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UCC
University College Cork, Ireland
Coláiste na hOllscoile Corcaigh









Trends in Mathematics Education

Conference Proceedings

Aula Maxima, Main Quadrangle,
University College Cork

26th November 2012





MICHIGAN STATE UNIVERSITY



IRISH RESEARCH COUNCIL
An Chomhairle um Thaighde in Éirinn

Trends in Mathematics Education

Conference Proceedings

Aula Maxima, Main Quadrangle,
University College Cork

26th November 2012

Funded by the School of Education, UCC and the Irish Research Council-funded Advanced Collaborative Research Award: *Re-imagining Initial Teacher Identity and Learning Study*

Paul Conway, Vanessa Rutherford, Michael Delargey (Eds.)
School of Education, Leeholme, O'Donovan's Road, University College Cork, Cork, Ireland.

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Welcome from Head of Education, UCC

Dear Colleagues/A Chairde,

I am delighted to avail of this opportunity to welcome delegates to the *Trends in Mathematics Education* conference here in UCC. In particular, we are pleased to host and contribute to the US National Science Foundation-funded FIRSTMATH project planning team meeting being held here in UCC this week.

The theme of the conference is of much interest in Ireland and elsewhere, given the heightened policy interest in mathematics education. This conference provides a valuable opportunity to consider how mathematics education research on teachers and teaching might inform policy and practice as well as point to new directions for mathematics pedagogy.

Recent research undertaken at the School of Education led by Dr Paul Conway on initial teacher education for post primary (*Learning to Teach Study*, LETS) teaching showed that there is much scope for developing student teachers' mathematics pedagogical knowledge but how best to achieve this in collaboration with schools merits further study, possibly in the form of research and development initiatives. The Teaching Council of Ireland is placing increasing emphasis on the need for teachers to learn and upskill throughout the career span and it is likely that action research and evaluation, involving opportunities for reflection at the level of the school and the classroom, will need to feature more significantly. To date, in Ireland we have been much better at focusing on system change than on the more complex but vital cultural transformation at the level of school and the classroom i.e. at the pedagogical level. This conference offers a wonderful opportunity to consider the latter and to ponder the kinds of interventions, initiatives and supports that are now needed to promote a more critical and nuanced approach to pedagogy in the mathematics classroom.

We look forward to being informed and inspired by the conference participants and to the new alliances that will be formed as a result of the event.

Le gach dea ghuí,

Prof. Kathy Hall
Head of School of Education, UCC

Foreword

Dear colleagues,

Welcome to UCC and the *Trends in Mathematics Education* conference.

At today's conference there are mathematics and teacher education researchers from ten of the countries involved in the design of the FIRSTMATH study (meeting in UCC 26-30th Nov), along with policy makers, teacher educators, researchers, teacher union leaders, teachers and student teachers from Ireland.

Today's conference provides a valuable opportunity to learn more about the FIRSTMATH study given its potential to inform both teacher education and mathematics education initiatives in Ireland and elsewhere. What is FIRSTMATH? FIRSTMATH is a cross-national study of novice teachers' (i.e. years 1-5) mathematical knowledge for teaching and the influence of previous preparation, school context and opportunities to learn-on-the-job, on that knowledge. The results of this study will provide much needed empirical evidence about the influence of school context and on-the-job opportunities to learn on mathematics teachers' knowledge, and on the nature of the knowledge that is useful in and for mathematics teaching students in diverse settings and school contexts. As such, the FIRSTMATH study can be viewed as both a site for researching mathematics education as well as a case study within teacher education. The latter emphasis signals its potential to inform the whole teacher education endeavour.

I would like to thank a number of people who made today's event possible: both Dr Vanessa Rutherford, Michael Delargey, in particular, as well as Stephanie Larkin, Angela Desmond and Prof. Kathy Hall all in the School of Education here in UCC; Prof. Teresa Tatto (PI on FIRSTMATH) and Prof. Jack Schwille, Michigan State University; teacher education and mathematics colleagues engaged with FIRSTMATH in Ireland to date (Dr Aisling Leavy, MIC; Dr Catharine Paolucci, NUIG, Michael Delargey, UCC; Dr Ann O'Shea, NUIM; Dr Elizabeth Oldham, TCD, Dr Vanessa Rutherford, UCC); education agencies including the Department of Education and Skills, National Council for Curriculum and Assessment (NCCA), Project Maths, the Teaching Council and, finally, colleagues presenting papers and posters listed in the programme.

Enjoy the conference.

Paul Conway
School of Education, UCC

FIRSTMATH STUDY: The *First* Five Years of *Mathematics* Teaching



UCC
Coláiste na hOllscoile Corcaigh, Éire
University College Cork, Ireland



MICHIGAN STATE UNIVERSITY



IRISH RESEARCH COUNCIL
An Chomhairle um Thaighde in Éirinn

in conjunction with the Trends in Mathematics Education Conference

Aula Maxima, Main Quadrangle
University College Cork

26th November 2012
12-6pm

Conference funded by the US National Science Foundation (NSF), the School of Education, UCC and the Irish Research Council-funded study: *Re-imagining Initial Teacher Identity and Learning* (Rii-TIL) Study.

AIMS:
To build a coalition of interest in the **FIRSTMATH** study given its potential to inform both teacher education and mathematics education initiatives in Ireland.

PARTICIPANTS:
20+ Leading Mathematics and Teacher Education Researchers on **FIRSTMATH** and *Key Stakeholders* involved in Mathematics and Teacher Education.

What is FIRSTMATH?
FIRSTMATH is a cross-national study of novice teachers' (i.e. years 1-5) mathematical knowledge for teaching and the influence of previous preparation, school context and opportunities to learn-on-the-job, on that knowledge.

IMPACT:
The results of this study will provide much needed empirical evidence about the influence of school context and on-the-job opportunities to learn on mathematics teachers' knowledge, and on the nature of the knowledge that is useful in and for mathematics teaching students in diverse settings and school contexts.

Programme: Trends in Mathematics Education Reform Conference

Aula Maxima, Main Quadrangle, University College Cork, Ireland
Monday 26th November 2012

12.00-1.15: Panel 1: Policy context for FIRSTMATH: Teacher knowledge

- *TIMSS, PISA and student achievement studies*: *Issues for mathematics education in Ireland*, Peter Archer, Education Research Centre (ERC), Dublin
- *TEDS to FIRSTMATH*, Maria Teresa Tatto, Michigan State University
- *Challenges of developing measures for cross-national studies: TEDS and FIRSTMATH as cases,* Michael Rodriguez, University of Minnesota
- Chair: Paul Conway, UCC

1.15-2.15: Lunch and poster session

- *Approaching mathematical problem solving in the Irish primary classroom: Using a constructivist framework to scaffold teachers' explorations*, John O'Shea & Aisling Leavy, MIC
- *Examining pre-service teachers' mathematical knowledge for teaching,* Catherine Paolucci, NUIG
- *Learning to Teach Study* (LETS), Michael Delargey, Paul Conway & Rosaleen Murphy, UCC
- *Noticing in the third level mathematics classroom*, Sinead Breen *et al*
- *FIRSTMATH*
- Effects of Calculators on Third Year Students' Achievement and Attitudes, Close *et al,* ERC

2.15-3.30: Panel 2: Problem posing and solving in mathematics education

- *Charting the development of problem posing in initial teacher education,* Aisling Leavy, Mary Immaculate College, Limerick
- *Teacher education and problem posing*, Sandra Crespo, Michigan State University
- *Teacher education and problem solving in mathematics*, Kiril Bankov, University of Sofia
- Chair: Michael Delargey, UCC

3.30-4.00: Tea / Coffee

4.00-5.30: Symposium: Cross-national studies of mathematics education reform

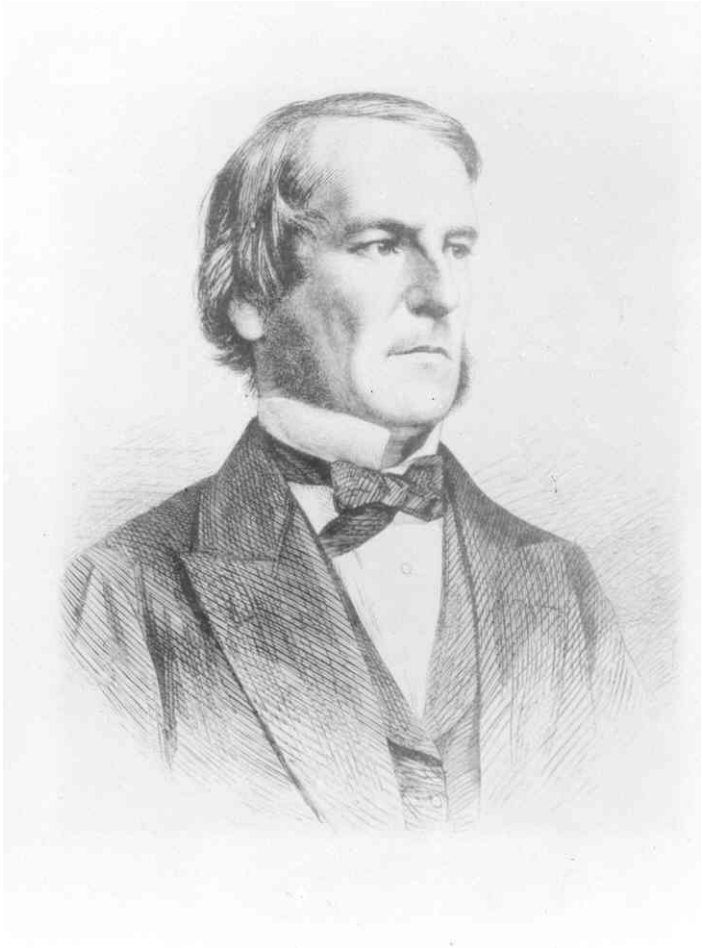
- *Perspectives on cross-national studies of education: Insights from the IEA experience*, Jack Schwille, Michigan State University
- *Mathematics education reform in Ireland: directions and dilemmas*, Elizabeth Oldham, Trinity College Dublin
- *Teacher evaluation: Beyond value-added models (VAM)*, Mark Reckase, Michigan State University
- Chair: Aisling Leavy, MIC Discussant: Paul Conway, UCC

530-5.35 Signing of MOU between School of Education, UCC and College of Education, Michigan State University

5.35-6.30: Reception @ Aula Maxima

The Enduring Legacy of George Boole, 1815-1864: First Professor of Mathematics at UCC

Dr. Vanessa Rutherford
School of Education, University College Cork



Source: BP/1/358, the George Boole Papers, UCC Library.

