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A Jaworskian analysis of four senior class primary teachers endeavouring to teach mathematics from a constructivist-compatible perspective.

By

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PhD Thesis

National University of Ireland, Cork

School of Education

September 2015

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Declaration

I hereby declare that this thesis is my own work and has not been submitted for another degree either at University College Cork or elsewhere.

_____

Joseph McCarthy

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Abstract

Title:

A Jaworskian analysis of four senior class primary teachers endeavouring to teach mathematics from a constructivist-compatible perspective

Author:

Joseph McCarthy

A constructivist philosophy underlies the Irish primary mathematics curriculum. As constructivism is a theory of learning its implications for teaching need to be addressed. This study explores the experiences of four senior class primary teachers as they endeavour to teach mathematics from a constructivist-compatible perspective with primary school children in Ireland over a school-year period. Such a perspective implies that children should take ownership of their learning while working in groups on tasks which challenge them at their zone of proximal development. The key question on which the research is based is: to what extent will an exposure to constructivism and its implications for the classroom impact on teaching practices within the senior primary mathematics classroom in both the short and longer term? Although several perspectives on constructivism have evolved (von Glaserfeld (1995), Cobb and Yackel (1996), Ernest (1991,1998)), it is the synthesis of the emergent perspective which becomes pivotal to the Irish primary mathematics curriculum.

Tracking the development of four primary teachers in a professional learning initiative involving constructivist-compatible approaches necessitated the use of Borko's (2004) Phase 1 research methodology to account for the evolution in teachers' understanding of constructivism. Teachers' and pupils' viewpoints were recorded using both audio and video technology. Teachers were interviewed at the beginning and end of the project and also one year on to ascertain how their views had evolved. Pupils were interviewed at the end of the project only. The data were analysed from a Jaworskian perspective i.e. using the categories of her Teaching Triad of management of learning, mathematical challenge and sensitivity to students. Management of learning concerns how the teacher organises her classroom to maximise learning opportunities for pupils. Mathematical challenge is reminiscent of the Vygotskian (1978) construct of the zone of proximal development. Sensitivity to students involves a consciousness on the part of the teacher as to how pupils are progressing with a mathematical task and whether or not to intervene to scaffold their learning. Through this analysis a synthesis of the teachers' interpretations of constructivist philosophy with concomitant implications for theory, policy and practice emerges. The study identifies strategies for teachers wishing to adopt a constructivist-compatible approach to their work. Like O'Shea (2009) it also highlights the likely difficulties to be experienced by such teachers as they move from utilising teacher-dominated methods of teaching mathematics to ones in which pupils have more ownership over their learning.

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Chapter 1: Introduction: thesis overview

1.1 Framing the study

This study revolves around the school settings of four primary teachers as they attempt to grapple with the theory of constructivism in their teaching of mathematics over a one-year period. The study takes place in two settings as two of the teachers teach in a disadvantaged suburb of Cork city while the other two teach in a predominantly middle-class suburb of the same city. The teachers taught either 5th or 6th class. The study is primarily concerned with the school world of the participants. It is sociocultural in that it recognises that schools are not vacuous spaces and that they are heavily influenced by wider societal factors. The study operates within the constructivist paradigm as, in ontological terms, it recognises the multiple, socially constructed realities of the participants (Mertens, 2005). In epistemological terms, the study acknowledges the interactive link between researcher and participants. It seeks to explicate the values of the researcher and participants who create findings together. There is an irony in that the theory of constructivism is put under scrutiny while the researcher operates within the constructivist paradigm. Initially, the study seeks to tease out what is meant by constructivism as a theory of learning and discuss its implications for classroom teaching in mathematics. As a result the main research question can be stated as follows:

- To what extent will an exposure to constructivist-compatible pedagogies impact on teaching practices within the senior primary mathematics classroom in both the short and longer term?

I will elaborate on this question and refine it as the thesis evolves.

1.2 Motivation and rationale for the study

The motivation for this study comes from my background as an in-service provider to teachers of primary mathematics over many years. From 1993 to 1998 I provided summer in-service courses to primary teachers under the auspices of the Irish National Teachers Organisation (INTO). In 1999 the primary curriculum in Ireland was revised and from 2001-2003 I worked as a tutor on secondment with the Primary Curriculum Support Programme (PCSP). The first year of this work involved delivering in-service to teachers in lecture style format, while at the same time exposing them to 'hands-on' activities in mathematics. In the second year I travelled to schools with the purpose of advising teachers on how to implement the revised curriculum. However, most schools just wanted advice on how to rewrite their school plan in mathematics, with a minority seeking advice on how to implement the revised methodologies of 'active learning', 'collaborative learning', 'using the environment' and 'problem solving'. The revised curriculum was meant to reemphasise the constructivist philosophy and I found myself querying whether these methodologies equated to a constructivist-compatible approach. From 2003- 2007 I worked as a lecturer in the area of mathematics education on secondment to Mary Immaculate College, Limerick. This work served to heighten my awareness and interest in constructivism as a philosophy of learning mathematics. I found myself remembering the first book I ever seriously read in mathematics education, namely The Psychology of Learning Mathematics (1971) by Richard R. Skemp. Although Skemp dealt with the theory of constructivism, an irksome question remained: How does constructivism manifest itself, if at all, in the Irish primary mathematics classroom? The answer to this question was not readily obvious to me but a seed had been sown which was to grow into a major study on the topic in the form of this thesis. In 2007 I returned to my role as principal of a DEIS (Delivering Equality of

Opportunity in Schools) school on the north side of Cork City. I now had responsibility for the teaching and learning of mathematics, and other subjects, in my school. A constructivist flame had been lit for me throughout my professional career and I decided that the best way to keep the flame burning was to enrol for further study in a PhD programme, which I did in 2008.

In this research I hope to design a supportive intervention, which will assist teachers in following a constructivist approach to their work. In other words, I hope to enable teachers move away from a solely teacher-led didactic approach. Schifter and Fosnot (1993) give a rationale for such a move away from traditional instruction when they state that "the conventional topic-by-topic, drill-and-practice pattern of mathematics instruction subverts student understanding of the principles that underlie mathematical order" (p. 340). My concern is teachers' professional development in mathematics and how to assist in the design of an initiative to support such development.

1.3 Significance of the study

Very few studies at doctoral level have been dedicated in their entirety to the study of constructivism in Irish primary mathematics classrooms. To date, I am aware of only one. That study was completed in 2009 by John O'Shea and was entitled 'Endeavouring to teach mathematical problem solving from a constructivist perspective: The experiences of primary teachers'. O'Shea looked at problem solving but in this thesis I take a broader look at what constitutes mathematics teaching and learning in Irish primary mathematics classrooms. For instance, I also pay attention to the norms or routines (Cobb and Yackel, 1996) which teachers establish in their classroom prior to the teaching of problem solving.

Constructivism is a theory which is vague on specifics. Although several authors (Brooks & Brooks, 1992; Jaworski, 1994; Brophy, 1996; Gagnon Jr. & Collay, 2001) attempt to tease out what a constructivist classroom involves, there still exists a need to explicate the implied practices for an Irish context. This thesis seeks to fulfil that need. Any findings will have implications for the theory itself, mathematics education policy and classroom practice.

It could be argued that constructivism is a theory of learning and that it is futile, therefore, to investigate it from a teaching point of view. Schoenfeld (2006) takes that stance but in this thesis I argue that teaching and learning are two sides of the same coin; if teaching methods can be improved then pupils' learning can be enhanced also. This is my rationale for exploring constructivist-compatible teaching. Such teaching seeks to engage children in challenging tasks that require the kind of mathematical thinking which moves beyond the merely procedural aspects of mathematics.

1.4 Thesis outline

This thesis follows a straightforward approach in terms of its outline and structure. The introduction serves to provide a theoretical and motivational context for the study. In chapter 2 I dissect the theory of constructivism in more detail. I situate my research in the zone between the individual and social aspects of learning and describe the various constructivist 'sects' which abound; radical, social and socio-constructivist and eventually, the emergent perspective. I acknowledge that the use of such terms is widely contested. I discuss whether constructivism excludes direct instruction. I draw brief attention to RME on account of its relevance to constructivism. Finally, I define what is meant by teaching to "big ideas" and

suggest such teaching as a way of helping pupils to make connections, thereby constructing their own knowledge.

This is followed by chapter 3 where I delve into current approaches to teaching problem solving internationally and in Ireland in particular. I consider the merits of the mathematization approach of the Dutch Realistic Mathematics Education movement. Closer to home I compare the problem solving approaches outlined in the 1971 and 1999 curriculum documents. I critique the merits of a problem solving approach to mathematics and discuss how such an approach might be assessed by bringing the Assessment for Learning initiative into focus. Finally, I discuss how to implement constructivist pedagogies. The contribution of several authors to such implementation is considered. These authors include Brophy (2006), Gagnon Jr. and Collay (2001), Jaworski (1996b), Simon and Schifter (1991) and Fosnot (2005).

In chapter 4 I discuss the difficulties attached to researching constructivist theory, which does not acknowledge the existence of an objective reality outside the mind of the learner. Yet, the question arises as to how one moves beyond the purely subjective. I outline my own research trajectory by reflecting on various research methodologies such as lesson study and design research. The relevance of such methodologies to this research is outlined. Having reflected on such methodologies, I opt for Borko's Phase 1 research design as the way forward. I describe the concomitant data collection methods and data analysis procedures. Ethical issues such as validity, reliability and generalisability are also discussed.

In chapter 5 I give a detailed account of the sixteen lessons I observed. Each of the four teacher participants taught four lessons for me. These lessons were videotaped.

In each of the lessons my data analysis involves applying a constructivist lens, meaning that I link aspects of the lessons to constructivist theory. How teachers ensure pupils' progression in lessons, pupils gaining ownership of their learning and the complexity of tasks are several of the issues which emerge. As part of my data analysis, I apply a rating system to the tasks to ascertain their complexity.

In chapter 6 I continue with the analysis of other sources of data collected. These sources include questionnaires and interviews with the participant teachers, which were conducted at the beginning and end of the project. A cross-section of children was also interviewed for their views on the project. As themes emerge I draw on Jaworski's (1996a) Teaching Triad as the analytical tool to categorise them. I also hypothesise by suggesting a fourth element to Jaworski's Triad to convert it into a Quadriad.

Chapter 7 is the concluding chapter. In this chapter I draw on the evidence of previous chapters to derive conclusions. I discuss the implications of the research for policy, practice and theory development. I also make suggestions for further research.

1.5 Area of study

Piaget and Vygotsky could be called the fathers of modern constructivism but neither has been too explicit on the implications of the theory for the classroom. If constructivism is to remain an active theory its consequences for the classroom need to be elucidated. It is in the battleground between theory and practice that this study takes place. The study takes the form of an intervention in four classrooms to promote constructivist-compatible approaches among teachers. The researcher

recognises that teachers work under constraints of time, class size and limited resources, while trying to teach a prescribed curriculum. However, this is all the more reason to investigate the viability of a theory like constructivism for the Irish primary classroom; otherwise the theory could become inert and destined to be confined to textbooks on teacher education courses.

Chapter 2: An exploration of constructivist teaching

2.1 Introduction

I commence this chapter by situating my research in the zone of learning which Elwood (2008) perceives as ranging from individual to social activity. I proceed to justify an investigation of constructivist teaching. This requires an analysis of what constructivism entails for classroom practice. I look at research on the didactic triangle or the interplay among teacher, student and mathematics. I distinguish between active learning and constructivism and argue that equating activity with learning is a misconception of constructivist approaches. I also look at the debate about whether or not constructivism excludes direct instruction. A case is made for telling pupils information at certain points in their learning. As an advocate of the RME movement, I examine it for its theoretical links to constructivism. In turn, the radical and social sects of constructivism are examined for their applicability to classroom learning with a compromise between the two, namely the emergent perspective, suggested as a possible way forward. The issue then becomes one of how best to organise classroom learning. I suggest teaching to ‘‘big ideas’’ and what this entails is both defined and explored.

2.2 Situating my research from a theoretical perspective

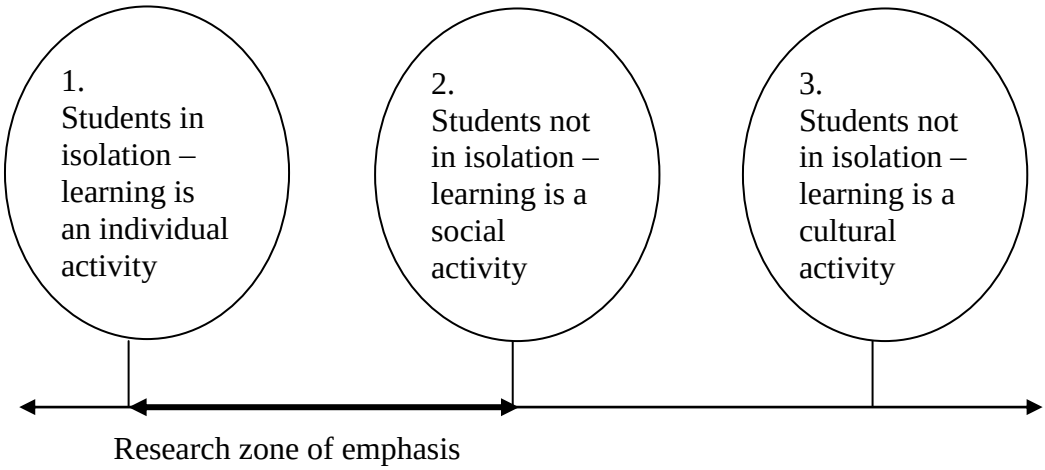


Figure 1: Theoretical models of learning (Source: Elwood 2008)

Figure 1 above is used by Elwood (2008) to illustrate that assessment and testing on individuals can be viewed as isolated activity, as social activity and as cultural activity. It can also be used to show differences in the main theories of learning. I draw on Elwood (2008) as her analysis adds clarity and definition in relation to language theory and her analysis grapples successfully with the boundaries of constructivism, social constructivism and sociocultural theory in a way that helps me to distinguish and highlight the theoretical focus of my study framework. In stage 1 on the continuum the learner is viewed as possessing knowledge which can be easily measured through external testing. This view of learning is in line with behaviourist psychology which emphasises a stimulus-response theory of learning. Here, a test item becomes the stimulus and the answer is the response. National and international assessments, such as TIMSS and PISA favour this type of approach as it allows for observable comparisons across countries. Elwood (2008) describes this stage as encompassing a local model of mind; in other words, mind inside the head and intrinsic to the learner. In stage 2 on the continuum the view of learning shifts to encompass Vygotsky's (1978) stance that human learning presupposes a specific social nature and a process by which children grow into the intellectual life of those

around them. This view of learning falls within social constructivist theories of learning whereby students learn by actively making sense of new knowledge, deriving meaning from it and fitting it with their existing knowledge maps or schema. Although social constructivists contend that learning is a social activity and that learners construct their own meaning, a symbolic view of cognition still prevails and mind is still located 'in the head' (Cobb, 1999, p. 135). Therefore, learning and meaning are co-constructed but eventually this learning gets placed back within the individual. Moreover, summative or formative assessments (the latter favoured by social constructivists) are still measuring something that is the property of the individual. Learning is still the internalization of external knowledge, and what the student can do alone having learned through social interaction (Vygotsky, 1978). Stage 3 on the continuum places learning, mind and assessment as constructs that are culturally generated and mediated. What is acknowledged is the essential relationship and interaction between learning; the assessment of that learning; the social, cultural and historical lives of teachers and students; and the economic and political contexts in which assessment functions. Teachers and students bring social, cultural and historical experiences to assessment situations and Elwood (2008) contends that to better comprehend students' performances on assessment we need to look into students' histories, into their *forms of life* (McGinn, 1997) and not into their heads. She states that it is by looking into their forms of life that we can start to understand their learning and why they respond to tasks in different ways.

The model of learning on display here brings into focus the socio-cultural perspective on learning. The socio-cultural view of learning is one that emphasises the socially constituted nature of individuals, i.e. that they cannot be considered in isolation from their social and historical contexts. In this view of learning Cobb (1999) states that

mind is viewed as situated between individuals *in social action*. Wertsch (1991) elucidates on a socio-cultural approach to mind as being one that gives an account of human mental processes which recognises the essential relationship between these processes and their institutional, cultural and historical settings. Hence, Elwood (2008) comments that mind is not local to the individual but situated in the cultural setting and within cultural relationships, and resides between individuals' interactions and reactions. Furthermore, a non-local view of mind suggests that the learner and teacher are entangled, and that learning is the product of the relationship between the teacher, the student and the assessment task.

Cobb and Yackel (1996) ambitiously attempt to reconcile the varying views of where the mind is located. They suggest that the sociocultural perspective gives rise to theories of the conditions for the possibility of learning, whereas theories emanating from the constructivist perspective focus on both what students learn and the processes by which they do so. They elaborate that constructivists might argue that sociocultural theories do not adequately account for the process of learning, whereas sociocultural theorists might reply that constructivist theories fail to account for the (re)production of the practices of schooling and the social order. Ball (1993) goes further in stating that the tension in teaching between individual construction and enculturation into wider social mathematical practices cannot be resolved once and for all. There is also difficulty in the language we use to describe theoretical standpoints. For instance, the Mathematics Teacher Guidelines (1999) describe constructivism as a sociocultural theory. However, the language of educational theory has changed so much since the 1990s that we have reached a stage whereby constructivism could be viewed as dealing with an individual's learning and not seen as sociocultural at all by some theorists due to the perceived lack of emphasis on

social dimensions of learning. For instance, Lave and Wenger (1991) attempt to avoid any reference to mind in the head, preferring to state that a learning curriculum unfolds in opportunities for engagement in practice. In this research I intend to take a pragmatic approach and view learning as fitting into the continuum as outlined by Elwood (2008) above. I use the phrase 'research zone of emphasis' to situate my research in the segment between stages 1 and 2. My emphasis is on teachers' and pupils' learning in the microcosm of the classroom while acknowledging the constraints imposed by wider societal forces. However, such forces are not the main focus of this research. In modern educational parlance, my inquiry could be deemed to be cognitive constructivist rather than sociocultural as I focus on individuals' interpretation of events within the classroom. However, interpretations of such terms continue to be a source of ongoing debate.

2.3 A rationale for investigating constructivist teaching

Not every author in mathematics education is an ardent fan of the application of constructivist theory to teaching. Schoenfeld (2006) is quite pessimistic about the research prospects for constructivist teaching. Indeed, he captures the breadth of interpretations of constructivist teaching when he states that it can mean "anything from diagnostic and prescriptive instruction to pretty much anything goes" (Schonfeld 2006, p. 201). He elaborates by remarking that constructivist teaching, despite its faddishness, was an oxymoron to begin with; and once an oxymoron, always an oxymoron. Presumably, Schoenfeld prefers to focus on constructivism as being limited to a theory of learning. He elucidates by describing what learning to become competent in a domain includes:

> The development of a knowledge base; the ability to employ domain-specific and general problem solving strategies; the development of productive metacognitive behaviours such as monitoring and self-regulation; the development of one's own

> identity, including membership in various communities of practice; the development of productive beliefs and dispositions; the ability to participate productively in the practices of the domain, including discourse practices.
>
> (Schoenfeld 2006, p. 201).

Comprehensive and all as this definition of learning is, it fails to answer one vital question: what is the teacher's role in all of this? For me, Schoenfeld has ignored the interdependence and interplay which occurs between teaching and learning. I prefer to take the view that one cannot describe learning without referring to the resultant implications for teaching. Otherwise, learning will be seen as the sole responsibility of the learner herself, even if appearing to work within a community of practice as outlined above. Such a view is somewhat isolationist. In some languages the word for teaching and learning is the same. In Dutch, for instance, the distinction between teaching and learning is made only by the use of a different preposition. The verb is exactly the same. *Leren aan* means teaching; *leren van* means learning. Fosnot and Dolk (2005) remark that when teaching and learning are so closely related, they should be integrated in learning/teaching frameworks. As a result, teaching should be seen as closely related to learning, not only in thought and language but also in action. "If learning doesn't happen, there has been no teaching. The actions of teaching and learning are inseparable" (Fosnot and Dolk 2005, p. 175).

Another justification for investigating constructivism from a teaching point of view comes from the guidance given to teachers in the revised mathematics guidelines published in 1999. The guidelines recommend the use of scaffolding as one form of instruction. This is where the teacher modifies the amount of support according to the needs of the child. Initially, the teacher may expose the child to various possible

methods of approaching a problem. Thereafter, "the teacher breaks down the task and makes the task manageable for the individual child, thus supporting the development of the child's own problem-solving skills" (NCCA 1999, p. 4). The guidelines also suggest that it is through social interaction that children can begin to appreciate the viewpoints of other people. The guidelines explicitly state that "sociocultural theory sees cognitive development as a product of social interaction between partners who solve problems together" (NCCA 1999, p. 4). Therefore, the role of significant others, such as peers, parents and teachers, in no particular hierarchy, in influencing children's learning is acknowledged. The terms used in the guidelines such as 'scaffolding', 'social interaction' and 'solving problems together' imply that the underlying philosophy of the mathematics curriculum is social constructivism, although the term itself is never used. It makes sense to tease out and explore the implications of such a philosophy for classroom teachers. Otherwise, terms like 'scaffolding', 'social interaction' and 'solving problems together' will remain as slogans but with no real, practical meaning for teachers.

My rationale, therefore, in exploring constructivism is to shed some light on a theory, which has been designed for learning but has not been given sufficient attention with regards to its implications for teaching. I view teaching and learning as being two sides of the one coin. "You can't have one without the other," says the old Sammy Cahn song. The Dutch seem to take the same view. This research hopes to explore constructivism and its implications but from the teacher's perspective. If constructivism is indeed a theory of learning it seems reasonable to explore the pedagogical knowledge, which teachers need to learn, if they are to adopt constructivist-compatible pedagogies in the classroom.

2.4 Research on the didactic triangle by Jaworski and other authors

A focus on constructivist-compatible pedagogies entails a survey of the interplay between teacher, pupil and the subject of mathematics. This interplay is known as the didactic triangle. In a special edition of the ZDM Mathematics Education Journal (Volume 44, 2012) several authors, including Jaworski, combined to share their research on the didactic triangle. As a model the didactic triangle linking mathematics, teachers and students has been used by researchers wishing to consider teaching-learning interactions in mathematics classrooms. The triangle originated in the work of Chevallard (1985) and later with Brosseau (1997). While acknowledging the seminal importance of the triangle as a way of describing the teaching and learning situation in the subject of mathematics, more recent research has focused on the inadequacies of the triangle as a means of capturing the complexity of what happens in mathematics classrooms. For instance, Chevallard expands on the original triangle by placing it within a circle called the 'noosphère'. The noosphère is defined as the bureaucratic universe that shapes schooling, which influences what happens in classrooms. The noosphère in turn is placed within a rectangle called the 'environnement' which reflects the cultural and contextual factors that result in the transformation of mathematics as practised to mathematics as taught (Schoenfeld, 2012). An example from Ireland could be the 'high stakes' Leaving Certificate examination which determines what is taught in classrooms, how students view mathematics and, depending on their grades, who goes to university.

Brousseau labels the formal school environment the *système didactique*. Within this school environment he wishes to create and study Didactical Situations that assist student engagement with rich mathematics. These Situations have a number of properties. "They are intended to be mathematically and pedagogically rich, so that

by engaging in them students will develop deep understandings of the mathematics" (Schoenfeld, 2012, p.589). The emphasis is not on the teacher 'telling' the pupils the required information but on the students being asked to do a task in which their engagement with the mathematics to be learned is central. The creation of such situations would also be an aim of my own research. Therefore, what is envisaged is a task which requires some element of investigation by the pupils and not just the performance of routine algorithms.

The 'didactical contract' is another core component of '*Didactique*'. This is the classroom version of the 'social contract' which is the set of largely covert rules that govern the interactions of students and teacher. This is akin to the term 'sociomathematical norms', as promulgated by Cobb and Yackel (1996) and discussed elsewhere in this chapter. Schoenfeld (2012) gives the example of a classroom being either focused on 'answer getting' or alternatively on mathematical 'sense-making'. If the former is emphasised students will obtain answers to given tasks and see no need to explore the mathematics further. If the latter is emphasised students will seek out underlying mathematical reasons as to why things operate the way they do, and they will feel compelled to explain their understandings. In this way a sociomathematical norm or contract is established in the classroom as to what constitutes an appropriate mathematical explanation. If the emphasis is on obtaining correct answers to set written tasks then the authority of the textbook may suffice. If the emphasis is on explanation, the classroom community will require students to give clear and unified mathematical justifications. Through their classroom experiences students not only come to understand the rules of the game but also to shape such rules. In turn, the rules of the game shape not only students' actions but also their beliefs about the nature of mathematics. Schoenfeld (2012) comments that

a central part of *Didactique* is a concern for the nature of the didactical contract in any classroom, and the creation of Situations that are favourable to the higher levels of mathematical engagement and learning. Schoenfeld (2012, p. 592) cites Brosseau (1997) in stating that any mathematical knowledge to be attained should be in its full richness –"not merely a statement of a mathematical concept, but its meaning, its uses, its connections to prior knowledge, the context in which it is likely to be encountered, the language commonly used to express it." Schoenfeld contends that French authors like Chevallard and Brosseau have more of a cultural construal of the didactical triangle than has typically been the case in the English-speaking world. However, several authors have sought to remedy this situation.

For instance, Rezat and Sträβer (2012) argue for the addition of a fourth vertex to the didactic triangle to turn it into a tetrahedron. The fourth vertex would concern the use of tools or artefacts in the teaching of mathematics. The authors contend that mathematics content is dependent on artefacts (embodiments) to assist the teaching/learning process. Such tools could be physical like mathematical textbooks, rulers, compasses, log tables and, of course, digital technologies. However, tools could also be non-physical such as, for instance, language, gestures and diagrams. Vygotsky is quoted by Rieber and Wollock (1997, p.85) as he introduces another element of sophistication when he distinguishes between psychological and technical tools:

> The most essential feature distinguishing the psychological tool from the technical one is that it is meant to act between mind and behaviour, whereas the technical tool, which is also inserted as a middle term between the activity of man and the external object, is meant to cause changes in the object itself. The psychological tool changes nothing in the object.

