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Nonlinearization and waves in bounded media: old wine in a new bottle

Michael P Mortell[1] **and Brian R Seymour**[2]

Abstract. We consider problems such as a standing wave in a closed straight tube, a self-sustained oscillation, damped resonance, evolution of resonance and resonance between concentric spheres. These nonlinear problems, and other similar ones, have been solved by a variety of techniques when it is seen that linear theory fails.

The unifying approach given here is to initially set up the appropriate linear difference equation, where the difference is the linear travel time. When the linear travel time is replaced by a corrected nonlinear travel time, the nonlinear difference equation yields the required solution.

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1. Introduction

The series of problems discussed here all involve small amplitude, one-dimensional, nonlinear hyperbolic waves in a medium of finite extent, and thus inherently involve wave reflections from the boundaries. Guided by experiments such as in Saenger & Hudson [1], our fundamental hypothesis is that, for small amplitude disturbances, the interaction of these nonlinear waves is negligible in calculating the main features of the flow, such as the presence of shocks. Then we can frame the hypothesis as: "the motion consists of non-interacting simple waves."

In all examples presented here linear theory fails; for unforced standing waves linear theory fails in the long term as it fails to predict a singularity, and for forced periodic oscillations linear theory fails to give a bounded periodic solution at resonance. The presence of shocks in the flows is central to the observations.

We consider standing waves, a self-sustained oscillation, periodic resonance and its evolution, all in the context of a gas contained in a long straight tube. Similar resonance problems between concentric spheres and cylinders are also considered, making use of nonlinear geometric acoustics approximations. These problems have been solved previously by a variety of techniques. The contribution made here is a simple observation that allows for a simpler method of solution to each problem: "correct the travel time in linear theory by an appropriate *nonlinear travel time*," when allied to "the motion consists of non-interacting simple waves."

The genesis of this idea goes back to Whitham [2] who considered the supersonic flow past a projectile. He introduces "the fundamental hypothesis that linearized theory gives a valid first approximation to the flow *everywhere* provided that in it the approximate characteristics are replaced by the exact ones, or at least a sufficiently good approximation to the exact ones." This is referred to as Whitham's nonlinearization technique, see Whitham [3], Landau [4] and Lighthill [5].


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We extend these ideas to reflected waves in a bounded medium by using approximate nonlinear characteristics to calculate the nonlinear travel time, which depends on the signal carried, to replace the linear travel time, i.e., the time for a wave to complete a round trip in the bounded medium. This calculation is carried out on the basis that the motion in the medium is the superposition of two non-interacting simple waves.

These observations allow us to propose a simple and unified approach to the solution of the various problems.

2. Basic equations

The oscillations of a polytropic gas in a closed tube are described in terms of the equations of conservation of mass and momentum relating the velocity $u(x,t)$ and density $\rho(x,t)$. In Eulerian coordinates they are:

$$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x}(u\rho) = 0, \ \frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + \rho^{-1}\frac{\partial p}{\partial x} = 0. \tag{1}$$

Pressure and density are measured from their values in a reference state (p_0, ρ_0), so that

$$\frac{p}{p_0} = (\frac{\rho}{\rho_0})^{\gamma} = (1+e)^{\gamma} = 1 + \gamma e + \frac{\gamma(\gamma-1)}{2}e^2 + ... \tag{2}$$

where $e(x,t) = \rho/\rho_0 - 1$ is the condensation, and $c_0 = \sqrt{\frac{\gamma p_0}{\rho_0}}$ the associated sound speed.

Linear Theory

When $|e| \ll 1$ and $|u| \ll c_0$, the corresponding linear equations are

$$\frac{\partial e}{\partial t} + \frac{\partial u}{\partial x} = 0, \text{ and } \frac{\partial u}{\partial t} + c_0^2\frac{\partial e}{\partial x} = 0. \tag{3}$$

Eliminating e, u satisfies the linear wave equation:

$$\frac{\partial^2 u}{\partial t^2} - c_0^2\frac{\partial^2 u}{\partial x^2} = 0. \tag{4}$$

Velocity, pressure and density are nondimensionalized with respect to $(c_0, \rho_0 c_0^2, \rho_0)$, and (u, p, ρ) are considered as functions of length and time $(Lx, Lc_0^{-1}t)$, where L is the tube length. Then equation (4) becomes

$$\frac{\partial^2 u}{\partial t^2} - \frac{\partial^2 u}{\partial x^2} = 0, \ 0 \le x \le 1, \ t > 0. \tag{5}$$

The general solution to equation (5) is

$$u(\alpha, \beta) = f(\alpha) + g(\beta), \ e = f(\alpha) - g(\beta), \tag{6}$$

where f and g are arbitrary functions and

$$\alpha = t - x, \ \beta = t + x - 1, \tag{7}$$

are the linear characteristics. Note that $\alpha = t$ on $x = 0$; $\beta = t$ on $x = 1$.

