Distinguishing the energy and non-energy actions in balancing energy markets

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ABSTRACT

In the European context, balancing energy markets are established to optimise transmission system operator balancing actions closer to real-time. These actions aim to match total generation and consumption subject to a suite of security constraints (e.g., reserve requirements). However, there is no clear border between those actions that are taken due to the reserve requirements (non-energy actions) and those that are primarily taken to supply the demand mismatches (energy actions). To recognize the effect of non-energy actions, existing methods require comparing the results of counterfactual optimisation problems in which the non-energy-action-related constraints were deliberately omitted. This paper proposes a one-off solution enabling TSOs to distinguish energy actions from non-energy ones in the balancing market scheduling problem. By decomposition of the dual variables and clustering the constraints as proposed in this paper, there is no need to solve repetitive counterfactual optimisation problems. Case studies show that in addition to the non-energy actions caused by non-energy-based balancing requirements, the proposed method is able to recognize the energy actions that should be taken due to the non-energy root causes. This feature enables TSOs to efficiently retrace the effect of non-energy actions on the energy-based dispatch instructions issued according to the balancing market schedule.

Nomenclature

Variables

$\Gamma^{Itself}_{j,t}$ internal constraints contribution on the total deviation allocated to generator j at time t in the BEM

$\bar{\theta}_{j,t}, \underline{\theta}_{j,t}$ DVs associated with the generation capacity limits

$\bar{\xi}_{f,t}, \underline{\xi}_{f,t}$ DVs associated with the transmission capacity constraints

$\Delta^{Itself}_{j,t}$ total deviation allocated to the generators in the BEM due to the internal constraints

$\Delta^{Network}_{j,t}$ total deviation allocated to the generators in the BEM due to the network constraints

$\Delta^{Reserve}_{j,t}$ total deviation allocated to the generators in the BEM due to the reserve constraints

$\Delta^{Rivals}_{j,t}$ total deviation allocated to the generators in the BEM due to the rivals' constraints

$\Gamma^{Network}_{j,t}$ network constraints contribution on the total deviation allocated to generator j at time t in the BEM

$\Gamma^{Rivals}_{j,t}$ rivals constraints contribution on the total deviation allocated to generator j at time t in the BEM

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λ_t energy marginal price

$\upsilon_{f,t}$ DVs associated with the transmission power flow constraints

$\Delta_{j,t}$ total deviation allocated to the generators in the BEM

$\alpha_{j,t}$ DVs associated with the ramp up constraints

$\beta_{j,t}$ DVs associated with the ramp down constraints

$F_{f,t}$ transmission line flows (p.u.)

$\gamma_{f,t}$ DVs associated with the power flow constraints in the transmission lines

$\mu_{j,t}$ DVs associated with the lower bound of reserve capability for each generation unit

$\underline{\mu_t}$ DVs associated with the lower bound of system reserve requirement

$\vartheta_{j,t}$, $\rho_{j,t}$ DVs associated lower bound of increasing/decreasing FEP deviations

$\mathrm{P}^{inc}_{j,t}$, $P^{dec}_{j,t}$ BEM allocated increasing/decreasing deviations from FEP to the generators (p.u.)

$\mathrm{R}_{j,t}$ BEM allocated reserve to the providers (p.u.)

1. Introduction

1.1. Motivation

Balancing markets ensure second-by-second system balance by addressing load and supply mismatches using balancing services like operating reserves. [1]. European countries have unified the day-ahead market across most jurisdictions and aim to establish a unified balancing market [2]. The goal is to secure balancing capacity (BC) in advance through a balancing capacity market (BCM) and activate balancing energy bids in a balancing energy market (BEM), ensuring transmission system operators have sufficient balancing services for real-time operation. As outlined in energy balancing guidelines (EBGL) [3], BEMs must reflect the true value of balancing energy to incentivize market participants to support system balance. To understand the implications of EBGL requirements on the design of BEMs, some important definitions need to be considered:

- "Balancing Energy" is defined as the amount of energy that is used by TSOs to make the balancing actions.
- "Balancing Capacity" is defined as the volume of the reserve capacity that the service provider has blocked (cannot be offered as energy products in ex-ante energy markets such as day-ahead or intraday markets) and the balancing service provider has promised to submit balancing energy prices in the BEM based on that blocked BC.
- "Final Energy Position" (FEP) is the amount of energy that has been declared by a BEM participant and shows its energy position in the ex-ante energy markets.

In BEMs, energy balancing actions (e.g., adjusting dispatch levels) address load changes. Non-energy balancing actions, like securing reserve margins and voltage support, are also crucial for power system security and stability.

Balancing energy providers must submit bids/offers for deviations from their FEPs. To activate balancing energy bids, a merit order must be established and an optimisation problem solved to minimise the total cost of deviations from FEPs, subject to security constraints. For example, in some European countries like Ireland, the BEM's objective is to minimise this total cost. Therefore, in addition to the bids/offers, a market operator must be notified with FEPs. As is required by EBGL, those providers who sold BC must [1] notify the market operator with a compatible FEP to their BC trades.

[1] In Single Electricity Market (SEM), balancing energy is obtained through co-optimisation of energy and reserve in the BEM, where the reserve price is zero in the optimisation objective. Balancing energy providers are compensated based on BEM outcomes, and those offering system services are paid under the DS3 program [4]. There are plans in SEM's future to establish a Balancing Capacity Market (BCM).

In optimisation problems, every binding constraint can increase the total cost of the objective function. EBGL mandates flagging balancing energy-activated bids to prevent them from impacting the balancing energy price [3]. The guideline aims to ensure that non-energy actions do not influence the pricing in the BEM, necessitating a method to distinguish between energy and non-energy actions.

Expecting higher imbalance prices can reduce the system imbalances [5]. However, due to unexpected events such as changes in transmission system topology [6], intermittency of renewable energy resources [7], and load forecast errors [8], there is always a chance for unexpected imbalances. Integration of renewable energies should reduce the overall cost of electricity [9], however, 1) strategic bidding behavior [10, 11], and 2) flexibility costs [12, 13] can compromise this reducing effect [14]. This paper aims to address the challenge of distinguishing between energy and non-energy actions in the BEM to establish an efficient pricing and settlement mechanism aligned with EBGL requirements.

1.2. Background

Market clearing involves solving an optimisation problem where outcomes depend on i) power system, and ii) generation unit constraints. Power system requirements necessitate having operational strategies as preventive actions [15]. To do so, offline studies are performed by TSOs to detect transmission bottlenecks, voltage stability issues [16, 17], and frequency stability requirements [18]. These studies provide inputs to the electricity market scheduling process as constraints. Examples include must-run units and security margins, which belong to (i). Generation units can also affect the feasible solution space by imposing their internal technical constraints [19], such as minimum/maximum power capability, ramping up/down limits, etc.

Reference [20] assessed the impact of system non-synchronous penetration levels and transmission constraints on total wind curtailment using repetitive simulations with PLEXOS, focusing on system constraints. They simulated different scenarios to evaluate the effect of system constraints on the outcomes of economic dispatch. In reference [21], the effect of relaxing the minimum required conventional generation in Northern Ireland was evaluated on the wind curtailment. In similar studies [22, 23], based on comparing the assumptions for the required constrained on conventional plants in Western Denmark, the effect of system constraints on the wind penetration level in Denmark was evaluated. Later on in reference [24], the cost-saving effects of system operating constraints relaxation have been evaluated. Their study considered the system constraint requiring a minimum number of conventional generation units for stability in Ireland. By relaxing this constraint in simulations, they analyzed its cost implications for future power system scheduling. They found that stability requirements, combined with minimum power constraints of conventional plants, increase operation costs and wind curtailment. However, they did not classify the constraint sets as proposed in this study or disentangle the contributions of these constraint categories to the total power allocated to conventional generation plants. Reference [25] proposed evaluating the effect of internal technical constraints on power system flexibility by relaxing these constraints one by one and assessing the impact on total unit commitment operation costs. The author used multiple off-line solutions for every single constraint to evaluate the overall contribution of a generation unit on the total operation cost.

The repetitive simulation approach involves comparing two separate optimisation problems to evaluate constraint effects. In the first, the intended constraint is excluded, while in the second, all constraints are included. Decision variables from each optimisation are then compared. However, constraint interactions can alter decision variables, making it difficult to isolate the impact of the intended constraint.

Reference [26] assessed generation units' strategic behavior by withholding fast ramping ability using a bi-level approach. The upper level focused on profit maximisation, while the lower level cleared the security-constrained economic dispatch problem. They demonstrated how ramping constraints, alongside other system constraints, could be used to manipulate market prices. However, they did not evaluate the impact of binding ramping constraints on the total power allocated to strategic bidders.

Costs from power system constraints (first category) are inevitable for end-users and are standard. However, with the rise of energy communities and self-renewable consumers [27], dealing with generation unit's internal technical constraints (second category) must be updated. In the future, self-sufficient consumers may resist bearing these costs in market clearing.

Reference [28] conducted a perturbation analysis on the look-ahead security-constrained economic dispatch optimality conditions. They derived a linear sensitivity matrix linking perturbed input data to the decision variables in the optimisation objective, addressing the impact of corrupted data on dispatch outcomes. Reference [29] proposed decomposing dual variables in security-constrained economic dispatch. They used this decomposed strong duality

Table 1
Comparison of the current study with the existing research works

Ref	Repetitive Simulation	Optimisation-based Approach	Addressing the System Constraints	Addressing the Generators Constraints	Discriminating Energy vs Non-Energy Actions
[20],[21]	✓	X	✓	X	X
[24]	✓	X	X	X	X
[25],[26]	✓	X	X	✓	X
[28],[29]	X	✓	✓	✓	X
This Study	X	✓	✓	✓	✓

formulation to map decision variables' contributions to constraints, highlighting how internal technical constraints of different technologies affect total power allocation to rivals. Reference [29] proposed a dual decomposition technique to trace the impact of various constraints on the total power allocated to a generation unit in the security-constrained economic dispatch problem. They used the decomposed strong duality constraint to determine the contribution of binding constraints to the allocated power. However, their study did not explore the technique's potential to capture contributions to other decision variables. The advantage of the optimisation-based method proposed in references [29, 28] over the repetitive simulation approach is that it eliminates the need to compare outcomes from separate optimisation problems. Later In reference [30], the dual variable decomposition technique was used to map offered prices to shadow prices in a dynamic security-constrained economic dispatch (SCED) problem. The authors applied this technique to the dual SCED problem to determine how offered prices contribute to shadow prices.

1.3. Contribution

This paper proposes an optimisation-based approach to distinguish energy-balancing actions from non-energy ones in BEMs. The reserve requirement, representing non-energy actions, is included as a decision variable in the objective function of the security-constrained economic dispatch problem. This paper extends the method proposed by reference [29] by adding the reserve requirement to the objective function of the security-constrained optimisation problem with a small penalty factor. The small cost for the reserve decision variable ensures it doesn't affect the BEM's optimal solution. This aligns with practices in some European markets, like Ireland, where reserve costs are excluded from the balancing market objective function. The proposed small cost term allows transmission system operators to trace the contribution of all binding constraints to the allocated reserve. Additionally, the contributions of binding constraints to the decision variables of the security-constrained economic dispatch problem are formulated to distinguish between energy and non-energy actions. Table 1 shows the knowledge gap and the contribution of this paper to the existing literature. This paper contributes to the existing literature as follows:

- Reference [29]'s decomposed strong duality approach is extended to assess the contribution of reserve requirements in allocating incremental/decremental power to balance responsible parties in the BEM.
- This paper proposes adding a decision variable to the security-constrained economic dispatch with a small penalty cost. This allows TSOs to trace how constraints associated with that variable contribute to all dispatch decision variables, advancing previous research in this area [29, 28].
- Classifying constraints, as proposed in this paper, helps TSOs distinguish between energy-balancing actions and non-energy actions. To the authors' knowledge, this issue has not been addressed in existing literature.

1.4. Paper Organization

The present paper is organized as follows: In Section 2, the formulation of a typical BEM is presented. Then in Section 3, the proposed theory is introduced and applied to the BEM model to distinguish the energy and non-energy actions. Section 4 presents the case studies followed by a discussion of the results. Finally, the conclusions are made in section 5.

