A Planar Reconfigurable Linear Rigid-Body Motion Linkage (RLRBML) with Two Operation Modes

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ABSTRACT: A planar reconfigurable linear (also rectilinear) rigid-body motion linkage (RLRBML) with two operation modes, i.e., linear rigid-body motion mode and lockup mode, is presented using only R (revolute) joints. The RLRBML does not require disassembly and external intervention to implement multi-task requirements. It is created via combining a Robert's linkage and a double parallelogram linkage (with the equal lengths of the rocker links) arranged in parallel, which can convert a limited circular motion to a linear rigid-body motion without any reference guide way. This linear rigid-body motion is achieved since the double parallelogram linkage can guarantee the translation of the motion stage, and the Robert's linkage ensures the approximate straight line motion of its pivot joint connecting to the double parallelogram linkage. This novel RLRBML is under a linear rigid-body motion mode if the four rocker links in the double parallelogram linkage are not parallel. The motion stage is in the lockup mode if all the four rocker links in the double parallelogram linkage are kept parallel in a tilted position (but the inner/outer two rocker links are still parallel). In the lockup mode, the motion stage of the RLRBML is prohibited from moving even under power off, but the double parallelogram linkage is still moveable for its own rotation application. It is noted that further RLRBMLs can be obtained from the above RLRBML by replacing the

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Robert’s linkage with any other straight line motion linkage (such as Watt’s linkage). Additionally, a compact RLRBML and two single-mode linear rigid-body motion linkages are presented.

KEYWORDS: Planar linkage; Linear rigid-body motion; Reconfigurable mechanism; Two operation modes; Lockup

1 INTRODUCTION

A linear (also rectilinear) rigid-body motion linkage is a linkage that is able to achieve linear rigid-body motion by transforming a rotational motion. It has a variety of potential applications including the linear motion guiding mechanism, linear actuator, linear positioning stage, drawing tools, and pseudo-rigid-body model of the compliant translational joint in MEMS/Precision Engineering [1-3].

There are a number of well-known planar straight line motion linkages using only revolute (R) joints (pins), such as Watt’s linkage, Robert’s linkage, and Chebyshev’s linkage [4], which have been applied extensively since they were proposed. For example, Watt’s linkage was used as part of an automobile suspension [5]. However, these linkages can only perform straight line motion for a single point on the coupler link. When linear rigid-body motion is needed, additional high-cost slide/screw/gear guiding mechanisms [1-2] or the well-known spatial Sarrus linkage (not planar mechanism) [6] are commonly employed.

There is an increasing need for developing manipulators that can rapidly adapt to changing environment and variable tasks like a transformer in manufacturing, packaging and food industries. A reconfigurable mechanism with multi-operation modes without disassembly and external intervention to implement multi-task requirements [7-10] may respond to this demand.

Two methods have been reported for the design of single-loop reconfigurable mechanisms with multiple operation modes. One intuitive approach is to construct a single-loop reconfigurable mechanism with multiple operation modes by combining two overconstrained mechanisms [9-10]. Another design approach for constructing single-loop reconfigurable mechanisms with multiple operation modes is to insert one or more joints into an overconstrained mechanism [11-12].

Based on the above advances, this paper focuses on the design of a novel planar *reconfigurable linear rigid-body motion linkage* (RLRBML) using only R joints without disassembly and external intervention. This RLRBML is constructed by combing a straight line motion linkage with a double parallelogram linkage. It is organised as follows. Section 2 describes the RLRBML with two operation modes. The kinematic analysis for the motion mode is then conducted in Section 3. Section 4 further analyses the lockup operation mode, which is validated by a prototype. In Section 5, variations of configurations are discussed. Finally, the conclusions are drawn.

2 DESCRIPTION OF THE RLRBML

A planar RLRBML with two operation modes, i.e., linear rigid-body motion mode and lockup mode, is shown in Fig. 1. The RLRBML involving only R joints is composed of a Robert's linkage (Fig. 1a) and a double parallelogram linkage (Fig. 1b) arranged in parallel. The RLRBML can convert a limited circular motion to a linear rigid-body motion without any reference guide ways. This linear rigid-body motion is achieved since the double parallelogram linkage can guarantee the translation of the motion stage, and the Robert's linkage ensures the approximate straight line motion of its pivot joint P connecting to the double parallelogram linkage. The coupler link 2 of the double parallelogram linkage has two degrees-of-freedom (DOF). Combining with the Robert's linkage, the DOF of the coupler link 2 decreases from two to one due to the inherent constraint of Robert' linkage. Here, the initial position (home position) of the pivot joint P is specified at the middle position.