Rezat and Sträβer (2012) comment that the central aim of tool use in didactical situations is to change the students' cognition of mathematics and not the mathematics itself and that, therefore, all tools used in the teaching and learning of mathematics, be they physical or not, can be considered psychological tools. According to Solomon et al. (2006) students use the tools and artefacts of culture to assist their conceptual development and express themselves more meaningfully. Since the notion of tools is easily tainted with the idea of something material Rezat and Sträβer (2012) prefer the broader notion of artefacts. Bringing in the fourth dimension of 'artefact' (artifact in American usage) means that the didactic triangle becomes a tetrahedron as follows in Figure 2 below:

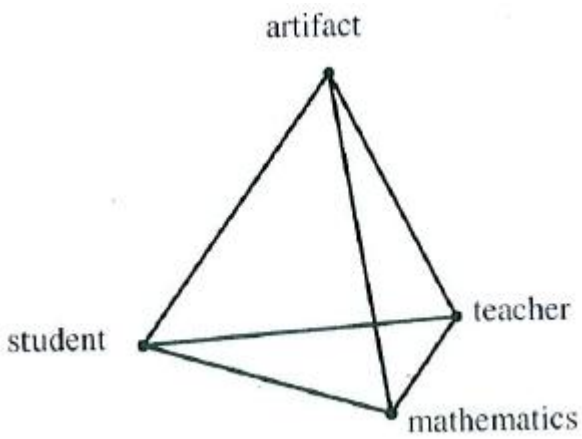


Figure 2: Tetrahedron model of the didactical situation

Straβer (2009) points out that it may be worthwhile to think of what I will term 'spheres of influence' surrounding the tetrahedron. An example would be the sphere containing the personnel and institutions interested in the teaching and learning of mathematics; the 'noosphere' to quote Chevallard's (1985) term. More recently, Geiger (2014) speaks of 'spheres of social context' (SSC) which are inspired by Chevallard's 'noosphere' but they differ in that they are peculiar to the types of interaction that take place in individual, small group and whole group settings. Geiger (2014) contends that social interactions, in harmony with available secondary artefacts, influence the transformation of students' understanding of mathematical

knowledge, as well as their ways of reasoning and sense making, in different ways according to the particular social setting in which learning is located. Therefore, his SSCs are not concentric and independent entities but rather that SSCs interact.

Rezat and Sträβer (2012) comment that Geiger's extension of the tetrahedron model draws definite attention to social settings in the classroom and their effect on instrumented learning but that it does not include societal and institutional influences. To overcome this shortcoming of the model they draw on Engestrom's (1998) model of the activity system from the perspective of cultural-historical-activity theory. Engestrom sees activity as a collective systemic formation that has a complex meditational structure. The attraction for Rezat and Sträβer is that less visible social mediators of activity- rules, community and division of labour- are depicted at the bottom of the model. They contend that artefacts play a crucial role in the system because they serve to focalise the other aspects of the entire system. In turn, they derive a sophisticated 'socio-didactical tetrahedron' which is depicted in Figure 3 which follows on the next page. It can be seen that the 'socio-didactical tetrahedron' is a sophisticated three-dimensional representation and expansion of the original two-dimensional model (teacher-student-mathematics) as outlined by Jaworski (2012) at the beginning of this section.

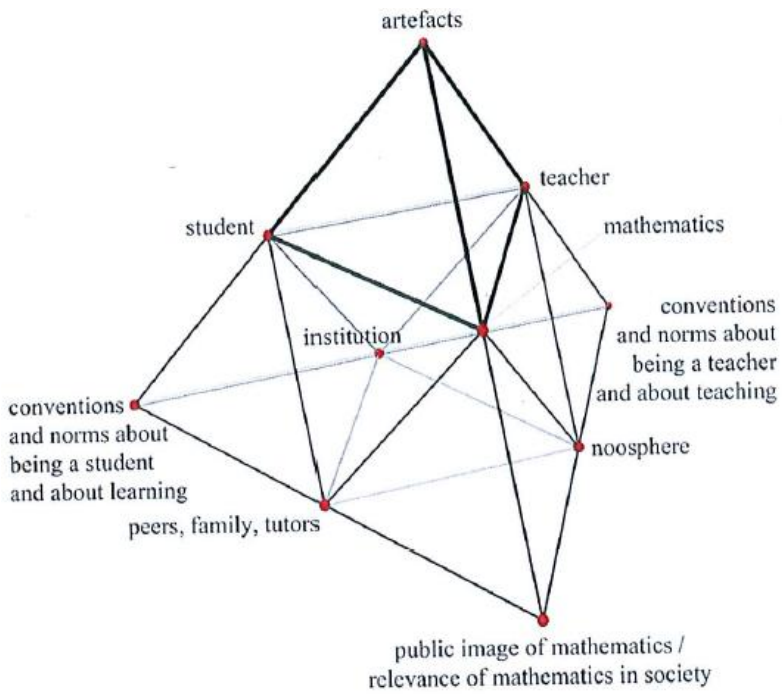


Figure 3: A socio-didactical tetrahedron (Source and StraBer 2012)

Rezat and Sträβer are quick to point out the limitations of their model of the socio-didactical tetrahedron. For instance, they state that a direct connection between the student and the public image of mathematics/relevance of mathematics in society is missing. However, I have to comment that these can be linked indirectly via the vertex 'mathematics' of the original didactic tetrahedron. In other words, the public image of mathematics is relevant in any discussion of the interplay between student and mathematics. Speaking of discussion, they also make the interesting point that they would place research by Yackel and Cobb (1996) on sociomathematical norms on the triangle linking artefacts with the conventions and norms about being a student and those about being a teacher. This is because they consider the role of 'discussion' in the negotiation of sociomathematical norms to be an artefact.

In this section I have considered various authors' contributions to the research on the didactic triangle and its expansions. I now wish to consider Jaworski's contribution as her work is a central component of this thesis.

2.5 Jaworski's contribution to research on the didactic triangle

In my opinion Jaworski has made two significant contributions to research on the didactic triangle. The first contribution concerns the development and analysis of teachers' work in classrooms. During her PhD research Jaworski came up with the 'Teaching Triad' as an analytical tool to look at the attempts of four secondary school teachers to make their mathematics lessons more investigative in line with a constructivist view of children's learning. As defined earlier the Teaching Triad consists of three categories: management of learning, sensitivity to students and mathematical challenge. Goodchild and Sriraman (2012) believe the triad helps in answering a question which drives developmental research in mathematics education: *How might teachers be empowered to become aware of and work on relationships among themselves, their students and the mathematics?* They further state that research and development activity (like Jaworski's) that has focused on problem solving, inquiry and investigation, and teachers' engagement with students in classrooms is basically concerned with students' engagement with mathematics, and the mathematical challenge they experience. They believe that researchers taking these issues as the focus for their enquiries address the fundamental relationships represented within the didactic triangle. I believe this is a fine tribute to Jaworski's work.

A corollary of Jaworski's Teaching Triad is that it can be used as a developmental tool in teacher education for those teachers seeking to follow a constructivist-compatible approach to their work. For instance, the notion of mathematical challenge reminds us of the Vygotskian zone of proximal development which purports that children should be challenged at the frontiers of their current knowledge. This connects with the idea of sensitivity to students as teachers have to

be aware of pupils' current knowledge levels and pitch their lessons accordingly. It also reminds us that pupils' interests should be borne in mind by teachers when they are planning lesson topics. Potari and Jaworski (2002) define *harmony* as the extent to which the degree of challenge in a lesson is appropriate to the particular cohort of students involved. Harmony involves achieving a balance between sensitivity and challenge. The third category of management of learning informs us that constructivist-compatible classrooms have to be set up in a certain way, usually involving groupwork, so that pupils can pursue lines of inquiry and develop their critical thinking skills. Leonard (2003) comments that constructivist lessons are often described as student-led learning where the teacher debriefs before and after but sets up a learning situation where the pupils discover the solution themselves. Therefore, the heart of constructivism in education is critical thinking. It can be seen that I am an enthusiast of Jaworski's Teaching Triad and I intend to use it in my own research both as a developmental and analytical tool. I will return to this issue in Chapter 4.

Another Jaworskian contribution is that she has expanded on the didactic triangle in her recent research. She suggests adding the role of researchers in the classroom, or didacticians, to use her term, as an additional node or adjunct to the didactic triangle. Her rationale is that teachers and didacticians share a reflexive relationship. She comments that although teachers' knowledge in practice goes far beyond didacticians' knowledge, the complementary knowledge of research and theory brought by didacticians provides stimulus and inspiration to which cohorts of teachers are able to respond. The relationship is reflexive in that teachers develop new approaches to working with their students such as using inquiry modes of learning. In tandem with this didacticians learn about how theories and research findings can and do "influence the practice of real teachers in real schools and

classrooms acting under all the constraints of institutional and political pressure" (Jaworski, 2012). Jaworski states that as a didactician herself, she is aware of the power of this collaborative knowledge and associated developmental practice in addressing approaches to educating students in mathematics. She offers the diagram (Figure 4) below as a way of representing the reciprocal influence of didacticians and teachers on the didactic triangle.

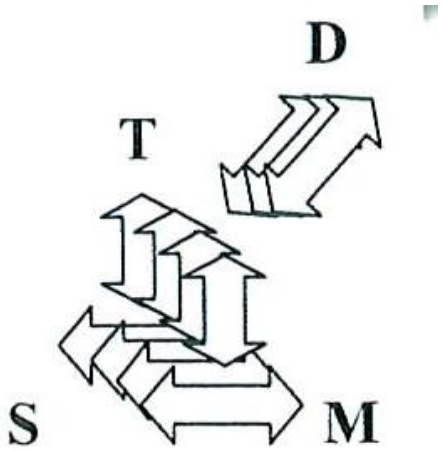


Figure 4: The didactic triangle for several teachers, their students and didacticians (Source: Jaworski 2012)

However, she is quick to point out that the above diagram might be seen to capture relationships at a particular point in time but that it does not recognize teaching development in any clear way and that it does not recognise elements of situation and context. To reflect elements of learning and development for both didacticians and teachers over time she offers the schematic representation as in Figure 5 which follows on the next page.

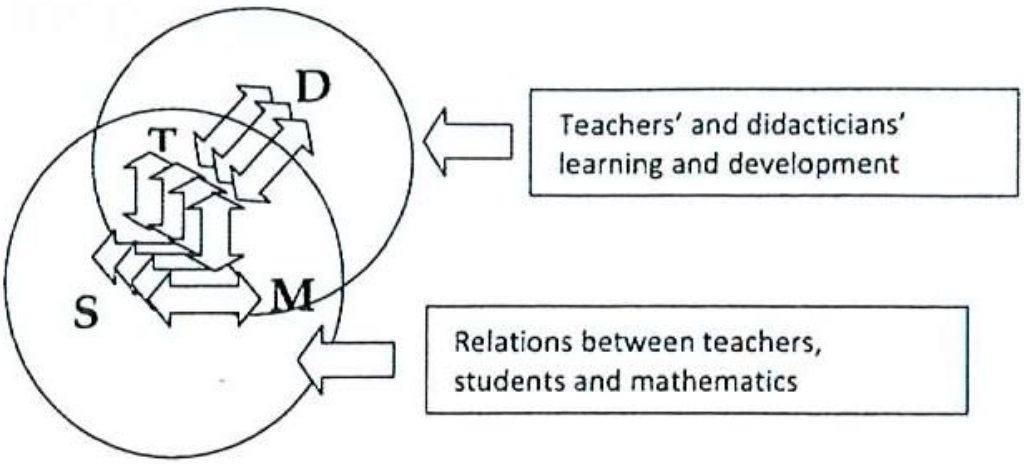


Figure 5: The expanded didactic triangle (Source: Jaworski 2012)

Jaworski states that the lower circle represents the traditional didactic triangle, connecting teacher, student and mathematics and attempting to characterise elements of the relationships involved within a community of teachers, their students and mathematics. It includes both the didactical and pedagogical thinking of the teacher in converting mathematics into classroom action, the interactions between teacher and students, the ways in which both teacher and students interact with mathematics and ways in which teachers themselves interact within the school context. It also encompasses the teaching and learning philosophies present and the sociocultural contexts in which the mathematics classrooms are located.

The upper circle is different in nature to the lower one. Whereas the lower circle strives to characterise situations, activity, events and relationships (what Jaworski calls the situational), the upper circle purports to represent the developmental processes which occur when teachers and didacticians inquire into all that is characterised in the lower circle. Therefore, the upper circle can be labelled as being developmental in nature. Jaworski states that the upper circle represents co-development between teachers and didacticians, a meta-dimension on the lower. It focuses on the learning of both groups as they participate in insider and outsider

research with clear learning outcomes to be achieved as both groups build new identities and increase their agency. In my own research I hope to comment further on developing Jaworski's work on the Teaching Triad in chapter 6. For now, I move the focus back to constructivist-compatible pedagogies.

2.6 Constructivism doesn't live on social interaction alone

As stated earlier, the mathematics guidelines view "cognitive development as a product of social interaction between partners who solve problems together" (NCCA 1999, p. 4). Indeed, this is very laudable. A few words of caution are needed though. Holt-Reynolds (2000) suggested in her case study of a prospective teacher, named Taylor, that such teachers may see constructivist pedagogies as strategies for activating children without necessarily building new learning. Put simply, "participation is not necessarily learning" (Holt-Reynolds 2000, p. 30). Taylor was inclined to accept all pupils' opinions in her classroom as being equally valid. She was not encouraging pupils to counter one another's opinions and search for disconfirming evidence. Indeed, Phillips (1995) used the adjective 'bad' to describe "the tendency within many forms of constructivist epistemology towards relativism, or towards treating the justification of our knowledge as being entirely a matter of socio-political processes or consensus, or toward the jettisoning of any substantial rational justification or warrant at all" (Phillips 1995, p. 11). Yet, Phillips describes the 'good' in constructivism as being the emphasis "that various constructivist sects place on the necessity for active participation by the learner, together with the recognition (by most of them) of the social nature of learning" (p. 11). For me, the essence of the argument is that social interaction requires active participation by the pupils for learning to take place. However, the word active here means cognitively active and not just physically active. Pupils may be physically active when they engage in discovery learning and hands-on activities, but they may not be

cognitively challenged. Fosnot (2005) goes so far as to say that many educators confuse discovery learning and hands-on approaches with constructivism. It seems to me that to turn social interaction into active cognitive participation requires mathematical challenge in the content of the tasks undertaken and exposure of pupils to opinions and methods, which rival their own. The issue of challenge later becomes one of the lenses in framing my classroom observations.

Prawat (1992, p. 37) uses the term 'naïve constructivism' to refer to the tendency to equate activity with learning. He describes this view as a misinterpretation of Dewey's ideas on activity approaches. Dewey (1938) was concerned that his views on experiential approaches led to a 'development from within' view of education. Instead, Dewey advocated that experiences must be carefully selected and structured. He emphasised the need for the educator to know where the experience is heading. This requires the teacher to draw on her subject knowledge to help pupils make sense of their present life experiences. The teacher attempts to connect her subject-matter knowledge with the child's experience. This reminds me of the Realistic Mathematics Education (RME) approach to children's learning which I later outline. An example from the teaching of place value might be a teacher providing manipulatives such as Dienes blocks to pupils to enable them to group in tens and units and not just in units alone. Dewey (1938) states that finding material for such learning is only the first step:

> The next step is the progressive development of what is already experienced into a fuller and richer and also more organised form, a form that gradually approximates that in which subject-matter is presented to the skilled, mature person. (Dewey 1938, p.74)

Further experiences are required, but they must contribute to the growth of subject-matter knowledge. This is the educative standard. Sometimes, in progressive educational environments, Dewey argued that there can be "little continuity from one activity to another or much of a sense of where an activity fits in the total scheme of things" (Prawat 1992, p. 370).

Many teachers use activities rather than ideas as their starting points when they are planning mathematics lessons. The tendency to equate activity with learning seems to rest on the belief by many teachers that pupil interest and involvement in the classroom "is both a necessary *and* sufficient condition for worthwhile learning" (Prawat 1992, p. 371). What I am suggesting here is that mathematical activities need to form part of a 'big idea', which the teacher has in mind for the pupils. For instance, if the children are being encouraged to group in tens and units using Dienes blocks the hope is that they would come to appreciate the base ten structure of our place value system when they move on to work with hundreds, tens and units and eventually thousands, hundreds, tens and units. Here the 'big idea' is place value. The activities only contribute to the 'big idea' and are not just an end in themselves. Place value is an authentic mathematical concept in that mathematicians assume it and use it all the time in working with our number system. It is a challenging concept for children also. I shall return to the issue of 'big ideas' in mathematics and their characteristics shortly.

2.7 Does constructivism exclude direct instruction?

The mathematics guidelines assert that "while direct instruction is very important in mathematics children also need to develop their own learning strategies" (NCCA 1999, p. 4). Direct instruction may appear to be in direct conflict with the earlier claim made in the guidelines that cognitive development is a product

of social interaction between partners who solve problems together. If direct instruction is so important the issue arises as to how children can also be given opportunities to develop their own learning strategies. Direct instruction has always been popular in mathematics teaching. Indeed the very influential Cockcroft Report (1982), subtitled Mathematics Counts, stated that mathematics teaching at all levels should include exposition by the teacher. However, Cockcroft sought balance by also stressing the need for other forms of interaction such as discussion, practical work, consolidation of skills, problem-solving and investigational work. Cockcroft also concluded that there was one aspect of exposition, which was insufficiently appreciated; this aspect was questioning.

> Questions and answers should constitute a dialogue. There is a need to take account of, and to respond to, the answers which pupils give to questions asked by the teacher as the exposition develops….. exploration of a pupil's incorrect or unexpected response can lead to worthwhile discussions and increase awareness for both teacher and pupil of specific misunderstandings or misinterpretations.
>
> (Cockcroft 1982, p. 72)

It follows that not only pupils' errors but teachers' errors also could bring about valuable learning experiences. It can be seen that Cockcroft took a very sophisticated view of what exposition by the teacher should entail. His view is more complex than the standard, closed three-part sequence of teacher Initiation, student Response and teacher Evaluation (IRE) of which Cazden (2001) writes. Cazden describes this pattern of discourse as being the oldest, "with a long and hardy life through many decades of formal Western-type schooling" (Cazden 2001, p. 30). Cockcroft's dynamic type of dialogue through questioning is essentially social

constructivist in that both parties; teacher and pupil, are engaged in the negotiation of meaning.

However, are there any circumstances in which simply telling pupils information could be compatible with social constructivist pedagogy? Love and Mason (1995) state that there are many circumstances in which it is not only proper and effective, but essential to tell people things. For instance, Love and Mason (1995) suggest that "telling people something, in expository or explanatory mode, can be of positive assistance, as long as what is said or explained is at the edges of what the pupils can do for themselves, rather than in the core" (Love and Mason 1995, p. 58). This is consistent with a social constructivist pedagogy in that the pupils in question may have misconceptions of which the teacher is aware or may have turned down a blind alley in their thinking and as a result the teacher attempts to help the pupils work at their zo-ped or at what Skemp (1995) terms their 'frontier zone' (Skemp 1995, p. 197). As Jaworski (1994, p. 62) cautions, "There has to be recognition of where a student stands and where she might reasonably reach". In such cases the teacher's dilemma can be summarised as 'to tell or not to tell'. Love and Mason (1995) point out that "it makes sense to tell people things when they are in a state to be able to hear, to relate to, to make connections with, and to assimilate what is being said and yet not be able to work it out quickly for themselves" (Love and Mason 1995, p. 34). To tell or not to tell is a real dilemma for teachers when they are under curricular and time constraints. In such situations, it makes sense to tell pupils information, which hopefully cultivates further clarity in their thinking. Indeed, it may be essential in maintaining the pupils' motivation and helping them avoid frustration.

So far I have advocated 'telling' in situations where scaffolding has failed and the teacher has no other option. 'Telling' can also occur at lower cognitive levels. Even

in classrooms where teachers espouse social constructivist principles pupils have to be organised so that they will work cooperatively and productively in groups. Through experience, a teacher knows which pupils will work best with one another. For instance, the teacher may be aware of personality clashes among pupils. A teacher may decide that a particular pupil would work better in one group than another. This may involve 'telling' the pupil that he would work better in an alternative group and discussing why so that the pupil can see the rationale behind his being moved. Another fundamental aspect of 'telling' occurs in the issuing of instructions to groups. Pupils have to be told of the nature of their activity, what equipment they may need and where they will be working. Such aspects of 'telling' may seem trivial but without the establishment of such routines or habits the classroom atmosphere becomes chaotic. Skemp (1995) describes habits as forming "an essential sub-structure of our daily life, since they free our conscious attention for the non-routine and problematic" (Skemp 1995, p. 82). 'Telling' pupils clearly what they have to do in their groups ensures a productive use of time and enables pupils to devote their energies to higher-order tasks such as problem solving.

Even within problem solving activities there may be occasions when a teacher will 'tell' a pupil what to do. Let me give an example from my teaching experience. One of my pupils was calculating the area of a floor 4.25m by 7.3m as part of a textbook problem solving exercise. My main objective for this pupil was to see if he understood the positioning of the decimal point in the answer. The pupil, not knowing his tables (basic number facts) very well, had written 5+5+5+5+5+5+5 to calculate 7X5. This twelve-year-old pupil had a legitimate but laborious strategy for calculating the tables. The dilemma I faced was whether or not to tell him the basic number facts as he worked through the computation. I decided to tell him the

answers to the computations he found difficult as time was an issue and I wanted to see where he would position the decimal point. As I suspected, he did not realise that tenths multiplied by hundredths led to an answer in thousandths with three digits after the decimal point. Further scaffolding was needed at that point so I would argue that my 'telling' of the tables preceded and was subordinate to my scaffolding of the place-value concept.

What I am suggesting here is that "low road transfer is also necessary in mathematics, so that the basic skills are automated in order to release the learner to use higher-order skills" (Open University, 1995, p. 120). Here I have illustrated 'telling' as the routinisation of the basic number facts. This aspect of 'telling' ensured the child and I had time to explore his understanding of the place-value concepts involved in decimal multiplication; such understanding being the primary objective of the lesson. Brophy (2006) puts it well when he states:

> It appears that transmission techniques are best used for efficiently communicating canonical knowledge (initial instruction establishing a knowledge base) and social constructivist techniques are best used for constructing knowledge networks and developing processes and skills (synthesis and application).
>
> (Brophy 2006 p. 534)

It may be that 'low road' knowledge, such as basic number facts, has to be regarded as a tool or even a cultural artefact, which can be used to release the pupil to engage in higher order cognitive processes such as problem solving. Furthermore, Hyslop-Margison and Strobel (2008) state that, in some circumstances, it is pedagogically acceptable to simply teach by lecturing, and lecture should not be entirely written out of a constructivist teacher's repertoire. They suggest that "lecture, or direct

instruction, is especially effective in classrooms where students already possess considerable subject knowledge" (Hyslop-Margison and Strobel 2008 p. 74). Although the authors appear to be writing about adolescent students the point is still the same – there are times when it is appropriate to tell students information. I believe it is important to state that practicalities, such as time constraints, dictate that it is impossible to set up group learning situations for all aspects of knowledge acquisition. It is therefore essential to rationalise which aspects of knowledge need telling and which need negotiation in group contexts.

In this section I have suggested that 'telling' is justifiable in certain contexts. It may succeed scaffolding when the teacher believes she has no option but to tell. However, it can also precede scaffolding in situations involving the establishment of classroom routines, such as grouping, in the automation of basic number facts and, in general terms, the promotion of low road transfer to free the mind for higher order processing. I leave my last words of qualification to Love and Mason (1995):

> For some reason, the idea of pupils making sense for themselves is often seen as incompatible with telling them things… they are in fact entirely compatible. The point about telling people things is to choose carefully *what* to tell and *when* to tell it.
>
> (Love and Mason 1995, p. 34)

2.8 The radical constructivist caveat

According to Von Glaserfeld (1990) radical constructivism is built on two principles:

1. Knowledge is not passively received but actively built up by the cognising subject.

2a. The function of cognition is adaptive, in the biological sense of the term, tending towards fit or viability.

2b. Cognition serves the subject's organisation of the experiential world, not the discovery of an objective ontological reality.

The first principle may be regarded as trivial constructivism as it is accepted by all constructivists. The second principle initially states that an individual learns by adapting. What he *knows* is the accumulation of his experiences to date. New experiences either add to previous ones or challenge them. Each individual organises his own experiential world, not just discovering some real world outside himself. The existence of an objective world is not denied, but it is stressed that it is only possible to know that world through experience. In other words, one knows what one has individually constructed. There may be justification at times to tell pupils information as Love and Mason (1995) have outlined. The radical constructivist caveat is that the teacher can never really be sure that the pupil has constructed meaning in the way the teacher has intended. The revolutionary aspect of constructivism lies in the assertion that knowledge cannot and need not be 'true' in the sense that it *matches* ontological reality, it only has to be 'viable' in the sense that it fits within the 'real' world's constraints that limit the cognising organism's possibilities of acting and thinking (Von Glaserfeld, 1987a).

Jaworski (1994) states that from a constructivist position we can never hope to construct a match with reality. Furthermore, because we can never know that reality – we can never know if and when a match is achieved. She elaborates by adding that "the best we can do is to construct mathematical concepts in such a way that they fit with our real-world experiences" (Jaworski 1994, p. 18). I have a proviso here:

mathematical concepts do not necessarily fit easily with our real world experiences. Consider the topic of complex numbers in mathematics, which revolves around the concept of iota or $\sqrt{-1}$ which does not exist in the 'real' world. However the concept of iota exists in the abstract mathematical world. It was brought into being by a mathematician thinking outside the constraints of the real world. The concept of iota then became real and part of our experience in that it could now be studied as an entity in itself. This would be akin to Treffers' (1987) vertical mathematization which is outlined later in section 3.5. When Jaworski (1994) mentions concepts fitting with our real world experiences, we have to remember that the *real* world in mathematics is often an abstract one. Through thought this abstract world can also become part of our experience. Therefore, mathematical concepts become richer when they are brought from the abstract world into the realm of real-world mathematicians who can give them further constructions and add to the field of inquiry.

Radical constructivism contends that one's construction of the experiential world is irrevocably subjective. This has been interpreted as a declaration of solipsism and as the denial of any 'real' world. Goldin (1990) believes that this interpretation is unwarranted. He states that constructivism has never denied an ulterior reality but it does say that this reality is unknowable.

2.9 Radical versus social constructivism

Critics (Chinn and Brewer (1993), Desforges (1995)) state that radical constructivism, as originally envisaged by Von Glaserfeld (1987b) ignores the role of social interaction in the construction of knowledge. Goldin (1990) holds that because other people contribute to a major part of an individual's experiential

environment, they have considerable power in determining which behaviours, concepts, and theories are considered 'viable' in the individual's physical and linguistic interactions with them. However these others "exist for the individual subject only to the extent to which they figure in the individual's experience – that is to say, they are for each subject what he or she perceives and conceives them to be" (Goldin 1990, p. 309). Whereas social constructivism believes in the social construction of meaning through negotiation in group settings, radical constructivism sees the individual as being the prime agent in abstracting meaning from such settings. Radical constructivism, therefore, sees no escape from subjectivity. Goldin (1990) uses the analogy that sharing meaning, ideas, and knowledge is like sharing an apple pie or a bottle of wine: none of the participants can taste the share another is having.

2.10 The teacher's dilemma

To follow the radical constructivist view to its extreme would be to fall into despair from a teaching viewpoint. If one cannot taste the share of knowledge another is experiencing then why teach anything at all, as it seems to be a futile exercise to attempt to experience another individual's construals? Earlier, I mentioned Brophy's (2006) statement that transmission techniques are best used for efficiently communicating canonical knowledge, involving initial instruction to establish a knowledge base. Again the radical constructivist approach is to query if there can ever be 'canonical' knowledge. Such an approach emphasises that knowledge is individually constructed and too subjectively gained to be regarded as canonical, as it cannot be imparted to others easily. Therefore, what brand of constructivism does the teacher follow? I refer to this as the teacher's dilemma.

