Measurement of production inefficiency in a technology and inefficiency heterogeneity setting

Ioannis Skevas, Lecturer, Department of Food Business & Development, University College Cork, College Rd, O'Rahilly Building, T12, Cork, Ireland, Tel: +353 21 4902750, Email: ioannis.skevas@ucc.ie

Abstract This study shows how the estimates of production inefficiency and of the marginal effects of its determinants can be distorted if not accounting for technology and inefficiency heterogeneity. This is achieved by employing a hierarchical stochastic frontier model with random parameters both in the production frontier and in the inefficiency distribution and comparing its results with a conventional frontier model. German dairy farming is used as a case study and estimation is performed in a Bayesian framework. The results reveal significant differences in the inefficiencies and the calculated marginal effects of its determinants across the two models. Specifically, it is shown that inefficiency is overestimated when heterogeneity is not accounted for. An inflation of the means and the variances of the marginal effects is also observed, with the latter result suggesting that technology heterogeneity dominates inefficiency heterogeneity. According to Bayes factors, the employed hierarchical frontier model is favored by the data when compared to the conventional frontier model.

Keywords Production Inefficiency · Heterogeneity · Marginal Effects · Bayesian Techniques

JEL Classification C11 · C23 · D24 · Q12

1 Introduction

Measuring the production inefficiency of decision-making units has received much attention in empirical research during the last decades. This is because the identification of shortfalls in the efficiency of production can be an informative tool for businesses regarding their relative performance and the scope for improvements. Equally important is the identification of the potential sources of inefficiency. Identifying production practices that contribute to efficiency enhancements can boost firms to adopt them in order to improve their performance. Additionally, identifying the impact of policies on firms performance can provide policy-makers with valuable information regarding the impact of their intervention tools. The importance of measuring firms' inefficiency and its sources as these are highlighted above, requires researchers to use quantitative tools that yield precise estimates. Nevertheless, it has been shown that estimates of inefficiency can be distorted if heterogeneity among individuals is not taken into account.

For instance, a lot of discussion in the efficiency measurement literature revolves around the need to account for technology heterogeneity among the decision-making units. This is because firms may adopt new technologies at a different pace depending on their financial capacity or manage their production according to their personal cognitive capacity (Tsionas 2002). For this purpose, the Stochastic Frontier Analysis (SFA) tool introduced by Aigner et al. (1977) and Meeusen and van den Broeck (1977) has been modified to account for technology heterogeneity and disentangle it from inefficiency. Within the SFA framework, the random-coefficients model offers a very flexible representation of the production technology by allowing the slope parameters of the frontier to vary across individuals thus forming a unique frontier for each of them (Kalirajan and Obwona 1994).

A plethora of empirical applications appear in the inefficiency measurement literature that account for technology heterogeneity among firms. For instance, Tsionas (2002) and Huang (2004) assumed that US electric utility companies operate under different technologies due to discrepancies in the timing that they adopt new technologies. Therefore, they both specified a random-coefficients frontier model to measure their inefficiency in a Bayesian framework, with their results revealing that inefficiency estimates are inflated if not accounting for technology heterogeneity. Karagiannis and Tzouvelekas (2009), measured the inefficiency of Greek olive-growing farms assuming different technologies due to differences in their environmental-social characteristics. Their modelling approach involved a random-coefficients production frontier model and the method of Simulated Maximum Likelihood (SML) to estimate it, with the results being compared to previous similar studies and not with a conventional frontier model. Furthermore, Assaf (2011) utilized a random-coefficients cost frontier model and Bayesian techniques to estimate the UK's airport industry inefficiency, assuming that technology differences stem from differences in staff training and business experiences. The empirical findings revealed that technological differences were indeed responsible for variation in airports' inefficiency levels. Njuki et al. (2019) hypothesized that US agricultural firms employ different technologies due to differences in the production environment and specified a random-coefficients production frontier to measure their inefficiency and calculate their productivity using SML. The results pointed towards significant differences in the inefficiency estimates across the random-coefficients and the conventional frontier model. Finally, Skevas (2019) used a random-coefficients production frontier model and Bayesian techniques to estimate the inefficiency of German dairy farms. The technology heterogeneity assumption was based on different management practices used by farms such as their levels of intensification. The results revealed that inefficiency is inflated when technology heterogeneity is not accounted for.

Irrespective of the case-study and the estimation method, all the above studies showed that the random-coefficients SFA model prevents an overestimation of inefficiency since it separates it from technology heterogeneity, which is not the case in the conventional SFA model. Although this is a very important result that fulfils the need to develop methods that yield precise inefficiency estimates, the related literature does not discuss the implications of the move from a conventional to a random-coefficients SFA model for the estimates of the inefficiency determinants. In fact, accounting for technology heterogeneity should not only result in different inefficiency estimates but could also alter the estimates of the utilized determinants of inefficiency.

Greene (2005) further discussed the need to account for heterogeneity not only in the technologies employed by firms but also in their inefficiencies. The need for the latter stems from the fact the effect of certain firm characteristics on inefficiency may differ across individuals due to unobserved heterogeneity. For this purpose, Greene (2005) introduced an SFA model that does not only allow the slope coefficients of the frontier to vary among individuals but also the coefficients with respect to the utilized inefficiency determinants. Recognizing the need to account for heterogeneity in both firms technology and inefficiency led several studies to adopt the model proposed by Greene (2005) in an attempt to get inefficiency estimates corrected for heterogeneity.