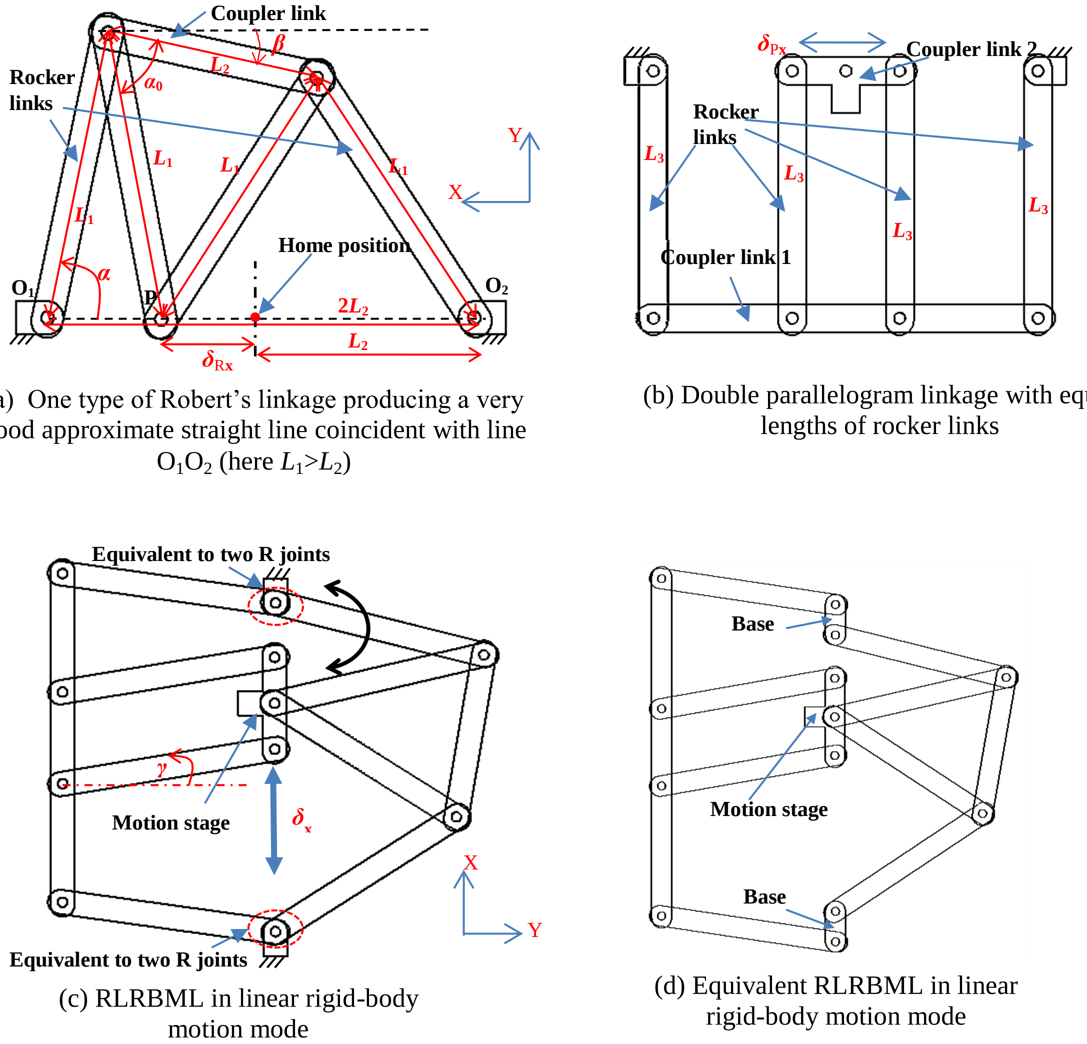


Figure 1. RLRBML and its two compositional units

This middle motion stage of this novel RLRBML is in a linear rigid-body motion mode if the four rocker links in the double parallelogram linkage are not parallel (Figs. 1c and 1d). Under this mode, the rotation of the inner two rocker links in the double parallelogram linkage is constrained to be the same as that of the outer two rocker links to cause a motion compensation perpendicular to the linear motion direction in the double parallelogram linkage. The motion stage is in the lockup mode if all of the four rocker links in the double parallelogram linkage are kept parallel in a tilted position, which will be detailed in Section 4.