I regard this work of Boole's on probability as being of the utmost brilliance and importance (Broad, 1917).

George Boole was born at 34 Silver Street, Lincoln, England on 2 November 1815. His father, John Boole, was a shoe-maker with a passion for science and mathematics especially in the application of math principles to the construction of telescopes, microscopes and other optical instruments. His mother, Mary Ann Joyce Boole, was employed as a midwife (Boole Papers, 1865).

At the tender age of one and a half, George was sent to Mrs. Holmes Dame's School. He subsequently attended a Preparatory School for the children of respectable tradesmen run by two ladies, Misses Clarke. He was next placed in the care of Mr Gibson, who conducted a small Commercial School in Mint Lane, Lincoln. At seven years old, he transferred to the primary school of the National Society run by John Walter Reeves. According to a companion of Boole, 'George Boole was a sort of prodigy among us and we looked upon him as a star of the first magnitude' (McHale, 1985:2). Boole developed a lifetime interest in literature and language. A voracious reader, he consumed books on Latin, history, literature, maths and classics. When he progressed to mathematics he made rapid progress. 'As a young boy his father would give him some Euclid to study and reward him with a cake or toast if he did it well' (Boole Papers, 1853). Pencilled in the *Sixth Book of Leslie's Geometry*, John Boole wrote, 'George Boole finished this book, November 1st, 1826'. This was the day before George's eleventh birthday (McHale, 1985:6).

The family were unable to support secondary education for George. Rather, from 1828 he attended the Commercial Academy of Mr. Thomas Bainbridge, Michaelgate [formerly Fish Hill], Lincoln. Here he studied Valpy, Virgil, and made further progress in maths by covering vast amount of ground in algebra equations. He assisted Bainbridge on taking classes and correcting exercises. Boole read best English authors and taught himself French, German and Italian. This multilingualism permitted Boole to read Continental scientific publications before they were published in English (Boole Papers, 1850; McHale, 1985). In addition, 'Boole was very fond of music especially plaintive airs', and 'he played the flute and the violin' (Boole Papers, 1853). His interest in music and art is attributable to his mother's influence. She insisted on teaching her children the love of beauty and truth (Harrison, 1993).

At sixteen Boole became the family breadwinner and was forced to withdraw from formal education. He accepted the position of Usher [Assistant Teacher] at a boarding and day school run by Mr Heigham, Doncaster. Whilst discharging his duties at this 'very respectable Wesleyan establishment', Boole unacceptably read maths on Sunday and did math problems in the chapel (McHale, 1985:17). He quit Doncaster and after a brief teaching period in Liverpool, Boole returned to Lincolnshire in 1833. He was employed as a teacher at the Academy of Mr Robert Halls in the village of Waddington, South of Lincoln. After a year, at the age of nineteen, Boole launched his own school, The Free School Lane. In 1838, Boole returned to Waddington Academy to take over from the proprietor. From 1840-1849, Boole ran his own boarding school for young gentlemen at 3 Pottersgate, Mister Yard, Lincoln (McHale, 1985).

In 1841 Boole founded a new branch of mathematics called *Invariant Theory*. He was awarded the first Gold Medal of the Royal Society of London in 1844 for a paper on Differential Equations. This paper was printed in the *Philosophical Transactions* of 1844. In August 1849 Boole was appointed Professor in Mathematics, Queen's College Cork. Since Boole had no degree certificates he forwarded testimonials to Cork from local clergy, from a student, from the Editor of the Cambridge Journal and from eminent scientists (Harrison, 1993). By 1850, Boole had 'eighty-nine students' in attendance at his classes (Boole Papers, 1850). In a letter from Boole in Cork to his sister Maryann Boole, he writes: 'The Queen's Colleges have begun to acquire a good reputation for teaching in Ireland' (Boole Papers, 1850).

In 1850, Boole met Mary Everest, daughter of Sir George Everest (after whom the Himalayan Mountain was named) and niece of Queen's College Professor of Greek. Mary met Boole while visiting Cork. They corresponded when she returned home and their friendship blossomed. Boole and she married in 1855. The couple had five daughters (Harrison, 1993).

Boole's contribution to mathematical logic and probability stamped him as a mathematical genius. He proposed the dependence of the theory of probability on an underlying mathematical theory of logic. He found logic to be a form of the branch of mathematics, known as mathematical analysis, which incorporates algebra and calculus

(Audi, 1999; Corcoran, 2003). He ultimately wrestled logic free from the sole possession of philosophy. His pamphlet on 'The Mathematical Analysis of Logic' was published in 1847. In 1848 he published a short paper on 'The Calculus of Logic'. Boole's 1854 seminal work on algebraic logic, *The Laws of Thought* used the symbolic language of mathematics to examine the nature of the human mind, the fundamental laws of human reasoning. According to Boole 'it is certain that [logic's] ultimate forms and processes are mathematical' (Boole, 1854:12). Broad commends this book as '. . . one of the most fascinating that I have ever read . . . it is a delight from beginning to end . . .' (1917: 81).

Boole's interest in astronomy led him to the marriage of pure maths and applied science. He recognised a connection between 'essence and form, logic and probability' (McHale, 1985:212). According to Boole, 'Europe's highest and best results are produced by union and mixture' of 'the continued exercise of reason … memory …imagination … mechanical employments … application' (Boole Papers). According to his biographer, McHale, the interesting assertion in Boole's application was that he 'was just as much a student of the human mind as he was a mathematician and that many of his discoveries were motivated by his interest in psychology' (McHale, 1985:217). After his death, a friend wrote to his sister Maryann Boole, mathematics was always a part of his life 'to the extent that he was engaged in laying the formulation of a new science having some relation to mathematical logic and mental philosophy but not identical with any of them' (Boole Papers, 1868).

Boole's ground-breaking texts laid out the underpinnings of Boolean algebra, which classifies the two-valued character (binary) of statements that may be either true or false. His new mathematics showed that binary numbers, combined through Boolean algebra, could be used to analyse electrical switching circuits and to design electronic computers. Digital computers and electronic circuits are designed to realize this binary arithmetic to the present day (Reville, 1996). Boole was conferred with the honorary title of LL.D., in consideration of his eminent services to the advancement of mathematical science by the University of Dublin in 1852. He was awarded the Keith Medal by the Royal Society of Edinburgh (1857), and in June of the same year he was elected a fellow of the Royal Society. In 1859 at the Oxford Commemoration he received the honorary degree of D.C.L. Between 1859 and 1860 Boole published a *Treatise on Differential Equations* and a *Treatise on the Calculus of Finite Differences.* These works enriched the discipline of mathematics.

Boole died prematurely on 8 December 1864 of a feverish cold and congestion of the lungs. He is buried in the churchyard of St. Michael's Church of Ireland, Blackrock, Cork. The Boole Window was installed by public subscription in the Aula Maxima, University College Cork. The Boole Library and the Boole Lecture Theatre complex were named in his honour. His most enduring legacy remains the subject of Mathematical theories.

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Various Representations of Boolean Operations

x \ y	$\wedge$ 0	$\wedge$ 1
0	0	0
1	0	1

x \ y	$\vee$ 0	$\vee$ 1
0	0	1
1	1	1

x \ y	$\rightarrow$ 0	$\rightarrow$ 1
0	1	1
1	0	1

x \ y	$\oplus$ 0	$\oplus$ 1
0	0	1
1	1	0

Figure 1. Truth tables

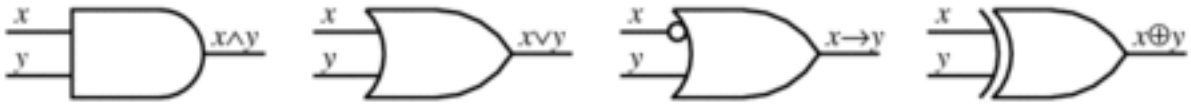


Figure 2. Logic gates

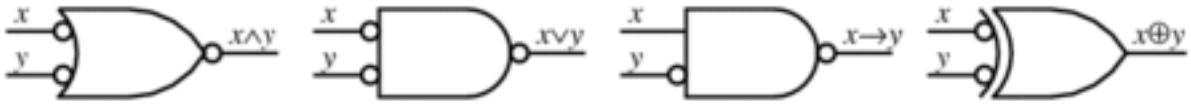


Figure 3. De Morgan equivalents



Figure 4. Venn diagrams



Source: *The Irish Times*

Criticism of Project Maths syllabus fails to add up

Jo Boaler

OPINION: SCHOOL MATHS should be one of the most useful subjects children learn, yet across the world thousands of children and young people leave school each year unable to use simple mathematical methods. Or worse – they are traumatised by their experiences in class.

This unacceptable state of affairs means that many adults are left vulnerable, not only to financial ruin but any situation involving mathematical thinking or reasoning.

Recently the newborn twin babies of actor Dennis Quaid nearly died because they were administered the wrong medicine. Nurses and pharmacists had been unable to distinguish between two similar mathematical labels.

Mathematics is important and widespread and it should be the right of all children to be given a high-quality mathematical preparation in school. Yet thousands leave school each year fearing or hating maths.

The reason for this is the way mathematics is usually taught in schools. Students spend hundreds of hours being shown a dry and narrow version nothing like the mathematics of the world and nothing like that used by mathematicians.

Mathematics exists in the petals of flowers, the rhythms of raindrops and the social networks that connect us; it is at the core of scientific and medical breakthroughs and it is a diverse and varied subject.

Ask mathematicians what mathematics is and they will generally tell you that it is the study and exploration of patterns. Ask school children what mathematics is and they will usually tell you that it is a vast collection of rules that have to be remembered.

Why are their descriptions so different? The reason is this: young people rarely experience real mathematics. Instead of posing questions, solving real and interesting problems, using and applying methods, investigating patterns and relationships, students spend their time watching a teacher demonstrate methods and then practising them. It is important for children to learn standard methods but this is just one small part of a very broad subject. The breadth of the subject is generally denied to children – at great cost. Ireland has taken an important step in introducing Project Maths and in streamlining the curriculum so that teachers have more time to teach mathematics in an authentic and interesting way.