2. Problem Formulation

The objective function of the optimisation problem in the BEM is to minimise the cost of deviations from the FEPs. In Eq. (1.1), $P^{inc}_{j,t}$,$P^{dec}_{j,t}$, and $R_{j,t}$ are referring to the incremental deviation, decremental deviation, and the allocated reserve to unit j at time t respectively. $O^{inc}_{j,t}$ and $O^{dec}_{j,t}$ are the commercial offer data of the units and $\varepsilon^{R}_{j,t}$ is a sufficiently small penalty factor that is added to the objective function to enable retracing the effect of each binding constraints to the final allocated reserve. It should be noted that $\varepsilon^{R}_{j,t}$ is set to a very small positive number that has no effect on the optimal solution.

$$\min_{P^{inc}_{j,t}, P^{dec}_{j,t}, R_{j,t}} \left\{ \sum_{j,t} O^{inc}_{j,t} P^{inc}_{j,t} + \sum_{j,t} O^{dec}_{j,t} P^{dec}_{j,t} + \sum_{j,t} \varepsilon^{R}_{j,t} R_{j,t} \right\} \quad s.t \tag{1.1}$$

$$\sum_{\ell \in N_L} L^{I}_{\ell,t} - \sum_{j \in N_G} \left(P^{pri}_{j,t} + P^{inc}_{j,t} - P^{dec}_{j,t} \right) = 0 \quad : \lambda_t \quad \forall t \tag{1.2}$$

$$F_{f,t} - \sum_{n \in N_N} \Omega^{SF}_{f,n} \left(\sum_{j} \Omega^{G}_{j,n} \left(P^{pri}_{j,t} + P^{inc}_{j,t} - P^{dec}_{j,t} \right) - \sum_{\ell \in N_L} \Omega^{L}_{\ell,n} L^{I}_{\ell,t} \right) = 0 \quad : \gamma_{f,t} \quad \forall f, t \tag{1.3}$$

$$-\bar{F}_f \le F_{f,t} \le \bar{F}_f \quad : \bar{\xi}_{f,t}, \underline{\xi}_{f,t} \quad \forall f, t \tag{1.4}$$

$$P^{pri}_{j,t} + P^{inc}_{j,t} - P^{dec}_{j,t} + R_{j,t} \le \overline{P}_j \quad : \overline{\theta}_{j,t}, \quad \forall j, t \tag{1.5}$$

$$\underline{P}_j \le P^{pri}_{j,t} + P^{inc}_{j,t} - P^{dec}_{j,t} \quad : \underline{\theta}_{j,t} \quad \forall j, t \tag{1.6}$$

$$P^{pri}_{j,t} + P^{inc}_{j,t} - P^{dec}_{j,t} + R_{j,t} - \left(P^{pri}_{j,t-1} + P^{inc}_{j,t-1} - P^{dec}_{j,t-1} \right) \le U_j \quad : \alpha_{j,t} \quad \forall j, t > 1 \tag{1.7}$$

$$P^{pri}_{j,t} + P^{inc}_{j,t} - P^{dec}_{j,t} + R_{j,t} - P^{0}_{j} \le U_j \quad : \alpha_{j,t} \quad \forall j, t = 1 \tag{1.8}$$

$$P^{pri}_{j,t-1} + P^{inc}_{j,t-1} - P^{dec}_{j,t-1} + R_{j,t-1} - \left(P^{pri}_{j,t} + P^{inc}_{j,t} - P^{dec}_{j,t} \right) \le D_j \quad : \beta_{j,t} \quad \forall j, t > 1 \tag{1.9}$$

$$P^{0}_{j} - \left(P^{pri}_{j,t} + P^{inc}_{j,t} - P^{dec}_{j,t} \right) \le D_j \quad : \beta_{j,t} \quad \forall j, t = 1 \tag{1.10}$$

$$\underline{R}_t \le \sum_{j \in N_G} R_{j,t} \quad : \mu_t \quad \forall t \tag{1.11}$$

$$0 \le R_{j,t} \quad : \underline{\mu}_{j,t} \quad \forall j, t \tag{1.12}$$

$$0 \le P^{inc}_{j,t} \quad : \vartheta_{j,t} \quad \forall j, t \tag{1.13}$$

$$0 \le P^{dec}_{j,t} \quad : \rho_{j,t} \quad \forall j, t \tag{1.14}$$

Eq. (1.2) forces the power balance and Eq. (1.3) gives the line flows using the generation shift factors. $L^{I}_{\ell,t}$ is the demand, $P^{pri}_{j,t}$ refers to the FEP, $F_{f,t}$ is the flow in line f at time t , $\Omega^{SF}_{f,n}$ is the generation shift factor matrix, and finally $\Omega^{G}_{j,n}$ and $\Omega^{L}_{l,n}$ are representing the generators and loads incident matrices. Line flow limits are preserved by Eq. (1.4) within the margins (i.e., $\bar{F}_f$). Eqs. (1.5) and (1.6) keep the allocated power within the capability limits ($\overline{P}_j$ is the maximum and $\underline{P}_j$ is the minimum generation capability of unit j). Eqs. (1.7) to (1.10) are satisfying the ramping up/down limits (U_j is the ramp up, D_j is the ramp down, and P^{0}_{j} is the power level for generation unit j at $t = 0$). Eq. (1.11) guarantees satisfaction of the minimum reserve requirements (i.e., $\underline{R}_t$) at time t. The presence of this constraint in the BEM is dependent on the day-ahead market structure. In the Island of Ireland, the day-ahead market is an unconstrained auction, therefore security constraints should be modeled in the BEM optimsation problem.

$$\Pi_{PV} = \left\{ P^{inc}_{j,t}, P^{dec}_{j,t}, R_{j,t}, F_{f,t} \right\} \tag{2.1}$$

$$\Pi_{DV} = \left\{ \lambda_t, \gamma_{f,t}, \bar{\xi}_{f,t}, \underline{\xi}_{f,t}, \overline{\theta}_{j,t}, \underline{\theta}_{j,t}, \alpha_{j,t}, \beta_{j,t}, \mu_t, \underline{\mu}_{j,t}, \rho_{j,t}, \vartheta_{j,t} \right\} \tag{2.2}$$

$$\Pi_{Cte} = \left\{ O^{inc}_{j,t}, O^{inc}_{j,t}, \varepsilon^{R}_{j,t}, L^{I}_{\ell,t}, B_f, P^{pri}_{j,t}, \underline{F}_f, \overline{F_f}, U_j, D_j, \overline{P}_j, \underline{P}_j, \underline{R}_t \right\} \tag{2.3}$$

Eqs. (1.12) to (1.14) keep the decision variables positive. Π_{PV} is the set of primary variables, while Π_{DV} and Π_{Cte} are representing the set of dual variables and constants respectively.

The value of $\varepsilon^{R}_{j,t}$ should be set sufficiently small to avoid any changes in the optimal basis and the resulting optimal solution but it does not need to be infinitesimal. The maximum permissible value for $\varepsilon^{R}_{j,t}$ depends on the value of decision variables in the optimal solution of the same problem while $\varepsilon^{R}_{j,t} = 0$. The range of validity for $\varepsilon^{R}_{j,t}$ can be determined through the sensitivity report of the linear programming model [31]. Although designating a small value for $\varepsilon^{R}_{j,t}$ can be perfectly fitted to the proposed approach, determining this maximum valid value allows increasing $\varepsilon^{R}_{j,t}$ from a small value to the maximum valid value. Thereby, this enables disregarding the reserve requirement as a decision variable with almost a zero coefficient in the objective function.

Since the problem is linear, thus convex, and the Slater condition holds, to obtain the dual variables, primal-dual transformation can be applied [32]. Primal feasibility is ensured by Eqs. (1.2) to (1.14), while the stationarity is satisfied by Eqs. (4.1) to (4.7). The dual feasibility inequalities are ensured by Eqs. (4.8). Lastly, the complementary slackness is maintained as per Eq. (5). This will apply the equivalence of KKT conditions and primal-dual transformation [33]. The objective function of the dual problem is represented by Eq. (3).

$$\begin{aligned}
\underset{\Pi_{DV}}{Max} \Bigg\{ & \sum_{t\in N_t,\ell\in N_L} L^{I}_{\ell,t}\lambda_t - \sum_{t\in N_t,j\in N_G} P^{pri}_{j,t}\lambda_t - \sum_{f,t}\gamma_{f,t}\left[\sum_{n\in N_F}\Omega^{SF}_{f,n}\left(\sum_{j}\Omega^{G}_{j,n}P^{pri}_{j,t} - \sum_{\ell\in N_L}\Omega^{L}_{l,n}L^{I}_{\ell,t}\right)\right] \\
& - \sum_{t\in N_t,j\in N_G}\overline{\theta}_{j,t}\left(\overline{P}_j - P^{pri}_{j,t}\right) - \sum_{t\in N_t,j\in N_G}\underline{\theta}_{j,t}\left(P^{pri}_{j,t} - \underline{P}_j\right) - \sum_{t>1,j\in N_G}\alpha_{j,t}\left(U_j - P^{pri}_{j,t} + P^{pri}_{j,t-1}\right) \\
& - \sum_{t=1,j\in N_G}\alpha_{j,t}\left(U_j - P^{pri}_{j,t} + P^{0}_{j}\right) - \sum_{t>1,j\in N_G}\beta_{j,t}\left(D_j - P^{pri}_{j,t-1} + P^{pri}_{j,t}\right) \\
& - \sum_{t=1,j\in N_G}\beta_{j,t}\left(D_j + P^{pri}_{j,t} - P^{o}_{j}\right) - \sum_{f\in N_F,t\in N_t,}\left(\overline{\xi}_{f,t} + \underline{\xi}_{f,t}\right)\overline{F}_f + \sum_{t\in N_t}\mu_t\underline{R}_t \Bigg\}
\end{aligned} \tag{3}$$

Stationary conditions and dual feasibility inequalities are ensured by Eqs. (4.1) to (4.7) and Eqs. (4.8) respectively.

$$O^{inc}_{j,t} - \lambda_t + \overline{\theta}_{j,t} - \underline{\theta}_{j,t} + \alpha_{j,t} - \alpha_{j,t+1} - \beta_{j,t} + \beta_{j,t+1} - \vartheta_{j,t} - \sum_{f\in N_F,n\in N_N}\gamma_{f,t}\Omega^{SF}_{f,n}\Omega^{G}_{j,n} = 0 \quad \forall j, t\neq 24 \tag{4.1}$$

$$O^{inc}_{j,t} - \lambda_t + \overline{\theta}_{j,t} - \underline{\theta}_{j,t} + \alpha_{j,t} - \beta_{j,t} - \vartheta_{j,t} - \sum_{f\in N_F,n\in N_N}\gamma_{f,t}\Omega^{SF}_{f,n}\Omega^{G}_{j,n} = 0 \quad \forall j, t = 24 \tag{4.2}$$

$$O^{dec}_{j,t} + \lambda_t - \overline{\theta}_{j,t} + \underline{\theta}_{j,t} - \alpha_{j,t} + \alpha_{j,t+1} + \beta_{j,t} - \beta_{j,t+1} - \rho_{j,t} + \sum_{f\in N_F,n\in N_N}\gamma_{f,t}\Omega^{SF}_{f,n}\Omega^{G}_{j,n} = 0 \quad \forall j, t\neq 24 \tag{4.3}$$

$$O^{dec}_{j,t} + \lambda_t - \overline{\theta}_{j,t} + \underline{\theta}_{j,t} - \alpha_{j,t} + \beta_{j,t} - \rho_{j,t} + \sum_{f\in N_F,n\in N_N}\gamma_{f,t}\Omega^{SF}_{f,n}\Omega^{G}_{j,n} = 0 \quad \forall j, t = 24 \tag{4.4}$$

$$\varepsilon^{R}_{j,t} + \overline{\theta}_{j,t} + \alpha_{j,t} + \beta_{j,t+1} - \mu_t - \underline{\mu}_{j,t} = 0 \quad \forall j, t\neq 24 \tag{4.5}$$

$$\varepsilon^{R}_{j,t} + \overline{\theta}_{j,t} + \alpha_{j,t} - \mu_t - \underline{\mu}_{j,t} = 0 \quad \forall j, t = 24 \tag{4.6}$$

$$\gamma_{f,t} + \overline{\xi}_{f,t} - \underline{\xi}_{f,t} = 0 \quad \forall f, t \tag{4.7}$$

$$\overline{\xi}_{f,t}, \underline{\xi}_{f,t}, \overline{\theta}_{j,t}, \underline{\theta}_{j,t}, \alpha_{j,t}, \beta_{j,t}, \mu_t \geq 0 \quad \lambda_t, \gamma_{f,t} \tag{4.8}$$

Since the problem is linear, and thus convex, the strong duality must hold which means the objective functions of the primal and dual problems must be equal as represented in Eq. (5):

$$\sum_{t\in N_t,\ell\in N_L} L^I_{\ell,t}\lambda_t - \sum_{t\in N_t,j\in N_G} P^{pri}_{j,t}\lambda_t + \sum_{f\in N_F,t\in N_T}\gamma_{f,t}\sum_{n\in N_N}\Omega^{SF}_{f,n}\sum_{j}\Omega^{G}_{j,n}P^{pri}_{j,t} - \sum_{f\in N_F,t\in N_t,}\left(\overline{\xi}_{f,t}+\underline{\xi}_{f,t}\right)\overline{F}_f$$
$$-\sum_{t\in N_t,j\in N_G}\overline{\theta}_{j,t}\left(\overline{P}_j - P^{pri}_{j,t}\right) - \sum_{t\in N_t,j\in N_G}\underline{\theta}_{j,t}\left(P^{pri}_{j,t}-\underline{P}_j\right) - \sum_{t>1,j\in N_G}\alpha_{j,t}\left(U_j - P^{pri}_{j,t}+P^{pri}_{j,t-1}\right)$$
$$-\sum_{t=1,j\in N_G}\alpha_{j,t}\left(U_j - P^{pri}_{j,t}+P^{0}_{j}\right) - \sum_{t>1,j\in N_G}\beta_{j,t}\left(D_j - P^{pri}_{j,t-1}+P^{pri}_{j,t}\right) + \sum_{f\in N_F,t\in N_T}\gamma_{f,t}\sum_{n\in N_N}\Omega^{SF}_{f,n}\sum_{\ell\in N_L}\Omega^{L}_{l,n}L^I_{\ell,t}$$
$$-\sum_{t=1,j\in N_G}\beta_{j,t}\left(D_j + P^{pri}_{j,t}-P^{o}_{j}\right) + \sum_{t\in N_t}\mu_t\underline{R}_t = \sum_{j,t}O^{inc}_{j,t}P^{inc}_{j,t} + \sum_{j,t}O^{dec}_{j,t}P^{dec}_{j,t} + \sum_{j,t}\varepsilon^{R}_{j,t}R_{j,t} \tag{5}$$