The two operation modes mentioned above can be achieved as long as the lengths of all the links in the RLRBML meet the conditions as indicated in Figs. 1a and 1b where the link parameters of the RLRBML are: L_1, L_2, (L_1>L_2) and L_3. Please note the lengths of the coupler links in the double parallelogram linkage can be of any non-zero value as long as the parallelogram conditions are satisfied and the input-output relation of the RLRBML is not affected. In the linear motion mode, the input and output are $\alpha-\alpha_0$, which denotes the change of the orientation angle of one rocker link of the Robert's linkage with regard to the home position (Fig. 1a), and δ_x, which represents the linear rigid-body motion of the motion stage (Fig. 1c), respectively. The detailed kinematic analysis for the RLRBML will be shown in the next section.

3 KINEMATIC ANALYSIS OF MOTION MODE

The DOF of the novel RLRBML (Fig. 1c) in its motion mode can be easily calculated to be one using the following well-known DOF equation:

$$\text{DOF} = 3 \times (m-1) - n \times (3-1) = 3 \times (10-1) - 13 \times (3-1) = 1 \quad (1)$$

where m=10 is the number of all effective links including the base, and n=13 is the number of R joints as explicitly shown in the equivalent configuration of the RLRBML in Fig. 1d.

As indicated in Fig. 1a, if the pivot joint P is at its home position, let $\alpha=\alpha_0$ and $\beta=\beta_0=0$. Here, α is the orientation angle of one rocker link of the Robert's linkage. β is the orientation angle of the coupler link of the Robert's linkage (herein $\beta \leq 0$ for $\delta_{\mathrm{Rx}} \geq 0$ and $\beta > 0$ for $\delta_{\mathrm{Rx}} < 0$, δ_{Rx} is the straight line displacement of the pivot joint P along the X-axis).

Using the triangle geometry relationship in Fig. 1a and assuming the perfect straight line motion of the Robert's linkage between O_1 and O_2, one can derive

$$\delta_{\mathrm{Rx}} = L_2 - 2L_1 \cos\alpha \quad \text{for } L_2 \geq \delta_{\mathrm{Rx}} \geq -L_2 \tag{2}$$

where $\alpha = \alpha_0 - \beta$ for $\begin{cases} 90^\circ \geq \alpha \geq \alpha_0 \text{ if } L_2 \geq \delta_{\mathrm{Rx}} \geq 0 \\ \alpha_1 \leq \alpha < \alpha_0 \text{ if } -L_2 \leq \delta_{\mathrm{Rx}} < 0 \end{cases}$, $\cos\alpha_0 = \dfrac{L_2}{2L_1}$ and $\cos\alpha_1 = \dfrac{L_2}{L_1}$.

Note that the closer to the home position, the straighter the motion of the pivot joint P is; and the larger the ratio of L_1 to L_2, the better the overall straight line motion of the pivot joint P between O_1 and O_2 is.

In addition, the orientation angle, β, of the coupler link of the Robert's linkage can be obtained as follows:

$$\beta = \alpha_0 - \alpha \tag{3}$$

where $\alpha_0 = \arccos(\dfrac{L_2}{2L_1})$.

For a given rotational angle, γ, of the inner rocker in the double parallelogram linkage in the motion mode, the linear motion (δ_{Px}) of the coupler link 2 along the X-axis in the double parallelogram linkage can be calculated as (Fig. 1c)

$$\delta_{\mathrm{Px}} = 2L_3 \sin(\gamma) \quad \text{for } -2L_3 \leq \delta_{\mathrm{Px}} \leq 2L_3 \tag{4}$$

Therefore, the linear rigid-body motion, δ_{x}, of the motion stage of the RLRBML is obtained as

$$\delta_{\mathrm{x}} = \delta_{\mathrm{Rx}} = \delta_{\mathrm{Px}}$$
$$\Rightarrow \delta_{\mathrm{x}} = L_2 - 2L_1 \cos[(\alpha - \alpha_0) + \alpha_0] = 2L_3 \sin(\gamma) \quad \text{for} \quad -\min(L_{2,} 2L_3) \le \delta_{\mathrm{x}} \le \min(L_{2,} 2L_3) \tag{5}$$

