CORA Cork Open Research Archive
Cartlann Taighde Oscailte Chorcaí

Title	Damage calibration of a beam using wavelet analysis and image processing
Authors	Pakrashi, Vikram;O'Connor, Alan J.;Basu, Biswajit
Publication date	2006
Original Citation	Pakrashi, V., O'Connor A.J., Basu B., 2006. Damage Calibration of a Beam using Wavelet Analysis and Image Processing. In: Bridge Engineering Research in Ireland (BEI), Third Symposium on Bridge and Infrastructure Research in Ireland. Dublin, Ireland.
Type of publication	Conference item
Download date	2026-11-02 00:37:23
Item downloaded from	https://hdl.handle.net/10468/259

UCC
University College Cork, Ireland
Coláiste na hOllscoile Corcaigh

ucc
Coláiste na hOllscoile Corcaigh, Éire
University College Cork, Ireland

CORA
Cork Open Research Archive
Cartlann Taighde Oscailte Chorcaí

Pakrashi, V., O'Connor A.J., Basu B., 2006. Damage Calibration of a Beam using Wavelet Analysis and Image Processing. In: Bridge Engineering Research in Ireland (BEI), *Third Symposium on Bridge and Infrastructure Research in Ireland*. Dublin, Ireland.

CORA Cork Open Research Archive http://cora.ucc.ie

Damage Calibration of a Beam Using Wavelet Analysis and Image Processing

Vikram Pakrashi, Alan O' Connor and Biswajit Basu

Abstract

Efficient damage detection and calibration of structures have gained great importance in recent times in terms of health monitoring and maintenance programmes. Wavelet analysis based damage detection and calibration from the deflected shape of beams are theoretically known to be a simple and efficient way of assessing damage. However, the measurement of the static or dynamic deflected shape of a vibrating beam is often difficult. The use of sophisticated devices to measure such spatial characteristics suffer from the disadvantage of high cost of the instrument and its unavailability. This paper considers a simply supported aluminium beam with an open crack and presents a video camera based inexpensive laboratory study to assess the damage using wavelet analysis. The vibrating deflected shape recorded by the camera has been processed using image processing methods and an intelligent pattern recognition procedure for the quantification of such the dynamic deflected shape at a particular instant of time. Wavelet analysis was subsequently performed on the damaged deflected shape to successfully identify the location of the damage and estimate the degree of damage for different crack depth ratios. The image analysis based detection is found to be a novel, easy and an inexpensive technique and the method is seen to have a potential for unmanned online structural health monitoring process.

Keywords: Image Processing, Open Crack, Structural Health Monitoring, Wavelet

Vikram Pakrashi: Vikram Pakrashi is currently pursuing his PhD in the Department of Civil, Structural and Environmental Engineering, Trinity College Dublin, Ireland. His research interests include structural health monitoring and control, damage detection and wavelet analysis.

Alan O' Connor: Alan O' Connor is a Lecturer in the Department of Civil, Structural and Environmental Engineering, Trinity College Dublin, Ireland. His research interests include structural health monitoring, reliability analysis, degradation of materials and bridge vehicle interaction.

Biswajit Basu: Biswajit Basu is a Professor in the Department of Civil, Structural and Environmental Engineering, Trinity College Dublin, Ireland. His research includes time-frequency analysis, earthquake engineering, structural health monitoring, nonlinear dynamics and transportation engineering.

1. INTRODUCTION

Identification of change in natural frequencies and modeshapes of a freely vibrating damaged beam with respect to its undamaged state is a popular method for damage identification. These changes are often quite small and the method does not perform efficiently when measurements are contaminated by noise. Analysis of damaged modeshapes by wavelet transform provides a better and more robust methodology for the identification of damage. A sharp change in the wavelet coefficients at different scales near the damage location indicates the presence and the location of the damage and the magnitude of the local extrema of the wavelet coefficients at the location of damage is related to the extent of the damage. The principles behind such wavelet based damage detection are associated with the detection of singularities in a function or in any of its derivatives. Information about the beam in its undamaged state is not required.