But the reforms are provoking a backlash from some quarters, with suggestions that essential content has been eliminated and young people will not be mathematically prepared. Critics are overlooking an essential point: children were not well-prepared before when they were taught reams of content they did not understand and that often seemed to be completely meaningless.

We know a lot from research about student learning of mathematics. There is a vast and well-respected research base policymakers can draw from when making decisions about curriculum and teaching reforms.

That research base tells us that more mathematics content is not better. In fact, it is detrimental to student understanding and meaning-making. Research also tells us that when teachers cut down on content and choose

to teach mathematics with depth, giving students experiences of problem-solving and reasoning, students do better on standardised tests and take mathematics to higher levels.

I conducted an 11-year research study in which I followed students through two different mathematics teaching approaches in England, one similar to the traditional Irish approach and one more similar to the approach promoted by Project Maths. I found that students who worked in the Project Maths approach – covering less content but working in depth on problems, considering why methods worked and which methods were more appropriate – not only scored higher grades on the national GCSE examination but ended up in more professional jobs as adults.

In interviews, the adults who had studied lots of content by simply practising methods, told me that maths was irrelevant to their jobs and lives, and as soon as they left school they forgot the vast majority. By contrast those who worked on less content but in more depth understood mathematics, enjoyed their mathematical experiences and went on to use mathematics competently in their lives.

Public debates about maths teaching are important, but they would be greatly enhanced if they drew from the research base.

When critics write that Project Maths is failing because students don't learn enough content, they miss something very important – that we are considering living, breathing young people who need to understand mathematics, to experience it with meaning and to enjoy mathematics, not empty containers that simply need to be filled up to the brim with content.

When critics do not consider the nature of the learning experience, they overlook the fact students who engage in problem-solving rather than procedure-repetition are working on a harder mathematics, one which requires them to think for themselves, to make connections between different mathematical areas and to reason.

Is Ireland dumbing down the curriculum in Project Maths? Far from it. Instead it is asking students to engage in authentic mathematics and introducing them to the richness and power of a subject critical not only to their lives but to the future of Ireland.

Jo Boaler is professor of mathematics education at Stanford University in California. She is the author of *The Elephant in the Classroom: Helping Children Learn and Love Maths* (Souvenir Press, May 2009).

Source: Reproduced from T*he Irish Times,* Wednesday, 29 August 2012; http://www.irishtimes.com

'Project Maths is simply not challenging enough'

Cora Stack, Margaret Stack, Ted Hurley and John Brennan,

OPINION: IN AN opinion article in The Irish Times on August 29th on Project Maths, Prof Jo Boaler, a mathematical educationalist, argued students do better "when teachers cut down on content and choose to teach mathematics with depth".

That same Project Maths, introduced in 2009, is a new secondary level mathematics curriculum. It came about after a review by the National Council for Curriculum and Assessment (NCCA) amid concern over the uptake of higher level mathematics, particularly in the Leaving Certificate.

Its aims were to provide students with a greater understanding of mathematical concepts, with increased use of contexts and applications that enable students to relate mathematics to everyday experience.

However, several academics who have reviewed the course as rolled out believe this initiative is ill-conceived and will be severely damaging not only to mathematics but to engineering, technology and all the sciences. This would adversely affect our reputation, our economy and investment in the country.

In the new syllabus, calculus has been reduced by at least 40 per cent (with most of integration removed), while theory and applications of vectors and matrices have been eliminated. Note that the concept of a vector is one of the most fundamental for any practising engineer. The treatment of sequences and series has been reduced very significantly. Also, the assessment of advanced mathematical techniques in examinations has been considerably reduced.

All this amounts to a major repositioning downwards of the Project Maths curriculum compared to the old Leaving Certificate higher course.

In her piece, Prof Boaler does not refer to any benchmarking exercises of the new Project Maths course to compare it to curriculums in other countries.

In Singapore, an expert in mathematics education, Prof Peng Yee Lee, of the National Institute of Education, is of the opinion that breadth, depth and academic challenge are important, and "whether you like it or not, calculus is the gateway" to advanced maths. Singapore is regarded as having one of the best mathematics education systems globally.

It is accepted that Project Maths and bonus points may have some success in attracting more students to the honours stream, and we would welcome this. However, we cannot accept Project Maths as currently constituted offers the best foundation for students wishing to continue studying engineering, science or mathematics. For those wishing to pursue these areas at third level, the content is not suitable nor challenging enough.

Some of the basics of the language of mathematics have been foregone and, without these, deep understanding is impossible. Could a language be advanced with its alphabet and grammar diluted? Students' problem-solving abilities, highly dependent on mathematics, are going to suffer.

Many topics that engineers and mathematicians need are completely missing from the Irish mathematics education system at second level in the new course.

In Singapore there is an obligatory syllabus for students wishing to pursue mathematics, engineering and science at third level. It is full of the topics that have been omitted from or considerably de-emphasised in Project Maths.

To study engineering, science or mathematics at a university in Canada, it is compulsory to take very challenging courses in advanced mathematical functions, calculus and vectors – the most demanding of these is the course entitled "Calculus and Vectors".

When the content of the most advanced mathematics curriculums at upper secondary level in Scotland and England and Wales are examined, they are full of the very topics that have been completely removed from our old top level honours maths curriculum.

In Holland, for the senior cycle, a Project Maths-type syllabus is available at two levels, as rolled out here. The important difference is that Holland offers a more demanding course, compulsory for students pursuing the sciences, engineering and mathematics, at two levels. These have the more challenging topics, covering calculus, theory and applications of vectors, and theory of sequences and series – essential to supporting a seamless advance from secondary level to university for engineering, science and mathematics or indeed applied mathematics at this highest level.

In short, we are competing with countries which insist future potential students of engineering, science and maths take topics being deleted from our Leaving Cert. This should be a matter of major concern for policymakers, the NCCA and for Minister for Education Ruairí Quinn.

So what about our "knowledge economy" and future investment in Ireland? Our concerns are that if Project Maths as rolled out thus far becomes the norm, students entering third level science, mathematics and engineering will not have had exposure to some of the key mathematical topics they will need. That makes for a much steeper and perhaps impossible learning curve.

Ireland's knowledge economy is centred on science, technology and engineering, and these sectors have an expanding need for people with strong maths backgrounds. People seeking employment in those sectors will need to know more sophisticated mathematics to compete. The sooner students are introduced to such training, the better.

Project Maths appears to be moving in the opposite direction. The possible long-term consequences of neglecting to quickly reform this project will include detrimental effects on the education sectors concerned. Such neglect will severely damage the reputation of our universities and thus Ireland's international competitiveness. In a country linked to such distinguished theorists as William Rowan Hamilton, George Boole and George Gabriel Stokes – to name a few – we should not deny our best and brightest the curriculum so many of our international peers embrace.

Dr Cora Stack is a lecturer in mathematics at Institute of Technology, Tallaght, Dublin; Prof Margaret Stack is professor of mechanical engineering at the University of Strathclyde; Prof Ted Hurley is former head of the department of mathematics at NUI Galway, and John Brennan is a teacher of mathematics at Ballinteer Institute, Dublin, and textbook author.

Source: Reproduced from *The Irish Times*, Monday, 10 September 2012; http://www.irishtimes.com

Paper and Poster Presentations

TIMSS, PISA and Student Achievement Studies: Issues for Mathematics Education in Ireland

Peter Archer[1]
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Introduction
This presentation attempts to provide an overview of participation by Ireland in international and national assessments of achievement. While the main focus will be on assessments of mathematics, occasional reference will also be made to assessments of other areas, especially English reading, for comparative and other purposes. The presentation also includes some information from a survey, completed in conjunction with the latest cycle of PISA in 2012, of teachers experience of and opinions about Project Maths.

Irish involvement in international assessments is summarised in Table 1. Similarities and differences between the assessments will be discussed in the presentation. It is important to note that, although reports of TIMSS and PIRLS are due to be published on December 11 next, results cannot be shared at the session on November 26 because of a strict embargo.

National assessments of mathematics in primary schools were first carried out in the 1970s and have been carried out at fairly regular intervals since then although the approach to testing and the classes tested have changed on a few occasions since then. The last national assessment of mathematics in 2009 focussed on second and sixth classes and was combined with an assessment of English reading using the same students. The next assessment is expected to take place in 2014.

[1] www.erc.ie

Table 1. International assessments of student achievement involving Ireland (1991-2011)

Year	Study	Areas Assessed	Population(s)
1991	International Association for the Evaluati of Educational Achievement Reading Literacy Study (IEA/RLS)	Comprehension of Narrative, Expository Texts and Documents	9- and 14-year olds
1991	Second International Assessment of Educational Progress (IAEPII)	**Mathematics**, Science	9- and 13-year olds
1995	Third International Mathematics and Science Study (TIMSS)	**Mathematics**, Science	3rd/4th class, (primary schools); 2nd/3rd year (secondary schools
2000	Programme for International Student Assessment (PISA)	Reading literacy (major domain), **Mathematical** Literacy and Scientific Literacy	15-year olds
2003	Programme for International Student Assessment (PISA)	**Mathematical** Literacy (major domain), Reading Literacy and Scientific Literacy	15-year olds
2006	Programme for International Student Assessment (PISA)	Science (major domain), Reading Literacy and **Mathematical** Literacy	15-year olds
2009	Programme for International Student Assessment (PISA)	Reading Literacy (major domain), **Mathematical** Literacy and Scientific Literacy	15-year olds
2011	PIRLS (Progress in International Reading Literacy Survey)	Reading Literacy	4th class (typically10-year olds)
2011	TIMSS (Trends in International Mathematics and Science Study)	**Mathematics** and Science	4th class (typically10-year olds)

Aims

National and international assessments are both designed to address questions about

- Overall levels of achievement in education systems
- Changes in achievement over time
- Strengths and weaknesses in the area being assessed
- Which, if any, home, school classroom (including teaching methods) factors are related to achievement.

International assessments also aim to provide opportunities for systems to compare their levels of achievement with those in other systems.

Both types of assessments can thus be seen as part of efforts to inform public debate and policy making, promote accountability, identify appropriate standards and ultimately raise these standards.

Methods

All of the assessments considered here are sample-based surveys to yield estimates for the population of students of interest. Because the selection of a random sample is rarely practical two-staged sampling strategies are used in which schools are selected first and then students (sometimes as part of the whole class to which they belong) are selected next. Schools are typically selected on a stratified basis and proportionate to size (large schools are more likely to be selected than smaller ones)

Statistical and other research techniques used in these studies have become increasingly sophisticated and include

- More robust approaches to estimating error(e.g., the error associated with clustering effects of two-stage sampling strategies)

- Complex rotated book designs that allow for wide coverage of the areas being assessed without making the test unduly long for individual students
- Item Response Theory
- Multi-level modelling.