All forms of constructivism seem to agree on the need to establish where the child is at in terms of their prior knowledge of a topic. For me, social constructivism comes to the fore in allowing the teacher to probe the child's understanding and to assist in the further processing of the child's thinking. In this way negotiated meaning, or 'taken as shared' knowledge, to use Cobb et al.'s (1995) phrase, is built up or constructed between the participants. Unless some form of common understandings is constructed between teacher and pupil the educative process is doomed to failure. Radical constructivism implies the need for an individual education plan for each child. This is laudable but the reality is that due to time constraints the teacher needs to feel reasonably confident that shared understandings are being assembled in the classroom. Without such common understandings the teacher will collapse under the theoretical pressure of radical constructivism in believing that he has to consult with each and every child about each and every aspect of knowledge acquisition. It seems to me that a social rather than a radical approach to constructivism has a better chance of informing actual classroom practice. That is not to state that social constructivism is the only show in town. Indeed, Cowan (2004) complicates the issue further by distinguishing between social constructivism and a socio-constructivist approach. I will briefly refer to the distinction between these two facets of constructivism whilst bringing the RME movement into focus.

2.11 Social constructivism and a socio-constructivist approach

Cowan states that through social constructivism "students can better construct their knowledge when it is embedded in a social process" (Cowan 2004, p. 4). He elaborates on a *socio-constructivist approach* by describing it as being a *type* of social constructivism, which is developed only in mathematics education. The tenets of this type are broadly similar to the characteristics of RME in that it is suggested that mathematics should be taught through problem solving, whereby students

interact with one another and the teacher. The theory draws on the Marxist idea of collective activity, wherein those who have more knowledge or are more skilled share that knowledge and skill with those who are less knowledgeable or less able in order to accomplish a task. Gravemeijer (1994) highlights two similarities between a socio-constructivist approach and RME. Firstly, both approaches have developed independently of radical constructivism. Secondly, in both approaches pupils are given opportunities to share their experiences with other pupils. Furthermore, mathematics is seen as a creative humanistic endeavour in which learning occurs as the pupils develop their own strategies and concepts in order to solve problems. De Lange (2006) states that the main difference between RME and constructivism is that RME is only applied to mathematics education, whereas constructivism is used in other subjects. However, Cowan (2004) contends, and I agree, that in RME the *integration* of mathematical strands is essential as RME is a holistic approach to mathematics, which incorporates several learning strategies within a mathematical problem solving foundation. I have to point out that in Ireland the word *integration* is normally reserved for the connection of mathematics with other subjects, whereas the word *linkage* is used for the connection of mathematical strands with one another. As far back as 1990, the Primary Education Review body recommended more frequent utilisation of integration in mathematics. This was to become one of the suggested methodologies in the revised curriculum of 1999. Linkage was also suggested as a methodology, with footnotes for it appearing on some of the curricular content pages. Returning to RME, Gravemeijer (1994) asserts that it offers heuristics for developing instructional activities for students, whereas a socio-constructivist approach does not offer heuristics, but seeks to find solutions through guided reinvention. Cowan (2004, p. 5) defines heuristics as “a method of solving problems by learning from past experiences and investigating practical ways of

finding a solution". In other words the socio-constructivist approach will not offer a ready-made formulaic solution but instead seek to derive one. This is similar to Vygotsky's notion of scaffolding, whereby pupils are assisted in working at the boundaries of their current reasoning, so that they can construct new knowledge. However, in RME it may not be the teacher who provides the scaffolding, but rather the other pupils with their ideas and strategies. In the co-construction of the zone of proximal development (zo-ped) the children are permitted to explain their strategies to the class and for the class to test their hypotheses, without the teacher pushing to seek a collective solution. Ernest (1995) and Wood et al. (1993) state that an awareness of the social construction of knowledge suggests more of a pedagogical emphasis on discussion, collaboration, negotiation and shared meanings. This is a practical manifestation of the theory of the social construction of knowledge. In Elwood's (2008) framework, as outlined in section 2.2, Ernest's (1995) and Wood et al's. (1993) ideas would appear to the centre-right of the continuum as they emphasise that learning is both a social and cultural activity. As Gergen (1995, p. 30) remarks, "Knowledge is in continuous production as dialogue ensues. To be knowledgeable is to occupy a given position at a given time within an ongoing relationship". The teacher is not meant to be the oracle constructing the knowledge for the pupils. Therefore, the teacher and pupils co-construct their evolving mathematical identities based on their mathematical experiences in the classroom. I hoped to witness and support this type of teacher interaction with pupils in this classroom research. Scaffolding evokes a construction metaphor in that Cowan (2004) states it has five major functions:

1. Provide support for the learner;
2. Function as a tool or *methodology* (to use a phrase more familiar to Irish teachers);

3. Extend the range of the learner;
4. Allow the attainment of tasks not otherwise thought achievable;
5. Works best when used judiciously.

Cowan (2005) describes the type of classroom scenario in which he believes scaffolding evolves. Initially, a teacher tends to do most of the work, but gradually the teacher and the learners share responsibility. As the learners become more competent, the teacher steadily withdraws the scaffolding so that the learners can perform independently. In construction parlance it evokes for me an image of a ladder being removed from a structure once it has been completed. The key to this structure is that the scaffolding keeps the children at the zo-ped, which is reframed as the children enhance their competency and improve their understandings. Cowan says he observed such practices during his own observations over a two week placement in RME classrooms. At the beginning of lessons the teacher seemed to be speaking for lengthy periods at the blackboard, slowly proceeding through algorithmical concepts step by step. As pupils' understandings developed the teacher-talk ebbed and became less evident. Cowan graphically describes the difficulties he experienced as he endeavoured to stop himself putting words and explanations into the children's mouths. He elaborates further:

> I also had to make myself play devil's advocate, which is a role I have trouble playing at times when I know that a correct answer has been arrived at; I tend to accept the answer without thinking about the process in which it resulted.
>
> (Cowan 2004, p. 5)

Little did I know that I too would later experience such difficulties in this classroom research. Theoretically speaking, Cowan sees RME as a brilliant concept with the potential to revolutionise the way we teach mathematics. He cautions, however, that

in practice RME can be very difficult to adopt, because it tests the faith the teacher has in her students to operate without the need for a tight framework or regime. He points out that during the first two days of his placement he found it difficult to deliver a lesson using RME, as he experienced anxiety when it came to leaving the children alone "long enough to free radical" (Cowan 2004, p. 6). It reminds me of Cobb's comment that 'trust' is one of the most important elements of a constructivist classroom. However, over time Cowan found it easier to let the pupils talk for most of the lesson in place of the teacher. This allowed the children to hypothesise and experiment with their strategies; thereby taking the focus off the 'right' and 'wrong' normally associated with mathematics. Furthermore, Cowan had the benefit of RME lesson guides, which allowed the teachers involved more time to concentrate on methodology as opposed to content. Cowan contends that RME aids the process of more of the class understanding a concept as opposed to less of the class. However, he adds the proviso that teachers have to be mathematically secure in their own understanding of concepts as they have to be able to interpret children's strategies without necessarily imposing their own viewpoint too soon.

2.12 Constructivist theory at large in the classroom

In looking at constructivist theory a practical problem now arises: how does constructivism actually manifest itself in classrooms? What is needed is an eclectic form of constructivism, which allows for several forms of knowledge and knowledge acquisition. Herscovics (1989) describes such an approach as 'rational constructivism'. For instance, the radical perspective is a psychological constructivist's view of an individual pupil's activity, as she engages with and contributes to the development of communal processes. However, the social perspective is an interactionist view of collective classroom activity. As a researcher,

do I focus on individual or communal activity to gain evidence of constructivism in action? Fortunately, there is a view called the emergent perspective, which seeks to combine both theories and in so doing justifies the need to look for both individual and socially mediated learning in classrooms. O' Shea (2009) gave a comprehensive elaboration of the emergent perspective and it is to this work I now turn. He cites Cobb and Bauersfeld (1995, p. 176) who state that the coordination of interactionism and psychological constructivism is the defining characteristic of the version of social constructivism that is referred to as the emergent perspective. This perspective highlights social processes and views knowledge acquisition as comprising both individual and social constituents and contends that these cannot be viewed as distinct in any meaningful way. Wilson (1996) comments that the difference between the rational and social perspectives is that radical constructivists emphasise how individuals create more sophisticated mental representations using information, manipulatives and other resources whereas social constructivists perceive learning as augmenting one's ability to participate with others in meaningful activity. To put it simply, the social constructivist's view is similar to the old adage that 'two (or more) heads are better than one'. Yet, from my own classroom experience, I have witnessed occasions when a pupil does not want to be helped by another pupil or the teacher; preferring to work a problem out by himself. Obviously, in this scenario, the pupil perceives the solution to be close at hand and the personal motivation required to solve the problem outweighs the need for social bonding. Tobin and Tippins (1993) surmise that the emergent perspective in a synthesis of radical and social perspectives, which claims that knowledge is both personally constructed and socially mediated. Windschitl (1999 p. 34) puts it well in declaring that learning is both an act of individual interpretation and negotiation with others. Therefore, in researching mathematics classrooms one has to ascertain if pupils are showing

evidence of being cognitively challenged at an individual level and/or engaged productively in discussion at group level.

O' Shea (2009) states that according to the social constructivist, the constructive processes are subjective and developed in the context of social interaction. Pupils gain mathematical knowledge through participation in the social practice of the classroom, rather than through the discovery of external structures, which exist independently of them.

Cobb and Yackel (1996) and Stephen and Cobb (2003) declare the emergent perspective to be a version of social constructivism. It draws on constructivist theories, which see learning as a series of cognitive reorganisations of the individual (von Glaserfeld, 1995) and interactionist theories, which perceive learning as a social accomplishment (Bauersfeld 1992). Therefore, the emergent perspective tries to reconcile radical and social constructivism. Cobb and Yackel (1996, p. 177) elucidate when they state that pupils reorganise their learning "as they both participate in and contribute to, the social and mathematical context of which they are part." They argue that mathematical knowledge is both an individual and a social construction and that both dimensions of learning complement one another. O' Shea (2009, p. 31) justifiably quotes Ernest in this regard:

> The two key features of the account are as follows. First of all, there is the active construction of knowledge, typically concepts and hypotheses, on the basis of experience and previous knowledge. These provide a basis for understanding and serve the purpose of guiding future actions. Secondly, there is the essential role played by experience and interaction with the physical and social worlds in both the physical action and speech modes. This experience constitutes the intended use of the

> knowledge, but it provides the conflicts between intended and perceived outcomes which lead to the restructuring of knowledge, to improve its fit with experience.
>
> (Ernest, 1991, p. 72)

Voigt (1992) elaborates on the relevance of the emergent perspective for mathematics education research, when he states that both cultural and social processes are integral to mathematical activity. Mathematical learning opportunities occur when pupils compare other solutions to their own and thereby try to make sense of their own solution in the broader perspective. To reiterate, the business of doing maths is both a social and an individual activity. The teacher has a pivotal role in initiating and guiding the formation of norms during mathematical activity, but the individual pupil also has an active role. Cobb and Yackel (1996) have done major research on the development of social norms and sociomathematical norms in the classroom and I now give a summary of their research.

2.13 Social norms and sociomathematical norms

Cobb, Yackel and Wood (1989) researched the prevalence of regularities in collective or communal classroom activity which they considered to be jointly established by the teacher and students as members of the classroom community. These negotiated regularities were given the term ‘social norms’. Obviously, the structure of such norms varies from classroom to classroom but Cobb, Yackel and Wood (1989) found that in one second grade primary classroom the teacher had to renegotiate classroom social norms when she wished to move from teacher-led discussions to ones in which the pupils attempted to articulate their own understandings. Since first grade these pupils had been used to inferring the responses the teacher had in mind rather than verbalising their own constructions. Therefore, examples of social norms for whole-

class discussions that the teacher highlighted as overt topics for negotiation included explaining and justifying solutions, attempting to make sense of explanations given by others, indicating agreement and disagreement, and questioning alternatives in situations where a conflict in interpretations or solutions had become obvious. Erickson (1986) and Lampert (1990) state that; in general, social norms can be seen to delineate the classroom participation structure.

It can readily be seen that the social norms for whole class discussions mentioned above are not unique to mathematics but can be applied to almost any subject. In their later research Cobb and Yackel (1996) shifted their focus of analysis to the normative aspects of whole-class discussions during students' *mathematical* activity. This shift involved looking at students' mathematical solutions and analysing what counts as a different mathematical solution, a sophisticated mathematical solution, an efficient mathematical solution and an acceptable mathematical explanation. The first three of these four norms entail a taken-as-shared sense of when it is opportune to contribute to a classroom discussion. The fourth norm of what counts as an acceptable mathematical explanation and justification concerns the actual process of making a contribution. McClain and Cobb (2001, p. 238) state that previous analyses of classrooms attempting reform recommendations "indicate that acceptable explanations and justifications typically have to be interpretable in terms of actions on mathematical objects that are experientially real to the listening students rather than in terms of procedural instructions". Hence, "sociomathematical norms differ from general social norms that constitute the classroom participation structure in that they concern the normative aspects of classroom actions and interactions that are specifically mathematical" (McClain & Cobb, 2001, p. 237). Cobb and Yackel (1996) conjectured that when students participated in the renegotiation of

sociomathematical norms their dispositions towards mathematics improved and they constructed specifically mathematical beliefs and values that enabled them to become increasingly autonomous members of classroom mathematical communities. The definition of autonomy also shifted from being a characteristic of individual activity to being a characteristic of an individual's participation in a community of learners (Lave and Wenger, 1991). This is in line with the emergent perspective on constructivism which contends that "negotiation is a process of mutual adaptation as that gives rise to shifts and slides of meaning as the teachers and students coordinate their individual activities, in the process constituting the practices of the classroom community" (Cobb and Yackel, 1996, p. 186).

It can be seen that the development of sociomathematical norms is a fruitful way of investigating practices in mathematics classrooms which attempt to adopt a reformist agenda. In chapter one I outlined Simon and Schifter's (1991) approach to investigating constructivist classrooms. Their approach involved utilising a Levels of Use (LoU) measure and an Assessment of Constructivism in Mathematics Instruction (ACMI) measure. The LoU measure involved researchers assessing the adoption of certain strategies by teachers; such as the use of manipulatives, diagrams and alternative representations. The ACMI measure was designed to rate teachers epistemologically on a scale of 0 (nonuse) to 5 (implements instruction with colleagues based on a constructivist view). I now wish to diagrammatically compare (see Table 1 below) the ACMI, sociomathematical norms and Jaworskian Teaching Triad analyses of teachers' constructivist-compatible approaches. For my own study I favour the Jaworskian analysis for the following reasons:

1. In 2008 the NCCA Primary Curriculum Review Group found that teachers had difficulty engaging pupils in cooperative group activities and had asked

for support in the implementation of methodologies other than direct instruction. What is currently needed, therefore, is a shift in teachers' approaches. Under Management of Learning, one prong of the Teaching Triad, I believe there is adequate scope to support and analyse such movements.

2. The use of ACMI and the negotiation of sociomathematical norms seem to require intensive time to be spent in classrooms; often by more than one researcher. Jaworski worked alone in developing the Teaching Triad as part of her PhD thesis while working full-time as a university lecturer, and I believe her approach is worth imitating given my own constraints working full-time as a primary principal.
3. ACMI rates teachers on a constructivist continuum from 0 to 5. However, it seems to me that teachers would fluctuate on that continuum depending on the type of activities pursued in the classroom. In other words, some activities may not be constructivist-compatible in that there may be too much teacher-talk going on during them. As regards sociomathematical norms Cobb and Yackel (1996) declare that such norms develop in teacher-dominated classrooms as well as in classrooms pursuing a constructivist agenda. Their focus is on the acceptability or otherwise of pupils' solutions, as negotiated in the classroom following a constructivist emphasis. In Ireland, teachers have not been chasing a constructivist agenda to date. My emphasis is not on the acceptability of pupils' solutions, per se, but on the processes required to get teachers to move towards a constructivist-compatible approach. I believe Jaworski's categories of Mathematical Challenge and Sensitivity to Students imply that Vygotsky's notion of the zone of proximal development is central to learning and that activities should

be set up which enable the teacher to be responsive to pupils' learning needs. It is through these categories that I see great hope for teacher change and, therefore, I proceed on that basis. I will return to Jaworski's Teaching Triad in forthcoming chapters. For now, I continue to elaborate on the emergent perspective in terms of how it might manifest, from a teaching viewpoint, in the primary mathematics classroom.

Instruments Used in Assessing Constructivist Approaches	**Type and Duration of Study**	**Data Collection Methods**	**Goals for Pupils**	**Goals for Teachers**
Cobb & Yackel's (1996) Negotiation of Mathematical Norms	Longitudinal: one school year	Video recordings of both individual teacher interviews and classroom episodes	Pupil autonomy: awareness of acceptability of mathematical solutions	More pupil-teacher discussion. Teacher movement towards an inquiry form of practice.
Jaworski's (1994) Exploration of Teaching Triad	Longitudinal: one school year	Field notes. Audio and video-tapes of lessons. Interviews with pupils.	Pupils work at ZPD. Pupils work in groups. Pupils' opinions valued.	Teacher movement towards an investigative form of practice.
Simon & Schifter's (1991) Assessment of Constructivism in Mathematics Instruction	Longitudinal: Option of two school years	Teachers' writings and interviews with teachers	Learners actively construct their own understandings	Changes in teachers' methods and epistemology

Table 1: A methodological comparison of ACMI, sociomathematical norms and Teaching Triad analyses

2.14 Key features of constructivist classrooms from the emergent perspective

O' Shea (2009, p. 32) elaborates on Windschitl's (1999) view of the key features of constructivist classrooms from the emergent or hybrid perspective. These features link what is known about how pupils learn with the optimal classroom conditions required for them to learn:

- Teachers elicit pupils' ideas and experiences in relation to key topics, then design learning situations that encourage pupils to augment or restructure their current knowledge.
- Pupils are presented with frequent occasions to deal with complex, meaningful problem-based activities.
- Teachers make different information resources available to pupils as well as the conceptual and technological tools required to mediate learning.
- Pupils work collaboratively and are encouraged to participate in task-oriented dialogue with one another.
- Teachers endeavour to make their own thinking processes explicit to learners and encourage pupils to do likewise through speech, writing, concrete, pictorial or symbolic representations.
- Pupils are frequently requested to apply knowledge in varied and authentic contexts, to communicate ideas, interpret texts, predict outcomes and build arguments based on evidence instead of focusing exclusively on the grasping of pre-determined correct answers.
- Furthermore, teachers encourage pupils' reflective and autonomous thinking in relation to the above conditions.
- Finally, teachers adopt diverse assessment strategies to witness the evolution of pupils' ideas and to provide feedback on the processes as well as the products of their thinking.

Windschitl's (1999) view of the key features of constructivist classrooms from the emergent perspective has a lot in common with Brooks' and Brooks' (1999, p.17) list of the key features of constructivist classrooms which are as follows:

- Curriculum is presented whole to part with an emphasis on big concepts.
- Pursuit of student questions is highly valued.
- Curricular activities rely heavily on primary sources of data and manipulative materials.
- Students are viewed as thinkers with emerging theories about the world.
- Teachers generally behave in an interactive manner, mediating the environment for students.
- Teachers seek the students' points of view in order to understand students' present conceptions for use in subsequent lessons.
- Assessment of student learning is interwoven with teaching and occurs through teacher observations of students at work and through student exhibitions and portfolios.
- Students primarily work in groups.

Windschitl (1999) emphasises the use of 'task-oriented' dialogue during 'problem-based' activities. Although Brooks and Brooks (1999) do not have this emphasis they specify a variety of assessment strategies to which Windschitl (1999) alluded, but did not elaborate, such as the construction of exhibitions and portfolios. In such assessment formats the teacher needs to suspend being judgemental and instead become 'nudgemental' in urging pupils to reveal their understandings through dialogue. In terms of Elwood's (2008) continuum Windschitl (1999) and Brooks and Brooks (1999) affirm both the individual and social aspects of learning. However, it

is difficult to describe learning devoid of the cultural context in which it occurs. Therefore, the continuum cannot be regarded as absolutist in its categorisations as overlaps and intersections will occur.

To summarise both authors' suggestions, one could say that pupils' ideas must be listened to so that the teacher can devise appropriate learning experiences, involving the provision of suitable tools and resources, which may be required for pupil use. Pupils should cooperate with one another in problem solving situations; designing, testing hypotheses, debating and reflecting upon work done to reach justifiable conclusions. Pupils should be encouraged to engage in a process of metacognition (Schoenfeld 1987; Garofalo, Kroll & Lester 1987) i.e.to reflect on their thinking processes, to become better problem solvers. This may sound like an individual activity but it can also occur in groupwork as children become aware of their learning processes in a social context. As O' Shea (2009, p. 33) states: "Particular emphasis is placed on extension activities that arise out of constructivist learning situations to validate, extend, refine and predict the usefulness of the learning exercise in future situations".

2.15 A pragmatic approach: teaching to the ''big ideas''

I now wish to shift the focus away from how pupils acquire knowledge to how teachers can best organise activities to help pupils construct knowledge in ways which are meaningful for them. I will argue that teachers need to teach to the ''big ideas'' in mathematics. This is similar to the Brooks' and Brooks' (1999) suggestion above that teachers should teach 'whole to part with an emphasis on big concepts'. I will contend that teaching to the ''big ideas'' requires the teacher to adopt an approach with three characteristics: fluidity of content, authenticity and process

orientation. The acronym CAP (content fluidity, authenticity, process) is useful in remembering such an approach.

2.16 A rationale for teaching to ‘‘big ideas’’

In this section I propose to give a rationale for discussing the issue of ‘‘big ideas’’ in mathematics. Schifter and Fosnot (1993) state that teachers often feel overwhelmed at the start of an academic year at the thought of teaching the amount of content suggested in a prescribed syllabus. They declare that most primary teachers also concede to remembering vividly their own experience of experiencing mathematics as a “bewildering succession of math facts and computational procedures committed to memory” (p. 34). They maintain that from the viewpoint of an instructional paradigm informed by constructivist principles, the pressure to move rapidly through content and the absence of understanding of mathematical principle are associated phenomena. If teachers are not sure of the true purpose of their teaching, then covering a large quantity of content, as quickly as possible, seems to provide some fake reassurance that nothing salient will be omitted. However, the opposite is often the case: the traditional topic-by-topic, drill-and-practice routine of mathematics instruction tends to interfere with pupil understanding of the principles that underlie mathematical order. Schifter and Fosnot (1993, p. 35) used the phrase ‘‘big ideas’’ to refer to these same “central, organising ideas of mathematics – principles that define mathematical order”. They suggest, and I agree, that teachers need to understand these ideas themselves if they are to enhance student understanding. The question then arises as to how best to assist teachers in teaching to the ‘‘big ideas’’. Clarity is required as to what constitutes ‘‘big ideas’’. The “big ideas” are not necessarily meant to be synonymous with traditional mathematical topics. In their ‘Summer Math for Teachers Programme’, Schifter and Fosnot (1993) encourage teachers to

think in terms of the 'ideas' they want pupils to learn rather than the 'topics'. For instance, the Irish primary mathematics curriculum contains a topic, indeed a strand unit, called symmetry. However, symmetry itself contains many "big ideas" such as the equivalence of line symmetry with mirror image. Schifter and Fosnot (1993) give three introductory illustrations of "big ideas":

1. One can count groups of objects as well as the discrete units that comprise them. Logically, every counting system possesses this property. As a matter of convention, place-value systems use the same numeral to denote a unit, or a group of units, or even a group of groups of units, depending on the numeral's position in a numerical expression. Hence, in the base-ten place-value system, in the expression 44, the 4 furthest to the right signifies 4 units, while the 4 to the left of it signifies 4 groups of 10 units. The idea that a numeral can either refer to groups or units is met again when pupils learn that 10 units can be 'regrouped' into 1 ten or vice versa. From my own experience, as a mathematics in-service provider, teachers often concentrate on teaching place value in discrete chunks, such as hundreds, tens and units without necessarily making the connections among them visible for pupils. For instance, an expression such as 444 is viewed as 400 + 40 + 4 without necessarily understanding it as 4 (10 tens/100 units) + 4 (10 units) + 4 (units).
2. A second example of a "big idea" is the usefulness of subtraction and addition viewed as inverse operations. Initially, children see these operations as distinct actions involving 'taking away' or 'adding on'. As a result, they have difficulty seeing that some problems are solvable using either operation. For example, consider a problem such as: *I have 30 pens. I give one to each of the 28 children in my class. How many do I have left?* One can either count on 2 ones to the 28 to make 30 or subtract 28 from 30. The same inverse operation principle applies

to multiplication and division and understanding it is a significant prerequisite for comprehending equivalent equations. Furthermore, I have found it very useful to give pupils calculation exercises which enable them to realise that subtraction and division (or addition and multiplication) are not inverse operations e.g. 6-2 =4 but 4÷2 ≠6.

3. A third example of a "big idea" is that the amount represented by a fraction depends on its reference whole, so that a single quantity may be shown by different fractions, depending on the whole to which it pertains. Consider the following problem: *John had ¾ of a metre of fabric and used ½ of it to make a napkin. How much of a metre was required to make the napkin?* In this example, the amount of fabric used to make the napkin is represented both as 3/8 and ½ i.e. 3/8 of a metre, but ½ of John's fabric.

In line with a constructivist approach, teaching to the "big ideas" necessitates facilitating their construction by organising contexts for relevant mathematical explorations. Developing a "big idea" may involve seeing old strategies in a new way; previously unrelated rules and results may be brought together to share a common unity. No one can claim that this approach is easy for teachers to adopt. However, I now wish to supplement Schifter and Fosnot's (1993) work by identifying three central characteristics that I deem essential for a teacher wishing to teach to the ''big ideas''. In doing so, I am focusing on the nature of activities required to encourage pupils to engage with the subject of mathematics.

2.17 Fluidity of content

The first characteristic I wish to identify is fluidity of content. In teaching symmetry as outlined above, Windschitl (2002) suggests that pupils should be given opportunities to compare examples of symmetry in nature, examine the relationship between temporal symmetry in music and spatial symmetry in other art forms, or identify ratios in symmetrical patterns. What is striking here is that this is unlike the standard approach given to symmetry in textbooks, where students are usually asked to provide written examples of symmetry as they occur in two-dimensional shapes and letters of the alphabet. As Gammage (1996) remarks about curriculum, "It must be capable of expansion and relocation and not seen simply in terms of fixed content" (p. 147). Teachers need to be encouraged to broaden their concept of curriculum so that they can encompass more interesting activities for pupils rather than use the ones encountered in standard textbooks. In other words, teachers need to adopt a fluid and flexible approach to what constitutes curriculum. Therefore, my rationale for including *fluidity* is that the idea of it encourages teachers to move beyond the confines of traditional textbooks. As part of a reform agenda Ross et al. (2002) remark that "the teacher's conception of mathematics in the reform class is that of a dynamic (i.e., changing) discipline rather than a fixed body of knowledge" (p. 125, original brackets).

2.18 Authenticity

The second characteristic I wish to propose in teaching to the ''big ideas'' is authenticity. By this I question whether pupils engage in authentic mathematical activities in the same way as real mathematicians do. Real mathematicians attempt to solve problems and often have to struggle to find solutions. Again, as part of their reform agenda, Ross et al. (2002) suggest that student tasks should be complex, open-ended problems; embedded in real-life contexts and possibly not affording a

single solution. I find it interesting that they use the word *authentic* to refer to the assessment of such tasks also. By this they imply that professional mathematicians assess their progress on an ongoing basis, link it with the task in hand, and do not wait for a body of work to be completed as in summative assessment. Therefore, authenticity can refer not only to the tasks undertaken but to the embedded assessment of such tasks also.