Based on Eq. (5), the range of the orientation angle, α, of the rocker link in the Robert's linkage should then be modified accordingly as

a) Case $2L_3 \ge L_2$:

$$\begin{cases} 90^{\circ} \ge \alpha \ge \arccos(\dfrac{L_2}{2L_1}) \text{ if } L_2 \ge \delta_{\mathrm{x}} \ge 0 \\ \arccos(\dfrac{L_2}{L_1}) \le \alpha < \arccos(\dfrac{L_2}{2L_1}) \text{ if } -L_2 \le \delta_{\mathrm{x}} < 0 \end{cases} \tag{6a}$$

b) Case $2L_3 < L_2$:

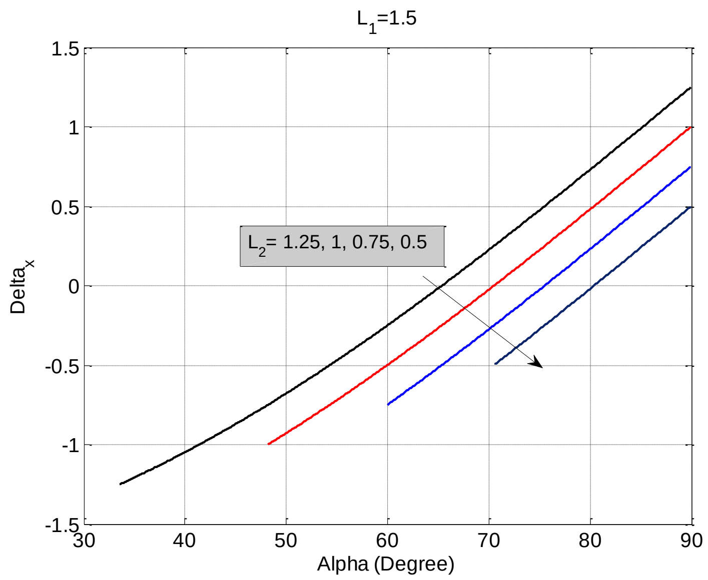
$$\begin{cases} \arccos(\dfrac{L_2 - 2L_3}{2L_1}) \ge \alpha \ge \arccos(\dfrac{L_2}{2L_1}) \text{ if } 2L_3 \ge \delta_{\mathrm{x}} \ge 0 \\ \arccos(\dfrac{L_2 + 2L_3}{2L_1}) \le \alpha < \arccos(\dfrac{L_2}{2L_1}) \text{ if } -2L_3 \le \delta_{\mathrm{x}} < 0 \end{cases} \tag{6b}$$

From the above analysis, it can be concluded that the link lengths, L_2 and L_3, contributes to workspace of the RLRNML.

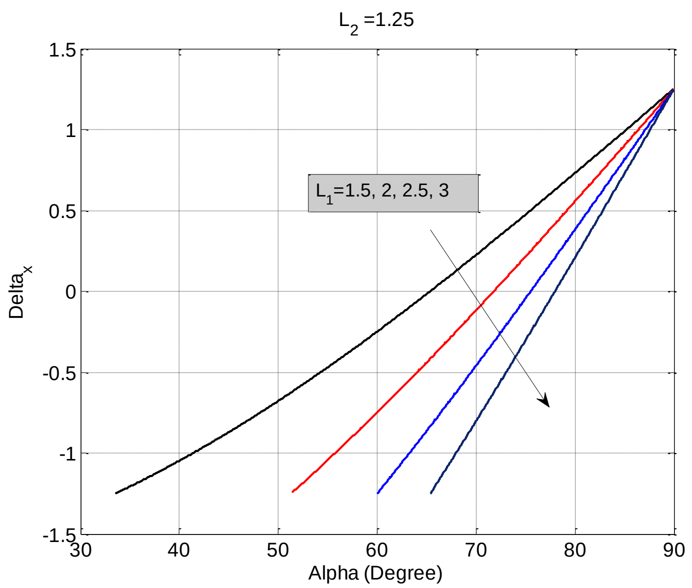
Using Eq. (5), the rotational angle of the inner rocker link in the double parallelogram linkage under the linear motion mode (Fig. 1c) can be derived as

$$\gamma = \arcsin(\frac{\delta_{\mathrm{x}}}{2L_3}) = \arcsin(\frac{L_2 - 2L_1 \cos\alpha}{2L_3}) \tag{7}$$