In particular, Agasisti and Johnes (2010) measured the inefficiency of Italian universities using a random-coefficients cost frontier, while also specifying random-coefficients for the inefficiency determinants which were allowed to impact both the mean and the variance of inefficiency. The heterogeneity assumption was mainly related to regional differences among universities. Estimation was performed using SML and the results revealed that the inefficiencies of Italian universities were overestimated when ignoring heterogeneity. Additionally, Barros and Williams (2013) also specified random-coefficients both in the (cost) frontier and in the inefficiency determinants with their model being estimated using SML. The case study concerned Mexican banks, which were assumed to be heterogeneous due to discrepancies in their sizes. As in the study of Agasisti and Johnes (2010), an inflation of inefficiency in the case where heterogeneity is not taken into account was found. Finally, Feng et al. (2018) measured the inefficiency of US electric utilities with the hypothesized heterogeneity being based on differences in firms' costs and emission intensities. To account for heterogeneity, a random-coefficients production frontier model was specified along with a random directional vector for inefficiency. Estimation of the model was carried out using Bayesian techniques and the discussion of results revolved around the distortions in the estimates of emissions' shadow prices when ignoring heterogeneity.

However, as in the case of the technology heterogeneity SFA studies, all the aforementioned studies that adopted the Greene (2005) approach discussed the consequences of accounting for both technology and inefficiency heterogeneity on the inefficiency estimates and not on the estimates of its determinants. Therefore, this study employs a hierarchical production frontier model that accounts for technology and inefficiency heterogeneity, and unlike previous studies, it is the first to discuss the implications for the estimates of the inefficiency determinants. For this purpose, a conventional SFA model is also considered and the differences in the estimates of the inefficiency determinants across the two models are discussed. Additionally, formal model comparison based on Bayes factors is performed. The following section presents the model, the Bayesian techniques used to estimate it and the utilized model comparison framework. The data and the empirical specification are then described. Presentation of the results follows and the final section concludes.

2 Model & Bayesian Estimation

2.1 Hierarchical production frontier

Let $i = 1, ..., N$ and $t = 1, ..., T$ indicate individuals and time observations, respectively. This study accounts for technology heterogeneity across individuals by specifying a hierarchical production frontier by means of random technology parameters:

$$y_{it} = f(\mathbf{x}_{it}^{'}; \boldsymbol{\beta}_i) + v_{it} - u_{it}, \tag{1}$$

where y_{it} is the logarithm of output, $\mathbf{x}_{it}^{'}$ is a vector of the logarithm of K production factors, $\boldsymbol{\beta}_i$ is the associated vector of random technology parameters, v_{it} is a two-sided noise component and u_{it} is the one-sided inefficiency term. Estimation of the hierarchical production frontier presented in equation (1) requires distributional assumptions on the involved random components. Starting with the two-sided noise component v_{it}, this study follows the typical procedure of assuming a Normal distribution with zero mean and precision (i.e. inverse variance) τ. Coming to the random technology parameters $\boldsymbol{\beta}_i$, the conventional assumption of Kalirajan and Obwona (1994) is made as follows:

$$\boldsymbol{\beta}_i \sim \mathcal{N}(\bar{\boldsymbol{\beta}}, \boldsymbol{\Omega}) \tag{2}$$

where $\bar{\boldsymbol{\beta}}$ is a vector of K parameters that represents the mean of the $\boldsymbol{\beta}_i s$ and $\boldsymbol{\Omega}$ is a $K \times K$ error precision matrix (i.e. inverse covariance matrix) for the distribution of the $\boldsymbol{\beta}_i s$.

In addition to the hierarchical specification of the production frontier presented in equation (1) that takes into account technology heterogeneity, a similar approach is followed to also account for inefficiency heterogeneity. Specifically, the one-sided inefficiency term is assumed to follow the typical truncated-Normal distribution as in Battese and Coelli (1995), which is further complemented by a random parameter specification for the mean of the distribution:

$$u_{it} \sim \mathcal{N}^{+}(\mathbf{z}_{it}'\boldsymbol{\delta}_i, \varphi) \tag{3}$$

$$\boldsymbol{\delta}_i \sim \mathcal{N}(\bar{\boldsymbol{\delta}}, \boldsymbol{\Psi}) \tag{4}$$

where $\mathbf{z}_{it}$ is a vector of L individual-specific socio-economic characteristics that can affect inefficiency, $\boldsymbol{\delta}_i$ is the associated vector of random parameters, φ is the precision parameter of the distribution of inefficiency, $\bar{\boldsymbol{\delta}}$ is a vector of L parameters that represents the mean of the $\boldsymbol{\delta}_i s$, and $\boldsymbol{\Psi}$ is a $L \times L$ error precision matrix of the distribution of the $\boldsymbol{\delta}_i s$. The last step needed to estimate the model in equations (1-4) is to specify the functional form $f(\mathbf{x}_{it}'; \boldsymbol{\beta}_i)$ from equation (1). A Cobb-Douglas specification is used so that the logarithm of output in equation (1) is a linear function of individual-specific parameters and the logarithms of inputs. The Cobb-Douglas specification is preferred against the more flexible translog for two main reasons: 1) given the large number of production factors associated with the subsequently utilized dataset, employing a translog specification would entail a very large number of individual-specific parameters and 2) the number of parameters to be estimated in $\boldsymbol{\Omega}$ (i.e. $K \times K$) in a translog specification would increase significantly as opposed to a Cobb-Douglas specification. For the above reasons, the majority of studies estimating random technology parameter frontiers also specify a Cobb-Douglas functional form (Kalirajan and Obwona (1994); Tsionas (2002), Huang (2004); Karagiannis and Tzouvelekas (2009)). A final note is that a constant term can't be included in both the $\mathbf{x}$ and the $\mathbf{z}$ vectors. This is because identification of two individual-specific and time-invariant random parameters is impossible. Hence, in the application that follows a constant term is only included in the vector $\mathbf{x}$.