The aspect of singularity detection through wavelets has been discussed in details by Mallat [1]. Gentile and Messina [2] have discussed the criteria for a proper selection of wavelet basis functions to efficiently identify the damage in a beam with an open crack in the presence of measurement noise and have demonstrated the performances of Gaussian and Symlet wavelets in the detection process. Loutridis et al [3], Chang and Chen [4] and Okafor and Dutta [5] have considered the problem of the identification of open cracks in a beam as well, but with a single type of wavelet basis function for analysing the damaged modeshapes. Melhem and Kim [6] have analysed the response of concrete structures and have shown the effectiveness of using wavelet transform over traditional Fourier transform in identification of damage. Spatial response data from beam structures have been successfully analysed by wavelets to detect damage by Wang and Deng [7]. Advantages of wavelet analysis over the usual eigenvalue analysis for a simply supported beam with a non-propagating open crack have been shown by Liew and Wang [8].

It is observed that although the detection of an open crack in a beam has been comparatively well dealt with theoretically, few experimental are present in the literature. One of the main reasons of such absence of experimental results is that difficulty associated with the measurement of the static or dynamic deflected shape of a vibrating beam. Sophisticated devices like the scanning laser vibrometer have been used before to obtain these shapes (Vanlanduit et.al [9], Okafor and Dutta [5]) in recent times. A disadvantage of using such devices is usually the high cost of the instrument and its unavailability. Comparatively inexpensive experimental studies on damage detection with wavelets from static deflected shape have been performed by Rucka and Wilde [10] and Patsias and Staszewski [11] using camera based optical measurement techniques. In this paper, a video camera based damage detection technique followed by an intelligent pattern recognition procedure developed by the authors has been adopted to successfully detect and identify the presence and the location of

an open crack in a simple supported aluminium beam by using wavelet analysis. The estimated degree of damage has also been found from the video camera based estimate.

2. Damage Model

A simply supported Euler Bernoulli beam with an open crack is modelled as two uncracked sub-beams connected through a rotational spring at the location of crack in the lumped crack formulation. The length of the beam is L with the damage located at a distance of 'a' from the left hand support of the beam. The crack depth is taken as c and the overall depth of the beam is h. The general arrangement of such a beam with an open crack is shown in Figure 1.The free vibration equation for both the beams on either side of the crack can be written as

$$EI\frac{\partial^4 y}{\partial x^4}+\rho A\frac{\partial^2 y}{\partial t^2}=0 \quad (1)$$

where E, I, A and ρ are the Young's modulus, the moment of inertia, the cross sectional area and the density of the material of the beam on either side of the crack. The displacement of the beam from its static equilibrium position is y(x,t), at a distance of x from the left hand support along the length of the beam at time t. The strains and stresses are concentrated at the crack tip and decay inversely proportional to the square root of the radial distance away from the crack tip (Carneiro [12]).

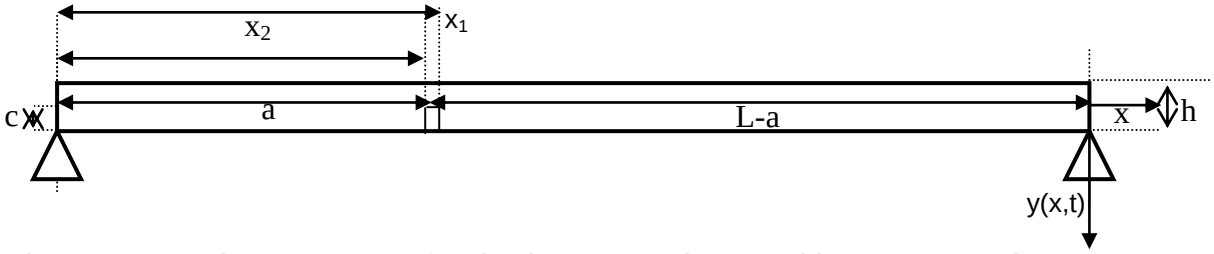


Figure 1. General Arrangement of a Simply Supported Beam with an Open Crack.