3. Decomposition Theory

3.1. Closed Form of the Decomposed Dual Variables

Having the mathematical model of the security-constrained economic dispatch in place, it is time to propose the theory that needs to be applied. First, the dual variables associated with the constraints of the optimisation problem must be decomposed. In Eq. (6), $C^T X$ is the objective function of a linear program (identical with the optimisation problem in a closed format). E and Θ represent shadow prices associated with equality and non-equality constraints, respectively.

$$\begin{aligned} &\underset{x}{Min}\ \left\{C^T X\right\} \\ &s.t. \\ &AX = B\ :\ E \\ &X \geq D\ :\ \Theta \end{aligned} \tag{6}$$

As seen in Eqs. (7.1) and (7.2), ε_i and θ_j, as the elements of Lagrange multipliers vectors E, Θ are decomposed to the partial derivatives of the terms forming the objective function of the linear program.

$$\varepsilon_i = \frac{\partial\left(C^T X\right)}{\partial b_i} = \sum_{z=1}^{n_\lambda}\frac{\partial\left(c_z x_z\right)}{\partial b_i} = \sum_{z=1}^{n_\lambda}\hat{\varepsilon}^z_i \quad \forall i = 1,...,n_\lambda, \quad c_z\in C, \quad b_i \in B \tag{7.1}$$

$$\theta_j = \frac{\partial\left(C^T X\right)}{\partial d_j} = \sum_{z=1}^{n_\mu}\frac{\partial\left(c_z x_z\right)}{\partial d_j} = \sum_{z=1}^{n_\mu}\hat{\theta}^z_j \quad \forall j = 1,...,n_\mu, \quad c_z\in C, \quad d_j \in D \tag{7.2}$$

It is proved in [29], having the result of the primal problem (i.e., decision variables), and the dual problem (i.e., the dual variables), the decomposed dual variables (i.e., $\hat{\varepsilon}^z_i$ and $\hat{\theta}^z_j$) can be obtained by an ex-post solution of a set of linear equations (see Eqs. (8.1) to (8.5)).

$$x_z = {c_z}^{-1}\left(\sum_{z=1}^{n_x}\hat{\varepsilon}^z_i + \sum_{z=1}^{n_\mu}\hat{\theta}^z_j\right) \quad \forall z = 1,...,n_x, c_z \neq 0 \tag{8.1}$$

$$c_z - \sum_{z=1}^{n_x} a_{z,i}\hat{\varepsilon}^\Delta_i - \hat{\theta}^\Delta_z = 0 \quad \forall z = 1,...,n_x, \quad \Delta = z \tag{8.2}$$

$$\sum_{z=1}^{n_x} a_{z,i}\hat{\varepsilon}^\Delta_i + \hat{\theta}^\Delta_z = 0 \quad \forall z = 1,...,n_x, \quad \Delta \neq z \tag{8.3}$$

$$\sum_{z=1}^{n_\lambda}\hat{\varepsilon}^z_i = \varepsilon_i \quad \forall i = 1,...,n_\lambda \tag{8.4}$$

$$\sum_{z=1}^{n_\mu} \widehat{\theta}_j^z = \theta_j \quad \forall z = 1, ..., n_\mu \tag{8.5}$$

Suppose N_x represents the total number of variables with non-zero coefficients in the objective function. Meanwhile, Z refers to the total number of decision variables, Γ represents the total number of constraints and $\bar{\Gamma}$ indicates the total number of binding constraints. Eq. (8.1) results in N_x equations, Eqs.(8.2) and (8.3) gives $Z.N_x$ equations in total, and finally Eqs. (8.4) and (8.5) provide $\bar{\Gamma}$ equations. The total number of unknown variables. i.e., decomposed dual variables equals $Z.\bar{\Gamma}$. To obtain a unique solution for decomposed dual variables from the set of linear equations represented by Eqs. (8.1) to (8.5), the total number of equations i.e., $N_x + Z.N_x + \bar{\Gamma}$ must be greater than or equal to the number of variables i.e., $Z.\bar{\Gamma}$. Since $\frac{N_x - 1}{N_x} \approx 1$, inequality $Z + 1 \geq \bar{\Gamma}$ must be hold. This indicates that the number of variables in the objective function must be greater than the number of binding constraints which holds true in power system studies. For instance, $F_{f,t}$ represents the line flow decision variable. As represented in Eq. (1.4), only one side of this constraint could be binding. Similarly, for $P^{inc}_{j,t}$ and $P^{inc}_{j,t}$, only one of the inequalities represented by Eqs. (1.5) and (1.6) could be binding. This statement holds also true for inequalities representing the ramp-up and ramp-down constraints for the generation units i.e., Eqs. (1.7) to (1.8).

3.2. Applying the Proposed Decomposition Technique to the BEM Optimisation Problem

In this section, the proposed decomposition technique is applied to the BEM optimisation problem represented by Eqs. (1.1) to (1.14). In this problem, the objective function contains only a subset of decision variables i.e., $P^{inc}_{j,t}$, $P^{dec}_{j,t}$, $R_{j,t}$, and all coefficients associated with $F_{f,t}$ are zero. Therefore, all of the dual variables in the primal problem will be decomposed into the dual variables corresponding to these decision variables. To distinguish the decomposed dual variables from the dual variables accents $|\widehat{\circ}|$, $|\widetilde{\circ}|$, and $|\overset{\frown}{\circ}|$ are applied to the dual variables set (as represented by Eq. (2.2) Π_{DV}). $|\widehat{\circ}|$ corresponds to the decomposed dual variables associated with $P^{inc}_{j,t}$, while $|\widetilde{\circ}|$ and $|\overset{\frown}{\circ}|$ correspond to the decomposed dual variables associated with the $P^{dec}_{j,t}$ and $R_{j,t}$ respectively. Eqs. (4.1) to (4.8) should be rewritten to hold the conditions forced by the linear set of equations as represented by Eqs. (8.2) to (8.3). To show the approach that should be taken to have the correct set of linear equations, Eq. (4.1) is extended to Eqs. (9.1), (9.3), (9.5), and (9.7). Similarly, Eq. (4.2) is also extended to Eqs. (9.2), (9.4), (9.6), and (9.8).

$$O^{inc}_{j,t} - \widehat{\lambda}^{m,h}_t + \widehat{\bar{\theta}}^{m,h}_{j,t} - \widehat{\underline{\theta}}^{m,h}_{j,t} + \widehat{\alpha}^{m,h}_{j,t} - \widehat{\alpha}^{m,h}_{j,t+1} - \widehat{\beta}^{m,h}_{j,t} + \widehat{\beta}^{m,h}_{j,t+1} - \widehat{\vartheta}^{m,h}_{j,t} - \sum_{f \in N_F, n \in N_N,} \Omega^{SF}_{f,n} \Omega^{G}_{j,n} \widehat{\gamma}^{m,h}_{f,t} = 0$$
$$\forall j, \quad t \neq 24, \quad m = j, \quad h = t \tag{9.1}$$

$$O^{inc}_{j,t} - \widehat{\lambda}^{m,h}_t + \widehat{\bar{\theta}}^{m,h}_{j,t} - \widehat{\underline{\theta}}^{m,h}_{j,t} + \widehat{\alpha}^{m,h}_{j,t} - \widehat{\beta}^{m,h}_{j,t} - \widehat{\vartheta}^{m,h}_{j,t} - \sum_{f \in N_F, n \in N_N} \Omega^{SF}_{f,n} \Omega^{G}_{j,n} \widehat{\gamma}^{m,h}_{f,t} = 0 \quad \forall j, \quad t = 24, \quad m = j, \quad h = t \tag{9.2}$$

$$-\widehat{\lambda}^{m,h}_t + \widehat{\bar{\theta}}^{m,h}_{j,t} - \widehat{\underline{\theta}}^{m,h}_{j,t} + \widehat{\alpha}^{m,h}_{j,t} - \widehat{\alpha}^{m,h}_{j,t+1} - \widehat{\beta}^{m,h}_{j,t} + \widehat{\beta}^{m,h}_{j,t+1} - \widehat{\vartheta}^{m,h}_{j,t} - \sum_{f \in N_F, n \in N_N} \Omega^{SF}_{f,n} \Omega^{G}_{j,n} \widehat{\gamma}^{m,h}_{f,t} = 0 \quad \forall j, \quad t \neq 24, \quad m \neq j, \quad h \neq t \tag{9.3}$$

$$-\widehat{\lambda}^{m,h}_t + \widehat{\bar{\theta}}^{m,h}_{j,t} - \widehat{\underline{\theta}}^{m,h}_{j,t} + \widehat{\alpha}^{m,h}_{j,t} - \widehat{\beta}^{m,h}_{j,t} - \widehat{\vartheta}^{m,h}_{j,t} - \sum_{f \in N_F, n \in N_N} \Omega^{SF}_{f,n} \Omega^{G}_{j,n} \widehat{\gamma}^{m,h}_{f,t} = 0 \quad \forall j, \quad t = 24, \quad m \neq j, \quad h \neq t \tag{9.4}$$

$$-\widetilde{\lambda}^{m,h}_t + \widetilde{\bar{\theta}}^{m,h}_{j,t} - \widetilde{\underline{\theta}}^{m,h}_{j,t} + \widetilde{\alpha}^{m,h}_{j,t} - \widetilde{\alpha}^{m,h}_{j,t+1} - \widetilde{\beta}^{m,h}_{j,t} + \widetilde{\beta}^{m,h}_{j,t+1} - \widetilde{\vartheta}^{m,h}_{j,t} - \sum_{f \in N_F, n \in N_N} \Omega^{SF}_{f,n} \Omega^{G}_{j,n} \widetilde{\gamma}^{m,h}_{f,t} = 0 \quad \forall j, \quad t \neq 24, m, h \tag{9.5}$$

$$-\widetilde{\lambda}^{m,h}_t + \widetilde{\bar{\theta}}^{m,h}_{j,t} - \widetilde{\underline{\theta}}^{m,h}_{j,t} + \widetilde{\alpha}^{m,h}_{j,t} - \widetilde{\beta}^{m,h}_{j,t} - \widetilde{\vartheta}^{m,h}_{j,t} - \sum_{f \in N_F, n \in N_N} \Omega^{SF}_{f,n} \Omega^{G}_{j,n} \widetilde{\gamma}^{m,h}_{f,t} = 0 \quad \forall j, t = 24, m, h \tag{9.6}$$

$$-\overset{\frown}{\lambda}{}^{m,h}_t + \overset{\frown}{\bar{\theta}}{}^{m,h}_{j,t} - \overset{\frown}{\underline{\theta}}{}^{m,h}_{j,t} + \overset{\frown}{\alpha}{}^{m,h}_{j,t} - \overset{\frown}{\alpha}{}^{m,h}_{j,t+1} - \overset{\frown}{\beta}{}^{m,h}_{j,t} + \overset{\frown}{\beta}{}^{m,h}_{j,t+1} - \overset{\frown}{\vartheta}{}^{m,h}_{j,t} - \sum_{f \in N_F, n \in N_N} \Omega^{SF}_{f,n} \Omega^{G}_{j,n} \overset{\frown}{\gamma}{}^{m,h}_{f,t} = 0 \quad \forall j, t \neq 24, m, h \tag{9.7}$$

$$-\overset{\frown}{\lambda}_{t}^{m,h}+\overset{\frown}{\overline{\theta}}_{j,t}^{m,h}-\overset{\frown}{\underline{\theta}}_{j,t}^{m,h}+\overset{\frown}{\alpha}_{j,t}^{m,h}-\overset{\frown}{\beta}_{j,t}^{m,h}-\overset{\frown}{\vartheta}_{j,t}^{m,h}-\sum_{f\in N_F,n\in N_N}\Omega^{SF}_{f,n}\Omega^{G}_{j,n}\overset{\frown}{\gamma}_{f,t}^{m,h}=0 \quad \forall j,t=24,m,h \tag{9.8}$$

To have the complete set of the required linear equations, Eqs. (4.3) to (4.7) must be extended using a similar approach. Eqs. (8.4) and (8.5) require that the summation of the decomposed dual variables corresponding to a dual variable equals that dual variable. As an example, Eqs. (10.1) to (10.3) represent the required equality constraints for λ_t, $\overline{\theta}_{j,t}$, and $\underline{\theta}_{j,t}$. To have the complete set of equations, similar equality constraints should be held for all of the dual variables (Π_{DV}) as represented in Eq. (2.2).

$$\sum_{m\in N_G,h\in N_T}\left(\hat{\lambda}_{t}^{m,h}+\tilde{\lambda}_{t}^{m,h}+\overset{\frown}{\lambda}_{n,t}^{m,h}\right)=\lambda_t \quad \forall t \tag{10.1}$$

$$\sum_{m\in N_G,h\in N_T}\left(\hat{\overline{\theta}}_{j,t}^{m,h}+\tilde{\overline{\theta}}_{j,t}^{m,h}+\overset{\frown}{\overline{\theta}}_{j,t}^{m,h}\right)=\overline{\theta}_{j,t} \quad \forall j,t \tag{10.2}$$

$$\sum_{m\in N_G,h\in N_T}\left(\hat{\underline{\theta}}_{j,t}^{m,h}+\tilde{\underline{\theta}}_{j,t}^{m,h}+\overset{\frown}{\underline{\theta}}_{j,t}^{m,h}\right)=\underline{\theta}_{j,t} \quad \forall j,t \tag{10.3}$$