Although detailed descriptions of these techniques are not part of the presentation, the advantages and challenges associated with their use are discussed.

Findings

Early national assessments of mathematics seemed to suggest that students were doing better in some aspects of mathematics (e.g., whole number operations) than others (e.g., problem solving). Similar patterns emerged in almost every subsequent assessment (international as well as national).

In terms of overall performance, measures of which were not part of national assessments in the 1970s and early 1980s, there is little or no evidence of changes in performance (positive or negative) over time with one exception. The exception is the cycle of PISA conducted in 2009 when the performance of the Irish sample showed a significant decline from previous cycles.

In all international assessments of mathematics in which Ireland has participated, the performance of Irish students was found to be at or above the average of all participating countries again with the exception of PISA 2009.

Across all assessments, the relationships between achievement and home /family factors tend to be much stronger than the relationships between achievement and school and classroom factors. This is sometimes interpreted pessimistically in the context of attempts to improve achievement. However, two points are worth noting; (1) some of the strong correlates of achievement that have been identified (e.g., parental involvement in school related activities) are arguably subject to influence and (2) some school and teacher factors do emerge in a fairly consistent way as significant correlates of achievement (e.g., disciplinary climate, home-school links, teacher participation in professional development, and assessment practices).

Significance

Studies of the kind considered here can and do impact positively on the education systems in which they take place. For example, the very fact of sustained involvement in such studies seems to often contribute to galvanising policy makers' efforts to raise standards. There are also many examples of systems making use of data from assessments to guide reforms in a variety of areas such as curriculum and teacher preparation.

It is, however, important to acknowledge that these assessments are not unproblematic. In particular, there are dangers associated with international assessments arising from the (understandable perhaps) preoccupation of some commentators with the league table aspects of such studies. It is also the case that, despite recent advances, methodological problems remain (e.g., with the measurement of change over time) that are not fully appreciated by the users and even the sponsors of this kind of research.

TEDS to FIRSTMATH

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This paper will discuss a program of international and comparative research to explore the opportunities to learn that best prepare high quality mathematics teachers before they begin to teach and on the first years of teaching. I will briefly describe the TEDS-M study which collected data in 2008-2009 from teacher education policy and programs in 17 countries. The study investigated how diverse policies have shaped pre-service teacher education programs across high achieving countries and searched for patterns that begin to explain future teachers' performance on assessments of mathematics teaching knowledge. Building on TEDS-M, FIRSTMATH is a collaborative, cross-national study of novice teachers' development of mathematical knowledge for teaching and is currently working with several countries to develop the methods and instruments to explore the connections between previous preparation and what is learned on the job as it concerns knowledge, skills and curricular content. In addition the study will investigate the degree to which standards, accountability and other similar mechanisms operate to regulate the support that beginning teachers of mathematics receive during their first years of teaching; and how all this connects with teaching practice and pupils' learning.

Aims
I will briefly describe the IEA TEDS-M study, including key results showing evidence that quality assurance strategies and opportunities to learn mathematics knowledge for teaching provided to future teachers in pre-service teacher education programs has been able to produce highly knowledgeable teachers, a result that is more marked in high achieving countries. I will then briefly describe the aims of FIRSTMATH and how we intend to make connections between these two studies.

Methods
Both studies, TEDS-M and FIRSTMATH, are the result of collaborative efforts of worldwide institutions to study the mathematics preparation of future primary and secondary teachers and how and whether this preparation serves them well once on the job. The two studies combined, explore if what future teachers learn in pre-service teacher education leads to more effective knowledge and practice of mathematics and mathematics for teaching.

Both studies are large scale and thus are breaking ground in the development of valid and reliable research methods that can be taken to scale. The methods include the development of viable research designs, including rigorous measurement and sampling strategies. For instance for TEDS-M we implemented a two-stage sampling design: (a) selected samples representative of the national population of institutions offering pre-service education to the target populations; (b) all programs in those institutions were included in the survey; (c) within institutions (and programs), samples of educators and of future teachers were surveyed. Samples had to reach the IEA rigorous sampling standards. In the case of FIRSTMATH we are in the process of developing the instruments to do the study (this is an important goal of this meeting), and in our next meeting we will develop the sampling design for the participating countries. Following TEDS-M methods, FIRSTMATH will use surveys to explore the influence of individuals' background and teacher education program characteristics (such as opportunities to learn, philosophy, selection policy, curricular policy, and program staff and resources) on future teachers' performance using assessments, observations and other strategies to better understand the mathematics knowledge and mathematics pedagogy knowledge of novice teachers. In every case the research design has to fit the particular context of the country while allowing for comparison to explore the effects of the same variables across all countries.

[2] http://www.educ.msu.edu/

Findings
In this paper I will briefly report on findings from future primary school teachers and their programs in "high achieving countries" in mathematics as indicated by international assessments in Chinese Taipei, Germany, Poland, Russian Federation, Singapore, Switzerland and the U.S. The data comes from the TEDS-M future teacher survey and from the survey of teacher education programs. The future teacher survey consisted of questions asking about background characteristics and opportunities to learn, and an assessment of mathematics knowledge for teaching which measured mathematics content knowledge and mathematics pedagogy content knowledge. The survey of teacher education programs consisted of questions asking information about the organization and content of the programs included in the study.

I will also report on the advances achieved by FIRSTMATH so far and on what we expect to be able to do as a result of our joint efforts.

Significance
FIRSTMATH just as TEDS-M will be the first cross-national study of the influence of previous preparation to teaching knowledge and practice in mathematics. Both studies together will be able to provide valid and reliable on an increasingly contentious area whether or not effective teachers are born or made. TEDS-M has collected data on the policies, curriculum, organization, processes and outcomes of teacher education from national probability samples of institutions, teaching staff and students in these institutions and has implemented the first international assessment of learning outcomes based on national samples in teacher education. FIRSTMATH will attempt to collect comparable data from novice teachers. These two studies will provide the most rigorous scientific evidence on the effectiveness of mathematics teacher preparation to date to offer policy makers well-grounded evidence of what works in mathematics teacher education. In addition and most importantly the study will work in collaboration with teacher educators and teachers on strategies to support teachers' developmental knowledge of teaching and learning to teach mathematics effectively.

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Challenges of Developing Measures for Cross-national Studies: TEDS-M and FIRSTMATH as Cases

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Aims

The design of the TEDS-M instruments took several phases, including an early item tryout, a formal pilot, and final instrument design. A formal process was developed to ensure coherence and consistency in item format and presentation for all items in all the questionnaires and on the mathematics knowledge and pedagogical knowledge test items across multiple languages and cultures. This paper is primarily methodological, including the application of modern measurement theories to solve practical problems with large-scale international survey research. This includes latent-trait methods such as confirmatory factor analysis using Mplus (Muthén & Muthén, 2009) and Winsteps to employ the Rasch scaling model (Linacre, 2011).

Methods

In all stages of the TEDS-M item development item analyses were conducted, including exploratory factor analysis, and correlations among and between items of similar and different constructs. The items functioned exceptionally well compared to prior experience with pre-existing items. Many items underwent revision based on item pilot reviews. A standard set of item-writing guidelines were adopted to assure consistency and coherence in all measurement aspects. For the final instrument, confirmatory factor analyses were conducted prior to scaling (Mplus), including analyses of factor structure across participating countries. In a small number of cases, this led to the elimination of a handful of items from the scaling procedure for some indicators of program opportunities to learn and beliefs, particularly regarding those items that functioned significantly different in some countries. Once the final scales were defined, the Rasch model was used to create score scales for study participants on the measures of opportunity to learn, beliefs, and knowledge. The Rasch analyses also provided additional scale quality information.

Items were solicited from the participating international research teams and many were developed to reflect the principles of mathematics knowledge for teaching. Some items were modeled after other international educational surveys, such as TIMSS, and from an earlier small-scale version of TEDS-M. Questions for opportunities to learn and beliefs were also obtained from related research instruments developed in the USA and Australia, and other countries. All items were reviewed by all participating countries and the TEDS-M management team.

Claims

The process of gathering information on the functioning of the TEDS-M measures according to the context and outcomes of teacher education internationally in some cases challenged the theory behind the method (such as whether the scale measure the same thing in every country). For example, using a confirmatory factor analysis approach to assess measurement invariance across countries was not possible because of some nuances in response patterns in some countries. The challenges and successes in building relevant measures are presented.

Significance

The development of sound measures is essential to the quality of scientific studies of teacher education. By developing valid and reliable measures TEDS-M has taken an important step in this direction.

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Approaching Mathematical Problem Solving in the Irish Classroom: Using a Constructivist Framework to Scaffold Teachers' Explorations

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The Irish primary mathematics curriculum is based upon a constructivist philosophy of learning. As constructivism is a theory of learning and not teaching, it requires teachers to identify the implications for teaching. This study describes the experiences of five primary teachers as they attempt to explore mathematical problem-solving from a constructivist perspective with primary school children in Ireland. The key question upon which the research is based is: to what extent will an understanding of constructivism and its implications for the classroom impact on teaching practices within the senior primary mathematical problem-solving classroom? Several perspectives on constructivism have evolved with the emergent perspective on constructivism being central to the Irish primary mathematics curriculum.
Following the involvement of five primary teachers in a professional development initiative involving constructivism in the context of mathematical problem-solving, case study was employed to record the teachers' experiences and the experiences of their students as they engaged in a constructivist approach to problem-solving in the classroom. These case studies reveal primary teachers' interpretations of constructivist philosophy and the implications for teaching in a primary mathematics classroom. The study identifies effective strategies for exploring mathematical problems from a constructivist perspective. The study also illuminates the difficulties in making the transition from utilising traditional methods of teaching mathematics to employing those teaching strategies that reflect constructivist philosophy.

Examining Pre-service Teachers' Mathematical Knowledge for Teaching

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Aims

The term pedagogical content knowledge was first introduced by Lee Shulman in 1986 and has since become a significant focus of international research in mathematics teacher education. This research has aimed to better define what this type of knowledge involves and has examined the role that it plays in the development of effective mathematics teachers. The more recent work of Ball, Thames and Phelps (2008) proposed that pedagogical content knowledge is a domain within the broader realm of *mathematical knowledge for teaching*. Within this realm, they also discuss a subdomain of content knowledge which they refer to as *specialised content knowledge.* This refers to the knowledge of mathematical content that is specifically required in order to teach the subject effectively.

