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Authors	Hao, Guangbo;Xie, Liyang;He, Xiuyun
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The Reliability Mathematical Model of Load-Sharing Parallel Systems based on Fatigue Cumulative Damage

GUANGBO HAO^{1*}, LIYANG XIE², and XIUYUN HE²

¹*Mechanical Engineering Department, Heriot-Watt University, Edinburgh, UK, EH144AS*

²*School of Mechanical Engineering & Automation, Northeastern University, China, 110004*

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Abstract: Conventional reliability models for parallel systems are not applicable for the analysis of parallel systems with load transfer and sharing. In this short communication, firstly, the direct reliability model of load-sharing parallel systems is presented based on the Miner cumulative damage theory, the total probability formula and addition principle. Secondly, the parallel system reliability is calculated using Monte Carlo simulation when the component life follows the Weibull distribution. The research results show that the proposed reliability mathematical model and Monte Carlo calculation method can analyze and evaluate the reliability of parallel systems in the presence of load transfer well.

Keywords: Load transfer; life distribution; Miner cumulative damage; parallel system; reliability

1. Introduction

Load-sharing parallel systems, namely $1/n$ load-sharing systems, are the simplest form of $k/n:G$ load-sharing systems. Their reliability problem is more complicated than that of the conventional parallel systems. The primary reason leading to complexity is the dependent failure caused by the load's redistribution [1]. The life distribution of a component changes due to the load's redistribution. Examples of load-sharing parallel systems include the multi-engine system in an airplane and the wire cables in bridges.

So far, the studies on the reliability analysis of load-sharing parallel systems are few. For convenience of analysis, Kececioglu [2] has advanced a reliability recurrence expression to convert and cumulate load action of all levels based on equivalent failure probability principle. References [3] presented the dependence-failure model of load-sharing parallel systems that applied the equivalent failure probability transform model when the component life follows Weibull distribution or exponential distribution (EP model in short). Amari [4] explored using the cumulative exposure model for reliability analysis of k -out-of- n load-sharing systems from a failure's point of view and proposed an efficient procedure to compute their reliability based on Markov reward models. In [4], an effective age of the component, which is the sum over all load durations multiplied by corresponding acceleration factors, was used for calculating the cumulative failure rate of the component.

However, these proposed reliability analysis methods have their limitations or complexity. For example, the EP model in [3] expresses damage degree in failure probability, which has been documented to be unreasonable by Xie [5]. Furthermore, the life distribution of component in the EP model is limited to be exponential. This paper focuses on presenting a simple and easy-to-understand mathematical model to evaluate

*Corresponding author: guangbohao@gmail.com

the reliability of load-sharing parallel systems using the Miner cumulative damage theory from a non-failure's point of view. And our method does not need to use the effective age of the component like in [4]. We develop a direct step-by-step reliability calculation method for load-sharing systems according to total probability formula and addition principle, which can provide an insight into the load-redistribution process. The Monte Carlo method is applied to calculate the reliability of a load-sharing parallel system following the Weibull life distribution or general life distributions.

Notation

T_i - Fatigue life of a component in a parallel system under stress level $S_i (i=1, \dots, n)$
 $f_i(t)$ - Probability density function of distribution of fatigue life T_i

2. The Reliability Modeling of Load-Sharing Parallel Systems

We present a method to calculate the reliability of a parallel system by breaking down the problem into several disjoint cases step by step, which can take into account the load-redistribution of the parallel system. In the following, we describe two cases when a parallel system composed of two components works at the time of t , as well as their reliability evaluation methods:

Case 1: both components work until the time t normally under the same stress level.

$$R_1(t) = \left[\int_0^t f_1(t) dt \right]^2 = [P(t/T_1 < 1)]^2 \quad (1)$$

Case 2: one component works to time t under varying stress levels and the other component fails at the random time of t_1 under a low stress level. If one component fails at the random time of t_1 in the time interval $[0, t]$, based on Miner cumulative damage theory and total probability formula [6], the reliability of the parallel system in this case can be calculated as:

$$R_2(t) = C_2^1 \int_0^t P(t_1/T_1 + (t-t_1)/T_2 < 1) \cdot f_1(t_1) dt_1 \quad (2)$$

