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Damage Equivalent Method of Fatigue Reliability Analysis of Load-Sharing Parallel System

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Abstract: Speaking of load-sharing parallel system like the multi-engine system and wire cable, dependence-failure must occur due to load redistributing, so the component life distributions changed. After analyzing the disadvantage of failure probability equivalent principle, transformed equivalent working time of different life distribution based on damage equivalent principle, and established the parallel system reliability model applying full probability formula. The established reliability model provides a new method for reliability analysis of load-sharing parallel system whose component life followed any distribution.

1. Introduction

In reliability engineering, it is common recognition to use redundancy technique to improve system reliability, so the *k*/*n* redundancy system is applied in some fields such as industry, military area and aeronautics. In addition, speaking of mechanical system, people have recognized that "dependence" is the universal characteristic of failure, and it usually leads to large error or wrong conclusion to process the reliability analysis and design in the condition of ignoring the failure dependence among components[1~2].

Load-sharing parallel system is the simplest form of *k*/*n* redundancy system, namely 1/*n* redundancy system. The reliability problem of load-sharing parallel system is more complicated than the traditional parallel system, the primary reason to lead to complexity is the dependent failure[3], which is induced by the load's redistribution, and the life distribution of component changes due to the dependent failure. The examples of load-sharing parallel system include the multi-engine system in an airplane and the wire cable in a bridge.

So far, the studies on the load-sharing parallel system are few, some scholars[4~5] analyzed the dependence-failure of the parallel system applied the failure probability equivalent principle when component life followed Weibull distribution or exponential distribution; literatures[6~7] studied the reliability of load-sharing parallel system comprising of two components or three components with nonidentical exponential distributions; Amari, etc provided a closed-form analytical solution for the reliability of tampered failure rate load-sharing *k*/*n* system[3], and H. Liu[8] presented a generalized accelerated failure-time model for load-sharing *k*-out-of-*n* system with arbitrary load-dependent component lifetime distributions.

However, these reliability analysis solutions are not complete, so the effective method is needed to estimate the reliability of load-sharing parallel system. This paper analyzes the dependent failure of load-sharing parallel system, and develops a damage equivalent method of reliability analysis, which easily extends to apply in *k*/*n* system.

2. Dependence-failure analysis of load-sharing parallel system

In a load–sharing parallel system, such as wire cable comprising of *n* wires (in Fig.1), the load on the failed component is redistributed among the surviving component. In a majority of cases, the load is equally distributed over all surviving components. If the total load is *L*, and there are *m* good components, then the load on each component is $z = L/m$. The equal distribution of load is appropriate when all components are of the same type. Hence, when the load is distributed equally, it is also reasonable to assume i.i.d components.

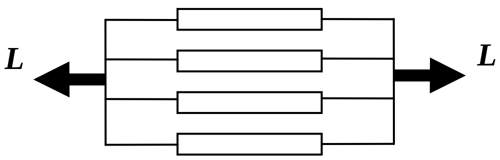

Fig.1 The load–sharing parallel system comprising of *n* wires

Let *n* be the total number of components in the system and z_i be the load on each of the surviving components when I components are failed. Hence, $z_0 = L/n$, $z_i = L/(n-i) = z_0 n/(n-i)$.

Now, the parallel system comprising of two components is discussed. If the two components are normal work, which bear equal load of $z_0 = 0.5L$, and the probability density function(pdf in short) of fatigue life *T* of two components are both $f_1(t)$. When a component fail at the time of t_1, the other one is working on and bears the total load of $z_1 = L$. Here, the work environment of working component begins to deteriorate and induces dependence-failure, and the probability density function of fatigue life *T* of working components becomes $f_2(t)$ (the general law is : the higher the load level is , the smaller the life variance and the life expectation are).

Therefore, the reason of dependence-failure of load-sharing parallel system is the redistribution of load, and the failure of one component influences the failure of other one, which leads to reduce the reliability of parallel system.