This study aimed to examine pre-service teachers' specialised mathematical knowledge at an early stage of their initial teacher education (ITE). An early assessment of such knowledge can offer valuable insight into the challenges faced by ITE programmes in developing future teachers' mathematical knowledge for teaching. This can help to inform planning and decisions regarding the overall content and structure of an ITE programme, in an effort to appropriately support and meet the needs of its developing teachers.

In particular, this study focuses on prospective teachers' understanding of relationships between mathematical concepts, understanding of the conceptual elements involved with a mathematical topic, and decisions regarding content and pedagogy for introducing new topics and/or concepts in mathematics. It explores the specialised mathematical knowledge required for teaching Algebra, and more specifically, it examines what prospective teachers' decisions about how to introduce students to the topic of Algebra can reveal about their specialised content knowledge.

Methods

The participants in this study included 52 student teachers in their second year of a four-year undergraduate mathematics teacher education program. These student teachers were beginning a series of six microteaching sessions in preparation for their first school-based teaching placement. For these sessions they were required to plan and teach a ten-minute lesson which would be videoed and discussed with a tutor.

The planning and teaching was done collaboratively in pairs, and for the first session, all pairs were asked to teach an introductory lesson on Algebra. The student teachers were given no further instructions about what to teach, but were encouraged to draw from their pedagogical studies of learning theory and teaching methodologies and their knowledge of the relevant content. In addition, they were advised to keep the lesson simple, not to plan too much in the ten-minute time frame and to aim only to initiate students' engagement with the topic.

There were 26 lessons in total, as they were all taught in pairs. Each lesson was video recorded. The videos were reviewed, coded and analysed to answer the question: What have these pre-service teachers chosen to include in a ten-minute introductory lesson on Algebra? Each lesson was analysed individually, but trends in the content choices and lesson structures emerged across the cohort. These trends were examined in an effort to determine what they could reveal about the development of these pre-service teachers' mathematical knowledge for teaching.

Findings
The content of the lessons varied with regard to specific details such as the chosen examples and exercises, the wording of definitions. However, the overall structure of the lessons, mathematical tasks, concepts and terminology were strikingly consistent across the groups.

In 23 of the 26 lessons, the student teachers began with a definition of Algebra that included some mention of variables, unknown quantities and solving equations. While varied in sequence, specific content and approaches, these lessons continued on from the initial definition to discuss using x to represent a number and offering some mention of algebraic expressions, variables, coefficients, terms, constants, combining like terms, solving equations and in a few cases, solving difficult multi-step equations such as $4 – 6x = 28$. All of this was done within the ten minute time frame.

Almost every pair planned a lesson that overwhelmed pupils with an extensive collection of new mathematical terminology and new mathematical ideas. This was in addition to the prior knowledge required by examples and exercises involving integer operations and the distributive property. Overall, the results of the analysis and the feedback sessions with the student teachers revealed that their knowledge of Algebra was more in line with what Ball, Thames and Phelps (2008) consider to be *common content knowledge*. This refers to knowledge about the subject that is relevant to teachers and non-teachers alike. It revealed a lack of the specialised knowledge of the mathematical content that is required to teach it effectively.

Significance
Two key issues are considered in relation to developing pre-service teachers' mathematical knowledge for teaching. The first focuses on the importance of developing pre-service teachers' knowledge of what mathematical ideas, skills and learning are involved with a particular concept. This includes their ability to unpack a mathematical concept in order to understand the network of mathematical knowledge involved.

The second issue focuses on the choices that pre-service teachers make about how to structure a students' initial engagement with a new topic. An awareness of the network of knowledge and ideas involved in a new concept is a key factor in a pre-service teachers' ability to identify and draw from pupils' existing knowledge and reasoning, in order to progress them toward new learning. It is essential for teachers to understand how this network can be organised to form an accessible foundation for a new topic if they want to help pupils to see new concepts as a natural extension of their previous learning.

Amidst the ongoing debate about the amount and type of mathematical knowledge required for effective teaching, this study highlights the need for ITE programmes to include coursework and experiences designed to develop the specialised type of mathematical knowledge that is required for teaching. It demonstrates that the mathematical knowledge of pre-service teachers early in their preparation for teaching is likely to be predominantly common knowledge of mathematics. In addition, it suggests that specific interventions are required to not only develop more specialised knowledge for teaching but to make future teachers aware of this distinction in their own mathematical knowledge.

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Learning to Teach Mathematics - Really!

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Introduction

This poster presentation draws on the Learning to Teach Study (LETS), the first of its kind on the Postgraduate Diploma in Education (PGDE) in Ireland, funded by the Department of Education and Skills (DES), which developed and implemented a study of initial teacher education in the PGDE in post-primary education. The mathematics component of the *Learning to Teach Study* (Conway et al, 2010) yielded four findings concerning how student teachers learned how to teach mathematics. These were

- The appeal and associated challenges of 'real-world' mathematics
- The dynamics and consequences of support in schools on learning to teach mathematics
- The mediating role of the methods module in Initial Teacher Education
- The challenge of using resources.

Attention in this paper is turned towards a discussion of the first finding in relation to so called 'real world' mathematics and its link to problem solving as this is a theme being discussed at the Trends in Mathematics Education Conference.

Reform-oriented mathematics: The Challenges of Project Maths Vision

There are many challenges in bringing student teachers around to the view of mathematics underlying Project Maths and in particular its emphasis on active learning methods, contexts and problem solving in the classroom. One such challenge, which poses many dilemmas for ITE, revolves around the difference between student teachers' experience of learning maths during their own schooling (what Lortie in 1975 termed the 'apprenticeship of observation') and what is being proposed as the future direction of mathematics in Ireland, with the current roll out of Project Maths[3]. At one level, this phenomenon is only about mathematics teaching but at another, it is one of the fundamental dilemmas of teacher education (Ball, 1996; Boaler, 2002): how should the next generation of teachers be educated and assessed in a reform-oriented era? Central issues to the teaching of any subject are conceptions of knowledge, assumptions about learning, teaching and assessment in the domain, and the role of curriculum resources. Project Maths has drawn significantly, although not exclusively, on both Realistic Mathematics Education (RME) and situated cognition (see Conway and Sloane, 2006; Lubienski, 2011). The Realistic Mathematics Education (RME) movement, which emphasises the notion of "mathematising", grew out of the work of Hans Freudenthal[4] a Netherlands based mathematician turned mathematics educator. It stresses the importance of problem solving, be it real world-focused or not but with an emphasis on the former, as both a source of learning and site in which mathematical ideas can be applied. The use of real-life materials fits very well with the philosophy of the Realistic Mathematics Movement (RME) which has become increasingly influential in mathematics curriculum development over the last few decades (Conway and Sloane, 2006; OECD, 2003). Goffree (1993, p. 89) writes that in RME "…reality does not only serve as an application area but also as the source of learning." The real world serves as a representation of a mathematical concept or technique. Schroeder and Lester (1989, p.33) write of this representation being a movement from the concrete, 'every day of things, problems and applications of mathematics" to the abstract world "mathematics symbols, operations and techniques", and/or vice versa. This raises yet another issue, that is, how student teachers and teacher educators understand and enact the 'real-world', as an important dimension of problem solving, in the context of maths teaching. In discussing mathematics teaching traditions, we refer to existing approaches, such as those

[3] Full details of the Project Maths initiative may be found at http://www.projectmaths.ie
[4] A classic text on RME is Freudenthal (1991).

described in the Lyons, Lynch, Close, Sheerin and Boland (2003) video study, as conventional and to Project Maths as reform-oriented mathematics. The reform-based ideals in mathematics education espoused by Project Maths advocate the use of contexts and real life examples as critical. To what extent do the pre-service teachers in this study understand and use the concrete, 'every day of things, problems and applications of mathematics" and move to the abstract world of "mathematics symbols, operations and techniques" (Schroeder & Lester, 1989, p.33) in their classroom teaching?

Method

LETS is an empirical research project into how students on the Post-Graduate Diploma in Education (PGDE) in University College Cork (UCC) develop teaching competence as post-primary teachers. The LETS study was undertaken over three years (2007-10) and involved the participation of an experienced research team from the School of Education, UCC. The team brought a variety of experiences and insights to the research task, and within the overall socio-cultural understanding that framed this study, were able to contribute their specialised knowledge in areas such as inclusion, equality and diversity, second language teaching, the teaching of mathematics and of science etc. The research team itself can be seen in this context as a community of learners, participating together in the task of achieving an understanding of the process of learning to teach. The principles of the interpretive research genre (Mertens, 2010) informed the LETS research project.

Table 1: Overview of Semi-Structured Interviews with Student Teachers

Part	Interview domains: January 2009	Interview domains: March 2009	Interview domains: May 2009
A	Background, previous experience, motivation to learn to teach	Update on progress learning to teach	Opportunity to learn to teach
B	Opportunity to learn to teach	Opportunity to learn to teach	Critical incidents in learning to teach and in school
C	Critical incidents in learning to teach and in school	Critical incidents in learning to teach and in school	Understanding of subject teaching, inclusion and reading
D	Understanding of subject teaching, inclusion and reading literacy	Understanding of subject teaching, inclusion and reading literacy	Future plans
E	Summary: SWOT 1	Summary: SWOT 2	Summary: SWOT 3

The methods used included semi-structured interviews, analysis of documents and a survey questionnaire (see Table 1 for overview of interview foci). Using a multiple-case study research design, seventeen student teachers were interviewed on three occasions over the course of an academic year. A survey focused on the prior experiences and beliefs (e.g. about learning) that student teachers bring to the PGDE, including a measure of their teaching efficacy and knowledge about reading literacy and inclusion, was completed in March 2009 by 133 of the 212 students of the 2008/2009 PGDE cohort (a response rate of 63%). For the purpose of this presentation we draw on data from interviews with those student teachers training to be mathematics teachers.