It is assumed that the effects of the crack are applicable in the immediate neighbourhood of the crack location and is represented by a rotational spring of equivalent local stiffness of K_t as shown in Figure 2.

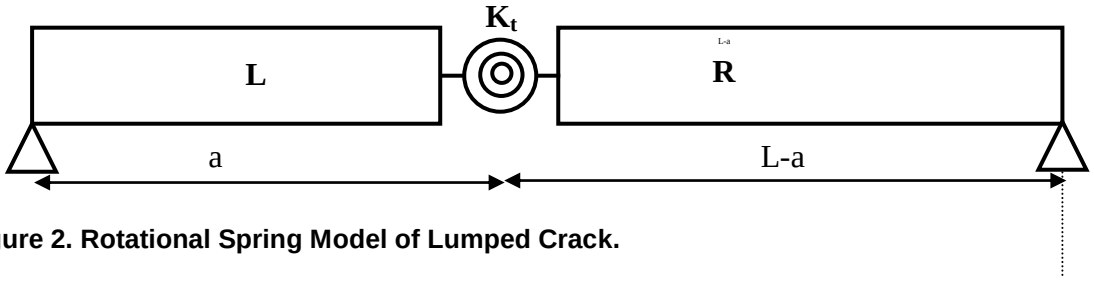


Figure 2. Rotational Spring Model of Lumped Crack.

Through the separation of variables in Equation 1 and solving the characteristic equation, a general solution of the modeshapes is found as

$$\Phi_L = C_{1L}\mathrm{Sin}(\lambda x)+C_{2L}\mathrm{Cos}(\lambda x)+C_{3L}\mathrm{Sinh}(\lambda x)+C_{4L}\mathrm{Cosh}(\lambda x)\ \ 0\le x<a \quad (2.1)$$

and

$$\Phi_R = C_{1R}\mathrm{Sin}(\lambda x)+C_{2R}\mathrm{Cos}(\lambda x)+C_{3R}\mathrm{Sinh}(\lambda x)+C_{4R}\mathrm{Cosh}(\lambda x)\ a\le x\le L \quad (2.2)$$

for the sub-beams on the left (L) and the right (R) side of the rotational spring respectively. The terms $C_{(.)}$ are integration constants arising from the solution of the separated fourth order differential equation in space. The term λ is expressed as

$$\lambda = (\frac{\rho A \omega^2}{EI})^{1/4} \quad (2.3)$$

where the natural frequency of the cracked beam is ω. Both displacement and moment at the two supports of the beam are zero. The continuity in displacement, moment and shear are assumed at the location of crack. A slope discontinuity is present at the crack location. The slope condition is modelled as

$$\Phi_R'(a) - \Phi_L'(a) = \theta L \Phi_R''(a) \quad (2.4)$$

In equation 2.3, the term θ is the non-dimensional crack section flexibility dependent on the crack depth ratio. As per Narkis [13] the function is considered to be a polynomial of the crack depth ratio in a non-dimensional form as

$$\theta = 6\pi\delta^2(h/L)(0.5033 - 0.9022\delta + 3.412\delta^2 - 3.181\delta^3 + 5.793\delta^4) \quad (2.5)$$

The term δ (=c/h) is the crack depth ratio (CDR). The boundary conditions are substituted in the general modeshape equation and the natural frequency of the cracked beam may be found by setting the determinant of the matrix derived from the system of equations to zero, expanding it and solving for the roots of λ numerically. The coefficient C_{1L} is normalized to unity, being consistent with the fact that for an undamaged beam the maxima of the first modeshape is equal to unity. It is observed, that an open crack introduces a singularity in the derivative of the damaged modeshape.