When pupils engage in standard textbook tasks, they experience the procedural aspects of mathematics without necessarily engaging in challenging, complex problem solving activities. As a result, they are often not working at what Vygotsky calls their 'zone of proximal development'. I draw attention to Vygotsky's work here as this research focuses on the broad area of constructivism but with a particular emphasis on social constructivism. Schifter and Fosnot (1993) advise that "teaching to the "big ideas" means facilitating their construction by providing contexts for relevant mathematical explorations" (p. 35). They add that some ''big ideas'' are never fully and finally comprehended. Rather, their construction is often provisional as they will be reorganised, acquiring additional content, as new areas of mathematics are explored. Again, this is the expansion and relocation of which Gammage (1996) referred to previously. It has to be conceded that this is not an easy approach for teachers to adopt. Teachers like to believe they are the 'content masters' in the classroom, and any approaches which take them out of their comfort zone, and into unknown territory, are not easily adopted. Teachers will need to journey into such unknown territory when they attempt to become problem solvers themselves during the pursuit of a mathematical enquiry raised spontaneously by a pupil. In this scenario the fluidity of the content demands that teachers join with pupils in becoming authentic problem solvers.

2.19 Process Orientation

The third characteristic I wish to identify in teaching to the ‘‘big ideas’’ is process orientation. This is where the teacher focuses more on how pupils work than on the products they create. The traditional products in mathematics are pages of sums which are marked as either correct or incorrect. The only input the pupil has in this scenario is to provide the required tick next to the sum. This is not very motivating for pupils, particularly the lower achieving pupils, who may not be succeeding in such mundane tasks. Brooks and Brooks (1999) suggested above that teachers should observe how pupils work and use exhibitions and portfolios to display their work. Such exhibitions and portfolios are an interesting blend of the formative and summative forms of assessment in that pupils engage in ongoing processes to arrive at such fixed products. Windschitl (1999) succinctly stated earlier that pupils need feedback on the processes, as well as the products of their work. Therefore, exhibitions and portfolios are not just an end in themselves, but a means of providing pupils with feedback. Such feedback can best occur through open dialogue between teacher and pupil and among pupils themselves. What I am proposing is that teachers create a classroom environment in which individuals are free to explore ideas, ask questions and make mistakes (Cobb et al. 1998). This is probably best done when pupils engage with one another in small groups. For the teacher who is new to this approach such groups may initially consist of pairs of pupils working together. As the teacher gains in confidence, along with the pupils, the groups may expand into four or five pupils. The important aspect is that the pupils are encouraged to mathematize, to quote Freudenthal’s term, for themselves rather than accept mathematics as a given. What I am emphasising here is that process in mathematics should become the focus. In other words the process of mathematizing should become the desired product of the lesson. This is not to say that the teacher engages

in what Prawat (1992) terms “naive constructivism” (p. 357). This is the tendency to equate activity with learning. Activity may be motivating for pupils, and this is welcome, but it does not necessarily mean that the pupils are learning. Therefore, the teacher has to become skilled in identifying if the activities being explored are challenging for the pupils.

2.20 Justifying the CAP approach

Teaching to the “big ideas” involves opening up mathematics in a way not catered for by traditional textbook teaching. I have identified three tenets of mathematical teaching, which I deem to be essential, if one is to ‘plough the mathematical field’. The tenets include adopting a fluid and flexible approach to *content*, choosing *authentic* activities from such content and shifting the focus to *process* as product (CAP approach). Bruner (1986), reacting to the ‘curriculum by objectives’ approach, stated that a subject is taught, not to produce little living libraries on that subject, but rather to get the student to think mathematically for herself i.e. to take part in the process of knowledge-getting. In other words, knowing is a process, not a product. Hence, I justify the inclusion of these tenets as they will assist teachers to open up their mathematical lessons in order to teach to the ‘‘big ideas’’ or principles that define mathematical order. The main idea here is that mathematics entails far more than the traditional framing of the subject in textbooks. Teaching to these ‘‘big ideas’’ will help pupils gain more ownership over their learning to negotiate meaning for themselves and this justifies the inclusion of such ideas under the umbrella of constructivist-compatible approaches, which is the central focus of this research. Pursuing these ideas ensures that pupils are presented with frequent occasions to deal with complex, meaningful problem-based activities, as advised by Windschitl (1999). I will return to such ideas in the final chapter.

2.21 Summary

In this chapter I began by situating my research in the zone between the individual and social aspects of learning and followed with a rationale for investigating constructivist teaching. I proceeded to discuss the various constructivist sects which abound; with a particular emphasis on radical and social constructivist theory. In reviewing the literature on constructivist theory I came to see learning as consisting of both individual and social acts of construction. Therefore, the emergent theory of constructivism appealed to me, as it seemed to represent a merger of both schools of thought. I examined how several authors such as Cobb and Yackel (1996), Simon & Schifter (1991), and Jaworski (1994) chose to approach research in constructivist classrooms. I outlined why Jaworski's approach appealed to me. Some practical dilemmas in adopting a constructivist approach were examined. For instance, I discussed the issue of whether or not a constructivist approach could include direct instruction. I made a case for direct instruction in situations where pupils had encountered 'blind alleys' and 'dead ends' in their thinking. I referred to RME as it is a good example of what a constructivist approach might look like in the classroom, despite Cowan (2004) stating that RME developed independently of constructivism. However, I have no doubt that RME was heavily influenced by the constructivist movement. I say this because RME gives pupils opportunities to work on authentic problems in group situations with pupils exercising a good degree of autonomy over their own learning, which is in line with a constructivist philosophy. Nevertheless, this admirable Dutch movement does not solve the problem of what teachers should do when confronted with a heavily prescribed curriculum. It was here that I turned to Schifter and Fosnot's (1993) research which suggested a move away from covering curriculum topics towards teaching to ''big ideas''. Their advice is strongly influenced by a constructivist approach to learning. In this chapter I

identified three characteristics which provide some guidance to deal with the nebulous notion of ‘‘big ideas’’ in mathematics. Such ideas need clarification if teachers are to adopt them in their teaching. The CAP acronym was useful here. In the next chapter, I discuss constructivism as it relates to mathematics in Ireland, and further afield, predominantly at primary level but at secondary level also as both levels are meant to link with one another as envisaged in the NCCA’s Bridging Framework (www.ncca.ie).

Chapter 3: A constructivist perspective on problem solving in the Irish mathematics curriculum

3.1 Introduction

In this chapter I delve into current approaches to teaching problem solving internationally and in Ireland in particular. I consider the merits of the mathematization approach of the Dutch Realistic Mathematics Education movement. Closer to home I compare the problem solving approaches outlined in the 1971 and 1999 curriculum documents. I critique the merits of a problem solving approach to mathematics and discuss how such an approach might be assessed by bringing the Assessment for Learning initiative into focus. Finally, I discuss how best to implement constructivist-compatible pedagogies.

I begin by stating that the academic year 2014/2015 is a bountiful one to be completing research in mathematics education. It is over three years since the Department of Education and Science (DES) published its national strategy, in July 2011, to improve literacy and numeracy among primary and post-primary pupils. The strategy was called Literacy and Numeracy for Learning and Life and in the press release which assisted the launch Minister Rory Quinn postulated:

> It is the government's belief that no child should leave school unable to read and write and use mathematics to solve problems. We know that there is currently much room for improvement and this strategy sets out the road map with concrete targets and reforms that will ensure our children, from early childhood to the end of second level, master these key skills.
>
> Minister for Education and Skills, 8 July 2011

The press release also stated that the strategy aimed to ensure that teachers and schools maintained a strong focus on literacy and numeracy skills, within a broad and balanced curriculum. The Government issued the following targets from the strategy to be achieved by 2020:

- At primary, increasing the number of children performing at Level 3 or above (the highest levels) in the national assessments of reading and mathematics by 5 percentage points
- Reducing the percentage performing at or below the lowest level (Level 1) by 5 percentage points
- At post-primary level, increasing the number of 15-year old students performing at Level 4 or above (the highest levels) in the Organisation for Economic Co-operation and Development's (OECD) Programme for International Student Assessment (PISA) test of literacy and mathematics by at least 5 percentage points
- Halving the numbers performing at Level 1 (the lowest level) in the PISA test of literacy and mathematics
- Improving early childhood education and public attitudes to reading and mathematics

DES, 2011

The publication of the strategy was ring fenced by the issuing of DES Circular 56/2011 which stated that the time allocated to literacy should be increased by one hour per week and the time allocated to mathematics should be increased by seventy minutes per week. Schools could integrate subjects to make up this time or use the existing discretionary two hour period in the curricular timetable allocation. The aspiration was that every child would leave school with a mastery of literacy and

numeracy. At this point a definition of numeracy, as envisaged in the strategy, is appropriate:

> Numeracy is not limited to the ability to use numbers, to add, subtract, multiply and divide. Numeracy encompasses the ability to use mathematical understanding and skills to solve problems and meet the demands of day-to-day living in complex social settings. To have this ability, a young person needs to be able to think and communicate quantitatively, to make sense of data, to have a spatial awareness, to understand patterns and sequences, and to recognise situations where mathematical reasoning can be applied to solve problems.
>
> DES, 2011, p. 8

Moreover, the importance of numeracy to a young person's life prospects and their participation in the economy and in society in general is reinforced throughout the strategy. Standardised tests were re-emphasised as being a major part of the assessment process for literacy and numeracy. DES Circular 138/2006 had introduced standardised testing at two points in the primary cycle. DES Circular 56/2011 requested that it be done at three points in the same cycle from 2012 onwards i.e. during May/June of 2nd, 4th and 6th class. The DES quotation above stresses the importance of problem solving in its definition of numeracy. Therefore, it is now appropriate to look at international research on problem solving.

3.2 International research on problem solving

In an oft-cited and influential article Schoenfeld (1992) states that problem solving has been used with multiple meanings which range from 'working rote exercises' to 'doing mathematics as a professional'. In illustrating the former he cites a problem set by Milne dating back to 1897: 'How much will it cost to plough 32 acres of land

at $3.75 per acre?' Although there are various ways to solve the problem, they all involve fairly basic computational skills. Accordingly, solving the problem hardly necessitates the use of higher order thinking skills. Schoenfeld quotes from the *Everybody Counts* report (1989) which recommended a renewed emphasis on

- seeking solutions, not just memorizing procedures;
- exploring patterns, not just memorizing formulas;
- formulating conjectures, not just doing exercises.

From this perspective, learning mathematics is empowering (Schoenfeld, 1992). Those who study mathematics are required to be flexible thinkers with a broad base of techniques and ideas for dealing with novel problems and situations. Here the reader will see that defining problem solving involves defining one's view of the epistemology or nature of mathematics knowledge. In quoting an earlier report by the National Council of Teachers of Mathematics (NCTM) entitled *Agenda for Action* (1980), which recommended that problem solving be the focus of mathematics, Schoenfeld (1992) declares that whereas 'back to basics' was the theme of the 1970s, problem solving was declared to be the theme of the 1980s. Yet, he also cautions that problem solving persisted as one of the most overworked and least understood terms of that decade.

Indeed, during the 1980s it came to be viewed, according to authors such as Stanic and Kilpatrick (1988), as consisting of a hierarchy of skills. Top of the hierarchy was the solving of non-routine problems. Next in the hierarchy was the solving of routine text book problems and subordinate to that was the acquisition of basic computational skills. It seems that in the Irish context it is the lower two levels of the hierarchy that gain most attention. In America in 2012 the National Centre for

Education Evaluation and Regional assistance issued a report entitled Improving Mathematical Problem Solving in Grades 4 through 8. These grades correspond to pupils aged 9 to 13 years in Ireland. The report made five recommendations for teachers:

1. **Prepare problems and use them in whole-class instruction.**

Teachers were requested to include both routine and non-routine problems. Routine problems were ones that could be solved by replicating previously learned methods in a step-by-step fashion. Such problems dominate Irish classroom teaching. Non-routine problems were ones for which there was no predictable, well-rehearsed approach or pathway suggested by the task, task instructions, or a worked-out example. Such problems are less prevalent in Irish classroom teaching and should form part of any reform agenda.

2. **Assist students in monitoring and reflecting on the problem solving process.**

This involves modelling for pupils how to monitor and reflect on the problem solving process by using *their* thinking about a particular problem.

3. **Teach students how to use visual representations.** As an enthusiast of Jerome Bruner I have long been an advocate of using this strategy as it is reminiscent of his iconic mode of representation.

4. **Expose students to multiple problem solving strategies**.

This recommendation involves asking students to generate and share multiple strategies for solving a problem. In my opinion, this strategy should form part of any reform agenda as it is in line with a constructivist philosophy seeking to empower students to take control of their learning and enable them to construct their own meaning. It can start at a simple level. Consider the routine problem of finding 35% of 200. With encouragement from the teacher pupils might consider finding 35/100

or 7/20 or 0.35 of 200. In line with recommendation number two above pupils can then reflect on how they completed the problem.

5. Help students recognise and articulate mathematical concepts and notation.

The advice here is to ask students to describe relevant mathematical concepts and notation. This recommendation reminds me of the advice given by Schoenfeld (1992) earlier that children should be encouraged to work as mathematicians do. Being familiar with mathematical language and notation is part of the students' enculturation process into the mathematician's world.

Therefore, it behoves me to discuss the type of problem solving Schoenfeld himself envisaged. To do this, I need to go to the other end of the continuum Schoenfeld (1992) mentioned earlier i.e. to doing mathematics as a professional. Schoenfeld (1992) comments that in becoming a mathematician he, and other colleagues, had undergone a process of acculturation, in which they had become members of, and had accepted the values of, a particular community of practice. In other words, they had become mathematicians in a deep sense as a result of a protracted apprenticeship into mathematics. This helps to situate Schoenfeld's (1990) view that "mathematics instruction should provide students the opportunity to explore a broad range of problems and problem situations, ranging from exercises to open-ended problems and exploratory situations" (p. 345). Working as a mathematician does not involve dealing with trivia; the problems encountered are often difficult and can take considerable time to solve. Schoenfeld (1990) conducted problem solving courses for students at college level. Roughly, one third of the time in these classes was spent with the students working on problems in small groups. In these classes Schoenfeld (1990) deferred teacher authority to the student community; both in withholding his own solutions to the problems and in developing in the class the critical sense of

mathematical argumentation to lead it, as a community, “to accept or reject on appropriate mathematical grounds the proposals made by class members” (p. 362). The justification for courses such as Schoenfeld’s comes from the research literature on practice change which reaches a consensus that “if teachers and prospective teachers are to provide new and different sorts of learning experiences to their students it is important that they have such experiences for themselves as learners of mathematics” (Crespo and Sinclair, 2008, p. 396). Since the new millennium the research literature has recommended that such experiences include problem posing as well as problem solving. Such a move is to be welcomed as it empowers students to take more control of their learning experiences. However, expecting pupils to pose problems spontaneously is not enough. English (1998) demonstrated that informal contexts, which are non-goal-oriented, are more conducive to the generation of diverse types of problems. She cites an example from her research of children being asked to look at a photograph of other children playing. This was followed by a request to make up a story problem which asked about something to be seen in the photograph. English (1998) found that children generated a more diverse collection of problems in this context than if they had been asked to generate a story problem that could be solved from a given number sentence (e.g. 12-8=4). This is a useful insight as converting a number sentence to a story problem is given prominence in the Irish curriculum (page 71). The use of the photograph as context led to an exploratory process which engendered the feeling of something being problematic enough to incite reflection and action. Crespo and Sinclair (2008) remark that this type of situation is essential for cognitive growth. However, they argue that there is a tension between the pedagogical and the mathematical fruitfulness or potential of problems. For instance, exploring Pythagorean triangles may be mathematically fruitful but teachers may decide that it is not a motivational topic for a particular

cohort of pupils. In research terms, Crespo and Sinclair (2008) call for further studies to understand this tension better. Such studies would help us learn more about how teachers balance pedagogically and mathematically interesting questions in their classroom, and ultimately learn how to help prospective teachers pose problems that have both characteristics.

3.3 Problem posing and its connection with problem solving

Since the 1990s several authors such as Silver (1994, 2013) as well as Crespo and Sinclair (2008) have been concerned with pupils' and teachers' efforts to problem pose as well as problem solve. Silver (1994) defines problem posing as "both the generation of new problems and the reformulation of given problems" (p. 19). Therefore, posing can occur before, during, or after the solution of a problem. Problem posing has been valued in the USA in such documents as the Principles and Standards for School Mathematics (NCTM, 2000). There it argues that the school curriculum should afford students opportunities to "formulate interesting problems based on a wide variety of situations, both within and outside mathematics" (p. 258). Although the term 'problem posing' is not used in the Irish Mathematics curriculum (1999) the Teacher Guidelines (1999) suggest that "children can invent problems for others to solve and, discuss the results". Silver (1994) gives a rationale for problem posing when he states that "contemporary constructivist theories of teaching and learning require that we acknowledge the importance of student-generated problem posing as a component of instructional activity" (p. 19). Silver (1994) outlines two types of problem posing. The first concerns problem formulation or reformulation. The solver changes a given statement of a problem into a new version that becomes the focus of solving. In this way the problem is 'personalised'. The operative question that inspires this form of posing is: how can I formulate this problem to

gain a solution? The second form of posing occurs when the objective "is not the *solution* of a given problem but the *creation* of a new problem from a situation or experience" (p. 20, original emphasis). Such problem posing can happen "*prior* to any problem solving, as would be the case if problems were generated from a given contrived or naturalistic situation" (p. 20). It can also happen *after* having solved a particular problem, when one might examine the characteristics of the problem to generate alternative related problems. Silver (1994) states that this kind of problem posing is reminiscent of the "looking back" phase of problem solving discussed by Polya (1945). Brown and Walter (1983) write about a version of this type of problem posing, in which problem constraints and conditions are scrutinised and changed through a process they refer to as "What-if?" and "What-if-not?". The operative question driving these kinds of problem posing is: What new problems are implied by this situation, problem or experience?

Despite the interest of the research community in problem posing because of its potential to improve problem solving, Silver (1994) comments that no clear, simple link has been established between competence in posing and solving. He calls for more research in that area. What can be stated is that "problem-posing tasks can provide researchers with both a window through which to view students' mathematical thinking and a mirror in which to see a reflection of students' mathematical experiences" (p. 25). Problem posing provides "a potentially rich arena in which to explore the interplay between the cognitive and affective dimensions of students' mathematical learning" (p. 25). Furthermore, the use of open-ended problem-posing tasks helps pupils to personalise and humanise mathematics through their lived experiences.

3.4 Realistic Maths Education (RME) and problem solving

One movement that encourages the use of pupils' lived experiences in the teaching of mathematical problem solving at both primary and secondary levels is Realistic Maths Education (RME) which originated in the Netherlands. The term RME is somewhat misleading as it implies that the mathematics involved is entirely based on real world circumstances; in other words a utilitarian view of mathematics is to the fore. However, this is not really the case. What is emphasised in RME is that students should learn mathematics by developing and applying mathematical concepts and tools in daily-life problem situations that make sense to them. This means that a real world view of mathematics is not imposed on the pupils; rather the pupils are presented with problems but also allowed a high degree of autonomy in how such problems are solved. The teacher uses any known cultural tools to assist pupils in their activities and takes on the role of facilitator in the process. Such a stance would be in line with a constructivist approach to children's learning. Den Heuvel-Panhuizen (2003) comments that on one level the term 'realistic' certainly concurs with how the teaching and learning of mathematics is perceived within RME but on another level the term is somewhat confusing. In Dutch the verb 'zichrealiseren' means 'to imagine'. In other words, the term 'realistic' refers more to the desire for students to be offered problem situations, which they can *imagine* (Van den Brink, 1984) than it refers to the 'realness' or authenticity of problems. However, the latter does not mean that the association with real life is not important. "It only implies that the contexts are not necessarily restricted to real-world situations. The fantasy world of fairy tales and even the formal world of mathematics can be very suitable contexts for problems, as long as they are real in the students' minds" (Van den Heuvel-Panhuizen 2003, p. 10).

3.5 Mathematics as mathematization

One of the basic concepts of RME is Freudenthal's (1971) view of mathematics as a human activity. For him mathematics was not the body of mathematical knowledge, but the activity of seeking out and solving problems and more generally, the activity of organizing matter from reality or mathematical matter – which he called 'mathematization' (Freudenthal, 1968). In succinct terms he declared what mathematics is about: "There is no mathematics without mathematizing" (Freudenthal 1973, p. 134).

This activity-based interpretation of mathematics also had significant consequences for how mathematics education was conceptualized. In particular, it affected both the goals of mathematics education and the teaching methods. According to Freudenthal, mathematics can best be learned by doing (Freudenthal 1971, 1973) and mathematizing is the central goal of mathematics education:

> What humans have to learn is not mathematics as a closed system, but rather as an activity, the process of mathematizing reality and if possible even that of mathematizing mathematics.
>
> (Freudenthal, 1968, p. 7)

Treffers (1987) suggested that there were two forms of mathematizing: 'horizontal' and 'vertical'. "In horizontal mathematizing, mathematical tools are brought forward and used to organize and solve a problem situated in daily life. Vertical mathematizing, on the contrary, stands for all kinds of re-organizations and operations done by the students within the mathematical system itself" (Van den Heuvel-Panhuizen 2003, p.12).In his final book Freudenthal (1991) adopted Treffers' distinction of these two ways of mathematizing, and articulated their

meanings as follows: to mathematize horizontally means to go from the world of life to the world of symbols; and to mathematize vertically means to move within the world of symbols. The latter suggests, for instance, making shortcuts and discovering connections between concepts and strategies and utilising these findings. Indeed, in this research 'connection-making' was to prove a significant pathway to understanding the findings. However, Freudenthal stressed that the differences between these two worlds are far from clear cut, and that, in his view, the worlds are not, in fact, distinct. Moreover, he found the two forms of mathematizing to be of equal value, and emphasised that both aspects could take place on all levels of mathematical activity. For example, even on the level of counting activities, both forms can occur.

This brief overview of RME is designed to illustrate the complexity of its approach. Gravemeijer (1991) and Streefland (1991) state that from the RME perspective school mathematics should be immersed in rich problem solving contexts that permit instruction to proceed from the reality of students' informal strategies. "Teaching in this kind of learning environment involves globally guiding students to be reflective and to develop increasingly abstract levels of mathematical reasoning that eventually lead to formal mathematization" (English 2008, p. 121). Therefore, it is naive to view RME as a movement solely based on real world situations. True, pupils' real world experiences of mathematics are taken into account, but these are only used as a springboard into deeper mathematical abstractions.

3.6 Influence of RME on Irish curricula

I believe I have demonstrated that the Realistic Maths Education movement is far more complex than just setting problems in real life situations. Certainly, children's

real life and often idiosyncratic experiences of mathematics are used as a springboard to jettison them into the world of mathematicians. However, the aim is to get children to a high level of abstraction, as expeditiously as possible; either through horizontal or vertical mathematization as outlined earlier. English (2008, p. 121) states that "there is evidence that students engaged in RME are especially successful in higher-level problem solving and reasoning, when compared to students who receive more traditional instruction." Next I will look at primary school mathematics pedagogy in more detail. I proceed by looking at problem solving and its links to constructivism.

3.7 Towards a definition of mathematical problem solving

Aistear, the Early Childhood Curriculum Framework (NCCA, 2009) refers to problem solving as children's ability to overcome obstacles that they meet while playing and undertaking activities. It must be remembered that Aistear is an early learning programme for children from birth to six years with active learning as a central methodology. The National Council of Teachers of Mathematics (NCTM) defines problem solving as engaging in a task for which the solution is not known in advance. Schoenfeld (1992, p. 10) declares that problem solving has had multiple and often contradictory meanings throughout the years. By way of example he quotes Webster's (1979, p. 1434) definition of a problem in mathematics as "anything requiring the doing of something" or a question that is "perplexing or difficult". It can be seen that such a definition implies that problems can range from performing simple pencil and paper calculations to more complex, demanding challenges. However, most of the literature surveyed justified problem solving as necessitating complexity. For instance, Francisco and Maher (2005) declare that providing students with the opportunity to work on complex tasks as opposed to simple tasks is crucial for stimulating their mathematical reasoning (Francisco and

Maher, 2005, p. 731). Moreover, the NCTM's Principles and Standards of School Mathematics comment that pupils should have "frequent opportunities to formulate, grapple with and solve complex problems that require a significant amount of effort and should then be encouraged to reflect on their thinking" (NCTM, 2000, p. 52). In a similar way, the Irish Primary Mathematics Curriculum encourages problem solving experiences which develop in children "the ability to plan, take risks, learn from trial and error, check and evaluate solutions and think logically" (NCCA, 1999, p. 35). It states that "problem solving is a major means of developing higher-order thinking skills" (NCCA 1999, p. 8). Therefore, it is logical to assume that problem solving inherently contains mathematical challenge for pupils. In this research the issue of mathematical challenge was to become one of the lenses through which I could observe classroom practice.

O'Shea (2009) tries to make explicit the link between problem solving and constructivist classroom practices. He comments that the curricula proposed by the NCTM and NCCA advocate that discussion and acceptance of the points of view of others are central to the development of problem solving strategies (O'Shea, 2009, p. 12). He states that both curricula are constructivist in origin and that therefore it makes sense to explore mathematical problem solving activities in which discussion plays a central role. In my opinion the Irish mathematics curriculum is even more definitive when it comments:

> It is in the interpersonal domain that children can test the ideas they have constructed and modify them as a result of this interaction. When working in a constructivist way children usually operate in pairs or small groups to solve problems co-operatively.

NCCA, 2009, p. 3

Although problem solving activities form a major focus of this thesis I also look at other classroom practices that encourage children to learn. These include mental and written warm-up activities which take place at the beginning of lessons. One of the difficulties with any classroom research is that focusing on one aspect of mathematics such as problem solving means that other aspects move out of focus. In this thesis I attempt to refer to these other aspects as and when they affect pupils' learning. From the participant teachers' point of view a focus on problem solving meant that they often neglected other aspects of the mathematics programme such as the teaching of standard algorithms. It could be said that problem solving became both an affordance and a constraint for them. It became an affordance in that they were able to explore novel problem solving situations with their pupils but it became a constraint in that they deviated from teaching standard algorithms which as Duffin (1986) points out tends to occupy 75% of teachers' time. Although Duffin writes about the English and Welsh situation I have no reason to believe that the Irish situation is any different.

3.8 Constructivism in the curriculum documents: definition of terms

In looking at constructivism in the Irish curriculum documents it behoves me to define some terms used in this thesis. The main term used in this study will be constructivism which refers not to a single theory but to a set of related theories that deal with the nature of knowledge. Scheurman (1997) states, "The common denominator linking these theories is a belief that knowledge is created by people and influenced by their values and culture" (p. 28). I have discussed constructivist theories and their associated concepts in chapter 2. It is my hope that the teachers in this study will create their own constructivist paths, which will be meaningful to them within their own values and school culture.