Findings

For the purposes of this poster we focus on the appeal and associated challenges of 'real-world' mathematics for student teachers. In doing so, we note pre-service teachers have discovered the limitations of traditional teaching, the challenge for mathematics teacher educators involved in teacher education is to show student teachers, that like science, mathematics *is just as close* to real life. Our findings suggest that student teachers are engaging with ITE coursework and are grappling with the importance of problem solving, particularly so called real world-oriented problems, in their mathematics teaching. Crucially, the student teachers are working toward what Chazan, Callis and Lehman (2007, p. 78-79) describe as a present-oriented approach to mathematics teaching. Such an approach helps students make connections between their lives and the mathematics they are learning in school. They are simply "finding mathematics in the world around us" (*ibid*., p. 79).

Significance

The student teachers interviewed as part of this study seem to have typically understood the notion of 'real' as simply being a tangible or concrete object that helped student understanding of a mathematical concept. None of the prospective teachers interviewed as part of this study present an interpretation of the concept as relating to the process of mathematising from a 'real-world' scenario. This process of mathematising is emphasised in the RME framework. Perhaps a recommendation for the ITE mathematics methodology module content and pedagogy may be that it should somehow move student teachers from a very simple understanding of real (concrete) to a more elaborate understanding of real as espoused in the RME framework and its emphasis on mathematising. The enactment of this recommendation will, we think, assist future mathematics teachers to be competent in the pedagogy required for the successful implementation of Project Maths and its emphasis on problem solving through the use of contexts.

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Noticing in the Third Level Mathematics Classroom

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Aims

In the academic year 2010/11 five mathematicians in five different third level institutions in Ireland initiated a group project in which each reflected on her own teaching. We endeavoured to follow John Mason's approach as outlined in 'Researching your own Practice: The Discipline of Noticing', (Mason 2002). In doing this we aimed as individuals to become more aware of events in our classrooms and in particular to recognise opportunities to act differently in our classes. We hoped that the Discipline of Noticing would facilitate reflection on our teaching and offer opportunities to analyse it. We aimed to share our practice with others in the group by circulating our accounts to the rest of the group and by discussing them during group meetings. We hoped that this collaborative work would inform our practice and in addition enable us to identify various phenomena in undergraduate teaching as we shared accounts and searched for similarities and differences between them. We have written previously about the challenges and benefits of using the Discipline of Noticing (Breen et al. 2011) in the third level classroom. In this presentation, we would like to focus on the question: To what do we attend whilst lecturing?

Methods

Each of the five mathematics lecturers endeavoured to write an account of a number of classes per week over the 2010/11 academic year. The modules involved ranged from first year pre-calculus and calculus courses to an MSc level course on topology, and included large and small class groups. We circulated the written accounts to each other every three weeks and we met twice each semester to discuss the accounts and the process. In addition, at the end of the year, each of us reflected on our experience of using the Discipline of Noticing.

Mason (2002, p.61) speaks of *disciplined rather than sporadic or serendipitous noticing*. He describes the features of the discipline as: keeping accounts; developing sensitivities; recognising choices; preparing and noticing; labelling; and validating with others. Central to his discipline is the creation of *brief-but-vivid* accounts which function as accounts of events rather than accounting for events. That is, the account should describe the event as objectively as possible while minimising evaluation and judgement so as to maximize the likelihood of it resonating with the experience of others. He suggests that the accounts should be used in *developing sensitivities by seeking threads among those accounts, and preparing oneself to notice more detail in the future.* (Mason 2002, p.87)

During the academic year, the five lecturers wrote accounts of events that they noticed in their classes, and produced between 20 and 50 accounts each. All then coded their own accounts using a grounded theory approach, and constructed a list of themes which emerged from this analysis. The fourth author then categorised these themes into eight categories. Both the themes and the categorisations were discussed and agreed by the whole group.

Findings

The eight categories that emerged from the analysis of the data were: student understanding; student engagement; instructional methods; asking questions and discovery; impact of environment on the lecturer; relationships, feelings and emotions; students' prior mathematical experience; and lecturer reflection and development. Each of these categories contains many concepts; for example the student understanding category includes accounts that focus on the problems of lecturers gauging students' understanding during lectures, accounts that concern students' understanding or misunderstanding of a particular topic, as well as accounts that relate to lecturers' concerns about depth of understanding versus breadth of knowledge. Many of the accounts related to student engagement. Some of the more frequent themes that arose in this category were those that involved lecturers monitoring students' engagement (or lack of it) during class, and efforts by the lecturer to promote student engagement. There are also connections between the categories, for example some accounts referred both to student engagement and student understanding. Similarly the accounts in the instructional methods category concerned particular techniques used by the lecturers in their classes and the aim of the techniques was almost always to encourage student engagement or understanding. Furthermore the asking questions and discovery category relates to instructional methods, but ones that were used and written about so often that they merited being in a category of their own. Other themes in this category related to efforts to encourage participation, and questions asked by students.

Significance

The value of reflection for teachers at primary and second level has been widely acknowledged (National Council of Teachers of Mathematics, 2010). In particular, Sheerin, Jacobs and Philipp (2011, p. 6) advocate the use of noticing by teachers and assert that it can lead to adaptive and responsive teaching, can allow teachers to learn from teaching (for example by attending to students' thinking), and can help them decompose practice into core activities and develop a language with which these can more readily be discussed. In contrast, there have been relatively few studies which involve mathematicians at third level reflecting on their own teaching. Indeed Speer, Smith and Horvath (2010) considered undergraduate mathematics teaching and noted that *very little research has focused directly on teaching practice - what teachers do and think daily, in class and out, as they perform their teaching work. (p.99)*. Our study has aimed to address the question of what lecturers notice while teaching. We have found that most of our accounts addressed the core categories of student understanding and student engagement.

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Effects of Calculators on Third Year Students' Mathematics Achievement and Attitudes

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Aims

Arising from concerns about the effects of calculators on the mathematics performance of students in Junior Cycle, our study aimed to:

- Examine, both before (Phase 1), and after (Phase 2), the introduction of calculators in the Junior Certificate mathematics syllabus , the performance of third-year students on calculator tests inviting varying levels of calculator usage:
  - A *calculator inappropriate test*, taken by all students without access to a calculator, containing items that students should be able to do without a calculator
  - A *calculator optional test*, taken by half the sample in each phase, with calculator access, and by the other half without access, and containing items that students should be able to do with or without a calculator
  - A *calculator appropriate test*, taken by all students in each phase, with access to a calculator, and containing items that would normally require a calculator (see Close et al., 2012).
- Examine, in both phases, the effects of calculator access vs. non-access during the testing of student performance on the calculator optional test, and to compare the results
- Examine the attitudes of students in both phases toward calculator usage and the relationship between attitudes and the performance of students on the three calculator tests
- Examine the relationship between performance of students on the three calculator tests in both phases and their performance on the corresponding Junior Certificate mathematics examinations.
- Examine the attitudes of mathematics teachers towards calculator usage in both phases, and describe their practices with respect to calculator usage during mathematics instruction.

The study is relevant to the extent that some stakeholders (e.g., Engineers Ireland, 2010) continue to argue that students in Junior Cycle and primary level should not have access to calculators during mathematics lessons on the basis that calculator access 'appears to lead to a decline on some aspects of numeracy' (p. 8).

Methods

In Phase 1, the three types of calculator-related mathematics tests and a questionnaire were administered to a nationally-representative sample of the final cohort of Third-year students to experience a mathematics curriculum in which calculators were not included. In Phase 2, the same three tests and a revised questionnaire were administered to a nationally-representative sample of Third year students in the third cohort to experience a mathematics curriculum (and association examinations) in which calculators were included. The mathematics teachers of participating students also completed questionnaires in both phases.

Items for the tests were located in textbooks, tests, and Junior Certificate examination papers (foundation, ordinary and higher levels), or were written when necessary, and were then assembled into tests for piloting. Test items fell into two cognitive categories specified in the syllabus: those that assessed *knowledge of mathematical facts, procedures and concepts*, and those that assessed *knowledge of application of mathematics in real-life contexts*. They included multiple-choice (one-third) and short constructed response items (two-thirds),

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and displayed an overall gradient of difficulty, whereby the calculator inappropriate test tended to be easier than the calculator optional test, which, in turn, was intended to be easier than the calculator appropriate test. The items focused chiefly on assessing the Junior Certificate syllabus content area of *applied arithmetic and measure* (because of its relevance for the use of real-life data), followed by *number systems and statistics* (as these are the most calculator sensitive topics accessible to most Third-year students), and to a limited extent on *algebra*. Two pilot studies were conducted: the first in a convenience sample of seven schools; and the second in a more representative sample of 15 schools. Reliability coefficients (Cronbach's alpha) for the tests ranged from .85 to.88.

The student questionnaire was designed to investigate variables that might be associated with student performance on the tests, and to provide background data on participating students. The questionnaire sought background information on students' calculator usage at home and at school in other subjects as well as mathematics, and asked about attitudes to mathematics in general, and towards calculator usage in particular. In Phase 2, additional questions were asked to investigate students' use of calculators in the revised syllabus. The teacher questionnaire sought to generate background information on teachers, information and views on calculator usage in mathematics classes, and teachers' perceptions of whether and when students are formally taught calculator and other skills. In Phase 2, teachers were also asked about benefits and problems associated with calculator usage in class, and about the extent to which they used specified calculator and number activities in their classes.

In Phase 1, 60 of 90 selected schools participated, while, in Phase 2, 73 of 100 schools took part. 1,453 students took part in Phase 1, and 1459 in Phase 2. Weights were computed and applied to compensate for the somewhat unequal distribution of students in different strata in the sample. The calculator tests were scaled using item response theory (IRT) methodology (see Close et al., 2008 for details).

Findings

Performance on both the calculator inappropriate test and on the calculator optional test (no calculator condition) declined slightly, though not significantly, between the two phases, indicating that the availability of calculators in the Junior Cycle syllabus did not have a significant negative effect on performance on aspects of mathematics for which a calculator was not deemed to be required (i.e., basic mathematical skills). On the other hand, performance improved significantly on the calculator appropriate test, which includes items most likely to bring calculators into play, which indicates that students' ability to use calculators to solve problems improved between the two phases. Nevertheless, the relatively low levels of performance in both phases on the calculator appropriate test (35.0% and 42.6% respectively) raises concerns about how well students can use mathematical knowledge to solve practical problems regardless of calculator availability. Predictably, in the calculator optional test, students with access to a calculator significantly outperformed those without access in both phases. Strong positive correlations were found in both phases between performance on the calculator tests and performance on the Junior Certificate mathematics examination (taken some six months after the calculator tests). In Phase 2, the mean score of ordinary level students on the calculator optional test with calculator access approached the mean score of higher-level students without access, suggesting that calculator access enables students taking ordinary level to perform at a higher level than they would otherwise attain, on the types of tasks assessed by the test.