Despite some minor modelling differences, taking into account both technology and inefficiency heterogeneity as this is done in the model in equations (1-4) has been the subject of a few studies including Greene (2005), Agasisti and Johnes (2010), Barros and Williams (2013) and Feng et al. (2018). However, the aforementioned studies only discussed the impact of accounting for both technology and inefficiency heterogeneity on the inefficiency estimates, with their main conclusion being that ignoring heterogeneity inflates inefficiency.

Nevertheless, the present study further argues that ignoring technology and inefficiency heterogeneity can also impact the magnitude and the variation of the marginal effects of the utilized socio-economic characteristics on inefficiency yielding misleading conclusions that are used by businesses/policy-makers.

To derive the marginal effects of the variables in $\mathbf{z}$ on inefficiency, one needs to calculate the derivative of inefficiency with respect to each $\mathbf{z}$ covariate. Given the truncated-Normal distribution imposed on inefficiency, its expected value is:

$$E(u_{it}) = \mathbf{z}_{it}^{'}\boldsymbol{\delta}_i + \varphi^{-1/2}\frac{\phi(\varphi^{1/2}\mathbf{z}_{it}^{'}\boldsymbol{\delta}_i)}{\Phi(\varphi^{1/2}\mathbf{z}_{it}^{'}\boldsymbol{\delta}_i)} \tag{5}$$

where $\phi(\cdot)$ is the standard Normal density function and $\Phi(\cdot)$ is the standard Normal cumulative distribution function. The derivative of the expected value of inefficiency with respect to the l^{th} variable contained in $\mathbf{z}$ is calculated as:

$$\frac{\partial E(u_{it})}{\partial z_{itl}} = \delta_{il}\Big[1 - \frac{\phi(\varphi^{1/2}\mathbf{z}_{it}^{'}\boldsymbol{\delta}_i)}{\Phi(\varphi^{1/2}\mathbf{z}_{it}^{'}\boldsymbol{\delta}_i)}\Big(\varphi^{1/2}\mathbf{z}_{it}^{'} + \frac{\phi(\varphi^{1/2}\mathbf{z}_{it}^{'}\boldsymbol{\delta}_i)}{\Phi(\varphi^{1/2}\mathbf{z}_{it}^{'}\boldsymbol{\delta}_i)}\Big)\Big] \tag{6}$$

An important characteristic of the marginal effect presented in equation (6) is that it does not only vary due to differences in the socio-economic characteristics of individuals, but also due to random variation in the associated parameters $\boldsymbol{\delta}_i$. At the same time, the marginal effect is calculated after the production frontier has been adjusted for technology heterogeneity as equations (1-2) manifest. Finally, it is noted in passing that the expected value of inefficiency in equation (5) is unconditional on the composed residual e_{it}, where $e_{it} = v_{it}+u_{it}$. Calculating the marginal effects of the $\mathbf{z}$ variables on the unconditional or the conditional expectation of inefficiency is still an open issue in the efficiency literature. For instance, Wang (2002) and Kumbhakar and Sun (2013) present the calculation of marginal effects unconditionally and conditionally on the composed residual, respectively. Given that in the present study the inefficiency scores are obtained unconditional on the composed residual, as it is the case with the data augmentation techniques used in the utilized Bayesian framework, the marginal effects are also evaluated on the unconditional expectation. This argument is similar to the one presented in Kumbhakar and Sun (2013) when justifying the use of the conditional expectation in a frequentist setting.

2.2 Bayesian estimation

A Bayesian method is used to estimate the model in equations (1-4). The parameters to be estimated are $\tau, \bar{\boldsymbol{\beta}}, \boldsymbol{\Omega}, \varphi, \bar{\boldsymbol{\delta}}$, and $\boldsymbol{\Psi}$. The $\boldsymbol{\beta}_i s$, $u_{it}s$ and $\boldsymbol{\delta}_i s$ are the latent data and can be obtained as by-products of the utilized sampler. The complete-data likelihood of the model is:

$$\begin{aligned} p(\{y_{it}\}, \{\boldsymbol{\beta}_i\}, &\{u_{it}\}, \{\boldsymbol{\delta}_i\} | \{\mathbf{x}_{it}\}, \{\mathbf{z}_{it}\}, \tau, \bar{\boldsymbol{\beta}}, \boldsymbol{\Omega}, \varphi, \bar{\boldsymbol{\delta}}, \boldsymbol{\Psi}) \\ &= \frac{\tau^{NT/2}}{(2\pi)^{NT/2}} \exp\left\{ -\frac{\tau}{2} \sum_{i=1}^{N} \sum_{t=1}^{T} (v_{it})^2 \right\} \\ &\times \frac{|\boldsymbol{\Omega}|^{N/2}}{(2\pi)^{NK/2}} \exp\left\{ -\frac{1}{2} \sum_{i=1}^{N} (\boldsymbol{\beta}_i - \bar{\boldsymbol{\beta}})' \boldsymbol{\Omega} (\boldsymbol{\beta}_i - \bar{\boldsymbol{\beta}}) \right\} \\ &\times \frac{\varphi^{NT/2}}{(2\pi)^{NT/2} \sum_{i=1}^{N} \sum_{t=1}^{T} \Phi(\varphi^{1/2} \mathbf{z'}_{it} \boldsymbol{\delta}_i)} \exp\left\{ -\frac{\varphi}{2} \sum_{i=1}^{N} \sum_{t=1}^{T} (u_{it} - \mathbf{z}'_{it} \boldsymbol{\delta}_i)^2 \right\} \\ &\times \frac{|\boldsymbol{\Psi}|^{N/2}}{(2\pi)^{NL/2}} \exp\left\{ -\frac{1}{2} \sum_{i=1}^{N} (\boldsymbol{\delta}_i - \bar{\boldsymbol{\delta}})' \boldsymbol{\Psi} (\boldsymbol{\delta}_i - \bar{\boldsymbol{\delta}}) \right\} \end{aligned} \tag{7}$$