Another term used in constructivist literature is the zone of proximal development (zo-ped or zpd). Vygotsky (1978) coined this term to refer to "the distance between the actual developmental level as determined by independent problem solving and the level of potential development as determined through problem solving under adult guidance or in collaboration with more capable peers" (p. 86). Indeed, teaching could be said to occur when assistance is offered at points in the zo-ped at which performance requires assistance. Therefore, careful assessment of the child's capabilities, relative to the zo-ped and the developmental level, is an ongoing requirement for the teacher. The teacher and more capable peers are crucial in supporting a pupil's learning. Such support has been called scaffolding, which is an idea developed by Vygotsky but labelled by Bruner (1967). Bruner (1986) argues that scaffolding must reduce the number of degrees of freedom that a child must manage in a task. He suggests that scaffolding should provide "the child with hints and props that allow him to begin a new climb, guiding the child in next steps before the child is capable of appreciating their significance on his own." (p. 132). He elaborates by stating that it is the loan of the adult's consciousness that gets the child through the zo-ped. However, does this mean that the child is being asked to seek some objective truth that the teacher has in mind? Surely, this is anti-constructivist in that it goes against the idea of the child constructing their own meaning? To get around this Cambourne (1988) describes scaffolding as "raising the ante" and he teases out what he means by suggesting the following steps in a scaffold:

1. Focus on a learner's conception
2. Extend or challenge the conception
3. Refocus by encouraging clarification
4. Redirect by offering new possibilities for consideration

It can be seen that Cambourne is re-emphasising the child as being in central control of their own learning. I will return to scaffolding in chapter 6 as it becomes significant in this study's data analysis of 'sensitivity to students', a category of Jaworski's Teaching Triad.

The 1971 Irish Primary Curriculum was based on a Piagetian view of learning whereby pupils were to be involved in their own knowledge construction. In other words, knowledge did not "result from a mere recording of observation without a structuring activity on the part of the subject" (Piaget 1980, p. 23). Discovery learning became the mantra of the day. Project work and hands-on approaches became fashionable. When the 1999 Revised Curriculum was launched it re-emphasised such approaches, but there was more of a recognition that learning is not solely an individual act of construction, but that pupils learn from significant others such as teachers, parents and peers. This meant that there was an acknowledgement that knowledge can be socially constructed. This was more in line with a Vygotskian than a Piagetian view of knowledge construction. It is interesting that the Vygotskian term "social constructivism" is never used in the 1999 curriculum. I am informed that a debate took place within the National Council for Curriculum and Assessment (NCCA) and that it was decided that the term "constructivist" rather than "social constructivist" would be used in describing the epistemology of the revised curriculum. I think this is regrettable because the Vygotskian view implies that children need resources and materials from the social world to enhance their learning.

This places an onus on the government to provide such resources. Unfortunately the present situation is that the government seems intent on letting primary education survive on a skeletal budget with 'no meat on the bones'. Symptoms of this are seen

with cutbacks in resource-material grants for learning support teachers and even book-rental grants for necessitous pupils. If pupils cannot afford books and other tools of learning it is very hard to envisage how pupils can be encouraged to engage fully in the educative process. Certainly, the social constructivist approach of encouraging children to work at their zone of proximal development or frontier zone will be a lot harder to implement and will remain more a theoretical than a workable proposition.

At the outset it has to be stated that the theory of constructivism is given scant attention in the primary mathematics curriculum. An overview is presented on pages three and four of the Teacher Guidelines, while approximately one third of page five of the Mathematics Curriculum itself refers to constructivism and guided-discovery methods. This sends the message that the theory is not nearly as important as the other aspects of the curriculum such as content, methodology and skills and that, therefore, teachers need not concern themselves with it too much. I believe this is a mistaken approach as teachers need grounding in the theory that informs their work if it is meant to inform their practice. Therefore, what little of the theory is presented seems to operate in somewhat of a vacuum.

For example, the Teacher Guidelines state:

> It is in the interpersonal domain that children can test the ideas they have constructed and modify them as a result of this interaction. When working in a constructivist way children usually operate in pairs or small groups to solve problems co-operatively (p. 3).

This is very laudable. However there is no chapter in the Guidelines or Curriculum itself dedicated to how teachers can actually implement such group work. Unless

such advice is forthcoming teachers are likely to remain in didactic mode, teaching mathematics to whole-class groupings. The Guidelines acknowledge that direct instruction is very important in mathematics, but that "children also need to develop their own learning strategies" (p. 4). This is unlikely to happen unless teachers feel confident engaging with children in small group situations.

Another example of the vacuum occurs with the following Teacher Guidelines statement: "This sociocultural theory sees cognitive development as a product of social interaction between partners who solve problems together" (p. 4). It follows by recommending "work on open-ended problems, where the emphasis is placed on using skills and discussion rather than seeking a unique solution" (p. 4). The difficulty I have here is that very little direction is given to teachers on how to locate these open-ended problems and which teaching methods to use with them. It is interesting that there is no book on problem solving recommended in the twenty four source references on pages seventy four and seventy five of the Teacher Guidelines. Even presuming that teachers source such open-ended, investigative problems it follows that teachers need guidance on how to get pupils to engage with these problems. The curriculum states that "the importance of providing the child with structured opportunities to engage in exploratory activity in the context of mathematics cannot be overemphasised" (p. 5). What is needed is more advice for teachers on the methods they need to employ to provide pupils with such structured opportunities. This involves teachers being shown how to act as facilitators in small group situations, where investigative problem-solving becomes the main modus operandi. This will not happen overnight and without adequate teacher in-service provision. The lack of guidance in sourcing open-ended problems and how to engage

with them were to become factors in this classroom research. I now discuss how problem solving evolved from the 1971curriculum to the 1999 version.

3.9 A comparison between problem solving in the 1971 and 1999 curricula

Problem solving did not appear as an explicit aim in the 1971 curriculum where it stated that the aims were "to kindle a lively interest in the 'subject', to give the child a grasp of basic mathematical structure and content and to lay a foundation for further work at post-primary level" (p. 125). Leading the child to a realistic level of skill in computation was also deemed to be an important aim. The 1999 curriculum was more explicit when it stated that one of its aims was "to develop problem solving abilities and a facility for the application of mathematics to everyday life" (p. 12). Concern with 'everyday life' mathematics was not the only aim as an appreciation of the "aesthetic aspects" was also mentioned (p. 12).

The 1971 mathematics curriculum was split into three sections: Number, Activities, and Language Development and Recording. Problem solving is not mentioned at all at the junior and senior infant level. The curriculum is heavily influenced by a Piagetian philosophy of learning. The presumption can only be that infants are perceived as not being at an appropriate stage of readiness to solve problems. In the curriculum for $1^{st}/2^{nd}$ and $3^{rd}/4^{th}$ classes the heading 'Problems' appears in the Number section. The curriculum states that problems involve the pupils in making judgements and also in applying and practising the skills discovered during number activities. These activities are meant to "give the pupils ample practice in attacking real-life problems in a sensible manner" (p. 162). Pupils are to devise problems related to their environment. The curriculum seems time bound when it states that "problems about marbles, conkers, pennies, chickens, eggs, apples, etc. set by the

pupil will be more meaningful to him and to his classmates than any textbook problem" (p. 163). Any curriculum is a product of its time and the previous statement reflects the importance of agriculture to the Irish economy in the early 1970s. The type of problems envisaged concerns those which involve the use of a frame or placeholder e.g. 12 - ? = 7. The pupil is meant to translate the mathematical sentence into a word problem or story and vice-versa. It is interesting that the 1999 curriculum places such problems in the 3rd/4th class sector to allow more time for this skill to be developed. The 1971 curriculum suggests that other pupils in the class "could criticise the story and discuss its suitability" (p. 164). A better choice of word might have been 'critique' but at least the curriculum suggests some element of problem reformulation or posing by the other pupils. When I delivered inservice on the 1999 curriculum during its introduction in the 2001/2002 school year I received feedback from some teachers that moving curriculum content from junior to more senior classes (as above) represented a 'dumbing down' of the curriculum. However, I would have to take the view that such moves were designed to allow pupils more time to gain and consolidate problem solving skills and even problem pose to some degree. Both the 1971 and 1999 curriculum agree that by 4th class pupils should be able to use all four operations in reducing practical problems to some form of mathematical statement. However, in the 1971 curriculum the example given as a practical problem is as follows; "I have 5 pieces of ribbon, each of which is 4 inches long. How far will they stretch if placed end to end along the picture rail?" (p. 190). I would contend that such a problem is not an example of a practical real life situation at all, but is instead a contrived attempt to give validity to a multiplication exercise.

In reviewing the 1971 curriculum I found it interesting that the heading 'Problems' disappeared in the 5th/6th class sector. It does recommend the application of the

number section, including social arithmetic, to both teacher-contrived and pupil-contrived problems. The only other guidance given is that project work, so prevalent in the 1970s, should alert pupils to mathematical possibilities in solving some problem that may arise from discussion. The example given is praiseworthy. As part of a project on transport pupils could investigate the speed factor as a cause of accidents. Bar charts, showing the stopping distance of cars travelling at various speeds, are suggested as a medium. In my opinion, this is a good example of a real life problem which transcends time. It also appeals to me as a problem that exposes pupils to a more investigative approach to mathematics.

The 1999 curriculum advocates such an approach also when it states that "the importance of providing the child with structured opportunities to engage in exploratory activity in the context of mathematics cannot be overemphasised" (p. 5). It goes on to comment that "the teacher has a crucial role to play in guiding the child to construct meaning, to develop mathematical strategies for solving problems, and to develop self-motivation in mathematical activities" (p. 5). Problem solving is suggested as a major means of developing higher-order thinking skills. Like the 1971 curriculum, practical applications of mathematics are emphasised but the phrase 'real life mathematics' is not used. Instead the suggestion is that "solving problems based on the environment of the child can highlight the uses of mathematics in a constructive and enjoyable way" (p. 8). The splitting of the 1999 curriculum into strands (number, algebra, shape and space, measures and data) and strand units has the advantage of allowing suggestions to be made for the practical applications of mathematics throughout the document. However, one of the difficulties in discussing problem solving in the 1999 curriculum is that the term appears at three levels. My contention is that there is problem solving as content (usually solving routine

textbook problems in the Irish context), problem solving as a methodology (teacher modelling how to solve problems as outlined on pages 35/36 of the Teacher Guidelines) and problem solving as a skill to be acquired (as outlined on pages 68/69 of the Teacher Guidelines). This is a microlevel translation of the didactic triangle which captures the interplay between teacher, student and the subject of mathematics. It can be seen that it is difficult to isolate any vertex of the triangle without considering the implications for the other two vertices. The microcase is illustrated in figure 6 below:

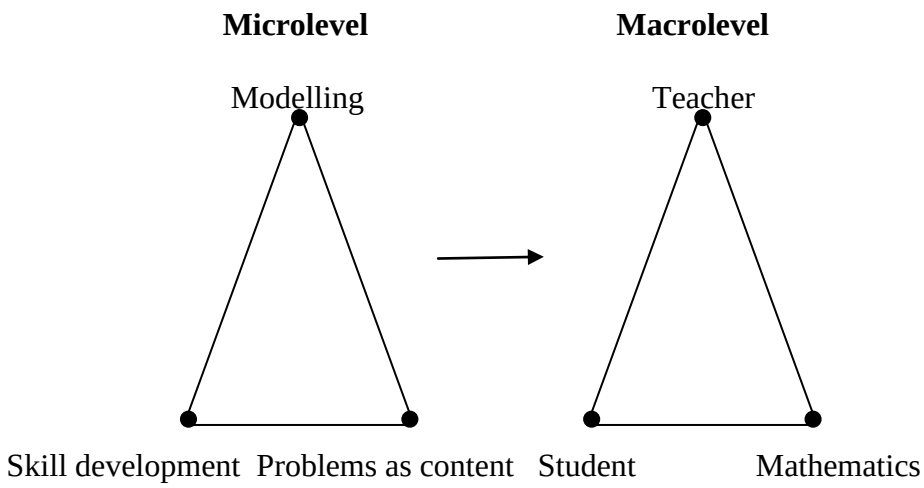


Figure 6: Problem solving as a microcase of the didactic triangle

3.10 Research reports on problem solving in Irish primary classrooms

Since the introduction of the revised mathematics curriculum in 1999, several research reports have commented on teachers' approaches to problem solving and curriculum implementation in general. Inservice on the mathematics curriculum did not occur until the school year 2001-2002. Having allowed three years for the curriculum to consolidate, two reports issued in 2005. One of these was an evaluation by the Inspectorate of the Department of Education and Science entitled 'Literacy and Numeracy in Disadvantaged Schools: Challenges for Teachers and Learners'. This publication was commonly known as the LANDS report. The report

is implicitly constructivist in recommending the use of activity methods, concrete materials and the correct use of mathematical language. The report also recommended addressing the further development of higher-order thinking skills in both literacy and numeracy. In the middle and senior classes, the Inspectorate had identified that pupils had a poor understanding of place value and poor estimation and problem solving skills. The report recommended that pupils should be encouraged to use a range of reasoning and problem solving strategies and that problem solving tasks based on the learning needs and experiences of the pupils should be provided. It also recommended the creation of linkage between all the strands of the curriculum.

The second report issued in 2005 was also conducted by the Inspectorate but did not confine itself to disadvantaged schools. It was concerned with the subjects of English, mathematics and visual arts. The report was entitled 'An Evaluation of Curriculum Implementation in Primary Schools'. It found that in the majority of classrooms problem solving was a feature of the lessons observed and that pupils were provided with a range of problems which promoted the specific skills of communicating, reasoning and connecting. However, in almost a third of classrooms there were deficiencies in the use of this teaching approach. These included the non-implementation of the school plan with regard to problem solving and an over-reliance on traditional textbook problems, which did not promote specific problem-solving skills. The report noted that active involvement by pupils was very limited in a quarter of classrooms. In those classrooms the pupils were engaged solely in paper-and-pen exercises which "does not reflect the constructivist approaches that are central to the curriculum" (p. 27). In more than two-thirds of classrooms observed there was an over-dependency on whole-class teaching, where teacher-talk prevailed and where pupils worked silently on individual tasks for excessive periods. The

report was more optimistic on the use of linkage than the one for disadvantaged schools but did note that the data strand could be better integrated with other subjects such as Geography and Science. As in the report for disadvantaged schools, it was recommended that greater emphasis should be placed on the development of estimation strategies at different class levels.

A third report on the primary school curriculum entitled "Mathematics in Early Childhood and Primary Education (3-8 years)" issued in 2014. Like the previous report on curriculum implementation (2005) it re-emphasises the theoretical perspectives underlying the primary school curriculum. Moreover, it brings the theoretical discourse up to date by commenting that cognitive and sociocultural perspectives provide different lenses with which to view mathematics learning and the pedagogy that can support it. It states that cognitive perspectives are helpful in focusing on individual learners while sociocultural perspectives are suitable when focusing on, for example, pedagogy. The report stresses that "learning mathematics is presented as an active process which involves meaning making, the development of understanding, the ability to participate in increasingly skilled ways in mathematically-related activities and the development of a mathematical identity" (p. 10). Learning is seen to be assisted by participation in the community of learners engaged in mathematization, in small-group and whole class dialogue. The term 'mathematization' is explored elsewhere in this thesis. The report states that the processes of mathematization should permeate all learning and teaching activities. These include connecting, communicating, reasoning, argumentation, justifying, representing, problem-solving and generalizing. The language used in the report is reminiscent of Lave and Wenger's idea of participation in communities of practice (1991) and Rogoff's (1990) notion of apprenticeship in learning communities. I welcome this elaboration as one of my criticisms of the 1999 curriculum is that it is

scant on detail concerning the theories underlying it. This report goes so far as to state that "the goal statements of a curriculum should be aligned with its underlying theory" (p. 12).

As regards higher-order thinking skills the report highlights that there are "concerns about the levels of mathematical reasoning and problem solving amongst school-going children, as evidenced in recent national and international assessments and evaluations at primary and post-primary levels" (p. 9). It suggests the use of learning paths in the teaching of mathematics. These paths are defined as sequences that apply in a general sense to children's development in the different domains of mathematics. In line with the ideas of the Realistic Mathematics Education movement, discussed elsewhere in this thesis, such paths are not meant to be fixed for each pupil, but rather characterised by fluidity and influenced by the role of context. The report recommends that educators should be assisted in the design and development of rich and challenging mathematical tasks that are appropriate to their children's learning needs. This assistance needs to commence at the teacher preparation level in colleges of education.

On the next page I present Table 2 which compares the three reports outlined above in terms of their underlying philosophy, the problem solving deficits they highlighted and the solutions which they proposed.

Report	Underlying Philosophy	Problem solving Deficits	Proposed Solutions
LANDS (2005)	Implicitly constructivist	Estimation and higher-order thinking skills	More problems based on pupils' real life experiences Create more strand linkage
An Evaluation of Curriculum Implementation (2005)	Explicitly constructivist	• Estimation skills • Disconnect between school plan and classroom practice • Over-reliance on textbook problems	Use the environment to contextualise learning Promote active learning
Mathematics in Early Childhood and Primary Education 3-8 years (2014)	Both cognitive constructivist and sociocultural	• Levels of higher-order thinking skills	Adaptable learning paths for pupils Promote mathematization Design challenging mathematical tasks

Table 2:DES and NCCA reports on mathematics teaching/learning 2005-2014

In summarising the three reports, it can be seen that there is an ongoing general concern with the levels of children's higher-order thinking skills. Solutions proposed include moving away from textbooks and basing mathematical tasks on children's real life experiences. It is interesting that Conway et al. (2011), referring to research by Lyons et al. (2003), state that "being able to justify solutions or displaying real life understanding of mathematics use was considered *not* very important by

teachers" (p. 120). The Evaluation of Curriculum Implementation Report (2005) found that there was an over-reliance on textbook problems. Such problems are often presumed to be based on real life experiences but often wear a thin veneer of such experience. Kilpatrick (1985, p. 4) refers to one such problem: If a 7-oz. cup of cola costs 25c, what is the cost of a 12-oz. cup? Kilpatrick (1985) states that this problem is meant to simulate a real problem the pupils might face. I have two difficulties with this interpretation. Firstly, when one buys beverages the cost of a larger cup is rarely in proportion to the cost of a smaller cup (due to economies of scale). Retailers extract extra money from the consumer by providing the larger cup at a 'discount' to the proportional price. I am reminded of the McDonald's Supersize Me campaign which received a lot of criticism for encouraging consumers to overindulge. Secondly, when one does the calculation the answer turns out to be 42.857142 cents which contains a recurring decimal. This amount of money cannot exist in real life. Kilpatrick (1985) puts it well when he states that the actual calculation exercise could be termed "the computational skeleton beneath the skin" of the problem (p. 4). The emphasis on teaching problem solving through real life contexts is discussed at length elsewhere in my critique of the Dutch RME movement in this thesis. Here it suffices to say that authors like Brown (2001), as well as Nicol and Crespo (2005) ask us to broaden our conception of what counts as real. Ross et al. (2002) state that reform mathematics should encompass student tasks which are "complex, open-ended problems, embedded in real life contexts" (p. 125). However, Brown (2001) requests that we reconceptualise what is 'real' in a more imaginative sense than what exists or what we can touch and see. He states that by doing this we not only legitimise more interesting connections between mathematics and the real world, but "we also suppress the need to seek real-world connections as a slave against an otherwise 'unreal' world of mathematics" (p. 191). Egan (1997) and Brown (2001)

reject the premise that making subject matter interesting and meaningful to students requires the need for it to be placed in real-life contexts. Egan (1997) suggests that school subjects need to connect with students' fascination with the limits of reality, their interest in heroic qualities that exceed their everyday lives, and their wish to connect with human intentions and emotions. As an example of pupils' interest in limitations, Egan points to their fascination with the human traits, information and numbers which abound in the *Guinness Book of Records.* Crespo and Nicol (2005) suggest two tasks as examples of activities that engage students' mathematical imaginations. The first is called the Mayan Dresden Codex. The codex artefact is a 3.5 metre band of paper which illustrates the Mayan place value system. In the activity pupils are exposed to Mayan culture and asked to work as archaeologists to discover the place value inherent in the scroll. Apparently, the Mayan numeration system operated in base twenty rather than base ten. The second activity is called Life in Flatland. It is based on a story written by Edwin Abbott in the 1800s under the pseudonym of A. Square. In this activity pupils are asked to imagine what life would be like in two dimensions. For example, pupils are asked to work in pairs whereby one pupil hides his or her eyes while the other pupil places a set of thin geometric shapes on the table. Then the first pupil bends down so that his eyes are level with the table and tries to distinguish the shapes. These activities show that although research continues into how best to solve textbook problems, other researchers as above have shifted the focus onto defining and promoting problems as tasks or activities which should engage pupils both intellectually and emotionally.

The Mathematics in Early Childhood and Primary Education Report (2014) promotes the use of fluid learning paths for pupils and includes the development of rich mathematical tasks which are challenging for pupils. These proposed solutions

remind me of Jaworski's Teaching Triad (1994) in that the teacher has to be conscious of how she manages the learning in her classroom so as to be aware of pupils' individual learning paths (sensitivity to students) and yet promote the use of rich tasks (mathematical challenge). In these circumstances the teacher is using the Teaching Triad as a tool to develop her practice. In this thesis I also use the Triad as an analytical tool to categorise teachers' practice as they attempt to move away from routine textbook tasks to engage in more challenging and varied mathematical activities, as was recommended in the Evaluation of Curriculum Implementation Report (2005). I am particularly interested in tasks which can be solved using various strategies or techniques. When pupils do routine textbook problems they develop an image of mathematics as being concerned with a single procedure or algorithm to obtain a correct answer. Therefore, as part of my reform agenda, I would like teachers to experiment with problems which have different solution paths and incorporate cognitive challenge as recommended in the Mathematics in Early Childhood and Primary Education Report (2014).

3.11 A description of Assessment for Learning

No discussion of mathematics pedagogy would be complete without reference to how mathematics should be assessed. Here I draw attention to a movement entitled Assessment for Learning (AfL). Such an initiative seeks to focus on the *process* of how pupils learn mathematics and not just the observable *products*. Often teachers will teach to the test to improve pupils' performance on paper without necessarily improving their mathematical understanding. Fortuitously, the National Council for Curriculum and Assessment (NCCA) currently advocates an AfL approach which takes the emphasis away from the type of summative assessments which occur at the end of an academic term. AfL is an initiative defined as "providing feedback to

learners on how to improve their learning" (www.ncca.ie). The idea is that assessment should not just happen at the end of an assignment (summative assessment) as a bolt-on but should be integral to the learning process. To ensure this Brown (2005) advises that "tutors should be able to concentrate equally strongly in giving feedback and on making evaluative decisions about performance" (p. 83). Therefore, information is not just gathered by the teacher but is shared with the learner on an ongoing basis throughout the teaching and learning process. AfL is not meant to replace summative assessments which will always remain in any education system. A metaphor could be that summative assessment is like taking a still photograph whereas AfL requires continuous film. It also requires that students are involved in assessing their own learning and are given strategies to assess how they are doing. One of the best methods of monitoring student progress and achievement is the use of judicious questioning by the teacher. What is envisaged is that the teacher's comments focus on what the student has done well and what needs to be improved, rather than listing errors which have been made. AfL involves the use of open or 'higher order' questions to apply what the student has learnt (e.g. Can you find similar patterns, themes or concerns in other areas of your work?) and to analyse what the student is learning (e.g. What makes you think that? Do you agree with this point of view?). The NCCA website uses the acronym HOT to refer to questions that encourage higher order thinking and LOT to refer to ones that promote lower order thinking. The latter are often closed questions (e.g. What is the sum of 26 and 15?) so I would also suggest using the acronym CLOT to refer to the closed lower order thinking which they inculcate, as it also conveys the lack of conversational flow inherent in such questions.

In my opinion, the type of ongoing or formative assessment envisaged by AfL sits well with a constructivist epistemology. AfL advocates the adjustment of teaching to take account of the results of assessment. If, for instance, such assessment takes the form of teacher observation and higher order questioning then the notion of scaffolding the pupil's learning so that she operates at her zone of proximal development becomes relevant. Indeed, the AfL section of the NCCA website states that students need to develop the capacity for self-assessment so that they can become independent learners with the ability to seek out and gain new skills, knowledge and understandings. Such an aim would be in line with a constructivist epistemology seeking to empower learners to take control of their learning.

3.12 Critique of a problem solving approach

According to Ross et al. (2002) such empowerment involves teaching *via* problems as opposed to teacher demonstration of specific problem solving methods. However, Sweller et al. remark that there is "no body of research based on randomized, controlled experiments indicating that such teaching leads to better problem solving" (Sweller *et al*. 2010, p. 1303). Moreover, Ross et al. (2002) emphasise teaching mathematical topics through problem-solving contexts and enquiry-oriented environments, which are characterised by the teacher helping students construct a deep understanding of mathematical ideas and processes by engaging them in doing mathematics. In her PhD thesis Kirwan (2012) quotes Kirschner et al. (2006) who state that underlying such a teaching approach is the expectation that students are challenged to solve authentic problems or acquire complex knowledge in information-rich settings, based on the assumption that having learners construct their own solutions leads to the most effective learning experience (Kirwan 2012, p. 224). Kirschner et al. (2006) point out that evidence from controlled studies almost universally supports direct, strong instructional guidance, rather than constructivist-

based minimal guidance. Sweller et al. (2012) go further in recommending that students need domain-specific schemas to become effective problem solvers and that mathematical problem solving skill is built up through the use of precise problem solving strategies, which are relevant to particular problems (Sweller et al. 2010, p. 1304). I believe that one of the great misnomers of constructivist approaches is the assumption that they automatically exclude direct instruction. Even the most ardent of constructivists would allow for some element of direct instruction and this issue has been discussed at length previously (see section 2.5). Advocates of constructivism believe that "the teacher's role is to challenge the learner's thinking… and not to dictate or attempt to proceduralize that thinking" (Savery & Duffy, 2001, p. 5) and request that instructional support not be provided unless there is "independent evidence that the learner cannot do the task or goal unaided" (Pea, 2004, p. 443). Moreover, such advocates refer to studies like that of Hmelo-Silver et al. (2007) "where students who construct solutions to tasks and problems achieve not only immediate learning but also longer-term transfer benefits" (Tobias & Duffy, 2009, p. 175).

3.13 Current questions

Brophy (2006) states that "social constructivist educators usually have much more to say about learning than about teaching" (p. 530). It is this gap in the literature that interests me. It is all very well to ponder on whether the curriculum comes from a constructivist or a social constructivist epistemology, but the real issue affecting teachers is how to teach in a constructivist-compatible approach whatever the epistemological variations. Such variations have been discussed in chapter 2. The strength of the literature to date is that there has been a strong focus on how children learn. The weakness is that there has been little focus on how teachers should teach

when they wish to adhere to a constructivist approach to teaching. Brophy (2006) 'hits the nail on the head' when he writes that "it is unrealistic to educate teachers to implement social constructivist principles without systematizing them into operational models of teaching" (p. 530). When I conducted a "deep see" trawl of the literature I was intrigued to find several authors who have tried to address the issue of constructivist-compatible *teaching* as opposed to *learning*. I now wish to give a brief summary of the work of such pragmatic authors.