The relation between δ_{x} and α under $2L_3 \ge L_2$ is also plotted in Fig. 2, which shows how the link lengths, L_1 and L_2, influence the performance of the RLRBML including the changing trends and workspace (Fig. 2a). The change of L_2 plays a very small role in the plotting trends but that of L_1 significantly affects the trends.



(a) Changing L_2



(b) Changing L_1

Figure 2. The relation between δ_x (Delta$_x$) and α (Alpha) under $2L_3 \geq L_2$

4 LOCKUP MODE ANALYSIS

If the four rocker links in the double parallelogram are all parallel to the horizontal direction (Y-axis), which is perpendicular to the direction of linear rigid-body motion of the motion stage, and the middle motion stage is just at its home position, singularity of the RLRBML occurs as shown in Fig. 3. In the singular configuration, the motion stage can undergo an infinitesimal motion if the outer rocker link in the double parallelogram linkage is locked, and the double parallelogram linkage undertakes a self-motion with all rocker links always parallel to each other if the rocker link in the Robert's linkage is locked.

When all of the four parallel rocker links in the double parallelogram linkage are moved to a tilted position (not parallel to the Y-axis) (Fig. 4a), then the middle motion stage is in the lockup mode. In this lockup mode, the four rocker links still undergoes a 1-DOF rotational motion determined by a rotational angle θ (Fig. 4a), while the Robert' linkage is converted to a structure. If one exerts a force on the motion stage, neither the Robert's linkage nor the double parallelogram linkage moves. Note that the closer $\sin\theta$ is to 1, the better the lockup effectiveness is.

The lockup mode can be verified theoretically. If there is an input force on the motion stage, during a very short time interval the motion stage must not be able to move due to the inherent constraints along the horizontal direction (Y-axis) and the direction parallel to the four rocker links in the double parallelogram linkage. Therefore, the input energy (input work) is zero due to no travel distance for the input force. And then, if the motion stage can move during the short time interval, there must be a non-zero output kinetic energy, which is conflict with the zero input work.

In this lockup mode, the double parallelogram linkage is equivalent to its variation with four parallel rocker links plus additional constrained link(s) (Fig. 4b), which may help us better understand the lockup mode. One coupler link (coupler link 2) in the variation cannot move due to the motion compensation between the inner parallelogram linkage and the outer parallelogram linkage.

Based on the above analysis, two motors can be used to fulfil the function of two operation modes of the RLRBML. One motor is employed to drive a rocker link in the Robert's linkage for achieving the linear rigid-body motion, and another is adopted to control a rocker link in the double parallelogram linkage for producing the lockup without disassembly and external intervention. It is noted that the double parallelogram linkage has the locking function when used along with a straight line motion linkage.

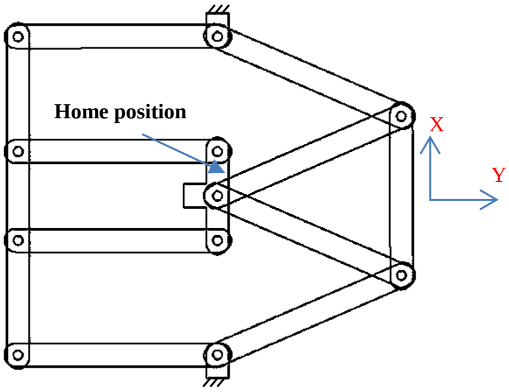
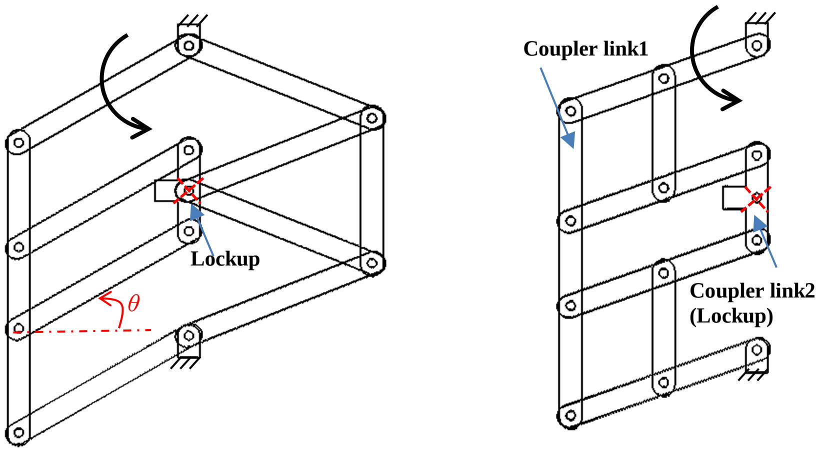