3. The damage equivalent transition

In order to analysis conveniently, D. Kececioglu[9~10]has advanced a reliability recurrence expression to convert and cumulate load action of all levels based on failure probability equivalent principle, and literature [4~5] has analyzed the dependence-failure of system applying to the failure probability equivalent principle. The method is as follows.

$$p_1 = \int_0^{t_1} f_1(t)\,\mathrm{d}t = \int_0^{t_{1P}} f_2(t)\,\mathrm{d}t \tag{1}$$

Where, t_{1p} is the equivalent time in the condition of pdf $f_2(t)$ and could be computed by the expression (1). Consequently, the failure probability of working component at the time of *t* equals to:

$$p = p_1 + \Delta p_{2\mathrm{P}} = \int_0^{t_1} f_1(t)\,\mathrm{d}t + \int_{t_{1P}}^{t_{1P}+t-t_1} f_2(t)\,\mathrm{d}t = \int_0^{t_{1P}} f_2(t)\,\mathrm{d}t + \int_{t_{1P}}^{t_{1P}+t-t_1} f_2(t)\,\mathrm{d}t = \int_0^{t_{1P}+t-t_1} f_2(t)\,\mathrm{d}t \tag{2}$$

Where, $\Delta p_{2\mathrm{P}} = \int_{t_{1P}}^{t_{1P}+t-t_1} f_2(t)\,\mathrm{d}t$.

But “the equivalent failure probability transition method” converts the time of different life distribution based on equivalent failure probability, which corresponds to express damage degree in failure probability and is unreasonable. Moreover, Xie Liyang[11~12] analyzed the fatigue reliability under program loading, and proposed “the equivalent damage transition method”, which satisfied the damage equivalent principle and is validated that it is better than “the equivalent failure probability transition method” by experiment.

The aim of research is to analyze the reliability of parallel system applying “the equivalent damage transition method”. The time of t_1, which the working component works to in the condition of life pdf $f_1(t)$, is converted to the time of t_{1E}, which the working component works to in the condition of life pdf $f_2(t)$.

According to Miner principle (it is in order to convenient discussing that the Miner principle is applied. In practice, the proper nonlinear cumulate damage model should be adopted for the sake of accurate computation):

$$\frac{t_1}{E_1(T)} = \frac{t_{1E}}{E_2(T)} \tag{3}$$

Where, $E_1(T)$ is the life expectation in the condition of pdf $f_1(t)$, $E_2(T)$ is the life expectation in the condition of pdf $f_2(t)$.

So, t_{1E} could be taken as equivalent time in the condition of pdf $f_2(t)$ after transition:

$$t_{1E} = \frac{t_1}{E_1(T)} E_2(T) \tag{4}$$

But the failure probability of working component is brought in the condition of pdf $f_1(t)$ at the time of t_1, which is $p_1 = \int_0^{t_1} f_1(t)\,\mathrm{d}t$. When the working component begins to work in the condition of pdf $f_2(t)$, the computation of failure probability increment Δp_{2E} should begin from the time of t_{1E}.

The failure probability of working component at the time of t equals to:

$$p_{12} = p_1 + \Delta p_{2E} = \int_0^{t_1} f_1(t)\,\mathrm{d}t + \int_{t_{1E}}^{t_{1E}+t-t_1} f_2(t)\,\mathrm{d}t \tag{5}$$

Where, $\Delta p_{2E} = \int_{t_{1E}}^{t_{1E}+t-t_1} f_2(t)\,\mathrm{d}t$, and the reliability of working component at the time of t equals to:

$$R_{12} = 1 - \left(\int_0^{t_1} f_1(t)\,\mathrm{d}t + \int_{t_{1E}}^{t_{1E}+t-t_1} f_2(t)\,\mathrm{d}t\right) \tag{6}$$

Let life pdf $f_1(t) \sim N(160, 40^2)$ (i.e. life follows the normal distribution with the mean 160 and variance 40), and life pdf $f_2(t) \sim N(100, 20^2)$ (i.e. life follows the normal distribution with the mean 100 and variance 20), t_1=120, which yields $t_{1E}=\frac{120}{160}\times 100=75$, $t_{1P}=\frac{(120-160)}{40}\times 20+100=80$.