3. WAVELET ANALYSIS

In a square integrable function space, a wavelet is considered to be a zero average function (Mallat [1]). Hence

$$\int_{-\infty}^{+\infty} \psi(x)dx = 0 \quad (2.6)$$

A wavelet family of functions may be obtained by considering

$$\psi_{b,s}(x) = \frac{1}{\sqrt{s}}\psi(\frac{x-b}{s}) \quad (2.7)$$

where s is the scale and b is the translation parameter. The continuous wavelet transform of a function f(x) in the same square integrable space can be represented as

$$Wf(b,s) = \int_{-\infty}^{+\infty} f(x)\frac{1}{\sqrt{s}}\psi^*(\frac{x-b}{s})dx \quad (2.8)$$

The Calderon-Grossman-Morlet theorem (Mallat [1]) requires a weak admissibility condition to ensure the completeness of the wavelet transform and to maintain energy balance. Mathematically, it is represented as

$$\int_0^{+\infty} \frac{|\hat{\psi}(\omega)|^2}{\omega} d\omega < +\infty \quad (2.9)$$

The identification of a discontinuity in a function or any of its derivatives can be linked with the number of vanishing moments of the wavelet basis function chosen for analysis. A wavelet has m number of vanishing moments if

$$\int_{-\infty}^{+\infty} x^k \psi(x)dx = 0, \ k=0,1,2,\ldots m-1 \quad (2.10)$$

For a wavelet with no more than m number of vanishing moments, it can be shown that for very small values of s in the domain of interest, the continuous wavelet transform of a function

f(x) can be related to the m^{th} derivative of the signal (Mallat [1]). For any wavelet ψ(x) with m vanishing moments, there exists a fast decaying function θ(x) satisfying

$$\psi(x) = (-1)^m \frac{d^m \theta(x)}{dx^m} \qquad (2.11)$$

Under this condition, the relationship between the continuous wavelet transform of f(x) and its m^{th} derivative can be expressed as

$$\lim_{s\to 0} \frac{Wf(b,s)}{s^{m+1/2}} = K \frac{d^m f(x)}{dx^m} \qquad (2.12)$$

where

$$\int_{-\infty}^{+\infty} \theta(x)dx = K \neq 0 \qquad (2.13)$$

Hence it is possible for a wavelet to detect singularities in a signal or its derivatives through the incorporation of a proper choice of basis function. Since, a deflected shape of a beam with an open crack contains singularities within itself or in any of its derivatives, wavelet transform is deemed to be a powerful way to identify the location of the damage. Due to the presence of the singularity, a wavelet transformed deflected shape would render a local extremum of the wavelet coefficient at the location of damage consistently at different scales. The measure of the local regularity in the neighbourhood of a point in a function can be related to the local Lipschitz exponent around that point. As per Mallat [1], a function f(x) in the square integrable space is pointwise Lipschitz $\kappa \geq 0$ at a point ν if there exists a K>0 and a polynomial p_{ν} of degree $\breve{m}$ such that

$$\forall x \in \mathbb{R}, \left|f(x) - p_\nu(x)\right| \leq K\left|x-\nu\right|^\kappa \qquad (2.14)$$

The term κ provides the degree of singularity in the neighbourhood of the point x. If the function f(x) is uniformly Lipschitz $\kappa < \breve{n}$ over an interval $[\breve{a}, \breve{b}]$, then there exists an $\breve{A}$ >0 such that

$$\forall (b,s) \in [\breve{a},\breve{b}] \times \mathbb{R}^+, \; |Wf(b,s)Z| \leq \breve{A} s^{\kappa+\frac{1}{2}} \left(1 + \left|\frac{b-\nu}{s}\right|^\kappa\right) \qquad (2.15)$$

Thus, the magnitude of the wavelet coefficients around a point can be related to the local Lipschitz exponent, and hence to the degree of singularity present at that point. This in turn indicates that the magnitude of the local extremum formed at the location of damage can be a descriptor of the extent of damage present at that point.

4. EXPERIMENTAL RESULTS

A simply supported aluminium beam of 1m length is employed for this purpose. The open crack is created by sawing a notch into the lower section of the beam of cross section 10mmx10mm. The location of the damage is situated at 0.3m from the left hand support of the beam. The beam is vibrated freely and is also subjected to a static weight (P=9 kg) at the centre. The general arrangement of the experiment is given in Figure 3. The initial excitation of the beam is provided by deflecting the beam manually about its static displaced position and then releasing it to vibrate freely. The CDR of the damage is taken as 0.5.