Then, applying Eq. (8.1) to the strong duality constraint enables us to comprehensively retrace the effect of all binding constraints on the allocated value to the decision variables. For instance, for the incremental deviation i.e. $P^{inc}_{m,h}$, where m refers to the generating unit and h represents the time; the equality constraint (11) must be held:

$$\begin{aligned}
O^{inc}_{m,h}P^{inc}_{m,h}=&\sum_{t\in N_t,\ell\in N_L}L^{I}_{\ell,t}\hat{\lambda}_{t}^{m,h}-\sum_{t\in N_t,j\in N_G}P^{pri}_{j,t}\hat{\lambda}_{t}^{m,h}-\sum_{f,t}\hat{\gamma}_{f,t}^{m,h}\left[\sum_{n\in N_F}\Omega^{SF}_{f,n}\left(\sum_{j}\Omega^{G}_{j,n}P^{pri}_{j,t}-\sum_{\ell\in N_L}\Omega^{L}_{l,n}L^{I}_{\ell,t}\right)\right]\\
&-\sum_{t\in N_t,j\in N_G}\hat{\overline{\theta}}_{j,t}^{m,h}\left(\overline{P}_j-P^{pri}_{j,t}\right)-\sum_{t\in N_t,j\in N_G}\hat{\underline{\theta}}_{j,t}^{m,h}\left(P^{pri}_{j,t}-\underline{P}_j\right)-\sum_{t>1,j\in N_G}\hat{\alpha}_{j,t}^{m,h}\left(U_j-P^{pri}_{j,t}+P^{pri}_{j,t-1}\right)\\
&-\sum_{t=1,j\in N_G}\hat{\alpha}_{j,t}^{m,h}\left(U_j-P^{pri}_{j,t}+P^{0}_{j}\right)-\sum_{t>1,j\in N_G}\hat{\beta}_{j,t}^{m,h}\left(D_j-P^{pri}_{j,t-1}+P^{pri}_{j,t}\right)-\sum_{f\in N_F,t\in N_t,}\left(\hat{\overline{\xi}}_{f,t}^{m,h}+\hat{\underline{\xi}}_{f,t}^{m,h}\right)\overline{F}_f\\
&-\sum_{t=1,j\in N_G}\hat{\beta}_{j,t}^{m,h}\left(D_j+P^{pri}_{j,t}-P^{o}_{j}\right)+\sum_{t\in N_t}\hat{\mu}_{t}^{m,h}\underline{R}_t
\end{aligned} \tag{11}$$

Eq. (11) represents the contribution of decomposed dual variables to the total incremental deviation (i.e., $P^{inc}_{m,h}$) allocated to the generators. When a binding constraint results in a non-zero dual variable, the associated decomposed dual variables will also take on values. As seen in Equitation (11), for instance if for generator m_1, at time h_1, (i.e., j_1, at time t_1), then $\hat{\alpha}_{j_5,t_3}^{m_1,h_1}$, takes a non-zero value. This means the binding ramp up rate constraint of generator j_5 at time t_3 contributes to the incremental deviation allocated to generator j_1 at time t_1. A similar explanation is valid for other decomposed dual variables. In fact, Eq. (11) is mapping the binding constraints to the decision variable $P^{inc}_{m,h}$. Similarly, for the decremental deviation i.e. $P^{dec}_{m,h}$, the decomposed strong duality must be as represented by Eq. (12):

$$\begin{aligned}
O^{dec}_{m,h}P^{dec}_{m,h}=&\sum_{t\in N_t,\ell\in N_L}L^{I}_{\ell,t}\tilde{\lambda}_{t}^{m,h}-\sum_{t\in N_t,j\in N_G}P^{pri}_{j,t}\tilde{\lambda}_{t}^{m,h}-\sum_{f,t}\tilde{\gamma}_{f,t}^{m,h}\left[\sum_{n\in N_F}\Omega^{SF}_{f,n}\left(\sum_{j}\Omega^{G}_{j,n}P^{pri}_{j,t}-\sum_{\ell\in N_L}\Omega^{L}_{l,n}L^{I}_{\ell,t}\right)\right]\\
&-\sum_{t\in N_t,j\in N_G}\tilde{\overline{\theta}}_{j,t}^{m,h}\left(\overline{P}_j-P^{pri}_{j,t}\right)-\sum_{t\in N_t,j\in N_G}\tilde{\underline{\theta}}_{j,t}^{m,h}\left(P^{pri}_{j,t}-\underline{P}_j\right)-\sum_{t>1,j\in N_G}\tilde{\alpha}_{j,t}^{m,h}\left(U_j-P^{pri}_{j,t}+P^{pri}_{j,t-1}\right)\\
&-\sum_{t=1,j\in N_G}\tilde{\alpha}_{j,t}^{m,h}\left(U_j-P^{pri}_{j,t}+P^{0}_{j}\right)-\sum_{t>1,j\in N_G}\tilde{\beta}_{j,t}^{m,h}\left(D_j-P^{pri}_{j,t-1}+P^{pri}_{j,t}\right)-\sum_{f\in N_F,t\in N_t,}\left(\tilde{\overline{\xi}}_{f,t}^{m,h}+\tilde{\underline{\xi}}_{f,t}^{m,h}\right)\overline{F}_f\\
&-\sum_{t=1,j\in N_G}\tilde{\beta}_{j,t}^{m,h}\left(D_j+P^{pri}_{j,t}-P^{o}_{j}\right)+\sum_{t\in N_t}\tilde{\mu}_{t}^{m,h}\underline{R}_t
\end{aligned} \tag{12}$$

And finally for the last decision variable i.e., $R_{m,h}$ the decomposed strong duality can be written as Eq. (13):

$$\begin{aligned}
\varepsilon^{R}_{m,h}R_{m,h} = &\sum_{t\in N_t,\ell\in N_L} L^{I}_{\ell,t}\widehat{\lambda}^{m,h}_{t} - \sum_{t\in N_t,j\in N_G} P^{pri}_{j,t}\widehat{\lambda}^{m,h}_{t} - \sum_{f,t}\widehat{\gamma}^{m,h}_{f,t}\left[\sum_{n\in N_F}\Omega^{SF}_{f,n}\left(\sum_{j}\Omega^{G}_{j,n}P^{pri}_{j,t} - \sum_{\ell\in N_L}\Omega^{L}_{l,n}L^{I}_{\ell,t}\right)\right] \\
&- \sum_{t\in N_t,j\in N_G}\widehat{\bar{\theta}}^{m,h}_{j,t}\left(\overline{P}_{j} - P^{pri}_{j,t}\right) - \sum_{t\in N_t,j\in N_G}\widehat{\underline{\theta}}^{m,h}_{j,t}\left(P^{pri}_{j,t} - \underline{P}_{j}\right) - \sum_{t>1,j\in N_G}\widehat{\alpha}^{m,h}_{j,t}\left(U_{j} - P^{pri}_{j,t} + P^{pri}_{j,t-1}\right) \\
&- \sum_{t=1,j\in N_G}\widehat{\alpha}^{m,h}_{j,t}\left(U_{j} - P^{pri}_{j,t} + P^{0}_{j}\right) - \sum_{t>1,j\in N_G}\widehat{\beta}^{m,h}_{j,t}\left(D_{j} - P^{pri}_{j,t-1} + P^{pri}_{j,t}\right) - \sum_{f\in N_F,t\in N_t,}\left(\widehat{\bar{\xi}}^{m,h}_{f,t} + \widehat{\underline{\xi}}^{m,h}_{f,t}\right)\overline{F}_{f} \\
&- \sum_{t=1,j\in N_G}\widehat{\beta}^{m,h}_{j,t}\left(D_{j} + P^{pri}_{j,t} - P^{o}_{j}\right) + \sum_{t\in N_t}\widehat{\mu}^{m,h}_{t}\underline{R}_{t}
\end{aligned} \tag{13}$$

Rearranging Eq. (11) reveals the contribution of each binding constraint to the final value of the incremental deviation allocated to each generation unit ($P^{inc}_{m,h}$). Eqs. (14.1) to (14.4) specify how binding network constraints contribute to the incremental deviation assigned to each generation unit. For instance, $\left[P^{inc}_{m,h}\right]^{\widehat{\bar{\xi}}^{m,h}_{f,t}}$ indicates the amount of incremental deviation allocated to generation unit m at time h due to the binding line flow constraint of the transmission lines. As illustrated in Eqs. (14.3) and (14.4), the granularity of the analysis can be increased to focus on a specific transmission line. For example, if one wishes to determine the impact of line f_1 maximum flow limit at time t_{10} on the incremental deviation assigned to generator m_1 at time h_{15} it can be readily calculated as: $\left[P^{inc}_{m_1,h_{15}}\right]^{\widehat{\bar{\xi}}^{m_1,h_{15}}_{f_1,t_{10}}} = -\left(O^{inc}_{m_1,h_{15}}\right)^{-1}\widehat{\bar{\xi}}^{m_1,h_{15}}_{f_1,t_{10}}\overline{F}_{f_1}$.

$$\left[P^{inc}_{m,h}\right]^{\widehat{\lambda}^{m,h}_{t}} = \left(O^{inc}_{m,h}\right)^{-1}\left(\sum_{t\in N_t,\ell\in N_L}\widehat{\lambda}^{m,h}_{t}L^{I}_{\ell,t} - \sum_{t\in N_t,j\in N_G}\widehat{\lambda}^{m,h}_{t}P^{pri}_{j,t}\right) \tag{14.1}$$

$$\left[P^{inc}_{m,h}\right]^{\widehat{\gamma}^{m,h}_{f,t}} = -\left(O^{inc}_{m,h}\right)^{-1}\sum_{f,t}\widehat{\gamma}^{m,h}_{f,t}\left[\sum_{n\in N_F}\Omega^{SF}_{f,n}\left(\sum_{j}\Omega^{G}_{j,n}P^{pri}_{j,t} - \sum_{\ell\in N_L}\Omega^{L}_{l,n}L^{I}_{\ell,t}\right)\right] \tag{14.2}$$

$$\left[P^{inc}_{m,h}\right]^{\widehat{\bar{\xi}}^{m,h}_{f,t}} = -\left(O^{inc}_{m,h}\right)^{-1}\sum_{f\in N_F,t\in N_t,}\left(\widehat{\bar{\xi}}^{m,h}_{f,t}\right)\overline{F}_{f} \tag{14.3}$$

$$\left[P^{inc}_{m,h}\right]^{\widehat{\underline{\xi}}^{m,h}_{f,t}} = -\left(O^{inc}_{m,h}\right)^{-1}\sum_{f\in N_F,t\in N_t,}\left(\widehat{\underline{\xi}}^{m,h}_{f,t}\right)\overline{F}_{f} \tag{14.4}$$

Eq. (15) quantifies the network reserve requirements' contribution to the incremental deviation allocated to generation unit m at time h. Specifically, it answers the question: "What is the contribution of the network reserve requirement to the allocated FEP deviations for a given generation unit in the BEM?" This is achieved by summing over the time t, providing a precise calculation of the network reserve requirement's impact on the allocated deviations.

$$\left[P^{inc}_{m,h}\right]^{\widehat{\mu}^{m,h}_{t}} = \left(O^{inc}_{m,h}\right)^{-1}\sum_{t\in N_t}\widehat{\mu}^{m,h}_{t}\underline{R}_{t} \tag{15}$$

Eqs. (16.1) to (16.4) give the contribution of the internal constraints of a generator on its own incremental deviation since $m = j$. Similarly, Eqs. (17.1) to (17.4) obtain the contribution of the rival generator constraints on the allocated incremental deviation to the generator m since $m \neq j$.

$$\left[P^{inc}_{m,h}\right]^{\widehat{\bar{\theta}}^{m,h}_{j,t}} = -\left(O^{inc}_{m,h}\right)^{-1}\sum_{t\in N_t,j\in N_G}\widehat{\bar{\theta}}^{m,h}_{j,t}\left(\overline{P}_{j} - P^{pri}_{j,t}\right) \quad ,m = j \tag{16.1}$$

$$\left[P^{inc}_{m,h}\right]^{\underline{\hat{\theta}}^{m,h}_{j,t}} = -\left(O^{inc}_{m,h}\right)^{-1} \sum_{t\in N_t, j\in N_G} \underline{\hat{\theta}}^{m,h}_{j,t} \left(P^{pri}_{j,t} - \underline{P}_j\right) \quad , m = j \tag{16.2}$$

$$\left[P^{inc}_{m,h}\right]^{\hat{\alpha}^{m,h}_{j,t}} = -\left(O^{inc}_{m,h}\right)^{-1} \left[\sum_{t>1, j\in N_G} \hat{\alpha}^{m,h}_{j,t} \left(U_j - P^{pri}_{j,t} + P^{pri}_{j,t-1}\right) + \sum_{t=1, j\in N_G} \hat{\alpha}^{m,h}_{j,t} \left(U_j - P^{pri}_{j,t} + P^{0}_{j}\right)\right] \quad , m = j \tag{16.3}$$

$$\left[P^{inc}_{m,h}\right]^{\hat{\beta}^{m,h}_{j,t}} = -\left(O^{inc}_{m,h}\right)^{-1} \left[\sum_{t>1, j\in N_G} \hat{\beta}^{m,h}_{j,t} \left(D_j - P^{pri}_{j,t-1} + P^{pri}_{j,t}\right) + \sum_{t=1, j\in N_G} \hat{\beta}^{m,h}_{j,t} \left(D_j + P^{pri}_{j,t} - P^{o}_{j}\right)\right] \quad , m = j \tag{16.4}$$

Eq. (17.3) gives the contribution of the ramp-up constraints of the rivals on the incremental deviation allocated to the generator m at time h. The effect of ramp-up constraint for every single generation unit at every time on the incremental deviation allocated to the generator m at time h is readily obtainable by using the right-hand side of the Eq. (17.3). As another example, Eq. (16.3) gives the contribution of minimum power constraint of the generator m to its own $P^{inc}_{m,h}$.