Figure 3. Singular configuration of the RLRBML



(a) Lockup mode of RLRBML

(b) Equivalent variation of double parallelogram linkage

Figure 4. Lockup mode of the RLRBML

A prototype, made of engineering plastics using a 3-D printer, was developed to show the two operation modes (Fig. 5), which has validated the function of the RLRBML. As shown in Fig. 5c, the motion stage was exerted a force but no motion was produced in the lockup mode.

(a) Motion mode (b) Singularity position (c) Lockup mode

Figure 5. Prototype of the RLRBML

5 CONFIGURATION VARIATIONS

Depending on the application purposes, the above RLRBML can be inverted by exchanging the base and the motion stage (Fig. 6) without changing the function of the mechanism. A more compact configuration (Fig. 7) for the proposed RLRBML is further conceived by an appropriate embedded arrangement.

A single-mode linear rigid-body motion linkage (Fig. 8) excluding the lockup mode can also be obtained by changing the length of the inner rocker links in one parallelogram linkage (i.e. the length difference, Δ, between the inner and the outer rocker links is not equal to 0). In addition, one can construct a single-mode linear rigid-body motion linkage (Fig. 9) by arranging two Robert’s linkages (or other types straight line motion linkages) in parallel and making the motion direction of the two pivot joints P_1 and P_2 parallel to the line connecting the two pivot joints.

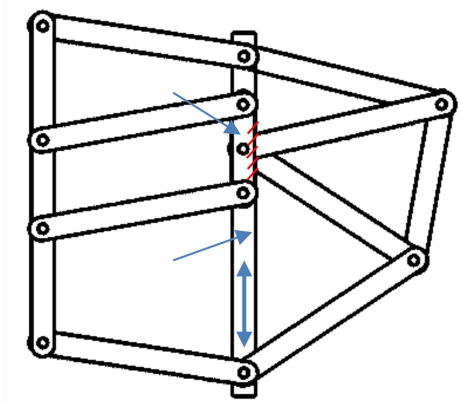


Figure 6. Inverted RLRBML

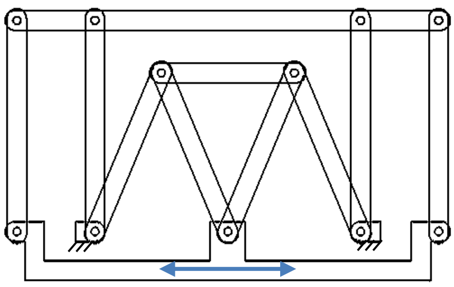
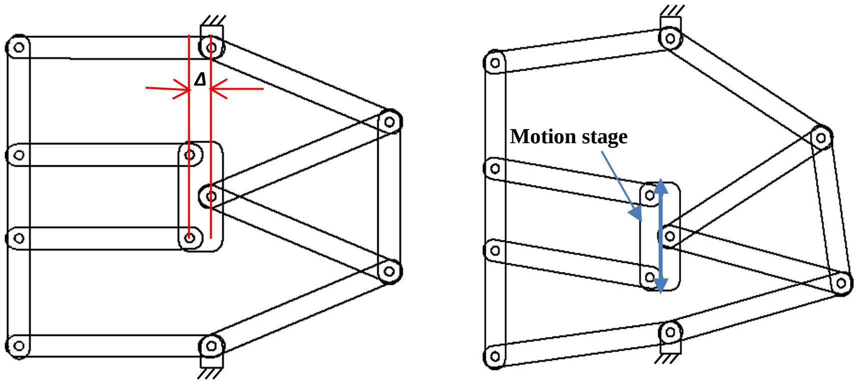