In Phase 2, 81% of students reported using a calculator often during mathematics classes (compared with just 1% in Phase 1). The proportions of students who agreed or strongly agreed that a calculator can help with performance and that its use should be allowed for classwork and homework in mathematics increased from about two-thirds in Phase 1 to over 90% in Phase 2. About three quarters of students in Phase 2 believed access to a calculator should not replace the need to learn pen-and-paper calculations. Teachers in the study saw improved speed and accuracy, and increased ability to progress through topics such as Trigonometry and Statistics, as the main benefits of calculator availability. They were less likely to see benefits in calculator usage for developing understanding of content and procedures.

Significance

The non-significant decrease in performance when students do not have calculator access for basic mathematics computations (calculator inappropriate test) should be monitored over time.

The relatively poor performance on the test of real-life problem solving (calculator appropriate test) in both phases was disappointing in both phases, and would be worth reviewing, in the context of the implementation of Project Maths in schools.
The low emphasis on use of technology – apart from scientific calculators – in Junior Cycle mathematics classes also gives rise for concern, and is something that should be addressed in the context of Project Maths, particularly in the context of using calculators to develop conceptual knowledge.
The introduction of standardised tests in Second year in post-primary schools from 2014 gives rise to the question of whether students should have access to a calculator for all parts of the proposed tests, or whether there should be a calculator-free section, where students' basic computational skills can be examined.

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Charting the Development in Problem Posing in Initial Teacher Education

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Problem solving is an important component of primary mathematics. Central to the activity of problem solving is the task of problem posing. This study examines the skills and predispositions of pre-service teachers when posing mathematical problems for use in primary classroom and charts the development of these skills across their teacher education program. At the end of their first semester of a teacher education programme pre-service teachers were asked to construct a mathematical problem for use in a primary classroom. One year later on completion of their teacher education program, and following targeted experience in problem solving and problem posing, 55 of these same pre-service teachers were presented with the same problem posing task. Analysis of the data involved categorization of the constructed problems according to problem type, features and mathematical content. Analysis of the problems revealed that the initial problems posed were single step problems with one possible solution, focused on procedures and operations relating to quantities and were of low cognitive demand. The later problems showed some increase in cognitive demand, were predominantly multiple-step and indicated a move from a computational emphasis. Despite these improvements there remained a focus on problems with a single solution and the move to multi-step problems reflected a greater emphasis on multiple procures rather than reasoning. Recommendations are made for practice in teacher education contexts to support pre-service teachers in developing the pedagogical skills required to pose mathematical problems and teach mathematical problem solving.

Teacher Education and Problem Posing

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Aims
The quality of mathematics teaching and learning is partially dependent on the mathematical tasks teachers use in their classrooms. Yet much of the research evidence suggests that in many countries the majority of the math problems and questions used in math classrooms focus on memorization and procedural understanding rather than on mathematical reasoning and inquiry. Even when teachers have access to high quality textbooks and curricula, they often transform potentially rich, worthwhile problems in ways that lower their cognitive demand (Henningsen & Stein, 1997). If teachers and prospective teachers are to provide rich and deep learning experiences to their students, it is important that they develop some principled ways of deciding on the relative worth of problems—what makes some problems better than others. More importantly, they also need experiences generating such problems themselves. In this presentation I share data from three of my past research projects (Crespo, 2003; Nicol & Crespo, 2006; and Crespo & Sinclair, 2008) to describe and theorize how preservice elementary teachers can grow their problem posing practices during teacher preparation.

Methods
This presentation offers a cross-projects synthesis of the impact of different types of problem posing experiences offered to preservice elementary teachers across different institutional settings, albeit all at the time when preservice teachers were taking math methods courses in their programs. Three problem posing activities featured in these problems included: (a) Classifying and adapting textbook problems (Nicol & Crespo, 2006); (b) Interactive problem posing with children (Crespo, 2003); and (c) Exploring situations to generate and evaluate problems (Crespo & Sinclair, 2008). Artifacts and interview data from a total of 44 participants are the data sources for this synthesis.

Findings
Left to their own resources, elementary preservice teachers across all three projects generated the prototypical types of math problems such as those documented in the research literature as focused more on memorization and procedures than on genuine mathematical inquiry. Their problem posing strategies can be characterized as follows:

Novice Approach	**Features**
Posing closed problems	- Problems are quick-translation story problems or computational exercises. Added questions if any at all test for speed and accuracy.
Posing simplified problems	- Adaptations narrow mathematical scope of original version of problem and the work of students. Added questions take the form of hints and lead students to a solution strategy or answer
Posing problems	- Problems are posed without solving beforehand or deeply understanding the mathematics. Added questions suggest unawareness of the mathematical potential and

blindly	scope of problem

With purposefully designed problem posing activities, and in a short period of time, prospective teachers in all three of these projects developed a wider repertoire of problem types and of problem posing approaches that included the following:

Beginner Approach	Features
Posing open problems	- Problems require students to explain their work and communicate their ideas. Added questions invite students to share, explain, and reflect on their thinking.
Posing mathematically challenging problems	- Problems introduce new ideas, push students' thinking or challenge their understanding. Added questions and adaptations scaffold rather than lead the student's thinking. Adaptations open the mathematical work and scope of the original version of problem rather than narrow it down.
Posing mathematically interesting problems	- Exploring and mathematizing situations to generate "interesting" problems. Using mathematics aesthetics criteria such as: surprise, novelty, simplicity, fruitfulness—to decide on the quality of generated problems.
Posing socially relevant math problems	- Exploring and mathematizing real world situations to engage students in understanding and addressing social issues with mathematics.

Collectively, these studies show that it is possible—within a short period of time—for preservice elementary teachers to make important gains in the range and quality of the mathematical problems and questions they generate and use in their mathematics teaching when they participate in purposeful problem posing experiences. The descriptive framework generated from these projects is offered as an initial blueprint for designing problem posing experiences for preservice teachers and for tracking growth in their problem posing strategies.

Significance

Much as it is possible for students to solve problems without engaging in significant mathematical activity, it is equally possible for teachers to pose questions that never really get to the mathematical significance of the situation. The studies synthesized here suggest that providing preservice and practicing teachers with opportunities to explore and discuss the mathematical potential of problems and evaluating their mathematical as well as pedagogical interest can be a positive step towards preparing teachers that will design rich mathematical opportunities for their students.

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Teacher Education and Problem Solving in Mathematics

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Abstract
Mathematics problem solving is an important part of school mathematics education as well as in teacher preparation programs. It is a demanding activity for both groups: students and teachers. The paper shortly discusses the main difficulties from teacher's point of view. The second part of the paper shares an experience of the Faculty of Mathematics and Informatics at the University of Sofia to prepare future teachers in mathematics problem solving. Some examples from MT-21 study are presented. The conclusion is that because of the common decline of future teachers' knowledge Bulgarian teachers are less and less prepared to teach problem solving. As a result, teachers teach solving problems from the textbooks that are routine, algorithmic, etc.

Aims
Paul Halmos (1980) wrote: "I do believe that problems are the heart of mathematics, and I hope that as teachers, in the classroom, in seminars, and in the books and articles we write, we will emphasize them more and more, and that we will train our students to be better problem-posers and problem solvers than we are." This citation determines the main aim of the paper, which is to bring the attention to the importance of mathematics problem solving in school education (Bankov, et al. 2005). The prerequisite for this is the existence of teachers that are able to teach. Therefore the programs for teacher preparation should include courses/training on mathematics problem solving. Another aim of the paper is to inform about the "Problem Solving Practicum" offered by the Faculty of Mathematics and Informatics at the University of Sofia and to present the recent situation in teaching problem solving in mathematics teacher preparation programs in Bulgaria.

Methods
The author uses his own experience in teaching mathematics problem solving for talented secondary students. He used to work with mathematically gifted students in extracurricular classes (Bankov, 1999) and trained them for mathematics competitions. In the beginning of his career as a university lecturer, the author was teaching problem solving strategies for future teachers. The observations he made and the experience gained during this period are a good basis for considerations about implementing mathematics problem solving in the preparation of future mathematics teachers. The analysis of the recent situation and the conclusions about the decline of the problem solving activities in Bulgaria are based on some findings from MT-21 study (Schmidt, et al. 2011).

Findings
Because mathematics problem solving is an important activity in teaching school mathematics and because this activity is demanding and needs special training, the Faculty of Mathematics and Informatics at the University of Sofia offers "Problem Solving Practicum" in the program for preparation of mathematics teachers. The Practicum lasts about 34 clock hours (60 minutes each) during the last (4-th) year of the teacher preparation program. It started about 25 years ago. The design of the Practicum is based on the instructional strategy of George Pólya (1945): (i) understanding the problem; (ii) designing a plan; (iii) carrying out the plan; (iv) looking back. In the beginning the Practicum was demanding for the attendance. Future teachers were introduced to problems from mathematics competitions, topics for extracurricular work with gifted students, etc. Later, the decrease of mathematics achievement in school which was clearly indicated in TIMSS (Bankov, 2007) impacted on the future teacher's knowledge. Students entered the teacher preparation program with less mathematics knowledge. The lost should have been compensated during the first years of the teacher preparation program and by the time to attend the Practicum future teachers were less and less prepared in mathematics. Therefore it was more and more difficult for them to get involved in the program of the Practicum. This is why the Practicum changed the topics during the years. Now it focuses mainly on solving problems from school textbooks. As a result: (i) future

teachers are less prepared for problem solving activities; (ii) when entering the profession, it is difficult for them to teach problem solving; (iii) most of them do not teach problem solving.

Significance

Because the teacher preparation programs in Bulgaria do not offer adequate training on problem solving, teachers in most of the Bulgarian schools do not teach mathematics problem solving. They teach solving problems from the textbooks that are routine, algorithmic, etc., i.e. do not offer any "surprises" to students and teachers. However, mathematics problem solving is taught in the extracurricular classes. Typically such classes can be found at the profile schools for mathematics. There are about 30 such schools in Bulgaria. Mathematics teachers in these schools are highly qualified. They have obtained skills to teach problem solving either by attending post-graduate course or by self-education. In these classes students are required to "invent" a process for solving demanding problems or to pose their own problems. To make this happen in most schools means to significantly raise standards and expectations, aiming to develop higher-order thinking skills and confidence in students' abilities to solve non-routine problems.