$$\left[P^{inc}_{m,h}\right]^{\hat{\overline{\theta}}^{m,h}_{j,t}} = -\left(O^{inc}_{m,h}\right)^{-1} \sum_{t\in N_t, j\in N_G} \hat{\overline{\theta}}^{m,h}_{j,t} \left(\overline{P}_j - P^{pri}_{j,t}\right) \quad , m \neq j \tag{17.1}$$

$$\left[P^{inc}_{m,h}\right]^{\underline{\hat{\theta}}^{m,h}_{j,t}} = -\left(O^{inc}_{m,h}\right)^{-1} \sum_{t\in N_t, j\in N_G} \underline{\hat{\theta}}^{m,h}_{j,t} \left(P^{pri}_{j,t} - \underline{P}_j\right) \quad , m \neq j \tag{17.2}$$

$$\left[P^{inc}_{m,h}\right]^{\hat{\alpha}^{m,h}_{j,t}} = -\left(O^{inc}_{m,h}\right)^{-1} \left[\sum_{t>1, j\in N_G} \hat{\alpha}^{m,h}_{j,t} \left(U_j - P^{pri}_{j,t} + P^{pri}_{j,t-1}\right) + \sum_{t=1, j\in N_G} \hat{\alpha}^{m,h}_{j,t} \left(U_j - P^{pri}_{j,t} + P^{0}_{j}\right)\right] \quad , m \neq j \tag{17.3}$$

$$\left[P^{inc}_{m,h}\right]^{\hat{\beta}^{m,h}_{j,t}} = -\left(O^{inc}_{m,h}\right)^{-1} \left[\sum_{t>1, j\in N_G} \hat{\beta}^{m,h}_{j,t} \left(D_j - P^{pri}_{j,t-1} + P^{pri}_{j,t}\right) + \sum_{t=1, j\in N_G} \hat{\beta}^{m,h}_{j,t} \left(D_j + P^{pri}_{j,t} - P^{o}_{j}\right)\right] \quad , m \neq j \tag{17.4}$$

3.3. Classifying the Main Contributors to the Allocated FEP Deviations

$\left[P^{inc}_{m,h}\right]^{Network}$ can be defined as the contribution of the network constraints to the incremental deviation allocated to the generator m at time h. As is represented in the right-hand side of Eq. (18.1); network constraints contribute to the decision variables of the primal optimisation problem by the dual variables associated with the power balance equality constraint ($\left[P^{inc}_{m,h}\right]^{\hat{\lambda}^{m,h}_{t}}$), line flows ($\left[P^{inc}_{m,h}\right]^{\hat{\gamma}^{m,h}_{f,t}}$), and line flow limits ($\left[P^{inc}_{m,h}\right]^{\hat{\overline{\xi}}^{m,h}_{f,t}}$ and $\left[P^{inc}_{m,h}\right]^{\underline{\hat{\xi}}^{m,h}_{f,t}}$).

$$\left[P^{inc}_{m,h}\right]^{Network} = \left[P^{inc}_{m,h}\right]^{\hat{\lambda}^{m,h}_{t}} + \left[P^{inc}_{m,h}\right]^{\hat{\gamma}^{m,h}_{f,t}} + \left[P^{inc}_{m,h}\right]^{\hat{\overline{\xi}}^{m,h}_{f,t}} + \left[P^{inc}_{m,h}\right]^{\underline{\hat{\xi}}^{m,h}_{f,t}} \tag{18.1}$$

$$\left[P^{inc}_{m,h}\right]^{Itself} = \left[P^{inc}_{m,h}\right]^{\hat{\overline{\theta}}^{m,h}_{j,t}} + \left[P^{inc}_{m,h}\right]^{\underline{\hat{\theta}}^{m,h}_{j,t}} + \left[P^{inc}_{m,h}\right]^{\hat{\alpha}^{m,h}_{j,t}} + \left[P^{inc}_{m,h}\right]^{\hat{\beta}^{m,h}_{j,t}} , m = j \tag{18.2}$$

$$\left[P^{inc}_{m,h}\right]^{Rivals} = \left[P^{inc}_{m,h}\right]^{\hat{\overline{\theta}}^{m,h}_{j,t}} + \left[P^{inc}_{m,h}\right]^{\underline{\hat{\theta}}^{m,h}_{j,t}} + \left[P^{inc}_{m,h}\right]^{\hat{\alpha}^{m,h}_{j,t}} + \left[P^{inc}_{m,h}\right]^{\hat{\beta}^{m,h}_{j,t}} , m \neq j \tag{18.3}$$

$$\left[P^{inc}_{m,h}\right]^{Reserve} = \left[P^{inc}_{m,h}\right]^{\hat{\mu}^{m,h}_{t}} \tag{18.4}$$

Similarly, the contribution of internal constraints to the allocated incremental deviation to a generator (i.e., $\left[P^{inc}_{m,h}\right]^{Itself}$) can be readily obtained by Eq. (18.2). In addition, Eq. (18.3) gives the contribution of rivals' internal constraints on the allocated incremental deviation to a generator (i.e., $\left[P^{inc}_{m,h}\right]^{Rivals}$). Eq. (18.4) is very handy to indicate how the allocated power to a certain generator can be increased in order to free up the head-rooms of the other generator enabling them to provide reserve service. Applying the same method to the decomposed strong duality equalitiy constraints corresponding to $P^{dec}_{m,h}$ and $R_{m,h}$, the contribution of the binding constraints to the remaining decision variables i.e., for the decremental deviation decision variable $P^{dec}_{m,h}$) and reserve $R^{dec}_{m,h}$ can be obtained.

3.4. Retracing the Effect of Reserve on the Incremental/Decremental Deviations Allocated in the BEM

In order to provide a correct and accurate answer to "what is the contribution of the network reserve requirement to the allocated FEP deviations to a certain generation unit in the BM", the following parameters are defined.

$$\Delta_{j,t} = P^{inc}_{j,t} - P^{dec}_{j,t} \tag{19.1}$$

$$\Delta^{Itself}_{j,t} = \left[P^{inc}_{j,t}\right]^{Itself} - \left[P^{dec}_{j,t}\right]^{Itself} \tag{19.2}$$

$$\Delta^{Rivals}_{j,t} = \left[P^{inc}_{j,t}\right]^{Rivals} - \left[P^{dec}_{j,t}\right]^{Rivals} \tag{19.3}$$

$$\Delta^{Network}_{j,t} = \left[P^{inc}_{j,t}\right]^{Network} - \left[P^{dec}_{j,t}\right]^{Network} \tag{19.4}$$

$$\Delta^{Reserve}_{j,t} = \left[P^{inc}_{j,t}\right]^{Reserve} - \left[P^{dec}_{j,t}\right]^{Reserve} \tag{19.5}$$

To measure the effect of each constraint set (i.e., *Network*, *Reserve*, *Itself*, and *Rivals*) on the total allocated deviation, the following parameters are defined:

$$\Psi^{Reserve}_{j,t} = \max\left\{\left(100\Delta^{Reserve}_{j,t}/\Delta_{j,t}\right), 0\right\} \tag{20.1}$$

$$\Psi^{Network}_{j,t} = \max\left\{\left(100\Delta^{Network}_{j,t}/\Delta_{j,t}\right), 0\right\} \tag{20.2}$$

$$\Psi^{Itself}_{j,t} = \max\left\{\left(100\Delta^{Network}_{j,t}/\Delta_{j,t}\right), 0\right\} \tag{20.3}$$

$$\Psi^{Rivals}_{j,t} = \max\left\{\left(100\Delta^{Rivals}_{j,t}/\Delta_{j,t}\right), 0\right\} \tag{20.4}$$

$$\Psi^{Total}_{j,t} = \Psi^{Network}_{j,t} + \Psi^{Itself}_{j,t} + \Psi^{Rivals}_{j,t} + \Psi^{Reserve}_{j,t} \tag{20.5}$$

These parameters quantify the deviation contributions that align with the overall deviation. It means, if $P^{inc}_{j,t} - P^{dec}_{j,t} \geq 0$, the operator ($\Psi$) quantifies the contribution of constraints that have positive contribution to the total deviation. For instance, if $\Delta^{Reserve}_{j,t} < 0$ and $P^{inc}_{j,t} - P^{dec}_{j,t} \geq 0$, reserve constraint does no contribution to the total positive deviation that has been allocated in the real-time. Based on the explained logic behind the definition of the parameters, the contribution of constraint sets aligned with the total allocated deviation can be obtained by Eqs. (21.1) to (21.3):

$$\Gamma^{Reserve}_{j,t} = \Psi^{Reserve}_{j,t}/\Psi^{Total}_{j,t} \tag{21.1}$$

$$\Gamma^{Network}_{j,t} = \Psi^{Network}_{j,t}/\Psi^{Total}_{j,t} \tag{21.2}$$

$$\Gamma^{Itself}_{j,t} = \Psi^{Itself}_{j,t}/\Psi^{Total}_{j,t} \tag{21.3}$$

$$\Gamma^{Rivals}_{j,t} = \Psi^{Rivals}_{j,t}/\Psi^{Total}_{j,t} \tag{21.4}$$

$\Gamma^{Reserve}_{j,t}$ (in percent) is the contribution of reserve requirement to the total deviation allocated in the BEM to the generator j at time t. As a result, the proposed theory classifies the root causes of allocating FEP deviations to the generators into four different categories: reserve, network, self, and rivals. Figure 1 illustrates the outcomes of the proposed decomposition theory for retracing the root causes of allocated FEP deviations in the BEM.

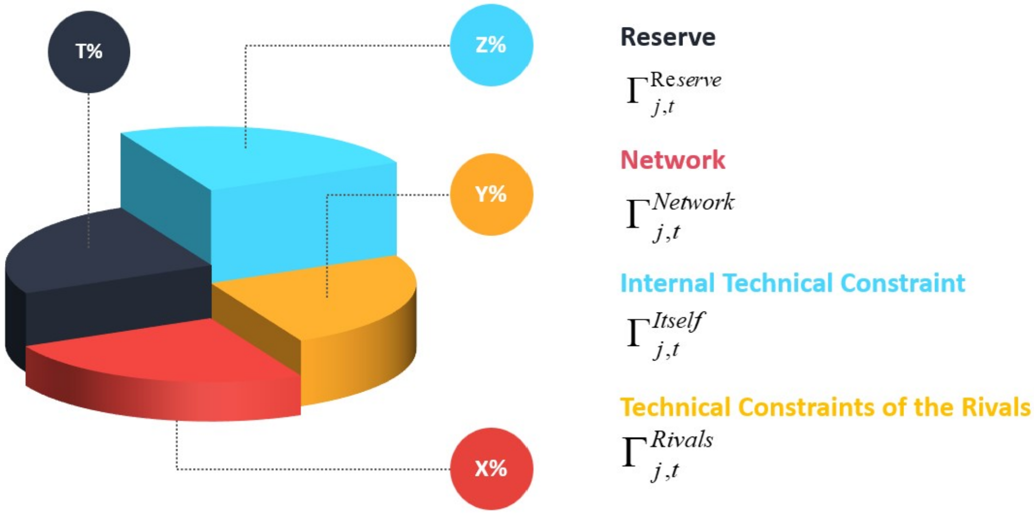


Figure 1: Contributing factors in the total allocated deviation with respect to the FEPs in the BEM

Table 2
Contribution of reserve requirement to the total deviation allocated to the generators [%]

Generator	t_8	t_9	t_{10}	t_{11}	t_{12}	t_{13}	t_{14}	t_{15}	t_{16}	t_{17}	t_{18}	t_{19}	t_{20}	t_{21}
j_1	100	100	100	100	100	100	100	100	96.16	64.13	0	96.18	0	0
j_2	0	0	0	0	0	0	0	0	0	0	0	0	0	0
j_3	0	0	0	0	0	0	0	0	0	0	0	0	0	0
j_4	0	1.08	8.58	8.58	8.58	0	0	0	13.49	22.65	100	29.28	100	100
j_5	0	0	0	0	0	0	0	0	0	0	0	0	0	0

4. Case Study and Discussion

In this section, the proposed theory is applied to IEEE 24 bus test system [34].

4.1. Scenario 1

In this scenario, the reserve requirement is 20% of the total load per interval. Appendix lists generator bids, offers in BEM with FEPs. Total demand is procured in ex-ante markets, but BEM constraints and reserves mean deviations must still be managed.

4.1.1. Analysing the Results

Figure 2 illustrates the total deviation ($\Delta_{j,t} = P_{j,h}^{inc} - P_{j,h}^{dec}$) with respect to the FEPs allocated to each generation unit. It is notable that no deviation has been allocated to $j \in \{j_6, j_7, j_8, j_{10}, j_{11}\}$. Table 2 contains $\Gamma_{j,t}^{Reserve}$ for all of the generators whose reserve requirement has contributed to their allocated FEP deviation in the BEM.