It is obvious that t_{1E} and Δp_{2P} do not equal to t_{1P} and Δp_{2E}, respectively. The curve of pdf $f_1(t)$ and the curve of pdf $f_2(t)$ are shown in Fig.2.

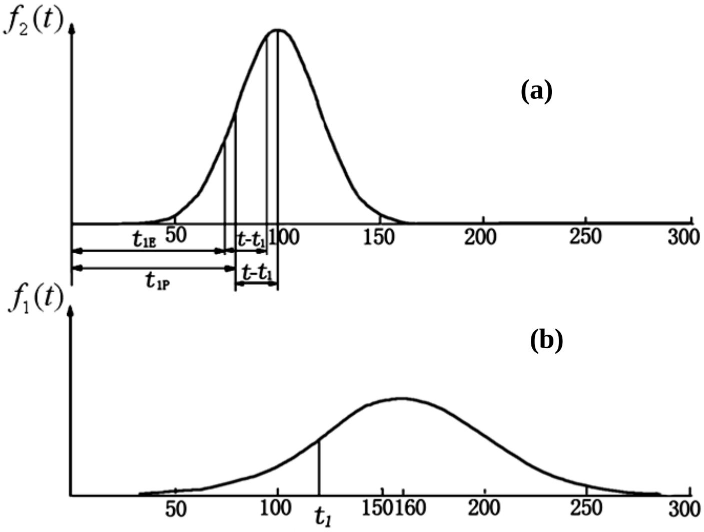

Fig.2 The life probability density curve

The Fig.2(b) shows, when t_1 is very little, the t_{1p} cannot be computed since the failure probability almost equals to zeros, but the equivalent time t_{1E} can be computed based on equivalent damage transition.

According to equivalent probability transition, the failure probability of working component is the area between the 0 and t_{1P}+t- t_1 under the curve of $f_2(t)$ at the time of t.

According to equivalent damage transition, the failure probability of working component is the area between the 0 and t_{1P} plus the area between the 0 and t_{1E}+t- t_1 under the curve of $f_2(t)$ at the time of t.

It is shown $\Delta p_{2E} < \Delta p_{2P}$ in the front half part of pdf $f_2(t)$, and $\Delta p_{2E} > \Delta p_{2P}$ in the latter half part of pdf $f_2(t)$.

4. The reliability modeling of parallel system

According to addition principle, there are two instances when the parallel system composed of two components works at the time of t: the one is that two components all work normally, another is that one component works normally and the other one fails at the time of t_1.

(1) If two components all work normally at the time of t, the reliability of parallel system will equal to:

$$R_1(t) = [\int_t^\infty f_1(t)\,\mathrm{d}t]^2 \tag{7}$$

(2) If one component fails at the time of t_1 (t_1<=t), its failure probability, between t_1 and $t_1 + \Delta t_1$, will equal to:

$$p_{t_1} = f_1(t_1)\Delta t_1 \tag{8}$$

In the precondition of one component failure, the reliability of working component at the time of t based on expression (6) equals to:

$$R = 1 - (\int_0^{t_{1i}} f_1(t)\,\mathrm{d}t + \int_{t_{1E}}^{t_{1E}+t-t_{1i}} f_2(t)\,\mathrm{d}t)$$

According to multiplication principle, the concurrence probability of one component working and the other one failure equals to:

$$p = p_{t_1} \times R = 1 - (\int_0^{t_{1i}} f_1(t)\,\mathrm{d}t + \int_{t_{1E}}^{t_{1E}+t-t_{1i}} f_2(t)\,\mathrm{d}t)\, f_1(t_1)\Delta t_1 \tag{9}$$