The first factor of the complete-data likelihood comes from the fact that each v_{it} follows a Normal distribution, while the second factor is due to the multivariate Normal distribution imposed on $\boldsymbol{\beta_i}$. The third factor is due to the truncated-Normal distribution imposed on each u_{it}. Finally, the fourth factor is because of the multivariate Normal distribution imposed on $\boldsymbol{\delta_i}$. The joint posterior density of the model's parameters and the latent variables is expressed as:

$$\begin{aligned} \pi(\tau, \bar{\boldsymbol{\beta}}, \boldsymbol{\Omega}, \varphi, \bar{\boldsymbol{\delta}}, \boldsymbol{\Psi}, &\{\boldsymbol{\beta}_i\}, \{u_{it}\}, \{\boldsymbol{\delta}_i\} | \{y_{it}\}, \{\mathbf{x}_{it}\}, \{\mathbf{z}_{it}\} \\ &\propto p(\{y_{it}\}, \{\boldsymbol{\beta}_i\}, \{u_{it}\}, \{\boldsymbol{\delta}_i\} | \{\mathbf{x}_{it}\}, \{\mathbf{z}_{it}\}, \tau, \bar{\boldsymbol{\beta}}, \boldsymbol{\Omega}, \varphi, \bar{\boldsymbol{\delta}}, \boldsymbol{\Psi}) \\ &\times p(\tau) \times p(\bar{\boldsymbol{\beta}}) \times p(\boldsymbol{\Omega}) \times p(\varphi) \times p(\bar{\boldsymbol{\delta}}) \times p(\boldsymbol{\Psi}) \end{aligned} \tag{8}$$

where the first factor is the complete-data likelihood specified in equation (7) and the second factor consists of the product of the prior distributions of the parameters to be estimated. As in Koop et al. (1995), Markov Chain Monte Carlo (MCMC) techniques are used to draw samples from the posterior. Furthermore, the latent data are integrated from the likelihood using data augmentation (Tanner and Wong 1987).

2.3 Alternative models, Bayes factors & posterior probabilities

In addition to the hierarchical production frontier presented in subsection 2.1, the typical stochastic frontier with fixed technology and inefficiency parameters (i.e $\boldsymbol{\beta}_i = \boldsymbol{\beta}$ and $\boldsymbol{\delta}_i = \boldsymbol{\delta}$) is also considered. From this point on, the former model will be called the 'Hierarchical Frontier' and the latter the 'Simple Frontier'. Following Kass and Raftery (1995), Bayes factors are used to infer which of the two models (M_1 or M_2) fits the data best:

$$BF = \frac{p(\mathcal{D}|\mathcal{M}_1)}{p(\mathcal{D}|\mathcal{M}_2)} \frac{P(\mathcal{M}_2)}{P(\mathcal{M}_1)} \tag{9}$$

where $\mathcal{D}$ represents the data, $p(\mathcal{D}|\mathcal{M}_1)$ and $p(\mathcal{D}|\mathcal{M}_2)$ are the marginal likelihoods of the two models and $P(\mathcal{M}_1)$ and $P(\mathcal{M}_2)$ are the prior probabilities of the models. Placing equal prior model probabilities, the calculation of Bayes factor reduces to the ratio of the marginal likelihoods of the two models:

$$BF = \frac{p(\mathcal{D}|\mathcal{M}_1)}{p(\mathcal{D}|\mathcal{M}_2)} = \frac{\int p(\mathcal{D}|\boldsymbol{\theta}_1, \mathcal{M}_1)\, p(\boldsymbol{\theta}_1|\mathcal{M}_1)\, d\boldsymbol{\theta}_1}{\int p(\mathcal{D}|\boldsymbol{\theta}_2, \mathcal{M}_2)\, p(\boldsymbol{\theta}_2|\mathcal{M}_2)\, d\boldsymbol{\theta}_2} \tag{10}$$

where $\boldsymbol{\theta}$ is a vector that contains all the parameters to be estimated in each model. The logarithm of each marginal likelihood can be obtained using the Laplace-Metropolis estimator as in Lewis and Raftery (1997):

$$\log\left[p(\mathcal{D}|\mathcal{M}_j)\right] \approx \frac{P}{2}\log[2\pi] + \frac{1}{2}\log\left[|\boldsymbol{H}^*|\right] + \log\pi[\boldsymbol{\theta}_j^*] + \log p[\mathcal{D}|\boldsymbol{\theta}_j^*] \tag{11}$$

where j is an index for the considered models, P equals the number of parameters in $\boldsymbol{\theta}_j$, $\boldsymbol{\theta}_j^*$ is an estimator of $\boldsymbol{\theta}_j$ that maximizes the likelihood $p(\mathcal{D}|\boldsymbol{\theta}_j^*)$ and $\boldsymbol{H}^*$ is the Hessian of the likelihood evaluated at $\boldsymbol{\theta}_j^*$. Finally, assuming that the set of models considered is exhaustive, posterior model probabilities are obtained using Bayes factors and the fact that the two posterior model probabilities sum to unity.