Nonlinear Theory

The sound speed is $a(e) = (1+e)^{(\gamma+1)/2}$, so that, in dimensionless variables, when $|e| \ll 1$,

$$a(e) = 1 + \frac{\gamma+1}{2}e + 0(e^2). \tag{8}$$

For an $\alpha-$ wave traveling to the right, with no $\beta-$ wave, $e = f(\alpha)$ by (6), and

$$\begin{aligned} \frac{1}{a} &= \frac{dt}{dx}|_\alpha = 1 - \frac{\gamma+1}{2}e + 0(e^2) \\ &= 1 - \frac{\gamma+1}{2}f(\alpha) + .. \end{aligned}$$

Integrating:

$$t = \alpha + x - \frac{\gamma+1}{2}xf(\alpha) + ..$$

with $t = \alpha$ on $x = 0$. Hence the approximate right-traveling nonlinear characteristic, $\alpha = \text{const.}$, is given by

$$\alpha = t - x + \frac{\gamma+1}{2}xf(\alpha) + .. \tag{9}$$

Similarly, the approximate left-traveling nonlinear characteristic, $\beta = \text{const.}$, is given by

$$\beta = t + (x-1) + \frac{\gamma+1}{2}(x-1)g(\beta) + .. \tag{10}$$

with $\beta = t$ on $x = 1$.

The fundamental assumption in the calculations for the nonlinear characteristics α and β in (9) and (10) is that the α and β waves do not interact to this order of approximation.

3. Standing wave in a closed tube

For a standing wave in a closed tube, the boundary conditions are

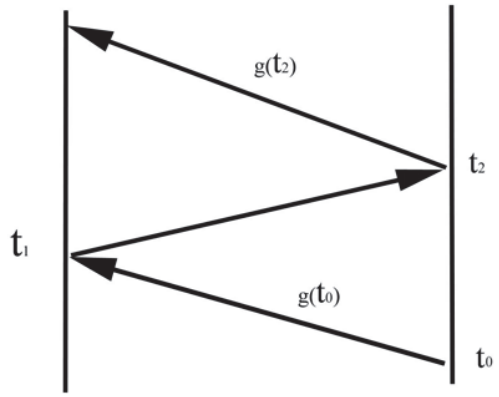
$$u(0,t) = 0,\ u(1,t) = 0. \tag{11}$$

In linear theory, using (6) and (7) in the boundary conditions (11) implies that $f(t) = -g(t-1)$ and $g(t) = -f(t-1)$, so g satisfies the linear difference equation

$$g(t) - g(t-2) = 0. \tag{12}$$

Then g, f and u have period 2 in time and maintain their initial form given at $t = 0$. This contradicts the results from Lax [6] who showed from the exact equations that a singularity must arise.

Consider a wave that leaves $x = 1$ at time $t = t_0$, is reflected at $x = 0$ at time $t = t_1$ and arrives back at $x = 1$ at time $t = t_2$. At each reflection the boundary condition is $u = 0$, hence from (6) at each boundary $g = -f$. The travel time down the tube and back is $t_2 - t_0$.



Characteristics between t_0 and t_2.

Nonlinear Travel Time

For the characteristic $\beta = t_0$ leaving $x = 1$, (10) and (11) imply that on $x = 0$

$$t_0 = t_1 - 1 - \frac{\gamma+1}{2} g(t_0) \text{ and } f(t_1) = -g(t_0). \tag{13}$$

For the characteristic $\alpha = t_1$ leaving $x = 0$, (9) and (11) imply that on $x = 1$

$$t_1 = t_2 - 1 + \frac{\gamma+1}{2} f(t_1) \text{ and } f(t_1) = -g(t_2). \tag{14}$$

Hence

$$t_2 = t_0 + 2 + \frac{\gamma+1}{2}[g(t_0) + g(t_2)] \text{ and } g(t_2) = -f(t_1) = g(t_0). \tag{15}$$

Hence the *corrected* nonlinear travel time is

$$t_2 - t_0 = 2 + (\gamma+1) g(t_0). \tag{16}$$

Substituting this into the linear difference equation (12) gives

$$g(t) - g(t + 2 + (\gamma+1) g(t)) = 0. \tag{17}$$

This represents a *simple wave* signal distortion on the boundary $x = 1$. A smooth initial function $g(t)$ will break to give a shock, agreeing with Lax [6]; see also Mortell & Varley [7].