Case 1

Figure 3 illustrates the actual reserve allocated to the generators. At $t = 8$ generator j_1 is allocated 0.12 p.u. reserve (see Figure 3), and the total deviation allocated to this generation unit is 0.2 p.u. (see Figure 2). Second row of Table 2, at $t = 8$, $\Gamma_{j_1,t_8}^{Reserve} = 100\%$indicating the grid's reserve requirement allocates a 0.2 p.u. deviation to generator 0.2 p.u. to the generator j_1 in the BEM. As expected, the total deviation in the BEM doesn't always align with the reserve allocation for a generator. For instance, optimising may involve increasing the output of one unit to free up capacity in another for providing reserves. Also, due to ramping constraints and intertemporal interactions, FEP deviations may

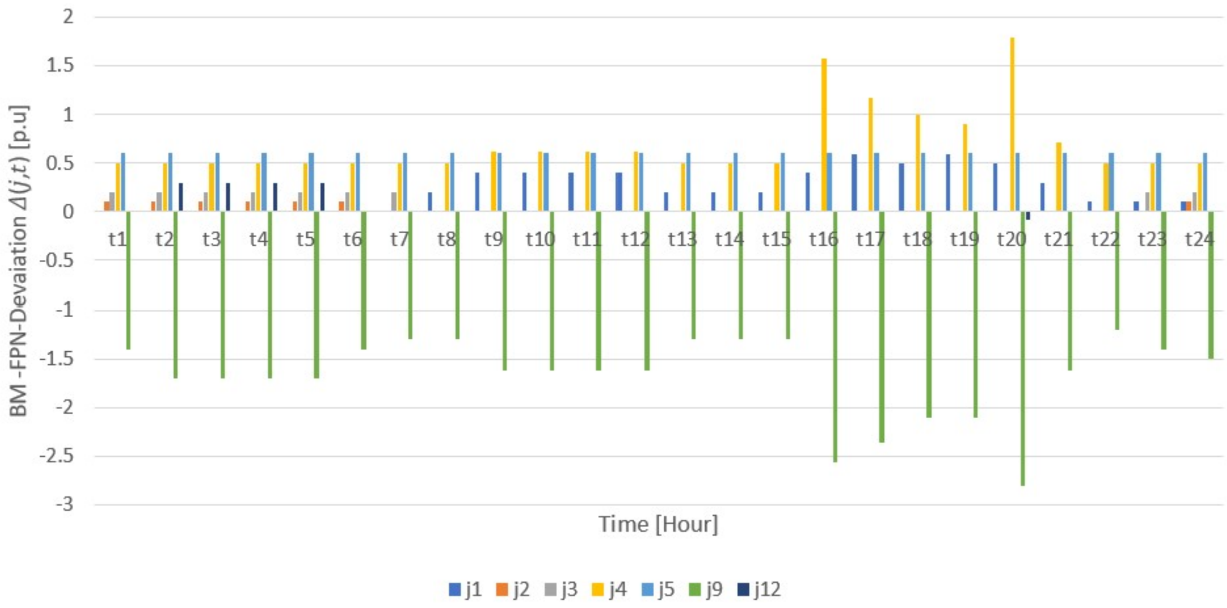


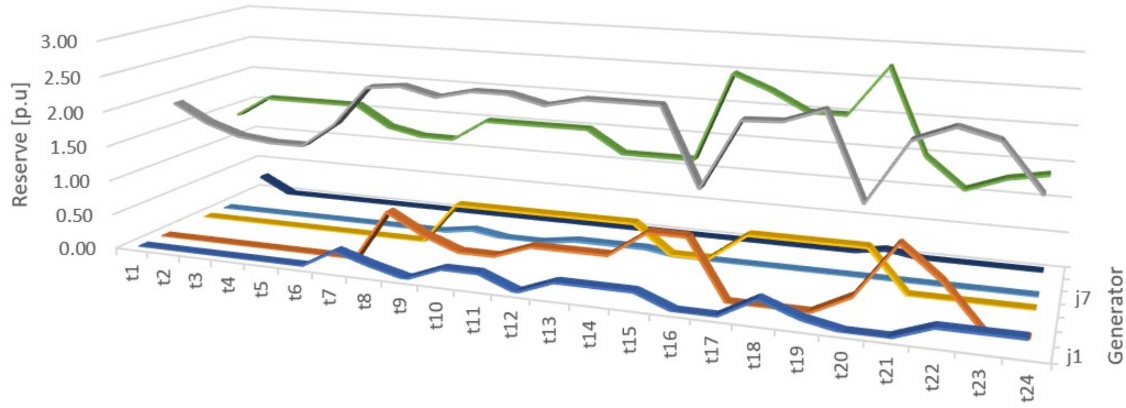
Figure 2: Total Deviation allocated to each generation unit in the BEM under scenario 1

occur at one time while reserve allocations occur at another. This influences the allocation of incremental deviations to generator j_1 at $t = 8$. Firstly Eq. (15) should be considered which is $\left[P^{inc}_{m,h}\right]^{\widehat{\mu}_t^{m,h}} = \left(O^{inc}_{m,h}\right)^{-1} \sum_{t\in N_t} \widehat{\mu}_t^{m,h} \underline{R}_t$. The value of the decomposed dual variables associated with the reserve constraint corresponds to the incremental deviation decision variable (i.e., $\widehat{\mu}_t^{m,h}$). Analyzing the optimisation results and applying the proposed decomposition theory reveals that $\widehat{\mu}_{t_9}^{j_1,t_8}$ takes a positive value, which means $P^{inc}_{j_1,t_8}$ has positive sensitivity to the binding reserve constraint at $t = 9$. In other words, increasing the reserve requirement at $t = 9$ will allocate more incremental deviation to generator j_1 at $t = 8$. In Case 1, a key observation is that within the BEM, FEP deviation can be assigned to a generator due to reserve constraints in a specific trading period. However, the allocated deviation may not necessarily match its reserve provision for that period.

Case 2

To bridge theory with practical understanding, another intriguing case can be explored. $\Gamma^{Reserve}_{j_1,t_9} = 100\%$ reveals that the root cause of the allocated FEP deviation ($P^{inc}_{j_1,t_9} - P^{dec}_{j_1,t_9} = 0.4$ p.u. see Figure 2) is the reserve requirement, however, looking at Figure 3, it is obvious that $R_{j_1,t_9} = 0$. To retrace the root cause of allocating an incremental deviation to j_1 at $t = 1$, firstly Eq. (15) should be considered.The value of the decomposed dual variables associated with the reserve constraint corresponds to the incremental deviation decision variable (i.e., $\widehat{\mu}_t^{m,h}$). It is ovserved that $\widehat{\mu}_{t_9}^{j_1,t_9}$ takes a positive value. This indicates $P^{inc}_{j_1,t_9}$ has positive sensitivity to the binding reserve constraint at $t = 9$. Slightly increasing the reserve requirement at $t = 9$, will aloate more incremental deviation to the generatorj_1 at $t = 9$. To examine the practical rationale behind attributing the reserve as the sole contributor to allocation $\Delta_{j_1,t_9} = P^{inc}_{j_1,t_9} - P^{dec}_{j_1,t_9} = 0.39578\,p.u.$, the status of generation units at $t = 9$ should be considered because $\widehat{\mu}_{t_9}^{j_1,t_9}$, indicates that the reserve requirement at $t = 9$ has caused allocating such deviation to j_1. It is obvserved that, $\sum_j(\Delta_{j,t_9} + P^{pri}_{j,t_9}) - \sum_j(\Delta_{j,t_8} - P^{pri}_{j,t_8}) = 2.38545\,p.u.$. This results in a 2.39 p.u. increase in total demand from $t = 8$ to $t = 9$requiring an additional 0.47709 p.u. reserve, which is 20% of the total increase in the load.

As seen in Table 3, the maximum power capability constraints are binding for both of j_7 and j_9 at $t = 9$. From Table 4, it is observed that $P_{j_7,t_9} = 1.42279\,p.u.$ and it is exactly $0.077721 p.u.$ below its maximum power capability (i.e., $\overline{P}_{j7}$). Again from Table 4, it is observed that $R_{j,t_9} = 0.077721\,p.u.$. It means the optimisation has not allocated



	t1	t2	t3	t4	t5	t6	t7	t8	t9	t10	t11	t12	t13	t14	t15	t16	t17	t18	t19	t20	t21	t22	t23	t24
j1	0.00	0.00	0.00	0.00	0.00	0.00	0.28	0.12	0.00	0.20	0.20	0.00	0.20	0.20	0.20	0.00	0.00	0.30	0.10	0.00	0.00	0.20	0.20	0.20
j3	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.74	0.46	0.28	0.28	0.46	0.46	0.46	0.83	0.83	0.00	0.00	0.00	0.28	1.02	0.60	0.00	0.00
j4	1.90	1.64	1.48	1.43	1.43	1.78	2.35	2.40	2.28	2.40	2.40	2.28	2.40	2.40	2.40	1.33	2.28	2.30	2.50	1.33	2.19	2.40	2.27	1.64
j5	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.60	0.60	0.60	0.60	0.60	0.60	0.21	0.21	0.60	0.60	0.60	0.60	0.00	0.00	0.00	0.00
j7	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.08	0.00	0.00	0.08	0.08	0.08	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
j9	1.40	1.70	1.70	1.70	1.70	1.40	1.30	1.30	1.61	1.61	1.61	1.61	1.30	1.30	1.30	2.57	2.37	2.10	2.10	2.80	1.61	1.20	1.40	1.50
j12	0.25	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.09	0.00	0.00	0.00	0.00

Time [hour]

j1 j3 j4 j5 j7 j9 j12

Figure 3: The actual allocated reserve to the generators in the BEM under scenario 1

Table 3
Assessing the binding MaxGen Constraints at t=9 [p.u.]

	j_1	j_2	j_3	j_4	j_5	j_6	j_7	j_8	j_9	j_{10}	j_{11}	j_{12}
$\Delta_{j,t} + P^{pri}_{j,t} + R_{j,t}$	1.52	1.52	3.5	5.91	6	1.55	1.55	4	4	3	3.1	3.5
$\overline{P}_j$	0.3957	1.52	3.5	2.9	1.2	1.55	1.55	4	4	3	3.1	3.5

Table 4
Reserve and power allocated to the generators j_7 and j_9 at t=8 and t=9 [p.u.]

	j_1	j_2	j_3	j_4	j_5	j_6	j_7	j_8	j_9	j_{10}	j_{11}	j_{12}
P_{j,t_8}	0.1957	1.52	1.397601	0.5	0.6	1.55	0.971992	4	2.7043	2.754707	3.1	3.5
R_{j,t_8}	0.1243	0	0.73886	2.4	0	0	0	0	1.2957	0	0	0
P_{j,t_9}	0.3957	1.52	3.037471	0.61806	0.6	1.55	1.472279	4	2.38624	3	3.1	3.5
R_{j,t_9}	0	0	0.462529	2.28194	0.6	0	0.077721	0	1.61376	0	0	0

$0.077721\,p.u.$ power to j_7 to set its headroom free for reserve procurement. In addition, $P_{j_9,t_8} - P_{j_9,t_9} = 0.31806\,p.u.$ which is equal to $R_{j_9,t_9} - R_{j_9,t_8} = 0.31806\,p.u.$. The results show a power reduction for j_9 to create additional reserve headroom. Notably, the freed capacity from at $t = 9$ from j_7 and j_9 totals $0.39578\,p.u.$ allocated to j_1. It means that the optimisation engine allocates $\Delta_{j_1,t_9} = P^{inc}_{j_1,t_9} - P^{dec}_{j_1,t_9} = 0.39578$ to open up the headroom in j_7 and j_9 to meet the reserve requirement.

Practically, a reserve requirement has triggered an energy-related action, yet no reserves were allocated to this unit. Thus, TSOs won't consider it a contributor to the system's reserves. Theoretically, contributing to a binding system

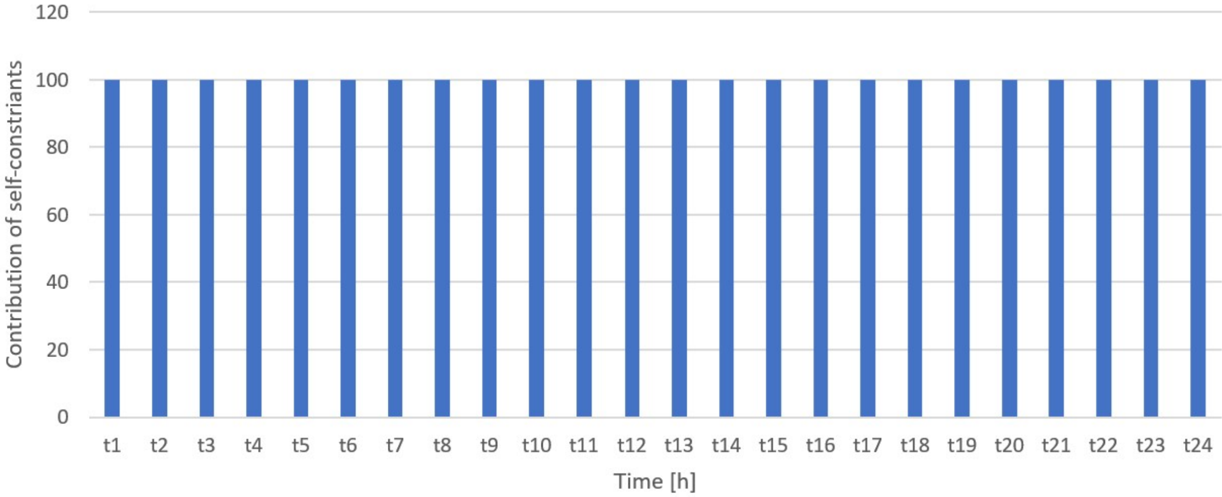


Figure 4: Allocated reserve to the generators in the BEM under scenario 1

requirement doesn't always equate to fulfilling the actual requirement. In this example, 100% of the positive FEP deviation is allocated to create headroom in other units for reserve service, demonstrating the advantage of the proposed theory over current approaches. In other words, for case 2, the TSOs' energy action in BEM (i.e., $P^{inc}_{j_1,t_9} - P^{dec}_{j_1,t_9} = 0.4$)is entirely a non-energy action.