3 Data & empirical specification

3.1 Data & model specification

The utilized dataset comes from the Farm Accountancy Data Network and is a panel of 6,732 observations of 748 specialized German dairy farms covering the period 2001-2009. The Cobb-Douglas specification, including also a time trend t, is written as follows:

$$\log y_{it} = \beta_{i0} + \sum_k \beta_{ik} \log x_{itk} + \gamma_i t + v_{it} - u_{it} \tag{12}$$

The output (y_{it}) consists of revenues from cow's milk, meat and crop products. Six inputs are used. The capital input (K) is specified as the deflated value of buildings and machinery, the labor input (L) as the total working hours and the land input (A) as the total hectares of utilized area. The intermediate input (M) represents the total deflated value of seeds and plants, fertilizers, pesticides, energy, veterinary costs, other crop-specific costs, forestry-specific costs, feed for pigs and poultry, contract work and other direct inputs. The animal input (S) is specified as the total number of livestock units and the feed input (F) as the deflated value of purchased feed[1]. The time trend (t) aims to capture technological progress/regress. Finally, the variables in the vector $\mathbf{z}$ in equation (3) that are allowed to impact inefficiency are the economic size of farms measured in European Size Units (ESU), the total amount of subsidies that farms receive (the majority of which are decoupled from production) and stock density, defined as livestock units per hectare. Summary statistics of the variables appear in Table 1. Prior to estimation, the data for output and inputs are normalized by their geometric means, while the time trend variable is normalized by its arithmetic mean.

Table 1 Summary statistics of the utilized variables

Variable	Mean	Std. dev.	Min	Max
y (€1,000)	133.17	151.92	13.60	3635.66
K (€1,000)	169.28	151.83	1.85	2617.92
L (1,000 hours)	3.27	3.09	0.97	77.00
A (hectares)	58.92	57.86	9.88	1058
M (€1,000)	45.30	50.06	6.59	1258.30
S (livestock units)	92.02	82.72	20.65	1579.50
F (€1,000)	19.62	29.21	0.05	740.92
ESU (100 ESU)	0.75	0.78	0.16	18.17
Subsidies (€100,000)	0.24	0.27	0.08	6.44
Density (livestock/hectare)	2.03	0.66	0.41	9.23

3.2 Prior specification

The priors for the parameters to be estimated are specified as follows:

[1]Price indices from EUROSTAT are used for the deflation of revenues and values of outputs and inputs.

⇒ A Gamma prior is used for the precision τ with $p(\tau) = \frac{b^a}{\Gamma(a)}\tau^{a-1}e^{-b\tau}$. The shape and rate hyperparameters are set equal to 0.001. The same prior distribution is used for the prior of the precision parameter φ ($p(\varphi)$). However, since this precision parameter corresponds to a latent equation, the shape and rate hyperparameters are set equal to 4 and 0.5, respectively, making this prior a bit more informative. The need to place a more informative prior on such a parameter is highlighted in Fernandez et al. (1997), who warn that a non-informative prior may lead to a posterior that is not proper. This is also emphasized in Van den Broeck et al. (1994) and Griffin and Steel (2007), who use a truncated-Normal distribution for inefficiency, as also the present study does.

⇒ A multivariate Normal prior is placed on $\bar{\boldsymbol{\beta}}$ with $p(\bar{\boldsymbol{\beta}}) = \frac{|\boldsymbol{P}|^{1/2}}{(2\pi)^{K/2}}\exp\Big\{-\frac{1}{2}(\bar{\boldsymbol{\beta}}-\boldsymbol{m})'\boldsymbol{P}(\bar{\boldsymbol{\beta}}-\boldsymbol{m})\Big\}$. The prior means $\boldsymbol{m}$ are set equal to zero and the precision matrix $\boldsymbol{P}$ is diagonal with its diagonal entries taking the value of 0.001. The same prior distribution and parameterization is used for the prior of the vector $\bar{\boldsymbol{\delta}}$ ($p(\bar{\boldsymbol{\delta}})$).

⇒ A Wishart prior is used for $\boldsymbol{\Omega}$ with $p(\boldsymbol{\Omega}) = \frac{|\boldsymbol{\Omega}|^{\frac{n-K-1}{2}}|\boldsymbol{V}^{-1}|^{n/2}}{2^{nK/2}\Gamma_K(n/2)}\exp\Big\{-\frac{1}{2}tr(\boldsymbol{V}^{-1}\boldsymbol{\Omega})\Big\}$, where n stands for the degrees of freedom, $\boldsymbol{V}$ is a scale matrix, Γ_K is the multivariate Gamma function and tr is the trace function. n is set equal to K and $\boldsymbol{V}$ is diagonal (I_K) as in Tsionas (2002). The same prior density and parameterization is used for the prior of the matrix $\boldsymbol{\Psi}$ ($p(\bar{\boldsymbol{\Psi}})$).

4 Results

The results reported in the section are based on 100,000 MCMC iterations. To prevent autocorrelation and dependence of posterior samples, the first 10,000 iterations are deleted and each 2^{th} is stored. The posterior moments of the elasticities (i.e. $\boldsymbol{\beta}$ and $\bar{\boldsymbol{\beta}}$) and the precision parameters as well as model comparison quantities from the simple and the hierarchical frontier models are presented in Table 2[2,3]. All output elasticities are positive with their magnitudes exhibiting some differences across the two models. Nevertheless, both models point towards livestock units being

[2]The hierarchical frontier is also estimated using a translog specification. However, most second-order terms are "statistically insignificant", while the posterior model probabilities present overwhelming evidence in favor of the Cobb-Douglas specification of the hierarchical frontier. The results can be provided upon request. Note also that the robustness checks with regards to the model specification and the performed test concern only the hierarchical frontier as results from the simple frontier are only presented for comparison.