Brophy (2006) outlines Graham Nuthall's seven principles for effective implementation of social constructivist teaching (See Appendix 1). What is interesting about his views is that they include aspects such as "ensure frequent repetition" and "repeat critical content" (p. 533). The interesting point here is that Nuthall does not see teaching as transmission as being mutually exclusive to a social constructivist approach.

Gagnon Jr. and Collay (2001) spoke of a constructivist learning design (CLD) "composed of six basic parts flowing back and forth into one another in the actual operation of classroom learning: situation, groupings, bridge, questions, exhibit and reflections" (p.xi). A more comprehensive outline is given as Appendix 2. The authors draw attention to the surface activation of students' prior knowledge before introducing them to new subject matter. They also stress the importance of the teacher providing questions, which instigate, inspire and integrate students' thinking and sharing of information. Teachers' questions usually fall into open or closed categories. This can be a useful indicator of whether pupils are being allowed to construct their own knowledge or being funnelled into the teacher's set knowledge.

Furthermore, Gagnon Jr. and Collay (2001) highlight the use of groupwork more than Nuthall does (Brophy 2006).

A third author who grappled with what a constructivist approach to teaching might entail is Jaworski. Her work was to have a profound effect on how I analysed my own classroom observations. In the classroom study, which formed the bulk of her PhD thesis, she found it useful to analyse teachers' modus operandi in terms of what she called "the Teaching Triad" (1996, p. 107). The three domains of the Teaching Triad were management of learning, sensitivity to students and mathematical challenge:

> Management of learning is manifested in a set of teaching strategies and beliefs about teaching which influence the prevailing classroom atmosphere and the way in which lessons are conducted. Sensitivity to students in inherent in the teacher-student relationship and the teacher's knowledge of individual students and influences the way in which the teacher interacts with, and challenges, students. Mathematical challenge arises from the teacher's own epistemological standpoint and the way in which she offers mathematics to her students depending on their individual needs and levels of progress.
>
> (Jaworski,1996, p. 108)

Jaworski has advanced my thinking in that she gives some guidelines as to what to look for in constructivist classrooms. It is useful to link her Teaching Triad categories with the advice given by Gagnon Jr. and Collay (2001). The following questions come to mind as regards observation of teachers:

1. Does the teacher manage learning in such a way that the classroom atmosphere is conducive to optimum learning? In this regard Gagnon Jr. and Collay (2001) would suggest the use of groupwork.

2. Is the teacher sensitive to pupils' needs? Gagnon and Collay (2001) would suggest helping the pupils to build mental bridges to enable them link their prior knowledge with the new subject matter.

3. Is there challenge for the pupils mentally in the work undertaken? It is here that Gagnon and Collay (2001) suggest that questions need to be inspiring for pupils and helpful to them in integrating their thinking.

A fourth set of authors endeavouring to shed some light on constructivist teaching approaches is Simon and Schifter (1991). They engaged in the Educational Leaders in Mathematics (ELM) Project (1989), which was conducted by the Summer Math for Teachers Programme at Mount Holyoke College, Massachusetts. The first aim of the project was "to create an innovative service program for precollege teachers of mathematics" (p. 309). A second aim was to study the effects of this program on teachers' thinking and practice. What is interesting, from a research methodological viewpoint, is how the authors "evaluate teachers' implementation of instructional strategies learned in ELM and their use of a constructivist view of *learning* as a basis for their instructional decisions" (p. 323).

To assess implementation of strategies, ELM adapted the Levels of Use (LoU) measure, developed by Hall et al. (1975), which consists of a structured interview and a five level classification scheme for rating teachers' responses. The five levels

were named: non-use, mechanical use, routine, refinement and integration. What is more interesting, from my viewpoint, is that ELM developed a new instrument, the Assessment of Constructivism in Mathematics Instruction (ACMI), which has a parallel format to the LoU. ACMI data were obtained during the same interview session and were rated according to the classification scheme shown as Appendix 3. Although the classification scheme is quite general, it does show that in following a constructivist epistemology a shift is required, whereby the focus moves from how the teacher is behaving in the classroom to how the pupil is learning. At the highest level (level V) collaboration among teachers is advocated as a way of advancing learning. This is certainly a challenge for me in this study as I seek to investigate teachers moving towards adopting constructivist-compatible pedagogies; firstly as individuals, and then as part of a community of practice.

LoU ratings were based on nine strategies, which were modelled during ELM instruction. The strategies are included as Appendix 4. As could be expected Simon and Schifter (1991) found that "changes in teaching strategies were more easily and more rapidly made than changes in teachers' views of learning with its concomitant effect on instruction" (p. 327). In other words, it is easier to bring about instructional shifts than philosophical ones. This is allied to the proviso that Fosnot (2005) brings to the discussion. She argues that "reform-based pedagogical strategies can be used without the desired learning necessarily resulting" (p. 279). This is because constructivism is a theory of learning, not a theory of teaching, and many educators who attempt to use such pedagogical strategies confuse discovery learning and "hands-on" approaches with constructivism. For instance, children may be observed engaging with a mathematical problem using manipulatives. However, this does not necessarily mean that they are operating at their zone of proximal development.

Holt-Reynolds (2000) gives another illustration of this misconception when she describes a prospective teacher named Taylor. In her classroom Taylor used active learning methods such as encouraging pupils to offer their opinions during English literature class. However, such opinions were not challenged by the teacher or other students. Taylor made the mistake of equating participation with learning. She needed to "see constructivist pedagogies as techniques for teaching, not merely as strategies for activating kids" (Holt-Reynolds (2000, p. 30). In other words, activation is an essential aspect of extrinsic motivation but it does not follow that activation will ensure pupils are cognitively challenged. Such challenge is at the heart of reform mathematics which seeks to move pupils beyond the mundane problems inherent in school mathematics textbooks. The primary curriculum and Project Maths programmes aspire to a 'minds-on' and not just a 'hands-on' approach to mathematics. Gagnon and Collay (2001) use the phrase 'mental bridges' to refer to the linking of prior knowledge with new subject matter. However, this needs to be an ongoing process so that pupils are constantly reinventing and reinterpreting their knowledge. Certainly, the use of sociocultural tools, like calculators, computers and the internet have a role to play in helping pupils to expand their 'mathematical horizons', to quote Ball's (1993) phrase. It can be seen that I am trying to weave several authors' writings into a constructivist framework which would enable me look at current practitioners' classroom teaching in terms of its relevance or otherwise to constructivist theory.

3.14 Summary

In summary, this chapter commenced with an overview of constructivist research on the teaching of mathematics internationally and in Ireland from a problem solving perspective. I looked at Piagetian and Vygotskian epistemologies in the old and revised primary mathematics curricula respectively. I outlined the serious lack of

guidance in the curriculum documents for teachers on how to implement and assess constructivist pedagogies. The contribution of several authors to such implementation has been considered. These authors include Brophy (2006), Gagnon Jr. and Collay (2001), Jaworski (1996b), Simon and Schifter (1991) and Fosnot (2005). In the next chapter I focus on the research design and methods required to investigate constructivist approaches in the classroom.

Chapter 4: Choosing the research genre and resultant data collection methods and analyses

4.1 Introduction

In chapter two, I proposed that teachers should teach to the ‘‘big ideas’’ if they wished to adopt a constructivist approach to teaching mathematics. I suggested that such an approach involves taking a fluid stance on what constitutes curriculum content, a quest to make activities mathematically authentic and a desire to focus more on process, rather than on content, as product in mathematics. In this chapter I will focus on the research methods required to explore constructivism. I reassert the usefulness of Jaworski’s (1994) Teaching Triad in categorising constructivist teaching. I describe my research as coming under Borko’s Phase 1 classification. I also compare my research to a fledgling design-based research approach. I use the adjective fledgling as my research lacks the iterative cycles involved in pure design research. I discuss the ethical considerations to which I had to adhere in conducting the research. The issues of validity, reliability and generalisability are also considered. Since I am approaching this study from a constructivist perspective, the problems associated with taking such a view are outlined. Finally, at a more practical level, I outline the data collection methods, types of analyses and state the limitations of this study.

4.2 My motivation for conducting this research

In 2005 the NCCA found that whole-class teaching was the organisational setting, which teachers reported most frequently using to support the Mathematics

Curriculum, followed closely by individual work. Limited use of pair work or group work was reported by teachers. This finding showed that although teachers had been given in-service during the school year 2001-2002, the organisation of classrooms had, by and large, not changed in the intervening three years to take account of the group investigative approach originally envisaged in the Teacher Guidelines. As one who acted as a tutor delivering the in-service in 2001-2002 I believed this finding to be very disappointing. Indeed, it has acted as a catalyst for me to carry out this research on how to enable teachers bridge the gap between a teacher-led whole-class approach and a teacher-as-facilitator investigative stance. In chapter two, I outlined a rationale for stating that the two approaches need not be mutually exclusive. Even within an investigative approach there will be occasions when the teacher has to address the whole class. For instance, the teacher may wish to revise some prior concepts, which she knows from experience the children need to solve a particular problem. There is a danger that in wishing to promote a constructivist approach the benefits of teaching as telling are dismissed (The Cockcroft Report-Mathematics Counts 1982, Love and Mason 1995). Indeed, I discussed this issue at length in section 2.7. The issue arises as to how best to get teachers to move along a constructivist trajectory without feeling they have to ditch all they held sacred in terms of whole-class teaching.

4.3 Categorising constructivist teaching: paradigm and assumptions

As the teachers in this study will create their own constructivist paths I am acknowledging that in ontological terms there will be multiple, socially constructed realities. In terms of epistemology I can foresee an interactive link between myself as researcher and the participant teachers, especially when this interplay involves possible changes of practice for the teachers involved. As regards methodology the

research will be primarily qualitative, dialectical in approach with 'thick description' of the contextual factors influencing the teachers' work. These assumptions should convey that I wish to work within the constructivist paradigm. From a sociocultural perspective it should be interesting to track how the relationship between the teacher participants and me changes over time and how it impacts on our evolving views of constructivism.

As I explained in chapter two different authors have chosen different ways to categorise constructivist teaching for research purposes. Jaworski (1994) is the author whose framework I draw upon in conceptualising teachers' mathematical practice. She used the term 'Teaching Triad' to refer to three domains, which in her view, captured the important elements of such teaching. The domains were named as 'management of learning', 'sensitivity to students' and 'mathematical challenge'. Jaworski (1994) elaborates as follows:

> Management of learning is manifested in a set of teaching strategies and beliefs about teaching which influence the prevailing classroom atmosphere and the way in which lessons are conducted. Sensitivity to students is inherent in the student-teacher relationship and the teacher's knowledge of individual students and influences the way in which the teacher interacts with, and challenges, students. Mathematical challenge arises from the teacher's own epistemological standpoint and the way in which she offers mathematics to her students depending on their individual needs and levels of progress (pps. 107-108).

The three categories are individual in identity, but are closely interrelated and Jaworski (1994) believes they have the potential to describe a complex classroom environment; provided the teacher involved is working to a constructivist agenda. Before I could analyse the data emerging on a constructivist-compatible approach to teaching I decided to take advice. In order to further develop my understanding of the origins of her work I contacted Barbara Jaworski by email, indicating my admiration for, and my use of, her work on constructivism. In 1991 she completed a

PhD thesis with the Open University entitled *Interpretations of a Constructivist Philosophy in Mathematics Teaching.* This study became the basis of her 1994 book called *Investigating Mathematics Teaching: A Constructivist Enquiry.* She generously agreed to speak to me by telephone and I rang her on 24th January, 2012 at 11.20am. We spoke for about thirty minutes. She gave the background to her own research on constructivism in the late 1980s in British classrooms. She stated that contextual factors are crucial. By this, she meant that it was easier in the late 1980s for teachers to adopt a constructivist approach to their work, as the curriculum was less prescribed than it is now. She raised a concept for me called *intersubjectivity;* a concept upon which she had elaborated in work she co-authored with Potari (2009). Jaworski and Potari (2009) state that in some studies of classroom interaction, the social dimension of learning has been seen in terms of intersubjectivity between participants (Cobb, Yackel & Wood, 1992; Jaworski, 1994; Steinbring, 1998; Voigt, 1996), a position which has been criticised by Daniels (2001) as limiting analysis. Daniels (p. 86) quotes Wertsch and Lee (1984) who comment that many of the psychological accounts, which attempt to discuss factors beyond the individual level "tend to equate the social with the intersubjective". The resultant criticism is that "the research focus stays within the interaction itself and does not address wider sociological factors with respect to which the interaction is meaningful" (Jaworski and Potari, 2009). However, intersubjectivity should be perceived as deeply sociocultural in its demonstrations- "a function of the setting, the activity, the actors, the texts, and so on" (Lerman, 1996, p. 137). Lerman argues for an integrated account, which brings the macro and micro together to enable us examine "how social forces, such as a liberal-progressive position, affect the development of particular forms of mathematical thinking" (Lerman, 2001, p. 89). He cites Wertsch, del Rio, and Alvarez when he states that "the goal of a sociocultural approach is to

explicate the relationships between human action, on the one hand, and the cultural, institutional, and historical situation in which this action occurs, on the other" (Wertsch, del Rio, & Alvarez, 1995, p.11, cited in Lerman, 2001, p. 89). Moreover, Jaworski (2009) suggests a unit of analysis comprising structures and systems on one level and daily classroom practices on the other. She quotes Engestrom (1998) who highlights "the middle level between the formal structure of school systems and the content and methods of teaching" (Engestrom, 1998, p.76 cited in Jaworski and Potari 2009, p. 221). Engestrom refers to this middle level of analysis as the 'hidden curriculum' which includes:

> grading and testing practices, patterning and punctuation of time, uses (not contents) of textbooks, bounding and use of the physical space, grouping of students, patterns of discipline and control, connections to the world outside school, and interactions among teachers as well as between teachers and parents.
>
> (Engestrom, 1998, p. 76, original brackets)

It can be surmised that embracing such a definition of the 'hidden curriculum' leads to a questioning of school and educational systems, as well as the place of family and friends in national political and economic systems. It is no surprise that Jaworski (2012) has broadened her research interests into activity theory. In my conversation with Jaworski, I asked if her Teaching Triad could be regarded as a tool for analysing classroom practice in constructivist situations. I was attempting to narrow the categories under which I might analyse such practice. Jaworski confirmed that her Teaching Triad could be used as an analytical tool. Jaworski (1994) found one counter-example in her research. When she tried to categorise the teaching of a teacher named Simon she failed to do so in terms of the Teaching Triad. She argued that this was because Simon was bound up in a transmission view of teaching, which did not allow for considerations of individual learners beyond their responses to what *he* offered. Furthermore, Simon's "planning and presentation of lessons

seemed to indicate an absolutist view involving the existence of invariant concepts, which it was his task to deliver, rather than of personal concepts which individuals could be encouraged to develop, share and negotiate" (pps. 183-184). The teacher's view seemed to be that knowledge was a fixed, immutable product, which could be communicated easily. It is here that I wish to refer back to 'teaching as telling' as outlined in the previous chapter. In a constructivist approach 'teaching as telling' can be somewhat justified as long as the teacher has the long term aim of engaging the pupils in an investigative approach to mathematics, which incorporates respect for their views on *what* mathematics to pursue and *how* it is to be pursued. Therefore, the issue is one of epistemology. The constructivist teacher allows for individualistic ways of knowledge construction, whereas the teacher as transmitter views knowledge acquisition as a fixed process requiring little choice on the learner's behalf. However, the caveat is that even the constructivist teacher will need to tell pupils information on occasions, because she knows that this will lead to further knowledge generation on pupils' behalf. The need to tell pupils information rather than let them discover it for themselves is often driven by time and curricular constraints placed on teachers. Such constraints may take the form of external assessments, such as standardised testing or entrance assessments to second-level schooling.

Another author who analysed teaching in Ireland from a constructivist perspective for his PhD thesis is O' Shea (2009). His PhD thesis was entitled *Endeavouring to teach mathematical problem solving from a constructivist perspective: The experiences of primary teachers.* O' Shea (2009) analysed the teaching of five teachers over a four month period in 2008. He adopted a case study approach. What

is interesting and informative for my work is O' Shea's (2009) analysis of teachers' mathematical practice in terms of two main categories:

1. *Teachers' didactic teaching style*

This included teachers' emphasis on rote memorisation of number facts, and their direct transmission approach to teaching problem solving, allowing pupils little freedom to construct their own methods.

2. *Teachers' constructivist approach to teaching and learning*

This was the category where teachers showed evidence that they encouraged pupils to choose a problem solving strategy and to justify it to the teacher and other pupils.

O'Shea (2009) found that themes emerged in his five individual case studies and across the case studies as a whole. These were "a focus on rote memorisation, mathematical problem solving from a constructivist perspective, as enrichment activity, and teaching students with different learning abilities from a constructivist perspective" (p. 91). He states that although the mathematics curriculum espouses constructivist principles, and he delivered in-service to the teacher participants, which reflected those principles; "the traditional understanding of and approach to mathematics teaching that was characteristic of the teachers inhibited the acceptance of a constructivist approach" (p. 244). O' Shea's (2009) research shows that it is extremely difficult to categorise teaching in absolute terms; as being purely constructivist, with the teacher acting in a facilitative role, or as being solely didactic with the teacher adopting a delivery mode. He found that all five teachers exhibited characteristics of both approaches. Therefore, in this research, I wish to remain open-minded to the possibility that teachers exhibit characteristics of both a constructivist and a didactic approach. In engaging with teachers' professional development Borko (2004) offers a model which I propose to use in this research as it encapsulates the approach I wish to adopt.

4.4 Borko's models of professional development systems

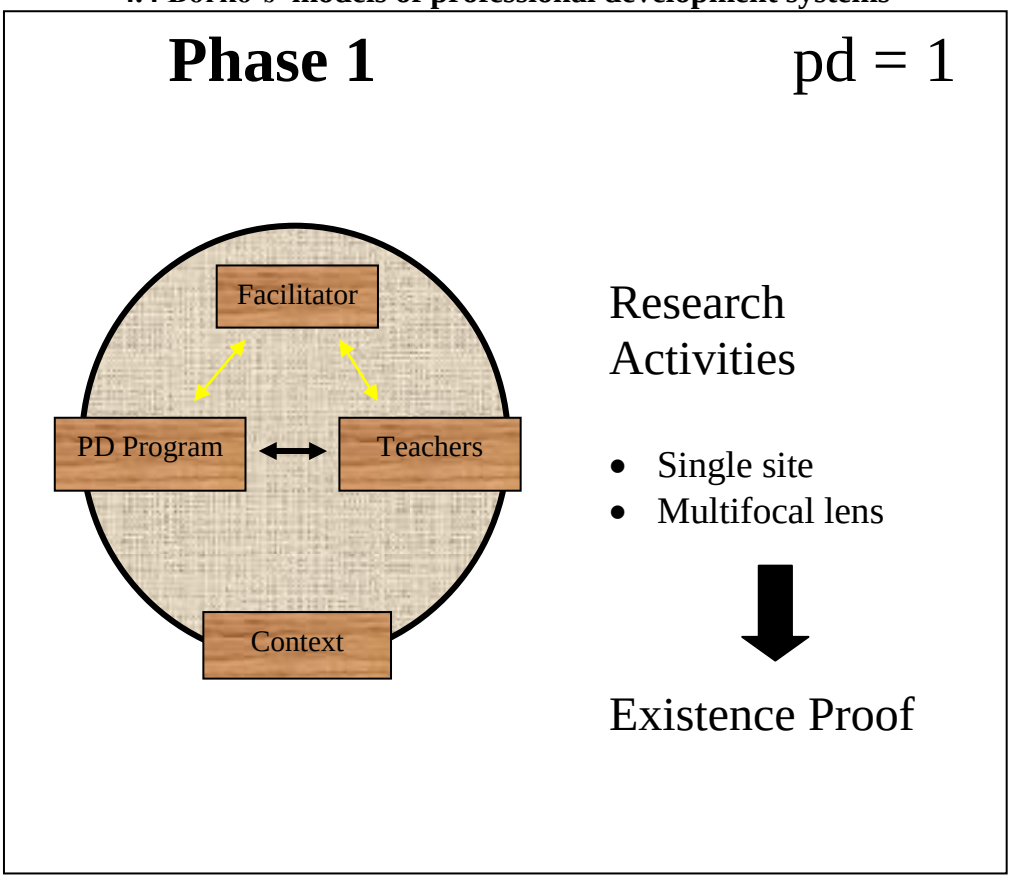


Figure 7: Elements of a Phase 1 professional development system (Source: Borko 2004)

Borko (2004) uses Figure 7 above to illustrate the key elements that make up any professional development system with the goal of creating an existence proof. An existence proof is defined as evidence that a professional development programme can have a positive impact on teacher learning. The key elements of such a system are:

- The professional development programme (represented as pd = 1)
- The teachers, who are the learners in the system
- The facilitator who guides teachers as they construct new knowledge and practices
- The context in which the professional development occurs

Phase 1 research activities focus on an individual professional development programme at a single site. Researchers usually study the professional development

programme, teachers as learners, and the relationships between these two elements of the programme. Borko (2004) states that the context and the facilitator remain unstudied ; although it has to be added that these elements have also to be described. In Phase 2, researchers study a professional development programme carried out by two or more facilitators at two or more sites, examining the relationships among facilitators, the professional development programme, and teachers as learners. It is interesting that Borko (2004) omits exploring the relationships among facilitators and among teachers as entities in themselves. Therefore, in Phase 2 the third element of facilitator is added. In Phase 3, the research lens widens to comparing multiple professional development programmes, each taking place at multiple sites. The researchers study the relationships among all four elements of a professional development system: facilitator, professional development programme, teachers as learners, and context. Therefore, in Phase 3 context is added as a fourth dimension. Again, as in Phase 2, the relationships among facilitators and among teachers as entities in themselves are not accentuated.

I now wish to elaborate on my research in terms of how it relates to Borko's Phase 1 research design.

- The professional development programme contains the following stages:

a) Four teachers are asked to familiarize themselves with the literature on constructivism in the Mathematics Curriculum (1999) and Teacher Guidelines (1999) Handbooks.

b) The teachers are interviewed to ascertain their views on the implications of constructivism for the mathematics classroom.

c) Teachers are asked to devise four lessons which would encompass such views in the mathematics classroom. As facilitator I would provide assistance in this

regard. Such assistance would involve providing relevant literature and advice where needed.

d) Teachers enact each lesson and each one is videotaped. At the end of each lesson, teachers load the videotapes onto their laptops to assist them in reviewing one lesson with a view to devising the next one. Teachers also review each lesson through a written reflection.

e) Teachers are interviewed for 5-10 minutes after each lesson and each interview is audiotaped. The purpose of the interview is to discuss the extent to which the lesson was constructivist-compatible and to explore the possible format of the next lesson. In each lesson teachers are encouraged to move beyond the confines of traditional textbooks and to choose activities which are open-ended in terms of the number of possible solutions or have various solution strategies. For instance, finding 35% of 200 has only one solution but various solution paths, whereas asking pupils to use percentages to compose a problem with 70 as the answer is open-ended. These activities are in line with the reform agenda set by Ross et al. (2002), the Evaluation of Curriculum Implementation Study (2005) and the Mathematics in Early Childhood and Primary Education Report (2014).

f) At the end of the project the teachers are interviewed as a group to determine if their views on constructivist-compatible approaches have evolved or changed. One year on, the group is reinterviewed to determine if the project has had any lasting impact on teachers' views of constructivist-compatible approaches.

- The four teachers, as learners in the system, are described as follows:

a) Anita qualified from Marino Institute of Education, Dublin in 2004 with a B.Ed. degree. In this project she was teaching in excess of 22 pupils in 6th class (12 year olds) in an all girls' school with disadvantaged status on the north side of

Cork city. She came highly recommended by her principal and when I spoke to her she was a willing volunteer for the project. She came to the project late as she replaced a teacher called Clarissa, who had initially volunteered, but had to drop out as she obtained a teaching position in another school.

b) Lisa qualified from Mary Immaculate College of Education, Limerick in 2005 with a B.Ed. degree. In this project she was teaching in excess of 22 pupils in 5th class in the same all girls' school as Anita. She also came highly recommended by her principal and when I asked her she became a willing volunteer for the project.

c) Claire qualified from Mary Immaculate College of Education, Limerick in 2006 with a B.Ed. degree. In this project she was teaching 23 pupils in 5th class (11 year olds) in a mixed-gender school in an affluent suburb of Cork city. Her principal had a high opinion of her teaching and when I spoke to Claire she was willing to take part in the project.

d) Aoife qualified from Mary Immaculate College of Education, Limerick in 2002 with a B.Ed. degree. She was the most experienced teacher participating in the research. In this project she was teaching 26 pupils in 5th class (11 year olds) in the same mixed-gender school as Claire. Indeed, the two teachers worked next door to one another and shared the same prefabricated building. Her principal had great praise for her teaching and when I spoke to Aoife she was more than willing to take part in the project.

- I, as facilitator, can be described as having a long background in teacher inservice. From 1992-1994 I co-delivered a summer inservice course entitled 'Developing Mathematical Thinking' under the auspices of Cork Education Support Centre (CESC). I also co-delivered a summer inservice course (1995-1999) entitled 'Learning Difficulties in Maths' which was a joint INTO/DES

initiative. After the mathematics curriculum was revised in 1999 I became a tutor with the Primary Curriculum Support Programme (PCSP) from 2001 to 2003 while on secondment from my role as principal of St. Patrick's Boys' N.S., Gardiner's Hill, Cork. This role not only involved delivering inservice but also visiting schools to assist in the implementation of the active learning methodologies envisaged in the revised curriculum. In 2003 I was seconded to Mary Immaculate College of Education, Limerick to lecture in the area of mathematics education. It was there that my interest in constructivist theory really deepened and this developed into a desire to do a PhD in the area. In 2007 I returned to my former role of principal of St. Patrick's Boys' N.S. and I have remained there since.

- The context of the research revolves around the implementation of the revised curriculum which is meant to espouse constructivist principles. The question arises as to how such principles impact, if at all, on classroom practice. If the answer is in the negative, then a second question arises as to what can be done to improve the situation. To assist in this research I have engaged four senior class primary teachers; one in 6th class and three in 5th class, as outlined above. The research site is the suburban area of Cork city. Two teachers are based in a disadvantaged primary school while the other two are based in a school in a far more affluent area. By pure coincidence, the four participant teachers are female.

4.4.1 Phase 1 existence proofs of effective professional development

Borko (2004) states that in Phase 1 systems researchers investigate the nature of the professional development programme, teachers as learners, and the relationship between teachers' participation in professional development and their learning. That is certainly a good summary of the projected path of this research. I intend to explore

the professional development programme itself, adapting it as necessary to suit the learning needs of the individual teachers. My emphasis is on how teachers can best organize their classrooms to be sensitive to pupils' needs and yet introduce children to appropriately challenging mathematical material. Jaworski's Teaching Triad is my frame of reference and I am cognizant of Vygotsky's (1978) construct of the zone of proximal development or zo-ped. The difficulty for teachers lies in introducing tasks which are challenging but not to the extent that pupils cannot identify or engage with them. My mission lies in providing any assistance the teachers require in devising such tasks. If at all possible, I do not wish to devise tasks for the teachers; preferring to let the teachers decide on what constitutes an appropriately challenging task for the cohort of pupils in their care.