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IEA: Creating an International Context for Mathematics Education Reform, 1964-2015

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For the last fifty years, the International Association for the Evaluation of Educational Achievement (IEA) has been engaged in creating an international context for seeing education and educational reform in a new light. IEA has developed, implemented and spread the word about a whole series of international assessment studies. At the core of this research are the IEA studies of mathematics in primary and secondary schools throughout the world. Between 1964 and 1995 there were three such studies, which were then transformed into trend studies with further data collection in 1999, 2003, 2007, 2011 and ,in the works, 2015. Various spinoffs have increased the richness and value of this research context, including the true longitudinal study embedded in the Second International Mathematics Study, the TIMSS studies of advanced mathematics in 1995, 2008 (and a third being planned), the TIMSS video studies of 1995 and 1999, and the first IEA teacher education study focusing on mathematics (TEDS-M 2008). Sixty-four educational systems have participated in TIMSS 2011, ranging from such stalwarts of IEA research as the Nordic countries, Australia, Germany, Japan, and the U.S. to systems which have begun to participate in more recent years such as Georgia, Ghana, Kazakhstan, Northern Ireland, Serbia, Tunisia and the Palestinian National Authority. Over the years the methodology and quality control for these studies have become increasingly sophisticated and able to guarantee the quality and accuracy of the results. Much effort has also been put into the organization and documentation of the databases to make secondary analyses not just possible but relatively easy to do, with data and documentation readily available on-line. In addition, every few years IEA sponsors an International Research Conference for presentation of the results of these secondary analyses. All of these efforts have enabled IEA to create an international context which consists of agreement or debate over answers to such questions as: (1) What is the intended content of school mathematics and how does it vary between and within countries? (2) What content is actually taught and how does it vary? (3) What of this content is actually learned by students who are representative of their countries? (4) What is the overall yield in school mathematics, defined as the percentage of an age cohort that reach specified levels of advanced mathematics learning? (5) How do students and teachers think about mathematics and how it is taught? (6) What is the actual variation in how mathematics is taught? (7) How do these indicators of the state of school mathematics change over time? (8) How well are beginning teachers prepared to teach mathematics? Examples will be given of the impact that answers to these questions have had on policy and practice in various countries.

Mathematics Education Reform in Ireland: Directions and Dilemmas

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Aims
The last four years have been notable with regard to mathematics education reform in Ireland because of the launch and ongoing rollout of Project Maths. Other papers at this conference describe Project Maths and examine aspects of the international and national scene in which it is set. For this paper, the chief aims are

- to set Project Maths in historical context by looking at second-level mathematics education reform in Ireland over the past fifty years
- to highlight both continuity and contrast with regard to the older reforms and those involved in Project Maths
- to identify dilemmas produced by the process and style of reforms.

"Reforms" here are equated with changes in the state curricula, as these are the major developments in the period. The fifty-year time-span covers the previous root-and-branch reform of second-level mathematics education in Ireland: the introduction of "modern mathematics" curricula in the 1960s.

A summary of key features of Project Maths is appropriate in order to provide a background to the arguments in the paper. The documentation and accompanying rhetoric specified increased emphasis on understanding, problem solving and applications (the latter notably in "relevant" real-life contexts); ambitious targets were set for uptake of Higher-level courses. Content was adjusted to reflect perceived current importance. Mechanisms to achieve the aims include phased rollout, with involvement of pilot schools; significant changes in assessment; and a professional development programme focused on teachers facilitating constructivist learning in their classrooms.

Methods
The chief framework used is the *three-level curriculum*, highlighted originally in the IEA's Second International Mathematics Study (SIMS) in the 1980s, and distinguishing between *intended*, *implemented* and *attained* curricula. Briefly, the intended curriculum is that prescribed by (typically) a Ministry of Education or curriculum council; the implemented curriculum is what is taught in classrooms; and the attained curriculum is what students have learnt. However, these broad definitions need to be made more precise for the current analysis.

The *intended curriculum* is taken to be *the outcome of a process* – discussion, consultation, piloting, and so forth – but to be *fixed* from the time at which it is introduced. According to this definition, applied to the Irish setting, assessment frameworks and sample examination papers are part of the intended curriculum; however, subsequent "real" examination papers are not, though their special position with regard to both implementation and attainment must be recognized. Textbooks – produced in Ireland by private enterprise – are also excluded. The intended curriculum can include teacher professional development when carried out by, or closely allied to, the agencies that decreed the curriculum during the period of its introduction. Overall, using this definition, implementation and attainment can be compared to original intention rather than to a moving target.

The *implemented curriculum* is taken to cover what teachers and students do in and for mathematics classes, and also the school settings and the broader culture in which they work. Factors such as the time allocated to mathematics in schools (a school-level decision in Ireland) are important. Broader cultural influences include the apparently prevailing didactical contract between Irish teachers, students and parents with regard to explicit focus on preparation for overly predictable examinations. Beliefs about the subject of mathematics, and about how it is learnt, that teachers and students bring to their work or study are further factors.

The *attained curriculum* obviously encompasses the mathematical knowledge and skill that students acquire in their schooldays. The dispositions and attitudes developed towards mathematics are also included.

As the paper addresses the evolution of intended mathematics curricula, their implementation (or non-implementation), and the resulting attainment over a fifty-year period, it involves historical research: for example, analysis of syllabus documents, examination results, examiners' reports, and findings from the realm of research in mathematics education. However, some relevant aspects – discussions at course committee meetings and mathematics teachers' meetings, for instance – are not well documented. As the author has been a player on the field, or at least a supporter on the touchline, for most of the period, this paper also draws upon her experiences of such aspects.

Claims / Findings

Claims and findings are presented under two main headings: *directions* (trends over the period, highlighting continuity and contrast), and resulting *dilemmas*.

Directions

As regards *intention*, the chief claim of this paper is that is that there is remarkable constancy in the rationales for reform over the years, and that the rationales are similar to that for introducing Project Maths. In particular, successive reforms concentrated on aiming to improve learning with understanding and greater uptake of Higher-level courses. Problem solving and applications of mathematics have also figured, although the focus is qualitatively different in Project Maths. Content was reviewed and updated to reflect contemporary understanding of mathematics and of students' needs. The earlier reforms in general did not document intentions with regard to teaching for understanding – methodology was considered the domain of the teacher rather than the state – but more recently this became a key aspect. As regards mechanisms to achieve the aims, assessment frameworks were often redesigned, and professional development played an increasing role.

However, *attainment* – cognitive and affective – has remained problematic. Assuming that the intentions are appropriate (for example, that learning for understanding is desirable), it would appear that there have been shortfalls in *implementation*.

A pertinent issue, therefore, is identifying the changes in mechanisms included in Project Maths that may lead to more faithful implementation. The more extensive programme of professional development should help to "upskill" teachers and may contribute to changing their beliefs about mathematics and learning. The rather dramatic redesign of the assessment framework – both addressing the issue of over-predictability and introducing extended context-based questions not familiar in the Irish system – provides a very strong incentive for change. However, the specified or desired changes on the three fronts of content, methodology and assessment – ongoing because of the rolling nature of the reform – pose a major challenge especially to teachers. Moreover, a long-term downward drift in time allocated to mathematics in schools, intrinsically problematic, may be exacerbated by the new structure for the Junior Cycle. Against this background, dilemmas facing participants are examined.

Dilemmas

For the *intended* curriculum, dilemmas are faced by those responsible for instituting reform.

- Process: a developmental rollout such as that for Project Maths allows for widespread involvement and incremental change; but it also increases the challenge posed to teachers because they are engaged in prolonged change and are teaching a different curriculum to each year-group in school.
- Content: learning with understanding takes more time than learning by rote, so if time is fixed or decreasing, some content must be eliminated; moreover, there are arguments for introduction of new material. How are demands for retention and inclusion for different purposes – academic, work-based and so forth – to be balanced?
- Methodology: while broadly supporting "reasons, not rules" in the classroom, research points to the fact that classroom organization is not a clear determinant of good learning. To what extent is it appropriate to emphasize one particular style – even with a view to introducing balance into a system seemingly over-wedded to a different approach?

- Assessment: if assessment does not change appropriately, reforms will fail; but how can a major shift be made without imposing undue stress on students and teachers?

For the *implemented* curriculum, teachers are currently facing allied and other dilemmas such as the following.

- Culture: many teachers struggle with the concept of setting up good learning situations not explicitly mirrored in examinations. Moreover, those who wish to teach for deep learning face students, parents and other stakeholders used to behaviourist-based drill.
- Content: as well as being unfamiliar and perhaps uncomfortable with new material, teachers face uncertainty about depth of coverage required.
- Methodology: some teachers' beliefs run counter to the approaches advocated. In any case they worry about the time taken to implement such approaches, and about their lack of skill in facilitating productive discussion.
- Assessment: teachers do not know how to pace teaching so as to prepare students for examinations. For the context-embedded questions, some are challenged by students' poor reading skills and limited vocabularies.

Significance

The historical study reveals that many of the problems are of long standing and have withstood numerous attempts to solve them. If we do not recognize their intractable nature, we underestimate the effort needed to bring about change. In this respect, balancing evolution and revolution is challenging. The very serious attempt at solution encapsulated in Project Maths has generated a high degree of stress – more, seemingly, than the usual amount accompanying curriculum reform; but perhaps some of it is necessary. Considerable time, and patience on behalf of both advocates and critics, may be needed before overall increases in attainment are forthcoming.

However, it is appropriate to end by noting changes in the reported balance of skills shown at Leaving Certificate by pilot-school students (hence those who are most familiar with Project Maths) in Summer 2012. Some changes, though not all, were in the desired directions. Are we there yet? No; but maybe we are on the way.

Teacher Evaluation: Beyond Value-added Models (VAM)

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Most value-added models are based on regression procedures that predict the achievement of students in a particular grade in school based on previous performance and the characteristics of teachers and schools. The teacher effect is the size of the regression weight on the teacher variable. This paper describes a different approach that treats the students as the equivalent of test items that are used to evaluate the teacher. Item response theory methodology is used to estimate the location of a teacher on a hypothetical latent trait called teacher competency. In order to apply this methodology, students are calibrated to determine their level of challenge for a teacher. Teachers who are able to bring challenging students to the desired level of proficiency are considered to be quality teachers. This approach has important implications for educational policy.