4. Damped standing wave in a closed tube

While the end $x = 1$ is again kept fixed, so $u(1, t) = 0$ and $g(t) = -f(t - 1)$, energy is allowed to radiate out through the surface at $x = 0$ via the interface condition

$$e(0, t) = -iu(0, t). \tag{18}$$

This implies that at $x = 0$

$$f(t) = -kg(t - 1), \; k = \frac{i-1}{i+1}, \; 0 \le i < \infty, \tag{19}$$

where k $(-1 < k < 1)$ is the reflection coefficient. Then (12) becomes the linear difference equation

$$g(t) - kg(t-2) = 0, \tag{20}$$

that implies $|g(t)| \to 0$ as $t \to \infty$.The signal is attenuated geometrically like $|k|^n$.

Nonlinear Travel Time

Using (19) the nonlinear travel time (16) becomes

$$t_2 - t_0 = 2 + (1+k)\frac{(\gamma+1)}{2}g(t_0), \tag{21}$$

producing the nonlinear difference equation

$$g(t) = kg(r),\ t = r + 2 + (1+k)\frac{(\gamma+1)}{2}g(r). \tag{22}$$

Consider the initial value problem:

$$g(t_0) = M\psi(t_0), \qquad 0 \le t_0 < 2, \tag{23}$$

where $M \ll 1$ is the Mach number. Then (22) implies

$$g(t_{n+1}) = kg(t_n) = k^{n+1}M\psi(t_0), \tag{24}$$

where

$$t_{n+1} = t_0 + 2n + \frac{k+1}{1-k}[1-k^{n+1}]\frac{(\gamma+1)}{2}]M\psi(t_0).$$

A shock forms when $\frac{dg(t_{n+1})}{dt_{n+1}} \to \infty$. But (24) implies that

$$\frac{dg(t_{n+1})}{dt_{n+1}} = k^{n+1}M\psi'(t_0)\frac{dt_0}{dt_{n+1}},$$

so that a shock will form when $\frac{dt_{n+1}}{dt_0} \to 0$ or

$$0 = \frac{dt_{n+1}}{dt_0} = 1 + \frac{k+1}{1-k}[1-k^{n+1}]\frac{(\gamma+1)}{2}M\psi'(t_0). \tag{25}$$

For a shock to form when $|k| < 1$ we require

$$\left|\frac{(\gamma+1)}{2}M\psi'(t_0)\right| > \left|\frac{1-k}{1+k}\right| = i \tag{26}$$

for some t_0 with $0 < t_0 < 2$. This says that there is a critical initial acceleration level, $\left|M\psi'(t_{cr})\right| = 2i/(\gamma+1)$, and a shock will only form for an initial acceleration above this level. Otherwise the signal is damped out before a shock can form. When $k = 1$ $(i = 0)$ there is always a shock; see Mortell & Varley [7].

5. A self-sustained oscillation

For the case $|k| > 1$ the signal is amplified geometrically. This is an example of a system that is linearly unstable to perturbations about the initial state. However, within nonlinear theory a shock always forms. Since shocks dissipate energy, this raises the possibility of a balance between the energy flowing in across the boundary and the shock dissipation.

As in the previous section, the linear solution satisfies the difference equation $g(t) = kg(t-2)$ so that in linear theory

$$g(t+2n) = k^n g(t),\ k > 1. \tag{27}$$

Hence g grows like k^n and is linearly unstable.

Within nonlinear theory the solution is governed by (22). Shocks form and the signal will eventually evolve to an $N-$ wave, or a series of $N-$waves, so will have a linear slope passing through the zeros of $g(t)$. The solution is found through the use of critical points of the nonlinear difference equation, see Seymour & Mortell [8] for details. We define a critical point, $t = r = t_c$, as a location where $g(t) = g(r)$; here $g(t_c) = 0$. If (22) is differentiated with respect to r and evaluated at $t = t_c$ we obtain a quadratic equation for $\lambda = g'(t_c)$:

$$\lambda(\lambda - \frac{2(k-1)}{(k+1)(\gamma+1)}) = 0. \tag{28}$$

The root $\lambda = 0$ yields the trivial solution $g(t) = 0$, while the nonzero root gives the periodic solution

$$g(t) = \frac{2(k-1)}{(k+1)(\gamma+1)}(t - t_c),\ g(t+2) = g(t), \tag{29}$$

with the mean condition $\int_0^1 g(y)dy = 0$. Substitution of (29) into (22) shows that this is an exact solution of the difference equation. The final periodic state is therefore piecewise linear, passing through alternate zeros of the initial signal function and joined by shocks which are of constant strength, $\frac{4(k-1)}{(k+1)(\gamma+1)}$, see Chu [9] and Mortell & Seymour [10].