Borko (2004) comments that most of the professional development community's work has been in Phase 1 research. These programmes are described as being relatively small but labour intensive. In most cases, the designers of the programmes are also the researchers and the participants are typically 'motivated volunteers' who wish to try out new ideas. I can certainly identify with Borko's comments in this regard. She comments that the resulting existence proofs are an important contribution to the field. Shulman (1983, p. 495) elaborates by stating that such projects "evoke images of the possible… not only documenting that it can be done, but also laying out at least one detailed example of how it was organized, developed and pursued". Such detail provides an important rationale for the inclusion of such projects in the research literature in the first place.

4.4.2 Differences between Phase 1 and Phase 2 programmes

Phase 2 research activities proceed directly from Phase 1. They are meant to consist of well-specified professional development programmes. Such programmes typically include materials and activities for teachers, descriptions of facilitator roles and teacher outcome measures. An example from the Irish context would be the introduction of Project Maths after the programme had been piloted in a sample of secondary schools at Phase 1level. Project Maths is discussed in more detail elsewhere in this thesis. Borko (2004) states that the main aim of Phase 2 research is to find out whether a professional development programme can be enacted with integrity in different settings and by different professional development providers.

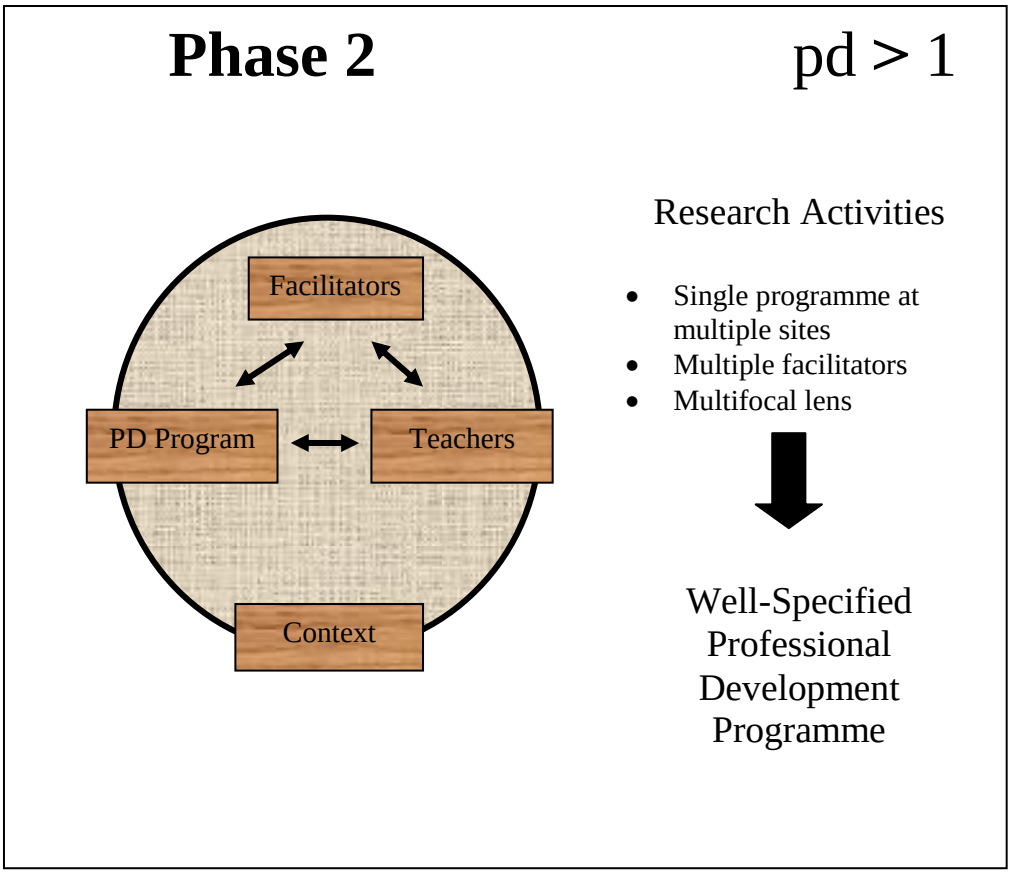


Figure 8: Phase 2 research Well-specified professional development programmes (Source: Borko 2004)

Figure 8 above gives a visual representation whereby "pd > 1" represents the focus on multiple sites and facilitators. The phrase 'enacted with integrity' is not meant to

imply the rigid implementation of a specific set of activities and procedures. There has to be "mutual adaptation", a term used by Berman and McLaughlin (1978), to account for the ways in which professional development programmes and their users change during the process of implementation. As they attempt to increase the scale of programmes, designers face a dilemma. On one side of the coin, mutual adaptation to local needs and conditions is essential if a programme is to be implemented effectively; on the other side, too much adaptation can mean that the overall intent of the programme is lost. Therefore, Phase 2 studies involve a tradeoff between fidelity and adaptation. They have to determine which elements of a programme need to be preserved to ensure the integrity of its underlying principles and goals. This complexity is not present in Phase 1 studies, like my own, as the programme is enacted in one location only by a single facilitator.

Another difference between Phase 1 and Phase 2 research activities occurs in the use of multiple facilitators. These facilitators will need pilot training in the use of programme materials, such as new curricular resources, so that they can anticipate problems which users might face in using such materials. They have to be able to respond when the goals of the programme appear to be in conflict with the expectations of the local participants. These problems occur in the scaling up of any programme. My research in Phase 1 involves designing problem solving activities for and with four teachers. It is a far more complex task to design such activities so that they can be used by several facilitators in different school contexts.

In this section I have compared Phase 1 and Phase 2 professional development programmes. The main differences occur in the scaling up of Phase 1 programmes to

allow for multiple facilitators and locations. The adaptation of any programme to suit local circumstances, without it losing its integrity, is a major challenge.

In setting a further research agenda, Borko (2004) suggests that Phase 1 research programmes should be explored for their applicability to other subject areas and grade levels. For instance, a mathematics programme with demonstrated effectiveness at primary level could be explored for its applicability to secondary level mathematics or to other subjects. However, I have to state that exploring mathematics programmes for their applicability to other subjects seems quite ambitious. Borko (2004) suggests that design experiments with their repeated cycles of design, enactment, analysis and redesign could be useful for exploring such applicabilities. Although, my own Phase 1 research programme is not well-defined enough to be labelled as typifying pure design research I believe the parallels should be explored and I intend to do that in the next section.

4.5 Motives for design-related initiatives

McCandliss , Kalchman and Bryant (2003) bemoaned the fact that U.S.congressional and Department of Education policy statements called for randomized controlled trials (RCTs) to be the primary source of scientific evidence relating to improving practice. They state that although traditional laboratory methods play a valuable role within a comprehensive approach to educational research, such policy statements can be counterproductive if they undermine support for methodologies - such as design research - that play a useful role in articulating the very questions and conjectures that serve as targets for randomized controlled studies. Brown (1992) envisioned a dynamic relationship between classroom-based and laboratory-based research, and her work provided specific examples of observations, conjectures, and artefacts that

might realistically be conveyed across these two research contexts. Brown's view is reinforced in the central organisational model for the National Science Foundation's Research on Learning and Education programme, which cultivates bi-directional flow of insights and agendas across research contexts. So for instance, investigations of brain systems and cognitive systems are seen as resting on a continuum with studies of social aspects of learning and learning in complex educational contexts.

The type of research in which I hope to engage is related to the genre of design research in such a complex context. I was interested in it because its raison d'être comes from the desire to increase the relevance of research for educational policy and practice. Studies in education need not be of purely academic interest. Design research has practical significance. Van Akker et al. (2006) state:

> By carefully studying progressive approximations of ideal interventions in their target settings, researchers and practitioners construct increasingly workable and effective interventions, with improved articulation of principles that underpin their impact. If successful in generating findings that are more widely perceived to be relevant and usable, the chances for improving policy are also increased. (p. 4).

Kelly and Sloane (2003) state that up to 1990 "there was little broadening of the scope of research methods in Ireland" (p. 30). They quote Sugrue and Uí Thuama (1994) in arguing that this paucity of research methods had "serious implications for the nature and conduct of research, the health of the teaching profession through in-service provision; for the quality of teaching and curriculum change; for the teacher educators and researchers as well as policy makers" (p.124). Therefore, a strong motive for engaging in design research is the need to link and improve both policy and practice. A second motive for design research comes from the need to develop empirically grounded theories, through combined study of both the process of learning and the means that support that process. If one adopts a sociocultural view

that accepts the need to better understand learning in context, then "research must move from simulated or highly favourable settings toward more naturally occurring test beds" (p. 4). A third motive relates to the need to improve the robustness of design practice. This can be done through making explicit the rationale for decisions made in choosing and establishing a design study. Such explicitness can advance subsequent design efforts.

It seems appropriate at this point to define what is meant by educational design research. Shavelson et al. (2003) describe design research as follows:

> Such research, based strongly on prior research and theory and carried out in educational settings, seeks to trace the evolution of learning in complex, messy classrooms and schools, test and build theories of teaching and learning, and produce instructional tools that survive the challenges of everyday practice. (p. 25).

It is worth noting here that the tools produced need not be concrete ones like an innovative set of manipulatives or piece of software, but could be a modified curriculum intervention or an altered assessment programme.

However, one of the difficulties in dealing with design research is that different researchers use different terminology to describe this emerging field. Akker et al. (2006) illustrate this point by giving four terms (not exhaustive) found in the literature:

- Design studies or design experiments;
- Development or developmental research;
- Formative research or formative evaluation;
- Engineering research.

4.6 Characterisation of design research

According to Akker et al. (2006) design research may be characterised as:

- **Interventionist:** the research aims at designing an intervention in the real world. My current research aims at developing an in-service provision for teachers which would assist them in employing constructivist-compatible methodologies in the classroom.
- **Iterative**: the research incorporates a cyclic approach of design, evaluation, and revision. It is not a case of 'anything goes'. My research would not involve a static intervention but one which evolves with participation and adapts accordingly. However, my research does not contain enough iterative phases in the intervention to be labelled as pure design research. In my research it is the individuals who construct their own trajectory. The intervention is loose and allows each individual to participate as they see fit in their own classroom.
- **Process oriented**: this implies that a black box model of input-output measurement is avoided; the focus is on understanding and improving interventions. I mentioned earlier that teaching to the ''big ideas'' in mathematics requires a focus on process-as-product anyway. In this research the intervention itself is kept under constant scrutiny. One way of looking at this is that the intervention is a case study in itself. As the intervention is under close examination it stands to reason that localised theories of instruction can be developed from it.
- **Utility orientated**: the merit of a design is measured in part, by its practicality for users in real contexts. This research will involve thick description of both the local school contexts and the teaching methods of the users or practitioners

involved. The intervention will be evaluated through a variety of data collection methods to be discussed later.

- **Theory orientated**: The design is (at least partly) based upon theoretical propositions, and field testing of the design contributes to theory building. I am fortunate in that this study hopes to elucidate constructivist approaches to teaching mathematics; especially as constructivism is the philosophy upon which the revised Primary Curriculum (1999) is based.

The LANDS study (2005) stated that guided discovery methods and the philosophy of constructivism underpin the mathematics curriculum, and the child is seen as an active participant in the learning process. Yet the NCCA (2005) evaluation of curriculum found that whole class teaching was still the norm in most schools. It is of real importance to ascertain which factors teachers perceive as either militating against their implementation of a constructivist approach or, to adopt a more positive stance, facilitate their implementation of a more child-active, discovery-based curriculum with an emphasis on process. Hence, there should be ample opportunities for local theory generation. This project will consist of a study of sixteen lessons (four teachers teaching four lessons each) . I note here that throughout this thesis I tend to use the terms ‘project’, ‘study’ and ‘research’ interchangeably to describe my work.

4.7 Ethical considerations

The teacher participants for this research were chosen in the following manner. I approached two principals I knew to see if any of their $5^{th}/6^{th}$ class teachers would be interested in volunteering to participate in the research as a way of developing their classroom practice through the use of constructivist-compatible approaches. I was

also hoping to learn more about such approaches. I did not enter this research with a set agenda in mind. However, in line with Jaworski's (2012) model of the expanded didactic triangle, as shown in section 2.3, I hoped that my experience as a teacher, principal and lecturer would be of some help to the participants in devising engaging and challenging activities for the pupils. Jaworski (2012) writes about the developmental processes which occur when teachers and didacticians inquire into classroom relationships. I certainly hoped that such development would be reflexive between me and the teachers. One danger is that the teachers would see me as some sort of expert willing to impart all they wished to know. This would be anti-constructivist and counterproductive for the generation of data on the developmental nature of the intended relationship. I wanted teachers to lay their own pathways to constructivist-compatible pedagogies.

As regards the principals involved, one was in charge of a designated disadvantaged school and the other was in charge of a school in a higher socio-economic area. I stressed to the principals that teachers' participation was to be entirely voluntary and that teachers could withdraw at any time. Thankfully, four teachers decided to participate in the project. One teacher withdrew from the project as she obtained a position in another school. However, I was fortunate that another teacher opted to take her place. Although I am a principal myself I did not choose teachers from my own school as it is possible that they would have felt obliged to take part in the research due to the power dynamic that exists between principal and staff. Gender was not an issue for me in choosing the participants as I was entirely dependent on who was teaching $5^{th}/6^{th}$ class at the time. As it transpired, all four teacher participants were female.

Any classroom research, which involves children as co-participants, needs to consider ethical issues such as informed consent. All parental and children's consent forms, which appear as appendices in this thesis, had to be vetted by the Social Research Ethics Committee (SERC) in University College, Cork (UCC) and hence they appear in their amended and final format. In my case the SERC made some very useful suggestions, which might also be helpful for other proposed researchers in the same area. For instance, the SERC suggested that the consent section for the pupils should not appear on the same form as that for the parents, as this would make it difficult for a child to refuse if their parents seemed happy to consent. The committee also advised that the right to withdraw from the research, without repercussions, should appear on all consent forms. The final version of the children's permission slip appears as Appendix 5 and the parent's consent form appears as Appendix 6. The letter of invitation to the teachers is called Appendix 7 and their consent letter is called Appendix 8. The protection of anonymity was another issue raised and all names in this thesis have been changed to avoid participants being identified. Obviously, this applies not only to the written thesis but also to original data transcripts. The committee suggested redrafting consent forms to give a more comprehensive view of the project, so that children would realise that samples of their work could be requested and that they could be asked to participate in an open group interview. As I proposed to use video technology to record lessons the committee again raised the issue of anonymity. I gave assurances that my supervisor, Dr. Paul Conway, and I would be the only people looking at the tapes, apart from the participants themselves, and that they would be destroyed within a three year period when analysis had been completed. I also redrafted and resubmitted my parental and children's consent forms. Having addressed all ethical concerns, I am pleased to report that I was granted approval by the SERC on 28th February, 2011 to conduct

this research. This approval is enclosed as Appendix 9. Teachers were interviewed individually via similar type questionnaires at the beginning (see Appendix 23) and end of the project (see Appendix 24). Also at the end of the project both teachers and pupils were asked for their views on the project via group interviews. The transcript of these interviews appears as Appendices 25 and 28 respectively. I chose this format as I believed participants would give their views more freely in a group, as opposed to an individual interview. The teachers were also interviewed via questionnaire (See Appendix 26) almost a year after the research finished in an attempt to ascertain if the project had had a long term impact on their views of practice. The transcript of their views appears as Appendix 27. I note that I was not evaluating the teacher participants' performance in the manner of a Department of Education and Skills inspector. Indeed, I was learning from the teachers about constructivist-compatible approaches just as they were learning from me. In that way, we were co-constructors of the report of this research and I am extremely grateful for their participation.

4.8 Validity

I wish to turn my attention to the issue of introducing validity to my work. Traditional experimental research introduces independent variables, which have little to do with either context or meaning. As a result, it can be extremely difficult to account for differences in findings when different groups of students supposedly receive the same instructional treatment.

Compared to traditional experimental research the challenge when conducting Borko's Phase 1 existence proof research is not that of replicating instructional innovations in exactly the same way in different classrooms. I mentioned that

teachers' professional growth is one of the goals of this intervention. One has to conceive of teachers as professionals who will adjust their plans based on their perceptions of pupils' understanding. Therefore, complete replicability is neither desirable nor, perhaps, even possible (Ball 1993; Simon 1995). Van Akker et al. (2006) suggest that design research aims for ecological validity, that is to say, the description of the results should provide a basis for adaptation to other situations. The supposition is that an empirically grounded theory of how the intervention works facilitates this requirement. The development of a local instruction theory that underpins the local instructional sequence is paramount. This theory can then "function as a frame of reference for teachers who want to adapt the corresponding instructional sequence to their own classrooms, and their personal objectives" (Van Akker et al., 2006, p. 45). My hope is that this research would serve as such a frame of reference for teachers wishing to pursue constructivist paths.

Hoadley (2004) argues that the rigour of design-based research is founded fundamentally on a close alignment of theory, research, and practice; what he calls 'systemic validity'. An explicit articulation of how these three elements are represented and united in a particular investigation is required. My theory is that teachers would adapt their teaching methods, if they worked towards a constructivist philosophy. This research is grounded in the classrooms of teacher participants as they struggle with the implementation of a constructivist approach. I wish to look at their practice and design this project around their emerging perspectives on constructivism. Messick (1992) states that an intervention must also have 'consequential validity'. This is where the researcher articulates how the intervention in hand might make a difference in achieving a well-defined and valued pedagogical goal. Working towards a constructivist approach is certainly a valued

goal as it links with the philosophy which underpins the entire revised Irish curriculum (NCCA, 1999). For me, well-defined means teachers adopting (primarily) problem solving approaches in the quest for a constructivist aspect to their work. In commenting on such endeavours it is impossible to omit the subjective 'I' when writing about one's own research. It is probably undesirable also as it is in the interplay between researcher and participants that richness of data is achieved. Jaworski (1994) states that she often found it difficult in her classroom research to separate her own thinking, both theoretically and methodologically, from her data analysis and reporting of this analysis. However, she further comments that she included an overview of her thinking and its development throughout the research, which contributed to conclusions drawn. This is the approach that I also hope to adopt in this classroom research. Jaworski cites Burgess (1985) who describes this style of writing as "an autobiographical approach" (Jaworski, 1994, p.76). She believes that it is in the research biography that verification ultimately resides. She cites Ball (1982) as follows:

> The research biography is in effect a representation of the research process both in terms of an account of the internal validity of research methods, standing as an autobiographical presentation of the experience of doing the research, and in itself as a commentary upon these methods it stands as a source of external validity, as a critical biography, a retrospective examination of biases and weaknesses. The research biography also represents what Denzin, 1975, calls sophisticated rigour, a commitment to making data, data elicitation and explanatory schemes as visible as possible, thus opening up the possibility of replication or the generation of alternative interpretations of data.
>
> (Ball, 1982 cited in Jaworski, 1994, p. 77)

The constructivist trajectory of four senior class teachers forms the major part of this research biography. In dealing with this journey, it has been extremely difficult to separate my reporting of the research from some of the teaching issues encountered;

details on such issues weaves through the teachers' stories. As a result, the teachers' choices of content and evolving classroom teaching methods are presented in great detail. I now turn to the issue of reliability and generalisability of the data collected.

4.9 Triangulation, reliability and generalisability

For data to give a comprehensive overview of a phenomenon it needs to be collected from as many sources as possible. Therefore, the sources of data for this study include videotapes of classroom lessons, questionnaires, teacher participant interviews, group interviews with teacher participants, group interviews with children and field or journal notes on classroom observations of both teacher participants and children. Most qualitative researchers try to employ at least two methods of data collection, as above; hence the term triangulation. Yin's (2006) advice is that one will always be better off using multiple rather than single sources of evidence. However, I am not saying that having several sources of data necessarily means that findings are going to be more robust. For instance, questionnaires can contain items that portray researcher bias and notes from classroom observations can also show that a researcher is looking for evidence of particular aspects of a phenomenon to occur. Triangulation tries to confirm inferences made from the findings of several research methods and approaches. However, Smith (2006) states that it "is less a method than a troublesome metaphor" (p. 465). This implies that triangulation has its critics. For instance, Silverman (1993) comments that the very notion of triangulation is positivistic, and that this is exposed most clearly in data triangulation, as it is presumed that a multiple data source is superior to a single data source or instrument. Patton (1990) suggests that even having multiple data sources, particularly of qualitative data, does not ensure consistency or replication. Guba and Lincoln (1989) note that triangulation should

not be used to gloss over legitimate differences in interpretations of data. They state that such diversity should be preserved in the final report so that the 'voices' of the least empowered are not lost.

In this research I take the view that there are multiple, socially constructed realities and that therefore such realities should be described using as many data collection methods as possible. In particular, children's views should be represented. This coincides with working under the constructivist paradigm. Reinking and Bradley (2008) state that data collection and analysis that produce convergent evidence from multiple sources through multiple methods produce findings, interpretations and recommendations that are more trustworthy and convincing, and thus more rigorous. Bell (2004) comments that there is a complexity associated with learning and that therefore we might be best served by exploring how far theoretical and methodological pluralism will carry us in better understanding, promoting, and sustaining innovation in education.

The issue of generalisability is a difficult one in design-related research for there are times when one has to focus on the particularisability (Erickson, 1986) of an intervention, so that its applications in a particular context can become clearer. There are other times when one believes one has gathered and analysed sufficient data to make more general statements i.e. to attend to the issue of generalisability. Thus the researcher is in somewhat of a conceptual dyad, paying attention to both the particulars of an intervention, but also being mindful of factors which could have more general applicability. In this research, for instance, the teacher participants will work under the constraint of having to find the time to explore investigative activities while the pressure to follow a prescribed curriculum simultaneously looms

large in their minds. The strategies they devise to cope with such pressure will certainly have more general applicability to other classrooms. Design researchers 0like to view classroom occurrences as paradigm cases of broader issues. It is this framing of classroom activities as exemplars that gives rise to generalisability. Obviously, this is not generalisability in the sense that the characteristics of individual cases are shunned and they are treated as interchangeable instances of the set to which assertions are said to apply. Rather, the theoretical analysis produced when coming to comprehend one case is deemed to be pertinent when interpreting other cases. As van den Akker et al. remark:

> Thus, what is generalized is a way of interpreting and understanding specific cases that preserves their individual characteristics...It is this quest for generalisability that distinguishes analyses whose primary goal is to assess a particular instructional innovation from those whose goal is the development of theory that can feed forward to guide future research and instructional design.
>
> (2006, p. 47)

4.10 Problems adopting a constructivist perspective

I categorically state that I am approaching this research from within the constructivist paradigm, whose ontology recognises the existence of multiple, socially constructed realities. Such a perspective is not without its critics. For instance, Hammersley (1993) warns that:

> ...[constructivist] research reports should be judged in aesthetic terms, in terms of their political correctness and/or in terms of their practical usefulness. Certainly, they cannot be judged in terms of their *validity*, in the sense of how accurately they represent events in the world, because constructivism denies the possibility of this... it becomes unclear how [constructivist] research differs from *fiction* or ideology, or if it does, why we should prefer it to these.
>
> (Hammersley 1993, cited in Jaworski 1996, 2nd ed., original emphasis, p. 78)

However, it has to be stated that this research biography will contain details of classroom incidents, the context in which they occur, the teacher's interpretation of the events, the reasons for the events' significance in terms of constructivist theory, and the linking of analysis to my experience as a practitioner and researcher. Such evidence for interpretations justifies their validity. That is not the same as stating they are correct, but it does mean that they have a reasonable basis and are worthy of consideration by others. I could be accused of creating a Hammersley fiction if I relied on my interpretations alone. However, it is the placing of the events into their situational contexts as comprehensively as possible which presents the credibility. "A reader needs to know on what basis interpretations are made in order ultimately to judge the validity of what is presented" (Jaworski, 1996, 2nd edition, p. 79). For instance, presenting one example from a lesson would hardly suffice as proof that a teacher was adopting an investigative approach to her work. However, I intend to take examples of practice from four different lessons taught by each of the four teacher participants. It should therefore be possible to see a pattern of interactions in which pupils investigate mathematical situations and lay the groundwork for the grasping of concepts. As the leading author in this research states:

> It is then possible to take these practical manifestations of aspects of theory and flesh out the theory. Initial theory gives starting points for observation and selection. Episodes selected are rich in details, which the theory is too narrow to predict. From this richness patterns emerge which not only substantiate the theory but make clearer what such theory means in terms of the practice of teaching and learning. This enhanced theory can then be reapplied to further practical situations for substantiation and enrichment. This process embodies a symbiotic, or dialectical, relationship between theory and practice.
>
> (Jaworski, 1996, 2nd edition, p. 80)

It can be seen that the process is basically reciprocal. On the one hand, constructions belong to the researcher himself. On the other hand, the researcher acknowledges the

experiences and interpretations of participants and negotiates analyses and conclusions accordingly. This is analogous to a pupil connecting personally constructed knowledge with that described by her teacher and peers in order to negotiate new meanings. Therefore, the researcher's conclusions are the result of a socially constructive process. Their validity is manifested in the thick description of the process followed. Then it is the reader who has the task of judging how convincing the account is.

Jaworski (1996) states that the problem of talking about constructivist research rests not in judgements about fiction, nor in problems of justifying validity of conclusions. The label 'constructivist' points out that nothing can be commented about the actual 'truth' of conclusions; therefore validity must acquire some other meaning. I have written above about other types of validity, such as systemic and consequential. There is a difficulty in talking about constructivism as if it were practice. 'Constructivist teaching' has the same difficulty. It will be remembered that Schoenfeld (2006) described 'constructivist teaching' as somewhat of an oxymoron. Jaworski (1996) describes constructivism as a perspective, a philosophy, even a theory, but not a practice. However, what I have in common with Jaworski is that this research attempts to tease out some of the practical implications of holding a constructivist view of knowledge and learning. Such research is justified as otherwise constructivism runs the risk of becoming an inert theory.

4.11 Data collection methods and data analyses

I wish to turn to the issue of which data collection methods and data analyses would best suit this design research intervention. Firstly, I need to outline the framing of the research, as well as the timescale and participatory issues.

4.11.1 Framing the research: timescale and participants

Initially, I proposed to conduct the research over the four month term from September to December 2010. However, the logistical difficulties of attempting to visit four teachers for four lessons each, whilst doing justice to my own position as headmaster of a disadvantaged primary school, ensured that the research carried on for the entire 2010/2011 school year. As mentioned earlier, pseudonyms were used for all participants. Two were $5^{th}/6^{th}$ class teacher participants named Aoife and Claire from a large, middle class suburban primary school and two were $5^{th}/6^{th}$ class participants named Lisa and Clarissa from a medium-sized, disadvantaged suburban primary school, with a third teacher, Anita on standby. All teachers had between four and eight years teaching experience. Their participation was subject to all teacher and pupil participants signing an informed consent agreement. As it transpired, Clarissa was observed for one lesson but subsequently gained employment in another school. Therefore, no reference is made to the data from Clarissa's first lesson, as Anita took her place for the desired four lessons.