[3]The posterior mean of $\boldsymbol{\Omega}$ from the hierarchical frontier model is presented in Table A1 in the Appendix.

the most influential input for farms. Additionally, the estimate with respect to time trend is the same across the two models and suggests that, on average, Dutch dairy farms face technological progress during the study period. The same conclusions as above are drawn by Skevas et al. (2018b) for the case of German dairy farms. The estimates with respect to the precision parameters can provide important information regarding the proportion of variation in output and inefficiency that is explained by the two models. Both precisions are higher in the hierarchical model. Given that variance is the inverse of precision, the last result suggests that unexplained variation in output and inefficiency is lower in the hierarchical model. This holds particularly for the case of output as the respective precision parameter is 7 times larger when compared to the simple frontier. This is a first indication of the superiority of the hierarchical frontier. Additionally, the estimates of the marginal log likelihood and the calculated posterior model probabilities clearly indicate that the hierarchical frontier fits the data better than the simple frontier.

Table 2 Posterior moments and model comparison for the two frontier models

	Simple Frontier			**Hierarchical Frontier**		
Variable	**Mean**	**Std. dev.**	**95% Interval**	**Mean**	**Std. dev.**	**95% Interval**
intercept	0.128	0.009	[0.110, 0.145]	0.062	0.008	[0.046, 0.078]
log_K	0.072	0.003	[0.066, 0.078]	0.029	0.008	[0.014, 0.044]
log_L	0.102	0.008	[0.087, 0.118]	0.044	0.014	[0.017, 0.071]
log_A	0.032	0.011	[0.011, 0.055]	0.174	0.017	[0.141, 0.208]
log_M	0.331	0.009	[0.314, 0.348]	0.145	0.013	[0.119, 0.171]
log_S	0.366	0.013	[0.340, 0.392]	0.403	0.019	[0.365, 0.440]
log_F	0.117	0.003	[0.110, 0.124]	0.159	0.008	[0.144, 0.175]
t	0.031	0.001	[0.029, 0.033]	0.031	0.002	[0.028, 0.035]
τ	49.365	2.612	[44.573, 54.671]	356.894	36.670	[292.298, 436.206]
φ	5.498	2.759	[0.976, 10.489]	6.055	1.568	[3.373, 9.158]
		Marginal log likelihood			**Marginal log likelihood**	
		3464.783			9853.422	
		Posterior model probability			**Posterior model probability**	
		0.000			1.000	

The two models also produce different inefficiency estimates. Average inefficiency across all individuals and time is 0.113 in the simple frontier and 0.077 in the hierarchical frontier. That is, when heterogeneity is not taken into account it is absorbed by the inefficiency component resulting in its inflation. This is a well-documented result in the literature (Tsionas (2002) and Barros and Williams (2013)). Figure 1 presents histograms of the individuals' inefficiency estimates produced by the simple frontier (left panel) and the hierarchical frontier (right panel). Obviously, lower inefficiency estimates are observed in the hierarchical model as the bulk of the observations are closer to zero. Note also that the maximum value of inefficiency is higher in the simple frontier. Additionally, a closer look at Figure 1 reveals a striking result; variation in inefficiency is higher in the simple frontier, which is unexpected given that only the hierarchical frontier accounts for heterogeneity in inefficiency by specifying the random parameters $\boldsymbol{\delta}_i$. However, given that the hierarchical frontier accounts for both technology and inefficiency heterogeneity, this result suggests that technology heterogeneity dominates inefficiency heterogeneity resulting in low variation in the individual's inefficiency scores.