Case 3

Between $t = 9$ and $t = 20$ in Figure 3 j_5 is assigned to provide reserve service. Its total deviation is 0.6 p.u. in all trading periods matching its allocated reserve (Figure 2) except at $t = 15$ and $t = 16$. At these times it provides 0.21 p.u. of the reserve and must generate at least 0.6 p.u. due to its minimum power capability. The last row of Table 2 indicates that at these trading periods, $\Gamma^{\mathrm{Re}\,serve}_{j,t} = 0\%$. This means the grid's reserve requirement didn't cause the FEP deviation for this unit. Other factors, like network or rival/self-constraints, led to these FEP deviations. Analysing $\Gamma^{Itself}_{j,t}$ (please see Figure 4) values reveal that the internal constraints of j_5 are the sole contributors to the allocated FEP deviations in all trading periods. Assessing the right-hand side of Eq. (18.2) indicates that, dual variables associated with the minimum power capability constraints could be the reason if they have taken non-zero values. It is observed that the mentioned dual variables take 0.6 in all trading periods. The question is whether this FEP deviation is an energy or non-energy action. Since the reserve requirement is the only non-energy action considered and its contribution is zero, the action instructed by the TSO to j_5 is purely an energy action, even though j_5 is providing reserve. Practically thinking, since the dual variable of the reserve is binding at all times, and generator j_5 is generating at its minimum power capability, based on the current procedure in TSOs, the FEP deviations allocated to j_5 could be categorized as non-energy actions. It means that in the BEM, a unit that had zero FEP, has been brought online at its minimum power capability, and it is providing reserve, however, the reason behind this action is not a non-energy requirement. One of its own internal technical constraints causes these FEP deviations. Practically, since the reserve's dual variable is always binding and j_5 is at its minimum power capability, the TSO could categorize its FEP deviations as non-energy actions. In the BEM, j_5 was brought online at minimum power for reserve, but the true cause of these deviations is its own internal technical constraints, not a non-energy requirement.

Case 4

Tables 5 and Table 6 show the contribution of self-internal technical constraints and rivals' constraints to the total allocated FEP deviation to generation units, respectively. Combining Tables 2, 5, and 6; results in for Figure 5 for j_4 at $t = 19$. Figure 5 illustrates that self/rivals constraints contributed 29%, 55%, 16%, respectively to the total deviations allocated to j_4. From Figure 2, $P^{inc}_{j_4,t_{19}} = 0.9053$ and $P^{dec}_{j_4,t_{19}} = 0$, while $P^{pri}_{j_4,t_{19}} = 0.785$.

To identify the root cause of allocating an incremental deviation to j_4 at $t = 19$ due to reserve requirements, firstly Eq. (15) should be considered. The negative value of $\hat{\mu}^{j_4,t_{19}}_{t_{12}}$ indicates that $P^{inc}_{j_4,t_{19}}$ is negatively sensitive to the binding

Table 5
The contribution of self-constraints to the total deviation allocated to the generation units [%]

	t_1	t_2	t_3	t_4	t_5	t_6	t_7	t_8	t_9	t_{10}	t_{11}	t_{12}	t_{13}	t_{14}	t_{15}	t_{16}	t_{17}	jt_{18}	t_{19}	t_{20}	t_{21}	t_{22}	t_{23}	t_{24}
j_1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	100	3.82	100	100	100	100	100
j_2	100	100	100	100	100	100	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	100
j_3	100	100	100	100	100	100	100	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	100	100
j_4	100	100	100	100	100	100	100	0	0	80.90	80.90	80.90	100	100	100	64.87	65.37	0	55.23	0	0	100	100	100
j_5	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100
j_6	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
j_7	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
j_8	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
j_9	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100
j_{10}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
j_{11}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
j_{12}	0	100	100	100	100	0	0	0	0	0	0	0	0	0	0	0	0	0	0	100	0	0	0	0

Table 6
Contribution of rival constraints to the total deviation allocated to the generators [%]

	t_1	t_2	t_3	t_4	t_5	t_6	t_7	t_8	t_9	t_{10}	t_{11}	t_{12}	t_{13}	t_{14}	t_{15}	t_{16}	t_{17}	jt_{18}	t_{19}	t_{20}	t_{21}	t_{22}	t_{23}	t_{24}
j_1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	35.87	0	0	0	0	0	0	0
j_2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
j_3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
j_4	0	0	0	0	0	0	0	100	98.92	10.52	10.52	10.52	0.00	0.00	0.00	21.64	11.98	0.00	15.49	0	0	0	0	0
j_5	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
j_6	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
j_7	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
j_8	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
j_9	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
j_{10}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
j_{11}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
j_{12}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

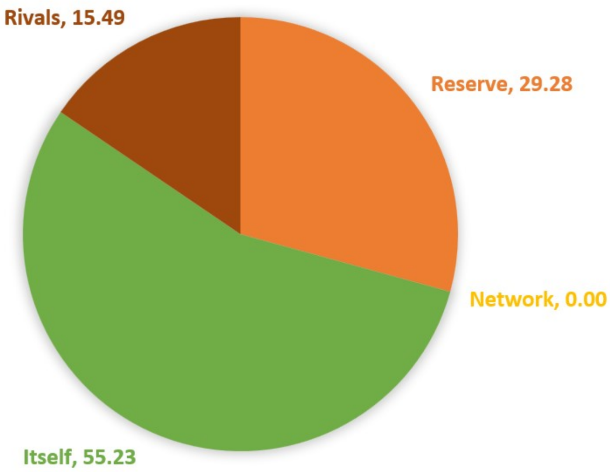


Figure 5: Contribution of reserve, network, self- and rivals' constraints on the total deviation allocated to generator 4 at time 19

reserve requirement constraint at $t = 12$. Thus, a slight increase in the reserve requirement at $t = 12$ would result in less incremental deviation being allocated to generator j_4 at $t = 19$.

To trace the root cause of the incremental deviation allocated to j_4 at $t = 19$ due to its internal technical constraints, the right-hand side of Eq. (18.2) should be assessed. Optimisation results show $\underline{\widehat{\theta}}^{m,h}_{j_4,t_{19}}$ has a positive value at ($m = 4$ and $h = 15$). indicating that $P^{inc}_{j_4,t_{19}}$ is positively sensitive to its minimum power constraint. Thus, if this unit's minimum power capability increased, more incremental power would be allocated to j_4 at $t = 19$.

Table 7
Contribution of rivals' constraints to the total deviation allocated to the generators [%]

	$\overline{P}_j$	t_{12}		t_{18}	
j_3	3.5	$P = 3.04$	$R = 0.46$	$P = 3.5$	$R = 0$
j_7	1.55	$P = 1.47$	$R = 0.08$	$P = 1.55$	$R = 0$

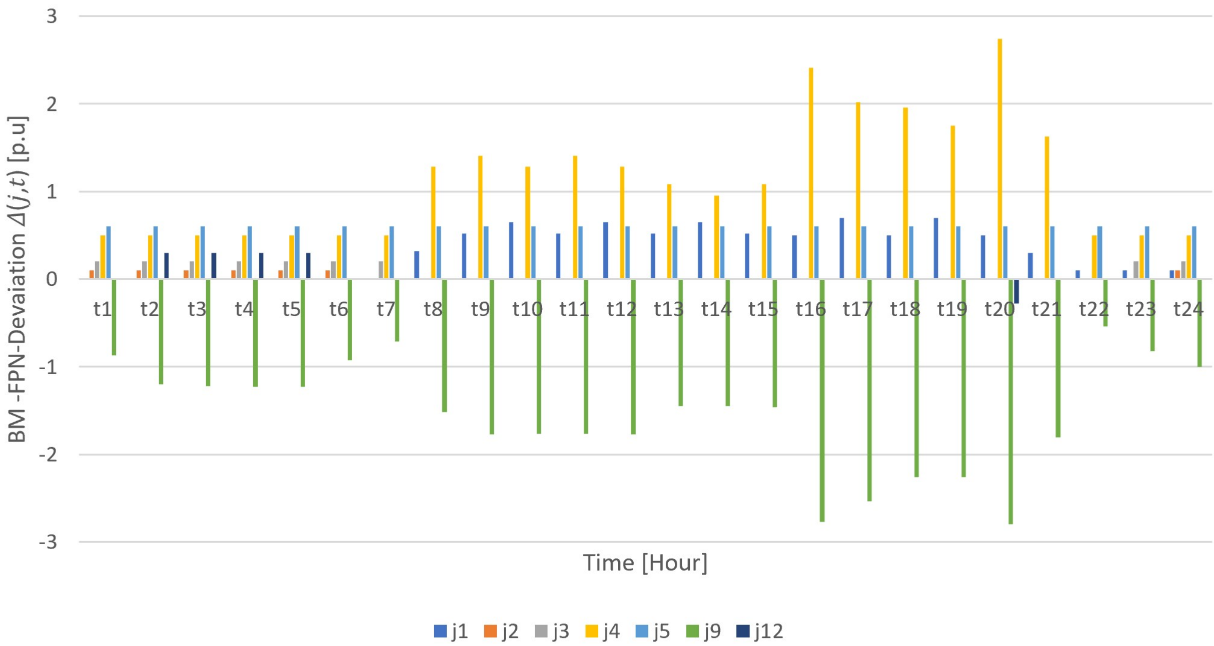


Figure 6: Total deviation allocated to the generators in the BEM under scenario 2

By assessing Eq. (18.3), it is revealed that binding maximum power capability constraints forj_3 and j_7 at $t = 12$ and $t = 18$ cause incremental deviation for j_4 at $t = 19$. Table 7 details the power and reserve supplied by j_3 and j_7 at these times.

4.2. Scenario 2

In this scenario, the reserve requirement is 20% of the total load in each interval. The bids, offers, and FEPs of each generator in the BEM are in the appendix, similar to the first scenario. Since the ex-ante markets did not fully procure the total demand, and the demand is 3% higher in the BEM than in the first scenario, deviations are necessary to meet the load, considering the BEM constraints and reserve requirements.

4.2.1. Analysing the Results

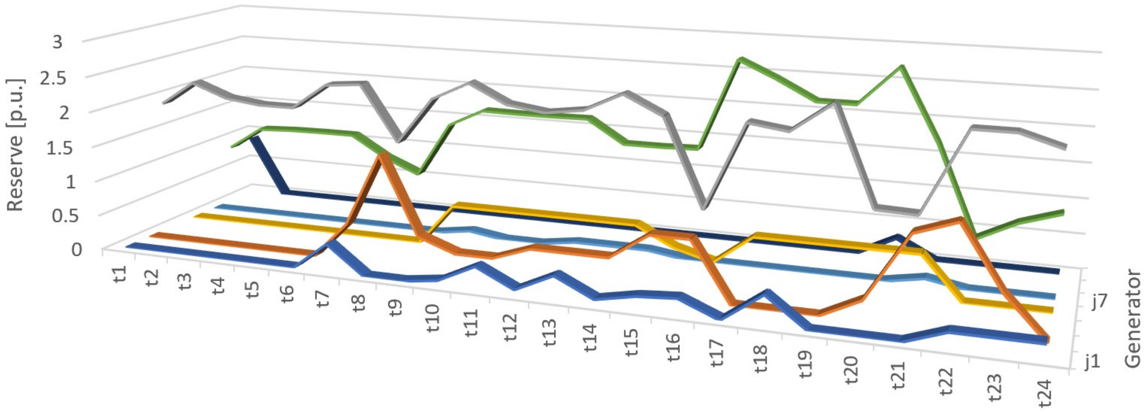
Figure 6 illustrates the total deviation ($\Delta_{j,t} = P^{inc}_{j,h} - P^{dec}_{j,h}$) for each generation unit's allocated deviations. Notably, no deviations were allocated to $j \in \{j_6, j_7, j_8, j_{10}, j_{11}\}$. Table 8 lists $\Gamma^{\text{Re}\,serve}_{j,t}$ for generators whose reserve requirement contributed to their allocated FEP deviations in the BEM.

In Figure 6, the yellow bars show the positive FEP deviation allocated to generator j_4. From $t = 8$ to $t = 21$, $\Delta_{j,t} = P^{inc}_{j,h} - P^{dec}_{j,h}$ and Figure 8 indicates that reserve requirement and network constraints are the main reasons for these deviations. Figure 7 confirms that j_4 is actually providing reserve.

Between $t = 8$ and $t = 21$, no internal constraints are considered as binding for j_4. Consequently, these actions may not categorized as non-energy actions. However, based on the proposed theory (as shown in Figure 8), the primary reason for allocating FEP deviation to this generator is the reserve requirement (87.29%).