According to the addition principle [6], the reliability model of the parallel system composed of two components (CD model) at the time of t can be expressed accurately as:

$$R_D^{(2)}(t) = R_1(t) + R_2(t) = [P(t/T_1 < 1)]^2 + 2 \int_0^t P(t_1/T_1 + (t-t_1)/T_2 < 1) \cdot f_1(t_1) dt_1 \quad (3)$$

Where, t_1 and $t-t_1$ are the durations of the component under low stress level S_1 and that under high stress level S_2 , respectively; T_1 and T_2 are the fatigue life of working component under low stress level S_1 and that under high stress level S_2 , respectively; and cumulative damage of working component in case 2 at the time of t is expressed as $t_1/T_1 + (t-t_1)/T_2$ directly.

Likewise, there are $n (n \geq 3)$ cases when the parallel system composed of n components works at the time of t . Similar to Equation (3), the reliability of a parallel system composed of n components can be calculated using the above step-by-step calculation as:

$$R_D^{(n)}(t) = C_n^0 [P(t_1/T_1 < 1)]^n + C_n^1 \int_0^t [P(t_1/T_1 + (t-t_1)/T_2 < 1)]^{n-1} f_1(t_1) dt_1 \quad (4)$$

$$+ \sum_{i=1}^{n-2} \{ C_n^1 C_{n-1}^1 \dots C_{n-i}^1 \int_0^t \dots \int_{t_i}^{t_{i+1}} [P(\sum_{j=1}^{i+1} (t_j - t_{j-1})/T_j + (t-t_{i+1})/T_{i+2} < 1)]^{n-(i+1)} \cdot f_{1,i+1}(t_{i+1}) \dots f_{1,2}(t_2) f_1(t_1) \cdot dt_{i+1} \dots dt_2 dt_1 \}$$

Where $f_{1,i+1}(t_{i+1}) = d[P(\sum_{j=1}^{i+1} (t_j - t_{j-1})/T_j > 1)] / dt_{i+1}$, $0 = t_0 \leq t_1 \leq \dots \leq t_{n-2} \leq t_{n-1} \leq t$.

It is noted that we can extend Equation (4) to calculate the reliability of $k/n:G$ load-sharing parallel systems by subtracting the last $k-1$ terms from Equation (4).

3. Monte Carlo Simulation to Calculate System Reliability

It is well known that the fatigue life of a component can follow the Weibull distribution $\text{Weib}(\alpha, \beta)$. It is noted that all parameters can be obtained by life test, and the parameter β of a Weibull distribution keeps constant due to the failure mechanism invariability of components.

In this paper, the Monte Carlo simulation method [7] is used to calculate the system reliability. Monte Carlo simulation results for two-component and three-component cases are shown in Figure 1 (a) and Figure 1 (b), respectively.

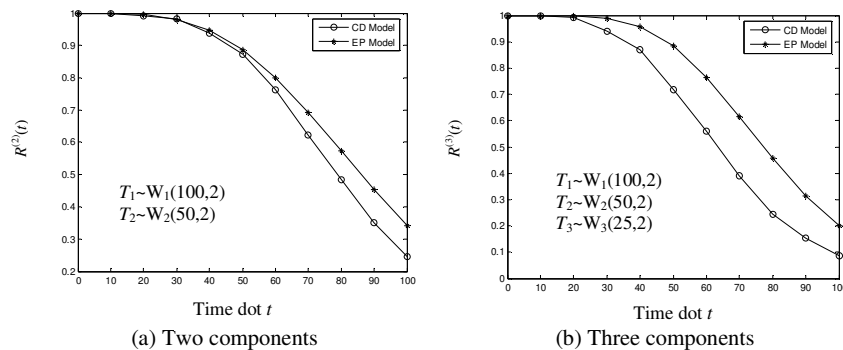


Figure 1: The Reliability Curve of Parallel Systems

Figure 1 illustrates that the reliability based on the existing EP model is larger than the reliability based on the proposed CD model, which means that the EP model is prone to produce overestimated reliability values. In other words, the design based on the CD model is prone to be a safe and conservative design.

4. Conclusions

This short communication presented easy-to-understand analytical formulas based on the Miner cumulative damage theory for analyzing the reliability of load-sharing parallel systems. The proposed reliability formulas were evaluated using the Monte Carlo simulation method. Compared to the existing EP model, the proposed model can produce safer designs for load-sharing parallel systems.

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