6. Damped resonance in a closed tube

The general solution is

$$u(\alpha, \beta) = f(\alpha) + g(\beta),\ e = f(\alpha) - g(\beta), \tag{30}$$

where in linear theory $\alpha = t - x$, $\beta = t + x - 1$. The boundary condition at $x = 0$, corresponding to an outflow of energy ($i > 0$), is

$$e(0, t) = -iu(0, t), \tag{31}$$

so that

$$f(t) = -kg(t-1),\ k = \frac{i-1}{i+1},\ 0 \le i < \infty,\ k \le 1. \tag{32}$$

At $x = 1$ there is periodic forcing

$$u(1, t) = Mh(\omega t), \tag{33}$$

where h is periodic with unit period, $\int_0^1 h(y)dy = 0$ and M is the Mach number. From the equations of motion, for a periodic response we require $\int_0^1 g(y)dy = 0$.

Linear difference equation

Let $y = \omega t$, then $g(y)$ satisfies

$$g(y) - kg(s) = Mh(y),\ y = s + 2\omega. \tag{34}$$

At $\omega = \frac{1}{2}$ this becomes

$$g(y) - kg(y-1) = Mh(y), \tag{35}$$

so there is no solution with unit period when $k = 1$. To consider problems at frequencies around resonance, define the detuning from resonance $\frac{1}{2}\Delta$ by

$$\omega = \frac{1}{2}(1 + \Delta), \tag{36}$$

then

$$g(y) - kg(s) = Mh(y),\ y = s + 1 + \Delta, \tag{37}$$

and the **nonlinear travel time** is

$$y = s + 1 + \Delta + \omega(1+k)\frac{(\gamma+1)}{2}g(s). \tag{38}$$

We define $G(y)$ and $H(y)$ as

$$G(y) = \Delta + \omega(1+k)\frac{(\gamma+1)}{2}g(y),\ MH = \mu\Delta + \omega(1+k)\frac{(\gamma+1)}{2}Mh, \tag{39}$$

where $0 < \mu = 1 - k < 1$ and now $\int_0^1 G(y)dy = \Delta$.

Then the **nonlinear difference equation** is

$$G(y) - kG(s) = MH(y),\ y = s + G(s). \tag{40}$$

on noting $g(y+1) = g(y)$. This is the *Dissipative Standard Mapping.*

Assuming $M \ll 1, |G| \ll 1$ and $|G'| \ll 1$, (40) reduces to the o.d.e.

$$G(y)G'(y) + \mu G(y) = MH(y), \tag{41}$$

subject to

$$\int_0^1 G(y)dy = \Delta,\ G(y+1) = G(y), \tag{42}$$

since $\int_0^1 g(y)dy = 0$, to determine the solution, see Seymour & Mortell [11] and Chester [12].

7. Evolution of damped resonance

The canonical equation is (40), where M is the Mach number of the input. For $M \ll 1$ we write

$$k = 1 - k_1 M^{1/2} \text{ and } \omega = \frac{1}{2}(1 + M^{1/2}\Delta_1), \tag{43}$$

so that (40) becomes

$$G(y) - (1 - k_1 M^{1/2})G(s) = MH(y),\ y = s + 1 + G(s). \tag{44}$$

We assume a long-time multiple scale expansion of the form

$$G(s) = M^{1/2}G_0(s,\tau) + MG_1(s,\tau) + ...,\ \tau = M^{1/2}s, \tag{45}$$

then at $O(M^{1/2})$,

$$G_0(s+1,\tau) - G_0(s,\tau) = 0. \tag{46}$$

At $O(M)$, to eliminate growth terms, G_0 satisfies the nonlinear kinematic wave equation with no secular growth:

$$\frac{\partial G_0(s,\tau)}{\partial \tau} + G_0(s,\tau)\frac{\partial G_0(s,\tau)}{\partial s} + k_1 G_0(s,\tau) = H(s), \tag{47}$$

with $G_0(s,0) = \Delta_1$.

If $k_1 = 0$, the linear equation gives

$$G(s) = M^{1/2} G_0(s,\tau) = \tau M^{1/2} H(s) = MsH(s), \tag{48}$$

which predicts initial linear growth. So the p.d.e. (47) is uniformly valid, see Cox & Mortell [13].