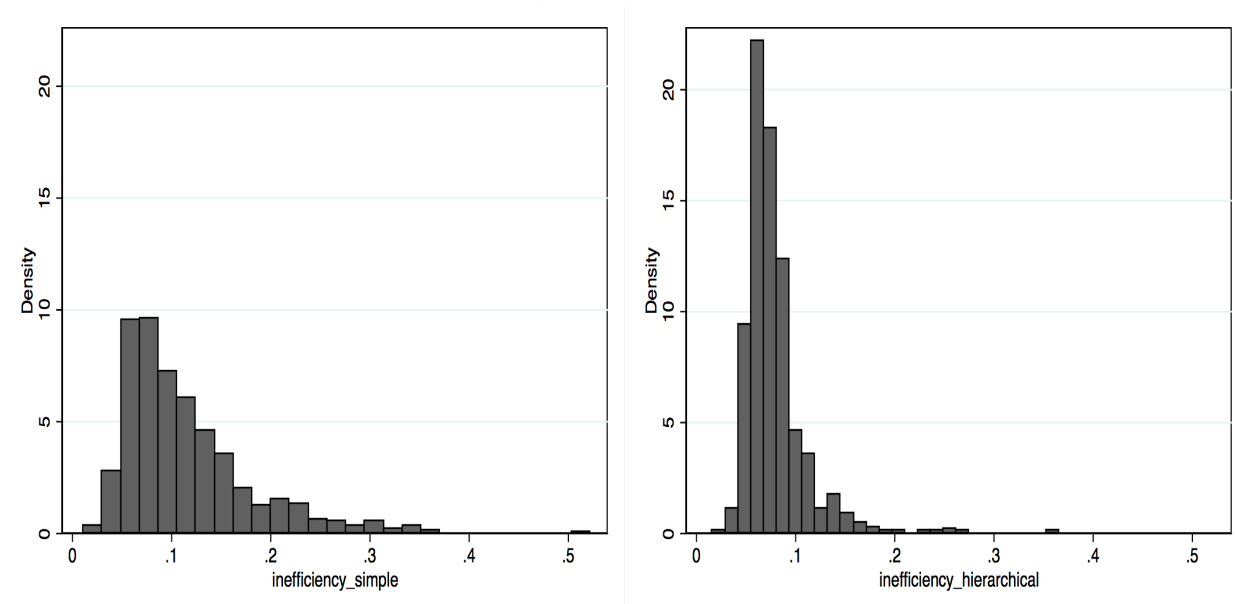

Fig. 1 Histograms of the inefficiency estimates in the two frontier models

The posterior moments of the determinants of inefficiency from the simple and the hierarchical frontier (i.e. $\boldsymbol{\delta}$ and $\bar{\boldsymbol{\delta}}$) appear in Table 3[4]. As mentioned earlier, the hierarchical frontier does not contain an intercept in its inefficiency specification due to identification issues arising when a model contains two individual-specific time-invariant random components.

[4]The posterior mean of $\boldsymbol{\Psi}$ from the hierarchical frontier model appears in Table A2 in the Appendix.

Table 3 Posterior moments of inefficiency determinants in the two frontier models

	Simple Frontier			Hierarchical Frontier		
Variable	**Mean**	**Std. dev.**	**95% Interval**	**Mean**	**Std. dev.**	**95% Interval**
intercept	0.529	0.229	[0.062, 1.148]	-	-	-
ESU	-2.179	1.948	[-7.776, -0.715]	-0.720	0.271	[-1.367, -0.289]
subsidies	1.595	1.453	[0.429, 5.917]	0.863	0.398	[0.198, 1.828]
density	-0.960	0.923	[-3.405, -0.289]	-0.995	0.320	[-1.771, -0.544]

The results reveal that ESU and density are negatively related to inefficiency, while subsidies exhibit a positive link. Such findings are also reported by Alvarez and del Corral (2010) and Skevas et al. (2018a) for dairy farms. The results further suggest that the parameter estimates of the inefficiency determinants are severely inflated in the simple frontier, which does not account for heterogeneity. The only exemption concerns the estimate with respect to the density variable, which is almost the same in the two models. Naturally, this inflation is also observed in the associated marginal effects of the inefficiency determinants. These are calculated according to equation (6) and their histograms are presented in Figure 2.

The left panel contains the histograms of the marginal effects from the simple frontier and the right panel the associated histograms from the hierarchical frontier. The average marginal effect of ESU on inefficiency across all individuals is -0.073 in the simple and -0.023 in the hierarchical frontier (upper left and right figures). Regarding subsidies, its average marginal effect is 0.053 and 0.024 in the simple and the hierarchical frontier, respectively (middle left and right figures). Finally, the marginal effect of density on inefficiency is -0.030 both in the simple and in the hierarchical frontier, on average (lower left and right figures). Presentation of histograms of the marginal effects facilitates making inferences regarding their variation across individuals. Note that the marginal effects in the simple frontier vary only due to differences in the utilized socio-economic characteristics of individuals, whereas in the hierarchical frontier vary also due to heterogeneity in the respective parameters (i.e. $\boldsymbol{\delta}_i$). As striking as in the case of inefficiency, the marginal effects in the hierarchical frontier are more concentrated around the reported means suggesting a lower variation when compared to those derived from the simple frontier. This is again because technology heterogeneity dominates inefficiency heterogeneity in the hierarchical frontier resulting in very low variation in the associated marginal effects.