The free vibration of the beam is recorded by an Olympus μ 800 digital camera. An appropriate deflected shape has been chosen by running the video using commercial software named Ulead Video Studio 6 and freezing a single frame, a method similar to that followed by Hartman and Gilchrist [14]. The frame is converted and saved as a bitmap image. The images are 240x320 pixels in size. The recording has been done in the presence of

controlled lighting and a white background to minimize ambient disturbances in the image. It should be noted here that a very high resolution image does not necessarily guarantee low noise in an image. Often the general imperfections of the underside of the beam are accentuated in high resolution images resulting in large file-space, increased computational time and false alarms.

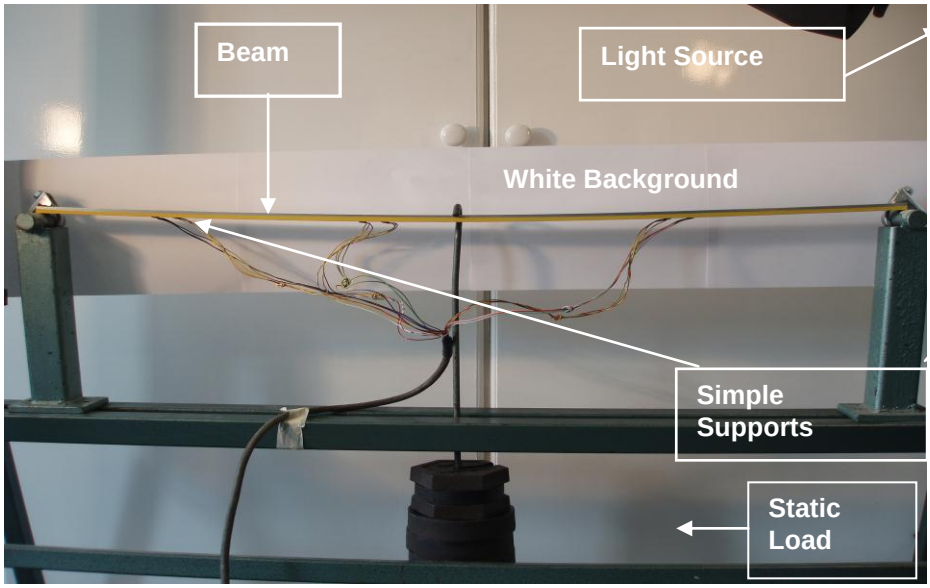


Figure 3. Experimental Setup for the Damage Detection of an Aluminium Beam Using Wavelet Analysis on Damaged Deflected Shape.

The saved image of the deflected shape is essentially a rectangular grid of pixels and the centre of a pixel occupies the integer co-ordinates in the grid so produced. The interior of each pixel can be further subdivided into a continuous spatial coordinate system by considering that the local origin for each pixel lies at the top left hand corner of the pixel, and not at the centre. Each pixel is associated with a vector of number describing its hue-saturation value. The gray-level threshold of the image is computed by minimizing the interclass variances of the black and white pixels using Otsu's method using MATLAB 7.0 signal processing toolbox. Pixels below the threshold assume a value zero and turn black, while the other pixels turn white. This enables to convert a complex matrix from the original image to a simpler binary black and white image. This transformation is important when the feature of interest is in contrast with its surroundings in the image. The damaged beam was tested to achieve such a contrast for the underside of the beam with its background. Figure 4-a shows a thresholded binary image representing the partial deflected shape of a beam with an open crack (δ=0.5) of 2mm width at a certain instant of time during the free vibration.