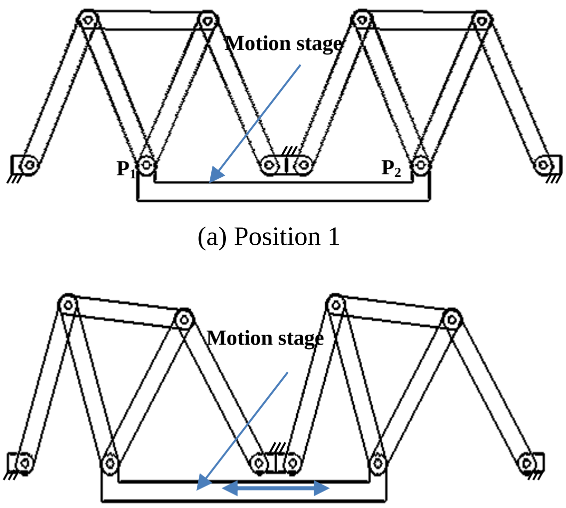
Figure 7. Compact RLRBML

It is noted that the proposed compact RLRBML (Fig. 7), the single-mode linear rigid-body motion linkage 2 (Fig. 9) and/or their combinations may be good candidates of pseudo-rigid-body models of large-displacement and high off-axis stiffness compliant translational joints over the whole motion range. The reason is that the Robert's mechanism based compliant translational joints (also referred to XBob in [13]) can produce large off-axis transverse stiffness, and the double parallelogram mechanism based compliant translational joints (also referred to folded-beam mechanism in [14]) can lend large off-axis torsional stiffness [15].



(a) Position 1 (b) Position 2

Figure 8. Single-mode linear rigid-body motion linkage 1



(a) Position 1

(b) Position 2

Figure 9. Single-mode linear rigid-body motion linkage 2

In fact, further RLRBMLs can be obtained from the above RLRBML by replacing the Robert's linkage with any other straight line motion linkage, such as Watt's linkage.

6 CONCLUSIONS

A RLRBML with two operation modes has been proposed and analysed in this paper. The variations of the RLRBML, including a compact RLRBML and two single-mode linear rigid-body motion linkages, have been presented.

In comparison with the existing linear rigid-body motion mechanisms, the proposed RLRBML is able to implement two operation modes for the motion stage: a) a linear rigid-body motion mode, and b) a lockup mode. In the lockup mode, the motion stage of the RLRBML is prohibited from moving even under power off, and the double parallelogram linkage is, however, moveable for its own rotation application.

The present RLRBML is a low-cost design due to its simple planar configuration and the use of only R joints which provide an alternative to some high-cost kinematic joints such as prismatic joints. In addition, there are extensive applications for obtaining linear rigid-body motion stemming from a rotation motion, generating a rotational motion from a linear motion input, or acting as the novel pseudo-rigid-body models of compliant translational joints.

Despite these findings, dynamic analysis and application of the RLRBML deserve further investigation in future. Other open issues may include the design and analysis of large-displacement compliant translational joints based on the proposed RLRBMLs or their combinations as the pseudo-rigid-body models.

ACKNOWLEDGEMENT

The authors would like to acknowledge Mr. Timothy Power and Mr. Michael O’Shea at the School of Engineering, University College Cork, Ireland, for their invaluable assistance in the development of the prototype.

NOTATION

L_1	Rocker link length in the Robert's linkage
L_2	Coupler link length in the Robert's linkage
L_3	Rocker link length in the double parallelogram linkage
m	The number of all effective links including the base
n	The number of R joints
α	Orientation angle of one rocker link of the Robert’s linkage
β	Orientation angle of the coupler link of the Robert’s linkage
γ	Rotational angle of the inner rocker link in the double parallelogram linkage under the motion mode
θ	Rotational angle of any one rocker link in the double parallelogram linkage under the lockup mode
δ_{x}	Linear rigid-body motion of the motion stage of the RLRBML
δ_{Px}	Linear motion of the coupler link 2 along the X-axis in the double parallelogram linkage
δ_{Rx}	Straight line displacement of the pivot joint P along the X-axis in the Robert's linkage

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