Therefore, based on full probability formula[13], in the precondition of one component working and one component failure, the reliability of parallel system is:

$$R_2(t)=C_2^1\int_0^t[1-(\int_0^{t_1} f_1(t)\,\mathrm{d}\,t+\int_{t_{1\mathrm{E}}}^{t_{1\mathrm{E}}+t-t_1} f_2(t)\,\mathrm{d}\,t)]\cdot f_1(t_1)\,\mathrm{d}\,t_1 \tag{10}$$

Consequently, the reliability of parallel system comprising of two components at the time of t can be expressed as:

$$R_{\mathrm{E}}^2(t)=R_1(t)+R_2(t)=[\int_t^{\infty} f_1(t)\,\mathrm{d}\,t]^2+2\int_0^t[1-(\int_0^{t_1} f_1(t)\,\mathrm{d}\,t+\int_{t_{1\mathrm{E}}}^{t_{1\mathrm{E}}+t-t_1} f_2(t)\,\mathrm{d}\,t)]\cdot f_1(t_1)\,\mathrm{d}\,t_1 \tag{11}$$

The expression (11) is the dependence-failure reliability model of parallel system of two components based on damage equivalent principle(ED model in short).

According to expression (2) and expression (11), the dependence-failure reliability model parallel system of two components based on failure probability equivalent principle can be expressed as:

$$R_{\mathrm{P}}^2(t)=R_1(t)+R_2(t)=[\int_t^{\infty} f_1(t)\,\mathrm{d}\,t]^2+2\int_0^t[1-\int_0^{t_{1\mathrm{P}}+t-t_1} f_2(t)\,\mathrm{d}\,t]\cdot f_1(t_1)\,\mathrm{d}\,t_1 \tag{12}$$

The expression (12) is the dependence-failure reliability model of parallel system of two components based on failure probability equivalent principle (EP model in short).

The relationship between reliability and time is shown in Fig.3.

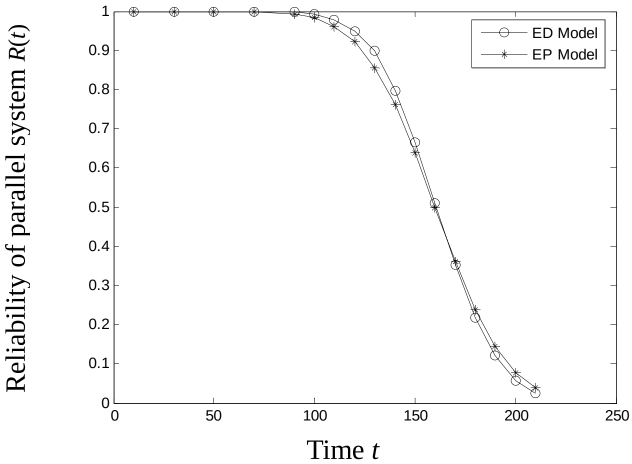

Fig.3 The relationship between reliability and time

based on ED and EP

It shows that the reliability of parallel system decrease along with the increase of time, and the reliability estimated by ED model is larger than the reliability estimated by EP model in incipient time, but the reliability estimated by ED model is smaller than the reliability estimated by EP model in subsequent time, which validates the above-mentioned analysis.

When the dependence-failure is not considered, $f_1(t)$ is the same to $f_2(t)$, and $t_{1\mathrm{E}}=t_1$. Therefore, the independence-failure reliability of parallel system comprising of two components equals to:

$$\begin{aligned}R_{\mathrm{I}}^2(t)&=[\int_t^{\infty} f_1(t)\,\mathrm{d}\,t]^2+2\int_0^t[1-(\int_0^{t_1} f_1(t)\,\mathrm{d}\,t+\int_{t_1}^{t_1+t-t_1} f_1(t)\,\mathrm{d}\,t)]f_1(t_1)\,\mathrm{d}\,t_1\\&=[\int_t^{\infty} f_1(t)\,\mathrm{d}\,t]^2+2\int_0^t[1-\int_0^{t} f_1(t)\,\mathrm{d}\,t]\cdot f_1(t_1)\,\mathrm{d}\,t_1\\&=[1-\int_0^{t} f_1(t)\,\mathrm{d}\,t]^2+2\int_0^{t} f_1(t_1)\,\mathrm{d}\,t_1-2[\int_0^{t} f_1(t)\,\mathrm{d}\,t]^2=1-[\int_0^{t} f_1(t)\,\mathrm{d}\,t]^2\end{aligned} \tag{13}$$

The computed result of ED model and the computed result of independence-failure model are shown in Fig.4.

