is now clear that if $\vec{\partial}\tilde{\Lambda}(X)\neq 0$ at some point then it cannot be transformed away. Therefore $\tilde{\Lambda}(X)=\tilde{\Lambda}(t)$, and may be removed by a change of phase of ψ. Thus the unique solution for Λ is $\Lambda=-m\phi$. We now see the geometrical meaning of the condition that the first derivative $\vec{\partial}\Lambda$ may be removed at a point by a change of coordinates: this derivative is equivalent to the connection, so the meaning is exactly the same as in the classical case. The covariance condition now simplifies to $-(m/2)\dot{\vec{a}}^2(t)+\hbar\,\partial_{t'}\tilde{\theta}(r,t')=0$. In this way we finally obtain the equation

$$\left[i\hbar\partial_t+\frac{\hbar^2}{2m}\vec{\partial}^2-m\phi(t,\vec{x})\right]\psi(t,\vec{x})=0, \tag{4}$$

and the transformation exponents

$$\theta(r,X)=-\frac{m}{\hbar}\dot{\vec{a}}(t-b)\cdot\vec{x}+\frac{m}{2\hbar}\int_0^t\dot{\vec{a}}^2(\tau-b)\,d\tau. \tag{5}$$

Until now we have considered the relation between two coordinate systems only. Is the resulting structure, Eq. (4) and the transformation laws (2), (3) and (5), consistent with the composition of transformations? That is, do we get the same result if we choose to break the transformation $X\mapsto X'$ into two steps with an intermediate system on the way: $X\mapsto X''\mapsto X'$? The answer is that the two final gravitational potentials differ in general by a time-dependent ($\vec{x}$-independent) additive term, while the two final wave functions differ by a time-dependent phase factor. This, however, poses no difficulty. The difference in the potentials is consistent with the freedom in their definition, while a time-dependent phase factor in the wave function does not change the state vector [in $L^2(\mathbb{R}^3,d^3x)$] at any time, and in the equation induces only another change of ϕ by an addition of a function of time. We mention as an aside that one could begin the whole analysis by classifying on group-theoretical basis all exponents $\theta(r,X)$ fulfilling this consistency condition. Such a classification has been achieved by one of us (J.W.) and will be published elsewhere.

We have thus shown that Eq. (4) is uniquely determined by Einstein's equivalence principle. In particular, we have shown that the principle implies equality of inertial and gravitational masses. The equation, of course, is standard, and has been discussed many times, but the derivation of its geometrical uniqueness within standard quantum mechanics is new. Within the path-integral formalism De Bièvre obtained earlier similar results for the Feynman propagator of a particle in gravitational field. His derivation is based on additional geometric structures (the Bargmann bundle over spacetime and an associated vector bundle). The propagator is assumed to have geometric properties with respect to these structures. As the connection of this formalism with the standard quantum mechanics is not explicit at this stage, it is not quite obvious what are the corresponding restrictions on the wave function. However, they must amount at least to some restrictions on the transformation properties of the wave function. On the other hand, in our derivation the phase of the wave function is completely free at the start. The equivalence principle and the adjustment of the phase yield both the dynamic equation and the transformation law in an extremely simple way.

Within standard quantum mechanics Kuchař [5] has derived the equation (in general covariant form) by canonical quantization of the geodesic motion. (Where in the process is the mass independence lost? It is, of course, when after going over to the Hamiltonian formalism, in which mass appears, the momentum loses any memory of the mass upon replacement by $-i\hbar\vec{\partial}$.) However, canonical quantization is a heuristic procedure (it is rather classical mechanics, which is believed, in principle, to be derivable from the quantum mechanics) and it is unable to decide the uniqueness question or to clarify the intrinsic structure at play. On the other hand, Duval and Künzle [5] work from start with a wave function of a particle. They show how the equation obtained by Kuchař may be derived by the minimal coupling principle if the wave function is assumed to have certain geometrical properties (is a section of a vector bundle associated with the Bargmann bundle over spacetime). The geometric structures introduced by them have been then adopted by De Bièvre in the paper mentioned above, and also by Christian [5], who makes it a basis for a construction of a Newton-Cartan quantum field theory of particles and gravitational field. While the

structures introduced by Duval and Künzle illuminate the geometry of the general covariant Schrödinger equation, they incorporate assumptions on the transformation properties of the wave function and on the form of the equation which we derive here from scratch.

Another approach to the question of the validity of equivalence principle in the quantum world has been proposed by Lämmerzahl [4]. He gives arguments to the effect that Eq. (4) is favored by a principle which he introduces and calls quantum equivalence principle. This principle formulates a condition for the possibility of the extraction of mass-independent characteristics from experimental results. However, there is no obvious connection of this principle with Einstein's geometrical idea and its compelling persuasiveness (in fact, Lämmerzahl avoids the covariance question completely). We do believe Lämmerzahl's results are important and interesting, but we see their role as being on the experimental side rather than as a theoretical paradigm. What we mean, more precisely, is this. Einstein's principle is a local principle. For a classical test particle, which is a local object, its content translates itself rather directly into experimental predictions. The quantum-mechanical wave function, on the other hand, is a nonlocal object, and there is no simple analogous translation—in general gravity cannot be eliminated on any hypersurface of constant time. Lämmerzahl's papers show how to extract experimental consequences of Einstein's equivalence principle from experimental data. Having said this, however, we also want to express disagreement with the opinion that nonlocality of the wave function precludes operational meaning of Einstein's principle. It may not be obvious how to reveal such meaning, but we can see no fundamental obstacle in the way. Measurements are done locally, which is the operational foundation of Einstein's principle.

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