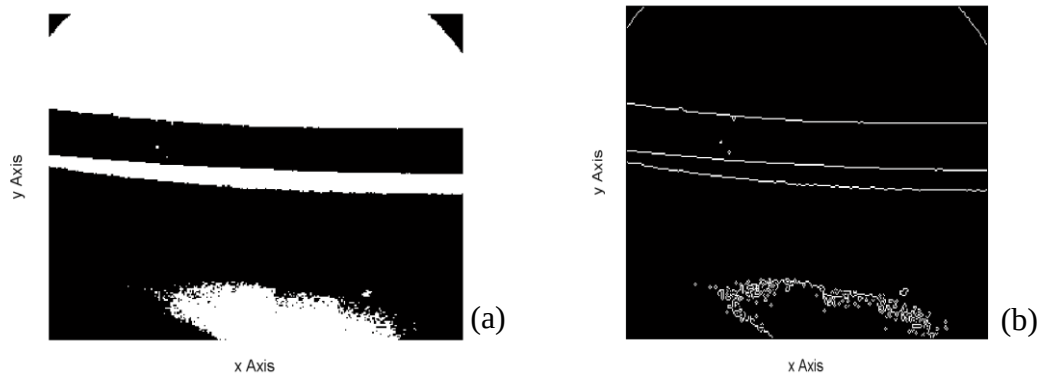


Figure 4. Binary Image and Edge detection of a Damaged Partial Modeshape with δ=0.5.

The black and white binary images are converted by thresholding and the edges of the images were found using the Sobel method (Sarfraz [15]) incorporating the MATLAB 7.0 signal processing toolbox. The Sobel method scans a binary image and returns the approximation to the derivative of the two-dimensional data. The edges are returned (Figure 4-b) as the points where the gradient of the image are locally maximum. This identified image contains noise and a number of definite features, which are not a part of the deflected underside of the beam. These spurious edges are present for all practical applications and thus the image needs to be post processed to identify the damaged deflected shape. A visual idea about the spurious feature can be helpful to pre-process the picture to make the computational effort of the pattern recognition scheme easier and the scanning region comparatively smaller. The lower edge of the beam has been extracted from Figure 4-b by an intelligent pattern recognition scheme. A scheme of the computer aided deflected shape recognition is as follows:

1. Acquire a video recording of the free vibration of the damaged beam.
2. Import it to the computer and run it using Ulead Video 6.
3. Select and freeze and appropriate frame representing the deflected shape from the video and convert it to a bitmap image.
4. Load the bitmap image in Matlab 7.0 workspace and convert the coloured image to a binary image.
5. Identify the edges of the binary image.
6. Given a co-ordinate pair of the vicinity of the lower edge of the beam, incorporate a vertical column by column search to recognise a pattern of a certain number of pixel coordinates with value 1 in between two large bands of zero. A pixel is considered to be discretised in space arbitrarily keeping the pixel value constant within a single pixel.
7. Identify the lower edge of the beam from the search.
8. Employ wavelet based damage identification on the measured deflected shape.

A wavelet analysis using Coif4 basis function and Hanning window is performed on the estimate of the deflected partial modeshape [16] and the result of the analysis is illustrated in Figure 5. It is observed that a definite local maximum is present in the analysed shape propagating at a range of scales consistently. The horizontal axis of the observed value is pre-calibrated against the beam length and the damage is seen to be detected around the neighbourhood of 0.3m from the left hand side of the beam.