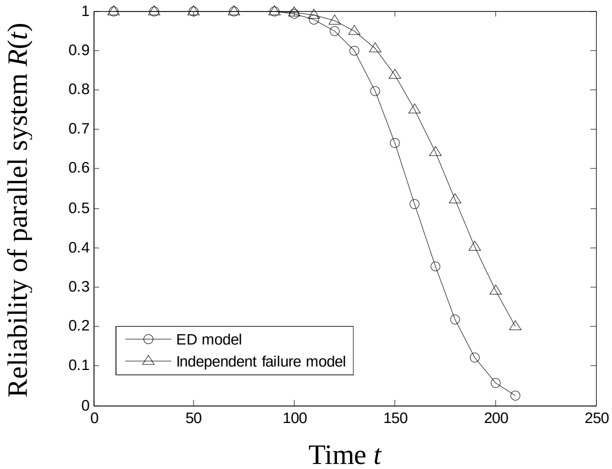

Fig.4 The computed results based on ED model and independent failure model in comparison

It shows that reliability of the parallel system considering dependence-failure is smaller than that reliability of the parallel system not considering dependence- failure.

Consequently, the reliability of parallel system composed of three components could be estimated by ED model, which can be expressed as expression (14):

$$R_{\mathrm{E}}^{3}(t)=R_{1}(t)+R_{2}(t)+R_{3}(t) \tag{14}$$

Where, $R_1(t)=[\int_t^{\infty} f_1(t)\,\mathrm{d}t]^3$, $R_2(t)=C_3^1\int_0^t[1-(\int_0^{t_1} f_1(t)\,\mathrm{d}t+\int_{t_{1\mathrm{E}}}^{t_{1\mathrm{E}}+t-t_1} f_2(t)\,\mathrm{d}t)]^2 f_1(t_1)\,\mathrm{d}t_1$,

$R_3(t)=C_3^1C_2^1\int_0^t\{\int_{t_1}^t[1-(\int_0^{t_1} f_1(t)\,\mathrm{d}t+\int_{t_{1\mathrm{E}}}^{t_{1\mathrm{E}}+t_2-t_1} f_2(t)\,\mathrm{d}t+\int_{t_{2\mathrm{E}}}^{t_{2\mathrm{E}}+t-t_2} f_3(t)\,\mathrm{d}t)]\times f_2(t_{1\mathrm{E}}+t_2-t_1)\,\mathrm{d}t_2\}\times f_1(t_1)\,\mathrm{d}t_1$,

$t_{2\mathrm{E}}=\dfrac{t_{1\mathrm{E}}+t_2-t_1}{E_2(t)}E_3(t)=E_3(t)(\dfrac{t_1}{E_1(t)}+\dfrac{t_2-t_1}{E_2(t)})$, $E_3(T)$ is the life expectation in the condition of pdf $f_3(t)$.

Likewise, the reliability of parallel system composed of *n* components could be estimated by ED model, which should be obtained by computer numerical computation.

5. Conclusion

The ED model could reflect the component dependence-failure of each other, and is accurate when component life follows Weibull distribution or exponential distribution, etc.

Comparing to the EP model, ED model could satisfy the damage equivalent condition, and could reflect the nonlinear principle of cumulate damage when the nonlinear cumulate damage model is applied. But the EP model cannot satisfy the damage equivalent condition, and its applicability is little.

Therefore, the ED model provides a new study method for the reliability analysis of load-sharing parallel system.

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