8. Resonance between concentric spheres

We consider sound waves in a spherical shell, $R_a < R < R_b$, generated by a periodically pulsating boundary at R_b with period τ_p. We nondimensionalize length with $L = R_b - R_a$, time with L/c_0, where $c_0 = \sqrt{\gamma p_0/\rho_0}$ is the ambient sound speed, and particle velocity with c_0. Then define $r_a = R_a/L$ and $r_b = R_b/L$ so $r_b - r_a = 1$.

The dimensionless governing equations are:

$$u_t + uu_r + b^{-1}cc_r = 0,\ c_t + uc_r + bc(u_r + 2u/r) = 0,\ r_a < r < r_b, \tag{49}$$

where $\omega = L/(\tau_p c_0)$, $u(r,t)$ is the particle velocity, $c(r,t)$ is the (dimensionless) sound speed and $b = (\gamma - 1)/2$ is a constant.

The rigid shell boundary condition on $r = r_a$ is

$$u(r_a, t) = 0, \tag{50}$$

while the oscillating boundary condition on $r = r_b$ is

$$u(r_b, t) = M\sin(2\pi\omega t), \tag{51}$$

where $M = u_0/c_0$ is the Mach number of the input.

The spherical wave general solution in linear theory is

$$u = \frac{1}{r}[f(\alpha) + g(\beta)],\ \alpha = t - (r - r_a),\ \beta = t + (r - r_b), \tag{52}$$

Following a wave leaving $r = r_b$ at time $t = t_0$, reflected from r_a at $t = t_1$ and then again at $r = r_b$ at $t = t_2$, the linear characteristics give the linear travel time

$$t_2 - t_0 = 2. \tag{53}$$

The boundary conditions then imply that

$$g(t_2) - g(t_0) = r_b M\sin(2\pi\omega t_2). \tag{54}$$

This is exactly as a straight tube with M replaced by Mr_b.

The nonlinear travel time, found by a nonlinear geometric acoustics expansion for the characteristics, see Whitham [3], with the restriction $L/R_a \ll 1$, or $1/r_a \ll 1$, is

$$t_2 - t_0 = 2 + g(t_0)\ln(r_b/r_a)(1 + \gamma). \tag{55}$$

The geometry enters through the term $\ln(r_b/r_a)$, and the result (55) should be compared with (16) for a straight tube.

Then (54) and (55) determine the nonlinear resonant oscillations when ω lies in the neighbourhood of resonance, $\omega = \frac{1}{2}$. At resonance

$$g(t)g'(t) = \frac{r_b}{1+\gamma}\ln(r_b/r_a)M\sin(\pi t), \tag{56}$$

$$g(t+2) = g(t), \text{ and } \int_0^2 g(y)dy = 0, \tag{57}$$

determine the solution, see Seymour, Mortell & Amundsen [14] and Galiev & Panova [15].

9. Resonance between concentric cylinders

We consider sound waves in a cylindrical shell, $R_a < R < R_b$, generated by a periodically pulsating boundary at R_b with period τ_p. In this case the linear geometric acoustics approximation, $L/R_a \ll 1$, produces the linear solution

$$u = \frac{1}{\sqrt{r}}[f(\alpha) + g(\beta)]. \tag{58}$$

This is used to calculate the linear difference equation and the nonlinear characteristics from which the nonlinear travel time is calculated. Hence the corrected nonlinear characteristics are

$$\alpha = t - (r - r_a) + f(\alpha)[\sqrt{r} - \sqrt{r_a}](1+b) + ..., \ \alpha = t \text{ on } r = r_a \tag{59}$$

$$\beta = t + (r - r_b) + g(\beta)[\sqrt{r} - \sqrt{r_b}](1+b) + ..., \ \beta = t \text{ on } r = r_b, \tag{60}$$

where $b = (\gamma - 1)/2$, see Whitham [3].

The boundary conditions are

$$u(r_a, t) = 0, \ u(r_b, t) = M\sin(2\pi\omega t), \ M \ll 1.$$

Then the linear difference equation is

$$g(t_2) - g(t_0) = \sqrt{r_b}M\sin(2\pi\omega t_2), \tag{61}$$

where $t_2 - t_0 = 2$ is the linear travel time. This is exactly as the previous linear (straight tube) equation with M replaced by $M\sqrt{r_b}$. There is no periodic solution for $\omega = 1/2$, the resonant frequency.

The approximate nonlinear characteristics give the nonlinear travel time as

$$t_2 - t_0 = 2 + 2g(t_0)[\sqrt{r_b} - \sqrt{r_a}](1+\gamma), \tag{62}$$

which can be compared with (16) for a straight tube.

The nonlinear solution is found from (61) and (62) as in (56) and (57), see Seymour, Mortell & Amundsen [14].

Acknowledgments

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