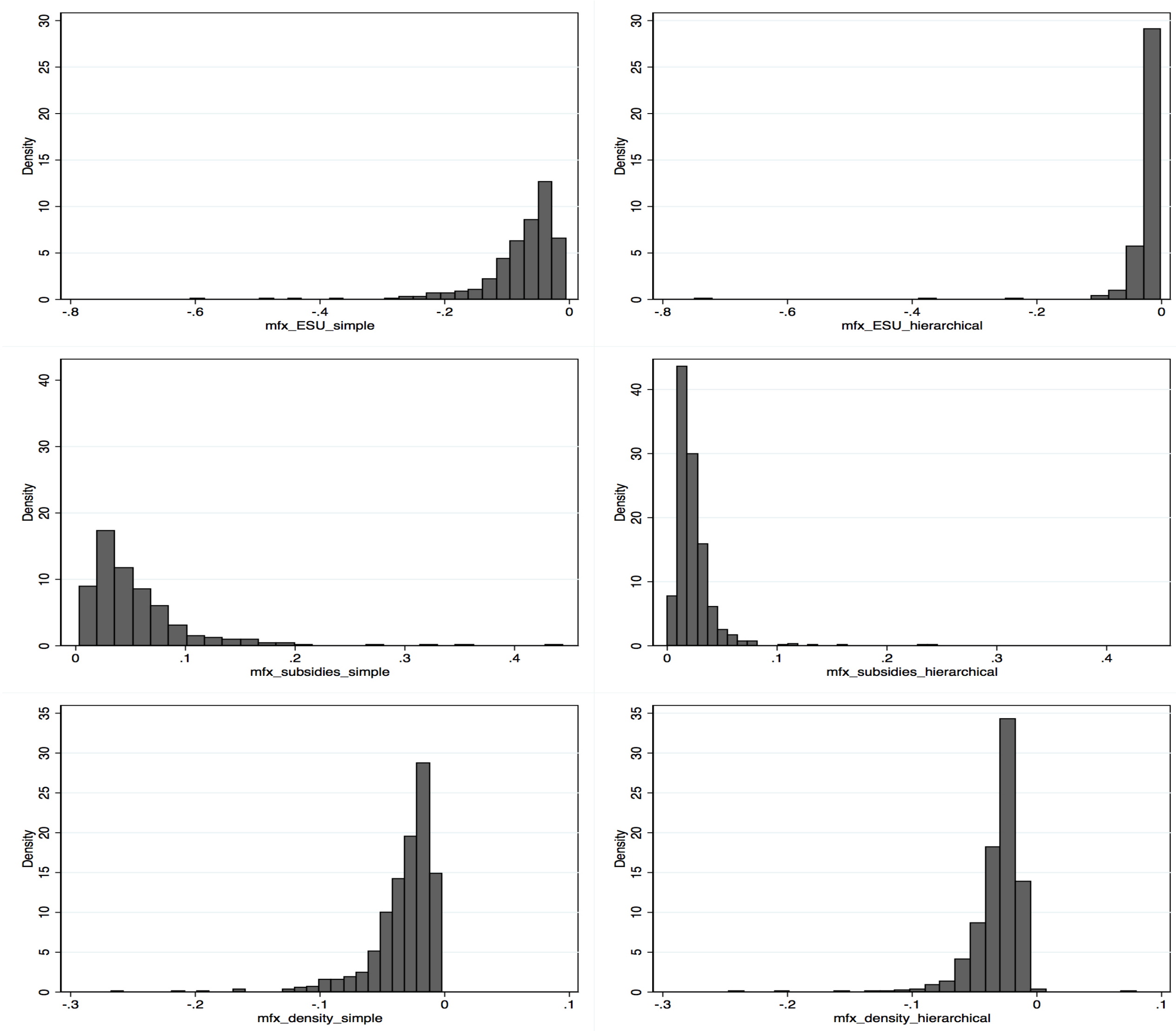

Fig. 2 Histograms of the marginal effects of ESU, subsidies and density on inefficiency across the two frontier models

5 Conclusions

The goal of this study is to present the distortions in inefficiency and in the estimates of its determinants when technology and inefficiency heterogeneity across individuals is not taken into account. For this purpose, a hierarchical frontier model with random parameters that allow for both technology and inefficiency heterogeneity across units is used, and its results are compared with a conventional frontier model that completely ignores heterogeneity. Data on German dairy farms are used and the two models are estimated using Bayesian techniques.

The empirical findings reveal that the inefficiency estimates are inflated if heterogeneity is not taken into account. This is because the inefficiency component also absorbs firm heterogeneity providing a warning to studies measuring the inefficiency of decision-making units that ignorance of heterogeneity can result in misleading inefficiency estimates. Additionally, there is evidence that technology heterogeneity dominates inefficiency heterogeneity. Specifically, although the employed hierarchical frontier model allows for inefficiency heterogeneity, the inefficiency estimates from the conventional frontier model exhibit higher variation. This implies that the main source of firm heterogeneity concerns technology differences rather than discrepancies in the way their socio-economic factors impact them.

The aforementioned distortions in the inefficiency estimates are also observed in the estimates of their determinants. Specifically, it is shown that the magnitude of the marginal effects of the utilized socio-economic characteristics on inefficiency are inflated if heterogeneity is not accounted for. Again, this finding provides a signal to studies searching for the sources of inefficiency that ignorance of heterogeneity yields imprecise results. The dominance of technology heterogeneity is also evident in the variation of the marginal effects as this is very low in the hierarchical frontier model.

Apart from the theoretical appeal of the hierarchical frontier model that comes from the rather realistic assumption of firm heterogeneity, it is shown that the model fits the utilized data better than the simple frontier model according to the derived posterior model probabilities. Although the data are old, the results are contemporary and relevant for researchers as they reveal the distortions and the related misleading inferences that can efficiency studies that ignore firm heterogeneity make. From a policy perspective though, it would worth mentioning that the results of this study regarding the inefficiency estimates are outdated for informing businesses or policy-makers. However, some policy implications can be provided based on the relationships between some of the utilized socio-economic characteristics and inefficiency. This holds particularly for subsidies and density because the provision of subsidies and measures targeting farmers' stock density remain a central theme in modern agricultural policy, with lessons from past experiences being of particular importance.

In terms of subsidies, two of the key instruments of the Common Agricultural Policy (CAP) for the Post-2020 period include the "Complementary Income Support for Young Farmers (CISYF)" and the "Complementary Redistributive Income Support for Sustainability (CRISS)". The former provides income support to young farmers in the form of annual decoupled payments per eligible hectare, and the latter includes redistribution of decoupled payments from bigger to smaller or medium-sized farms to improve sustainability (European Commission 2020). The results from this study point towards a positive link between subsidies (which almost exclusively consist of decoupled payments) and inefficiency, which can be because such income-support payments lower farmers' motivation to work efficiently. Therefore, this result provides a warning that decoupled payments, although boosting farmers income, can at the same time decrease the efficiency of production. Regarding density, the CAP Post-2020 in order to enhance farms' environmental performance aims to introduce a mandatory "greening" component of direct payments that will support sustainable management. However, in order to receive it farms will need to extensify their production through lower stocking density (European Commission 2020). The present study's empirical findings report a negative link among stock density and inefficiency. Hence, this result provides a signal that, although extensive schemes can improve the environmental performance, this can come at a loss of efficiency in production.

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