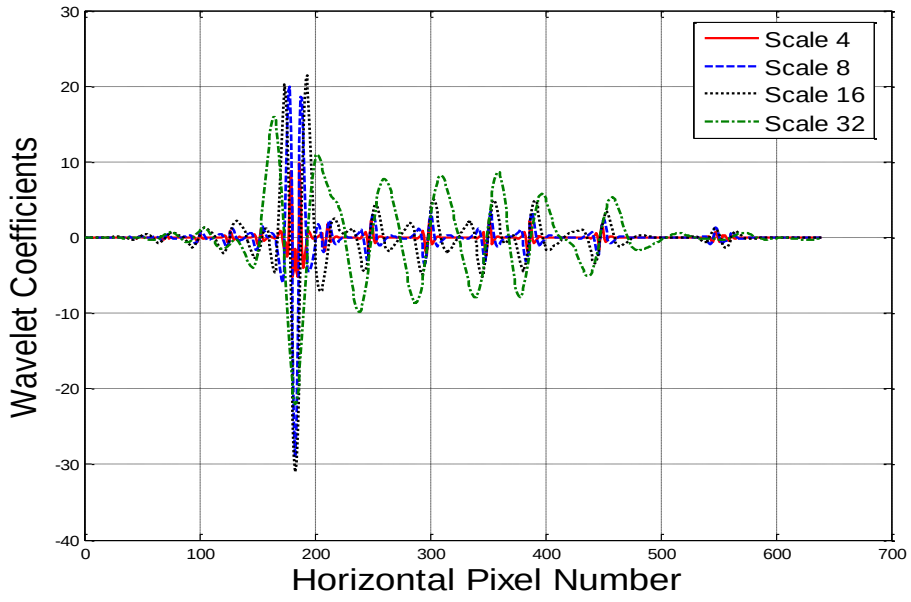


Figure 5. Experimental Identification of Damage Location.

5. Conclusion

A wavelet based damage detection scheme is experimentally validated on a simply supported aluminium beam with an open crack using a video camera based intelligent pattern recognition technique. The presence and the location of the damaged have been successfully identified. This image analysis based detection is found to be a novel, easy and an inexpensive method for damage detection. The proposed scheme is seen to have a potential for unmanned online structural health monitoring process.

6. Reference

1. Mallat, S. *A Wavelet Tour on Signal Processing*, Academic Press, New York, 2001.
2. Gentile, A and Messina, A. *On the Continuous Wavelet Transforms Applied to Discrete Vibrational Data for Detecting Open Cracks in Damaged Beams*, International Journal of Solids and Structures, 2003, 40, 295-315.
3. Loutridis, S., Douka, E and Trochidis, A. *Crack Identification in Double Cracked Beams using Wavelet Analysis*, Journal of Sound and Vibration, 2004, 277, 1025-1039.
4. Chang, C.C and Chen, L.W. *Vibration Damage Detection of a Timoshenko Beam by Spatial Wavelet Based Approach,* Applied Acoustics, 2003, 64, 1217-1240.
5. Okafor A.C and Dutta A, *Structural Damage Detection in Beams by Wavelet Transforms*, Smart Materials and Structures, 2000, 9, 906-917.
6. Melhem H and Kim H, *Damage Detection in Concrete by Fourier and Wavelet analyses*, ASCE Journal of Engineering Mechanics, 2003, May, 571-577
7. Wang Q and Deng X, *Damage Detection with Spatial Wavelets,* International Journal of Solids and Structures, 1999, 36, 3443-3468
8. Liew K.M and Wang Q, *Application of Wavelet Theory for Crack Identification in Structures*, ASCE Journal of Engineering Mechanics, 1998, February, 152-157
9. Vanlanduit S, Guillaume P, Schoukens J and Parloo E. *Linear and Non-linear Damage Detection using a Scanning Laser Vibrometer.*
10. Rucka M and Wilde K, *Crack Identification using Wavelets on Experimental Static Deflection Profiles*, Engineering Structures, 2006, 28, 279-288
11. Patsias S and Staszewski W.J , *Damage Detection using Optical Measurements and Wavelets*, Structural Health Monitoring, Vol 1(1), 2002, 5-22
12. Hartman A.M and Gilchrist M.D, *Evaluating Four-Point Bend Fatigue of Asphalt Mix Using Image Analysis*, ASCE Journal of Materials in Civil Engineering, January-February, 2004, 60-68
13. Narkis. Y, *Identification of Crack Location in Vibrating Simply Supported Beams*, Journal of Sound and Vibration, 1994, 172(4), 549-558.
14. Sarfraz M, *Computer Aided Intelligent Recognition Techniques and Applications*, 2005, John Wiley and Sons, Chichester, England
15. Carneiro S.H.S and Inman D.J, *Continuous Model for the Transverse Vibration of Cracked Timoshenko Beams*, Transactions of the ASME, Journal of Vibration and Acoustics, 2002, Vol. 24, 310-320
16. Pakrashi V, Basu B, O' Connor A, Identification of Open Cracks Using Wavelet Analysis, *Structural Engineering Convention, Bangalore, India,* December 14-16, 2005