Table 8
Contribution of the reserve requirement to the total deviation allocated to the generators under scenario 2 [%]

	t_8	t_9	t_{10}	t_{11}	t_{12}	t_{13}	t_{14}	t_{15}	t_{16}	t_{17}	t_{18}	t_{19}	t_{20}	t_{21}
j_1	0.00	0.00	8.43	0.00	8.43	0.00	8.43	0.00	0.00	0.00	0.00	0.00	0.00	0.00
j_2	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
j_3	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
j_4	87.29	87.29	87.29	87.29	87.29	87.29	87.29	87.29	87.75	87.29	87.29	87.29	87.29	87.29



	t1	t2	t3	t4	t5	t6	t7	t8	t9	t10	t11	t12	t13	t14	t15	t16	t17	t18	t19	t20	t21	t22	t23	t24
j1	0	0	0	0	0	0	0.4	0	0	0.07	0.33	0.07	0.33	0.07	0.18	0.22	0	0.4	0	0	0	0.2	0.2	0.2
j3	0	0	0	0	0	0	0.53	1.56	0.46	0.28	0.28	0.46	0.46	0.46	0.83	0.83	0	0	0	0.28	1.19	1.39	0.57	0
j4	1.9	2.24	2.05	1.99	1.99	2.35	2.4	1.62	2.27	2.53	2.27	2.2	2.27	2.53	2.27	1.07	2.27	2.2	2.6	1.29	1.27	2.4	2.4	2.24
j5	0	0	0	0	0	0	0	0	0.6	0.6	0.6	0.6	0.6	0.6	0.33	0.19	0.6	0.6	0.6	0.6	0.6	0	0	0
j7	0	0	0	0	0	0	0	0	0.08	0	0	0.08	0.08	0.08	0	0	0	0	0	0	0.1	0	0	0
j9	0.87	1.2	1.22	1.23	1.23	0.92	0.71	1.52	1.77	1.77	1.77	1.77	1.45	1.45	1.46	2.77	2.53	2.26	2.26	2.8	1.81	0.54	0.82	1
j12	0.89	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0.28	0	0	0	0

Time [Hour]

Figure 7: Allocated reserve to the generators in the BEM under scenario 2

4.3. Distinguishing the Energy and Non-energy Actions

As described in Section 3.4, $\Gamma^{Reserve}_{j,t}$ quantifies the contribution of system reserve requirements to the deviations allocated to a generation unit in the BEM. Since the only modeled non-energy action in this paper is the reserve requirement, positive values of $\Gamma^{Reserve}_{j,t}$ quantify the contribution of non-energy actions to deviations instructed to generation units based on their FEPs from the ex-ante energy markets.

Since the only non-energy action modeled in this paper is the reserve service, the reserve requirements are the sole non-energy action leading to deviations. $\Gamma^{Network}_{j,t}$ represents the contribution of network constraints (power balance, line flow values, and line thermal limits) to these deviations. $\Gamma^{Network}_{j,t}$, $\Gamma^{Itself}_{j,t}$, and $\Gamma^{Rivals}_{j,t}$ represent the contribution of energy actions to the deviations based on FEPs from the ex-ante energy markets. Future work will explore using $\Gamma^{Itself}_{j,t}$ and $\Gamma^{Rivals}_{j,t}$ to improve the BEM pricing mechanism.

Although this paper only considers reserve requirements as the non-energy needs of the power system, the proposed theory can also cover other requirements such as short-circuit current, dynamic stability (post-disturbance frequency, voltage stability, rotor angle stability), maximum wind/solar penetration, and minimum inertia.

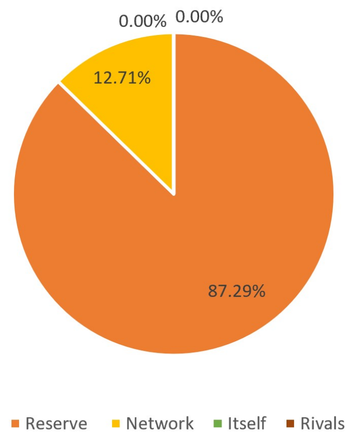


Figure 8: Contribution of reserve, network, self- and rivals' constraints on the total deviation allocated to generator 4 between t8 to t21

5. Conclusion

Balancing energy markets are established to optimise the essential balancing actions maintaining the security of the power system and matching the load and supply closer to real-time. Security-constrained economic dispatch is normally solved for short-term scheduling aiming to minimise the deviation cost with respect to the ex-ante energy market position while satisfying a suite of constraints. The constraints can be internal technical constraints of the generation units and/or the constraints associated with the network such as power balance and security requirements. These constraints can affect the actions that need to be taken in balancing energy markets. However, distinguishing the energy and non-energy actions is a difficult task because of the complexity of the balancing energy market optimisation problem and the interactions of those constraints. This paper proposes a method enabling TSOs to distinguish the balancing energy actions with energy root causes against security-oriented (non-energy) actions. It is shown that current approaches have some theoretical and practical shortcomings and cannot fully untangle the energy and non-energy actions in the balancing energy markets whereas the proposed method has a strong mathematical background, based on the theory of optimisation, and provides an accurate mechanism to understand the root causes of deviations instructed by balancing energy market schedule to the generation units. The proposed theory quantifies the contribution of every single binding constraint to the allocated FEP deviations. Although the magnitude of deviations allocated to generation units can be influenced by the assumed reserve requirements, the proposed methodology can distinguish between energy and non-energy actions under various assumptions. Accordingly, the contribution of reserve requirement as a constraint can be measured to distinguish the energy and non-energy actions. The proposed theory can be readily extended to cover other system security requirements by quantifying their contribution as non-energy actions in the balancing energy markets. It is shown that the proposed method can: 1) quantify the contribution of internal technical constraints of a certain generation unit to its allocated FEP deviations in the balancing energy market, 2) quantify the contribution of the technical constraints associated with the rivals of a certain generation unit to its allocated FEP deviations in the balancing energy market, 3)quantify the contribution of the network constraints in the allocated FEP deviations to every single generation unit in the balancing energy market, 4) distinguish the energy actions in the balancing energy markets from the non-energy actions.

Generation units can receive non-energy-based instruction for non-energy-based reasons. Based on the presented results, this kind of situation is fully retractable by the proposed method. In addition, the results show that whenever a generation unit receives purely energy-based instructions due to non-energy-based root causes, the proposed method is able to detect the non-energy-based root cause as well. The proposed methodology can have interesting practical applications in the context of highly renewable penetrated power systems where new system requirement has to be addressed in the power system scheduling such as flexibility ramping services. Application of the proposed theory to improve the pricing and settlement mechanism in the balancing energy market and system services is left for future works.

Table 9
Appendix-Generation unit Data

Generator	Bus	$\bar{P}_i$	$\bar{R}_i^U = \bar{R}_i^D$	$\underline{P}_j$
$i = 1$	$n = 1$	1.52	1.20	0.1
$i = 2$	$n = 2$	1.52	1.20	0.1
$i = 3$	$n = 7$	3.50	3.50	0.2
$i = 4$	$n = 13$	5.91	2.40	0.5
$i = 5$	$n = 15$	6.00	0.60	0.6
$i = 6$	$n = 15$	1.55	1.55	0
$i = 7$	$n = 16$	1.55	1.55	0
$i = 8$	$n = 18$	4.00	2.80	0
$i = 9$	$n = 21$	4.00	2.80	0
$i = 10$	$n = 22$	3.00	3.00	0.3
$i = 11$	$n = 23$	3.10	1.80	0.3
$i = 12$	$n = 23$	1.50	2.40	0.3

Table 10
Appendix-FEPs in scenario 1 [p.u.]

	t_1	t_2	t_3	t_4	t_5	t_6	t_7	t_8	t_9	t_{10}	t_{11}	t_{12}	t_{13}	t_{14}	t_{15}	t_{16}	t_{17}	t_{18}	t_{19}	t_{20}	t_{21}	t_{22}	t_{23}	t_{24}
j_1	1.32	1.12	0.92	0.72	0.52	0.32	0.12	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
j_2	0	0	0	0	0	0	0.63	1.52	1.52	1.52	1.52	1.52	1.52	1.52	1.52	1.52	1.52	1.52	1.52	1.52	1.52	1.52	0.55	0
j_3	0	0	0	0	0	0	0	1.4	3.03	3.22	3.22	3.04	3.04	3.04	2.67	2.67	3.5	3.5	3.5	3.22	2.31	0.85	0	0
j_4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0.52	0.79	0.79	0	0	0	0	0
j_5	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
j_6	1.45	1.05	0.39	0.35	0.61	0	0	1.55	1.55	1.55	1.55	1.55	1.55	1.55	1.55	1.55	1.55	1.55	1.55	1.55	1.55	1.55	0	0.96
j_7	1.55	1.55	1.55	1.55	1.55	1.55	1.5	0.97	1.5	1.55	1.55	1.47	1.47	1.47	1.31	1.31	1.55	1.55	1.55	1.55	1.18	0.85	1.47	1.55
j_8	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4
j_9	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4
j_{10}	1.94	1.88	1.94	1.92	1.86	2.11	2.77	2.75	3	3	3	3	3	3	3	3	3	3	3	3	2.96	2.63	2.73	1.99
j_{11}	3.1	3.1	3.1	3.1	3.1	2.83	3.1	3.1	3.1	3.1	3.1	3.1	3.1	3.1	3.1	3.1	3.1	3.1	3.1	3.1	3.1	3.1	3.1	3.1
j_{12}	0.4	0	0	0	0	1.1	3.5	3.5	3.5	3.5	3.5	3.5	3.5	3.5	3.5	3.5	3.5	3.5	3.5	3.5	3.5	3.5	3.5	1.1

Table 11
Appendix-Offered prices for incremental deviation in the BEM [monetary unit]

	t_1	t_2	t_3	t_4	t_5	t_6	t_7	t_8	t_9	t_{10}	t_{11}	t_{12}	t_{13}	t_{14}	t_{15}	t_{16}	t_{17}	t_{18}	t_{19}	t_{20}	t_{21}	t_{22}	t_{23}	t_{24}
j_1	12.52	12.31	11.96	12.49	12.17	12.51	12.79	12.86	12.07	12.63	13.10	13.25	12.61	12.06	12.49	12.63	13.07	12.01	12.53	13.20	12.56	13.14	12.53	13.20
j_2	20.64	20.33	22.07	21.96	22.00	21.78	22.18	21.21	21.04	20.73	20.71	20.79	20.97	20.67	21.17	22.05	22.01	21.67	20.80	22.07	21.37	22.08	20.80	22.07
j_3	17.82	16.99	15.93	15.83	17.17	15.87	17.11	16.86	16.63	16.77	16.30	16.28	17.71	17.73	15.92	16.56	16.42	17.29	16.34	17.77	17.49	17.66	16.34	17.77
j_4	13.57	13.00	14.14	13.44	14.50	14.45	12.94	13.39	13.68	13.14	14.30	14.40	14.70	13.45	13.14	14.09	14.65	14.46	14.43	13.46	14.78	13.36	14.43	13.46
j_5	19.24	19.05	18.88	18.61	18.19	18.30	19.15	18.15	18.70	19.52	18.87	18.91	17.83	18.84	18.18	18.06	19.21	19.48	17.86	19.52	19.57	19.43	17.86	19.52
j_6	19.57	19.40	19.87	20.39	20.21	19.53	19.89	20.17	20.30	20.95	20.82	20.04	20.12	20.43	20.16	19.46	20.37	21.30	19.67	21.33	19.74	19.86	19.67	21.33
j_7	26.19	26.62	25.35	26.36	26.22	26.14	25.91	26.67	27.21	26.16	27.29	27.28	26.46	26.98	25.94	25.87	26.44	25.52	26.90	27.19	25.46	25.98	26.90	27.19
j_8	26.03	25.95	26.31	26.47	26.78	26.03	25.54	26.94	25.48	25.83	26.41	26.31	26.34	25.47	25.44	25.40	25.87	26.00	26.20	25.52	26.42	26.98	26.20	25.52
j_9	12.08	11.67	11.85	11.55	11.96	11.89	10.63	10.60	11.72	11.90	12.14	11.23	12.38	12.14	10.84	12.11	11.84	12.56	11.34	12.25	11.84	11.15	11.34	12.25
j_{10}	28.55	28.94	28.97	27.99	27.33	28.48	29.08	28.66	29.01	27.23	28.55	27.70	28.68	27.81	27.33	28.62	27.34	27.62	27.37	29.03	27.78	27.36	27.37	29.03
j_{11}	22.97	21.66	22.46	23.21	23.37	23.09	23.52	22.41	23.01	23.51	23.53	22.88	22.22	22.97	22.41	22.19	23.41	21.76	22.16	22.10	22.84	22.31	22.16	22.10
j_{12}	15.53	16.99	16.95	16.75	17.37	16.70	15.96	17.26	17.41	16.75	15.58	15.48	15.60	16.96	15.41	15.80	16.26	15.75	16.06	15.74	16.26	16.86	16.06	15.74

Table 12
Appendix-Load data [p.u.]

Time	t_1	t_2	t_3	t_4	t_5	t_6	t_7	t_8	t_9	t_{10}	t_{11}	t_{12}	t_{13}	t_{14}	t_{15}	t_{16}	t_{17}	t_{18}	t_{19}	t_{20}	t_{21}	t_{22}	t_{23}	t_{24}
$\sum_{\ell \in N_L} \Omega^L_{l,n} L^I_{\ell,t}$ (total demand)	17.76	16.70	15.90	15.64	15.64	15.90	19.61	22.79	25.18	25.44	25.44	25.18	25.18	25.18	24.65	24.65	26.24	26.51	26.51	25.44	24.12	22.00	19.35	16.70
Buses	n_1	n_2	n_3	n_4	n_5	n_6	n_7	n_8	n_9	n_{10}	n_{13}	n_{14}	n_{15}	n_{16}	n_{18}	n_{19}	n_{20}							
% of total demand	3.8	3.4	6.3	2.6	2.5	4.8	4.4	6	6.1	6.8	9.3	6.8	11.1	3.5	11.7	6.4	4.5							

6. Appendix

The case studies are applied to IEEE 24 bus test system. Table 9 contains the generators' technical specifications and their connection topology. Table 10 contains the generation unit required data. In this paper, the optimisation problems are coded in GAMS and solved by the CPLEX solver. Table 11 contains the prices offered by generation units for incremental deviations with respect to the submitted FEPs. The detrimental offered prices are considered to be 0.8 of the prices represented in Table 11. Finally, 12 contains the total demand data and its distribution over the system buses.

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