4.11.2 The variety of data collection methods and data analyses

The following data collection methods and analyses were employed:

1. Initial semi-structured interviews based on questionnaires

I interviewed each teacher at the start of the project to ascertain their views on constructivism based on pages I had asked them to read from the Mathematics Curriculum (p.5) and Teacher Guidelines (p. 3-4) as I could not take it for granted that the teacher participants had any previous knowledge of constructivist approaches. This was done using a pre-lesson interview (PLI) questionnaire (see Appendix 23) which I had piloted with a teacher called Jim (pseudonym). The questionnaire was used as a device to encourage teachers to talk about their views on

constructivism. The interview was semi-structured with questions open enough to allow for individual interpretation. To this end, I am grateful to Jim, as I discovered that my initial questions were too closed and did not give adequate scope for a participant to air his views fully. As a result, I included more questions of the 'to what extent?' and 'what do you think?' variety. Cohen, Manion and Morrison (2009) state that open-ended responses often show up the gems of information that might otherwise be lost through closed questioning. From an ethical viewpoint, they also comment that such responses put the responsibility for and ownership of the data much more firmly into respondents' hands. Jim did not later engage with the project as one of my teacher participants. The actual four participating teachers were also interviewed as a group at the end of the project in June 2011 using a similar, but not identical, questionnaire. This was called the exit interview (EI) questionnaire (see Appendix 24). This questionnaire was designed to evaluate if teachers' views of constructivist approaches had changed throughout the project; hence the similarity of questions to those in the pre-lesson interview. A third and final questionnaire (see Appendix 26) was applied in May 2012, almost a year after the project had ended, to ascertain if the project had had a long term impact (LTI) on the teacher participants' practice.

Oppenheim (1992) states that semi-structured interviews are designed to develop ideas and research hypotheses rather than to gather facts and statistics. This is particularly relevant when one is researching a topic as broad as constructivism. Cohen et al. (2000) comment that interviews enable participants to discuss their interpretations of the world in which they live and to express how they regard situations from their own point of view. The danger is that an interviewer can intervene with his own views and cause bias to emerge. I attempted to remain as

impartial as possible during the interview process. Transcribing and analysing such interviews is also extremely time consuming. As interviews can be subjective in terms of what is deemed relevant by the interviewer it is important that other sources of data are collected also.

In terms of analysis I colour coded participants' responses to questionnaires to ascertain if their views on constructivism had changed during the course of the project. These views were then categorised under the three headings of Jaworski's Teaching Triad: Management of Learning, Sensitivity to Students and Mathematical Challenge. It has to be stated that these headings overlap. An instance of such overlap from the questionnaires came when I asked participants for their views on scaffolding. I categorised such views under the heading Sensitivity to Students. It could be argued that scaffolding fits under either of the other two elements of Jaworski's Triad. Therefore, it can be seen that I had to categorise the data based on personal interpretation of which element suited best. In the pre-lesson and exit interview questionnaires I had to apply Jaworski's Teaching Triad as the analytical tool as reported in chapters 5 and 6. However, the long term impact questionnaire from May 2012 is blatantly Jaworskian in format as I had the benefit of hindsight and knew that the Teaching Triad categories were probably the most useful way of analysing the data.

2. Open post-lesson audio-taped interviews with videotaping of lessons as background.

As stated earlier, I observed each of four teachers for a total of four lessons each. Each lesson observed was videotaped using a Cisco Systems Flipshare Mino HD recording device. The advantage in using this device was that it could record sixty minutes of footage (adequate for most lessons) and had an attachment for loading

such footage onto a computer. The disadvantage with this model is that Cisco Systems has withdrawn from the market for selling these devices and as a result online support and updates ceased on 31.12.2013. Immediately after the observed lesson the teacher was interviewed for five to ten minutes. The only set question for the interview was 'how do you believe the lesson went for you?' The purpose of the question was to initiate the process of tracking the teacher participants' emerging views on constructivist approaches and to provide advice where needed. Here I took the counsel of Kvale and Brinkmann (2009, p. 167) who recommend that an interviewer should be open-minded so that (s)he "hears which aspects of the interview topic are important to the interviewee, listens with an evenly hovering attention, and is open to new aspects that can be introduced by the interviewee and follows them up". In terms of analysis these interviews were important to the design of the project as they determined the type of design intervention required. The analysis therefore consisted of identifying the topics of concern to the teacher whose lesson had just been observed and providing appropriate advice on how to proceed. For instance, if a teacher's concern was that the mathematical activity chosen was too closed I suggested a more open-ended task. Most of this advice was instantaneous but I had the benefit of reviewing the videotape later that day to see if any further advice was needed. This advice could then be communicated by phonecall or text message. The oral interviews were recorded using an Olympus VN-3000PC digital voice recorder. These post-lesson interviews (POLI) were an invaluable source of data because one of the participants, Aoife, chose not to write written reflections on her lessons and, as a result, I was dependent on her oral testimony for data collection. I had also intended that at a later stage both the observed teacher and I would look at the videotape together. In the Learner's Perspective Study (2001) Clarke had used an insightful means to gather a teacher's

views. Clarke asked the teacher to fast-forward the videotape to an aspect of the lesson they found interesting. This ensured that salient aspects of the lesson for the teacher (and not just the researcher) were highlighted. Unfortunately, the practical constraints of having to return to my school, as well as the participating teachers having to carry on teaching their pupils, meant that a dual viewing of the videotape was not possible. However, after each lesson I loaded the videotape onto the teacher's laptop and they were encouraged to view it and include their insights in their post-lesson reflections written in their teacher journals.

3. Teachers' written reflections

Teachers were also encouraged to keep notes on aspects of lessons which had proved insightful for them in gaining an understanding of constructivism. I had hoped that this would be a useful way of collecting data when teachers were teaching off-camera. However, the teachers chose to use their journals solely to record their written reflections on the videotaped lessons. This would later indicate that they did not engage extensively with constructivist approaches once the camera was not present. The abbreviation used for the reflections on the lessons is RL so, for instance, LRL1, refers to Lisa's reflection lesson 1. As regards analysis, the written reflections would later be categorised under Jaworski's Teaching Triad and form part of the synthesis of the research on teachers' evolving views of constructivist approaches presented in chapters 5 and 6. In real time the reflections helped the teachers and me to plan the format of the next lesson to be observed. These observational lessons were scheduled for times and dates convenient to both teacher participants and researcher.

4. Researcher's journal entries

I kept a journal and included comments in it for each lesson observed. In terms of analysis these comments helped me to co-design a constructivist path with each

individual teacher participant. Taking notes on classroom observations is not without its dangers. Cohen, Manion and Morrison (2007) warn that the presence of the observer might bring about different behaviours. Indeed, interviews with the children later confirmed this to be the case with some of the participant teachers. Furthermore, the researcher is in danger of 'going native' or "becoming too close to the group to see it sufficiently dispassionately" (Cohen, Manion and Morrison, 2009). Therefore, the notes I took were used in conjunction with my viewings of the videotapes to enable me to reflect on both the practice and theory of constructivist approaches and to assist me in deciding which questions to raise and the type of dialogue to pursue with each teacher participant in the pre- and post-lesson interviews. As a result, my views on teachers' engagement with constructivist practices evolved lesson by lesson. I could be described as a participant observer of the lessons without being an interactive participant in them. There were times that I wanted to intervene in the lessons e.g. to pursue a pupil's line of questioning. However, in line with Cohen, Manion and Morrison (2009) above, I believed that in doing so I would lose my own sense of perspective. The entries quoted in this research are appropriately dated.

5. Pupils' work samples

I believed that when the teacher participants gave their views on constructivism during the project validity would be enhanced if such views could be supported by relevant work samples from the pupil participants. These samples would help teacher participants to reflect on their practice and observe if pupil understanding was improving in line with the relevant changes in teacher work practices. In terms of analysis I chose the work samples based on the Vygotskian construct of the zo-ped to illustrate the mathematical challenge required of the pupils in the tasks

undertaken. Mathematical challenge is also one of the elements of Jaworski's Teaching Triad which appears in chapters 5 and 6 on data analysis.

6. Focus group interviews

In line with a social constructivist view of knowledge construction I had hoped to hold several focus group interviews throughout the project in which all teacher participants could share their views on constructivism. However, the logistics of getting all teacher participants together at the same time after school hours proved very difficult. Therefore, I managed to hold only one focus group interview at the end of the project in June 2011. At the very least, this interview yielded data on the challenges faced by the teacher participants as they engaged with constructivist-compatible approaches to their work (see Appendix 25 for a transcript of the interview). It also helped me to theorise on how best to prepare teachers for such a pedagogical assault on a constructivist obstacle course. However, I was able to hold a second focus group interview, almost a year later, in May 2012 to ascertain the long term impact (if any) on teacher participants' practice. I took the 2011-2012 academic year out of my studies due to my wife suffering a protracted illness. However, my supervisor, Dr. Paul Conway, suggested that a long term impact interview could be a useful way of getting back into my studies. I thought this was a good idea and set up the group interview. In referring to the data I used the abbreviation LTIQ to refer to the long term impact questionnaire (see Appendix 26) and the long term impact interview transcript appears as Appendix 27.

7. Pupil interviews

Seeing as constructivism is a theory of learning I believed it was imperative to obtain some of the pupil participants' views of the project. This was done by interviewing groups of three pupil participants, of varying abilities, from each of the four classrooms. The interview was open and in it I just asked the pupils for their

impressions of the project and by extension their views of teacher behaviour during the project. Here I took Kvale and Brinkman's (2009) advice to avoid "long and complex questions and posing more than one question at a time" (p. 146). I did not propose to interview children individually, as this could have been intimidating for them. Eder and Fingerson (2002) draw attention to the power imbalance which exists between teacher and pupil and stress the need for the interviewer to avoid being associated with the classroom teacher. Therefore, I interviewed the children away from the classroom but in a room familiar to them, such as the learning support room or the computer room. Cohen, Manion and Morrison (2009) state that group interviewing "encourages interaction between the group rather than simply a response to an adult's question" (p.374). They also surmise that it is less intimidating for children than individual interviews. This proved to be the case as the children smiled and appeared relaxed during the interviews. Perhaps, they were also glad to be avoiding classwork. In giving their consent for the project the children (and their parents) knew they could be asked to take part in a group interview but that their participation was entirely voluntary. In analysing the children's views of the project I again applied the Vygotskian construct of the zo-ped to ascertain if there had been challenge for the children in the tasks undertaken. It will be remembered that I also applied this lens in choosing children's work samples. As an aside, the children gave interesting insights into how teachers deviated from their normal behaviour during the project and these are reported in chapter 6. In Table 3 below I offer a summary of the abbreviations I used in the research process and in Table 4 I give a chronology of the data collection. Please note that abbreviations may be combined also; so for instance, CPOLI4 would mean Claire's post-lesson interview for her fourth lesson.

Table 3: A list of data collection abbreviations

Abbreviation	Meaning
A	Anita
AO	Aoife
C	Claire
EI	Exit Interview
GI	Group Interview
L	Lisa
LTII	Long Term Impact Interview
PLI	Pre-lesson Interview
POLI	Post-lesson Interview
RL	Reflection on Lesson

Table 4:Chronological data collection timetable

Chronological order of data collection	
Initial semi-structured interviews	School year 2010/2011
Videotaping of lessons with post-lesson discussion	School year 2010/2011
Teachers compile written reflections	School year 2010/2011
Researcher compiles journal entries	School year 2010/2011
Collection of pupils' work samples	School year 2010/2011
First round of focus group interviews	June 2011
Interviews with pupil-groups of three	June 2011
Second round of focus group interviews	May 2012

4.11.3 Overview of data analysis procedures

I have already referred to the variety of data sources. In approaching the study from a constructivist perspective, I was aware that the researcher creates the categories

and concepts through interaction with the field-grounded theory approach, which attempts to build categories and concepts emerging from the data (Glaser & Strauss, 1967; Strauss & Corbin, 1990). Charmaz (2000) recommended adopting more of a constructivist approach to grounded theory that recognises that the categories and concepts are not inherent in the data, waiting for the researcher to discover them. Instead, it is the researcher who creates the categories and concepts as the result of interaction with the field and the questions that are asked. I looked at what was built through a critical constructivist lens, so that current theory on constructivist-compatible approaches could be affirmed, denied or reformulated in some way. It could be said that the process of induction used to build the data had to be subjected to a process of deduction to see if the data elaborated on issues pertaining to constructivist theory. Such issues included pupils' engagement, as individuals or in groups, at an intellectually challenging level, as manifestation of working at the zone of proximal development. Also included would be the issue of how the teacher scaffolded the learning to reach such a high level of engagement. Computer assisted software was not used in the analysis of data gathered. Instead, I used colour-coding to cross-reference data collected from classroom observations with that collected from teachers' journals and interviews. The categories of Jaworski's Teaching Triad were used as the dominant overarching analytical tool. My rationale was that I believed that the categories of the Teaching Triad were broad enough to encapsulate any data generated and any software searches for key words or phrases would not adequately convey the complexity of the classroom interactions. As mentioned above, the data emerging from the use of the Teaching Triad as an analytical tool still had to be compared to the literature on constructivist theory for affirmation or denial. A summary of the data analysis process showing the emerging themes is given below in Figure 9. I will refer to such themes in an aggregate way in chapter 5.

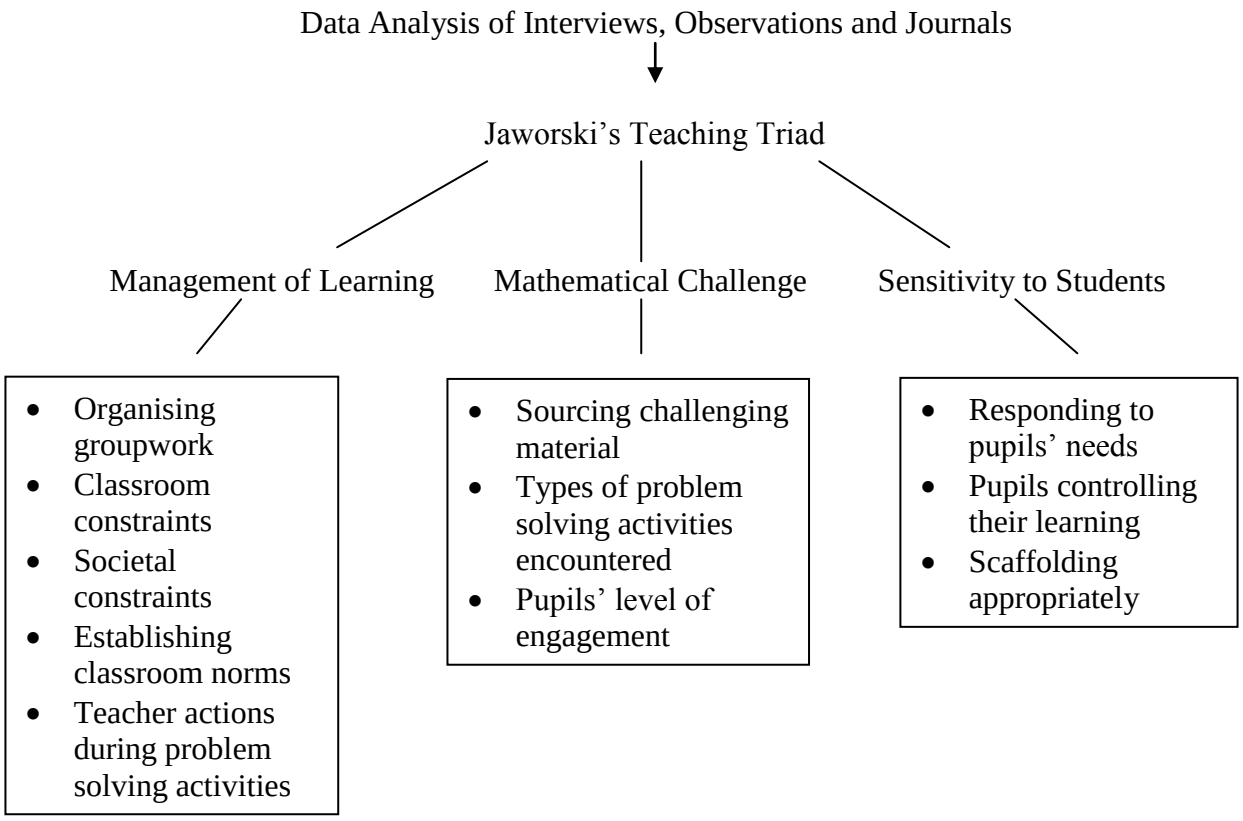


Figure 9: Data analysis of interviews, observations and journals

4.12 Limitations of the study

The study is designed to track the constructivist trajectory followed by four teacher participants in both an affluent and a lower socio-economic area over a one-year period. Initially, I had intended to carry out the research over a four month period as O' Shea (2009) had done, but this proved impossible given my own work commitments as headmaster of a primary school and the timetable constraints of the four teacher participants involved. Most Irish teachers like to teach mathematics in the mornings; when they believe pupils are more alert. This meant that afternoons had to be omitted as potentially suitable times to conduct classroom research. As I adhered to a constructivist paradigm, which recognises multiple, socially constructed realities the transferability of the data findings to other contexts would be limited. As mentioned earlier, I engaged with only four teacher participants; two in a disadvantaged setting and two in a middle class setting. However, I hoped that, in

general, other teachers would be interested in this research, as it sought to explore a constructivist-compatible approach, which underpins the primary curriculum.

4.13 Summary

In this chapter on methodology I have advocated exploring constructivist teaching under the genre of Borko's Phase 1 model of professional development. I compared Phase 1 and Phase 2 models with the former deemed more appropriate for this research. I have outlined the parallels with design research. I have decided to explore teaching at a micro level using Jaworski's Teaching Triad as an analytical tool. The issue of dealing with subjectivity in research was given particular attention. Different types of validity were mentioned; systemic, ecological and consequential. Reliability was defended through outlining the use of a variety of data collection methods and the limitations of the generalisability of the research to other contexts were indicated. Furthermore, my research question has now been refined to read as follows: From the perspective of Jaworski's Teaching Triad, to what extent do the participating senior primary class teachers adopt a constructivist-compatible approach to their mathematics pedagogy?

Chapter 5: Constructivism in the classroom: 'here comes the maths man'.

5.1 Introduction

In this chapter, I give a comprehensive account of the classroom research which took place using videotape evidence as background. I link classroom occurrences with constructivist theory for each of the four teacher participants: Lisa, Anita, Claire and Aoife, bearing in mind the emerging themes as outlined in chapter 4 under Jaworski's (1994) Teaching Triad analysis. In Lisa's 5th class I document pupils gaining ownership over their learning, the evolution of linkage, the need to increase complexity in lessons and how children, perceived as being of lower ability, can surprise teachers with their insights. I demonstrate that constructivist classrooms can be noisy places and that there is a need to focus on process as a product in itself. In Anita's 6th class I observe unanticipated solutions coming from pupils, how there is a need for a plenary session in maths lessons to share insights and I witness the development of a teaching methodology, which I call 'harvesting and sowing'. Anita learns where to intervene in lessons and where to lower the cognitive challenge to keep motivation constant. For Claire in 5th class the issue of maintaining progression in lessons dominates. She learns that introducing too many problem tasks in a lesson does not cultivate deep understanding of such tasks. There are potential opportunities on how to teach pupils to generalise patterns and strategies. She learns that there needs to be a balance between the individual and social construction of meaning in establishing classroom groupwork. For Aoife in 5th class the main issue which arises is teacher dominance. Aoife finds it difficult to hand control of learning over to the pupils. In lesson three her pupils give her feedback which makes her

reassess her dominance. Paradoxically, when she does relinquish control she doesn't realise that she still needs to guide pupils in the right direction and not adopt a hands-off approach. However, Aoife learns how to adopt new methodologies to suit investigational tasks; for instance in her final observed lesson she asks pupils to justify their solutions to their partners, which is in line with a constructivist view of learning. I now relate a constructivist lens to each of the sixteen lessons observed and detail what such a view revealed. I summarise the data collected using Jaworski's Teaching Triad as the analytical tool. Having outlined each teacher participant's four lessons, I also apply a grid to the lessons to appraise the affordances and constraints offered by the problem solving investigative activities. The grid is based on other authors' views of what a problem solving activity should encompass. Details of the grid are revealed in the next section.

5.1.1 A rationale for investigating problem solving tasks and their affordances and constraints

As this thesis is primarily concerned with an investigation of problem solving tasks I now intend to provide a rationale for such an investigation. In Professional Standards for the Teaching of Mathematics (NCTM, 1991) one finds consistent recommendations for the exposure of pupils to meaningful and worthwhile mathematical tasks. This means tasks should be truly problematic for pupils rather than a contrived way to have them practise a previously-demonstrated algorithm. Stein, Grover and Henningsen (1996) state that in such tasks students need to enforce meaning and structure, make decisions about what to do and how to do it and interpret the reasonableness of their actions and solutions. They comment that "such tasks are characterised by features such as having more than one solution strategy, as being able to be represented in multiple ways, and as demanding that students communicate and justify their procedures and understandings in written

and/or oral form" (p. 456). Here I am reminded of the relevance of Bruner's (1967) three modes of representation: the enactive, iconic or pictorial, and the symbolic. Given the current emphasis on the creation of instructional environments characterised by an increased emphasis on problem solving, sense making and discourse, a further examination of the assumptions underlying *how* such environments lead to the desired student outcomes seems to be in order. Carpenter and Fennema (1988) suggest a *student mediation model* which can be used to examine how instruction relates to student learning outcomes. This model suggests that teaching does not directly influence student learning but, rather, that teaching influences students' cognitive processes or thinking, which in turn, affects their learning. A mediating variable that is worth examining and describing is the nature of students' thinking processes and how these processes are changed when teachers try to establish enhanced instructional environments. Stein, Grover and Henningsen (1996) ask if authentic opportunities to think and reason are created when teachers use tasks that are problematic, that have multiple solution strategies, that demand explanation and justification, and that can be represented in various ways. In quite a reductionist approach, Doyle (1983) went so far as to define curriculum as a collection of academic tasks. He stated that tasks influence learners by directing their attention to particular aspects of content and by specifying ways of processing information. However, as a constructivist epistemology underlies this thesis it is important to me that any tasks chosen would encourage pupils to work at their zone of proximal development (zo-ped) as indicated by their level of engagement. Several other authors make suggestions on what constitutes *rich tasks*, the type which encourage pupils to work at their zo-ped, and also on what strategies can be employed in the solution of such tasks. For instance, Polya (1945) suggested 'trying a smaller case' as a useful strategy when a problem appears inaccessible. The

problem I later choose for Lisa which requires counting the squares in a four-by-four grid is an example of such a problem as one cannot solve it without looking at one-by-one squares. Boaler (2013) reasserts that tasks which require the exploration of patterns are inspirational for pupils. Therefore, this could be another useful criterion in appraising task worthiness. The number of operations needed to solve a task also gives some indication of the complexity of a problem. This is linked to what Polya (1981) calls the 'choice of a combination' i.e. a problem that requires the solver to combine two or more rules or examples given in class. Admittedly, this excludes problems outside the number strand which could be seen as a constraint; particularly as the 1999 curriculum emphasises problem solving in the other strands also. Furthermore, Ross et al. (2002) comment on the reform agenda set by the NCTM policy statements (2000) by also calling for broader scope in the curriculum. They emphasise the teaching of multiple math strands with increased attention on those less commonly taught such as probability. They decry the previous over-emphasis on numeration and operations. Yet, such operations are more than likely going to remain as part of any curriculum and their prevalence cannot be ignored. Ross et al. (2002) state in a reformist agenda student tasks are complex, open-ended problems embedded in real-life contexts and that many of these problems do not afford a single solution. Therefore, open-endedness, in the sense of offering multiple solutions, could be a useful criterion in appraising a problem. Such appraisal is indeed personal as different types of problems appeal to different types of mathematicians. However, in choosing certain factors over others I am appealing to Jaworski's (2012) notion of the fourth node to the didactic triangle as outlined in section 2.5. The reader will recall that she suggested adding the role of researchers in the classroom, or didacticians, to use her term, as the additional node or adjunct to the didactic triangle. Her rationale was that teachers and didacticians share a

reflexive relationship. She commented that although teachers' knowledge in practice goes far beyond didacticians' knowledge, the complementary knowledge of research and theory brought by didacticians provides stimulus and inspiration to which cohorts of teachers are able to respond. The relationship is reflexive in that teachers develop new approaches to working with their students such as using inquiry modes of learning. In tandem with this didacticians learn about how theories and research findings can and do influence the practice of real teachers in real schools and classrooms acting under all the constraints of institutional and political pressure (Jaworski, 2012). Jaworski states that as a didactician herself, she is aware of the power of this collaborative knowledge and associated developmental practice in addressing approaches to educating students in mathematics. Therefore, in offering mathematical activities to the teacher participants, or being offered activities by them, in turn, my quest is to see if such tasks help teachers move outside the traditional textbook towards a more reformist agenda in line with constructivist principles. Based on the authors mentioned above the following six criteria might be a reasonable way of looking at the affordances and constraints of any problem solving task:

1. Allows for several forms of representation (Bruner, 1967)
2. Allows for a smaller case to be considered (Polya, 1945)
3. Is open-ended in terms of possible solutions (Ross et al., 2002)
4. Involves a variety of mathematical operations (Polya, 1981)
5. Allows for engaging classroom discussion (Stein, Grover and Henningsen, 1996)
6. Involves the study of mathematical patterns (Boaler, 2013)

I now apply a constructivist lens to each of the sixteen observed lessons.

5.2 A constructivist analysis of Lisa's first lesson on 19.10.10

Lisa's first lesson began with a computational activity. This proved to be a trademark of her introductions to lessons. Presumably, the aim of this activity was to put pupils in the mood for mathematics and also enable them to practise their computational skills. Lisa drew the following grid on the Whiteboard:

X	**-**	**÷**	**+**	**=**
9	3	1	0	6
12	4	2	15	5

The pupils were then asked to use any numbers on the grid with any of the four standard operations to form other numbers on the grid. Pupils gave sample answers as follows: (3 x 4) + 3 = 15, 5 + 5 + 2 = 12, (6 + 0) x 2 = 12 and (3 x 6) – 3 = 15. Lisa inserted the brackets for the pupils on the whiteboard. I wrote in my journal entry on the lesson (19.10.10) that she probably missed an opportunity to explain the purpose of the brackets, which could have led on to an explanation of the priority of multiplication (and division) over addition (and subtraction). There is a need for teachers to see the "big ideas" like priority of operations, which can emerge from simple computational activities. However, this was Lisa's first lesson and it would be unfair to expect her to be alert to such opportunities at such an early stage in the research. This activity was followed by a second one which required pupils to logically place colours in four squares and two circles based on the following clues as shown in Figure 10 which follows:

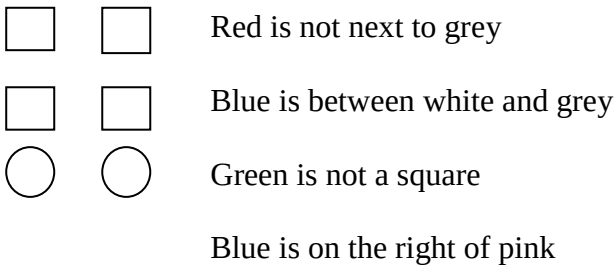


Figure 10 : The colour - positioning activity

Lisa showed the pupils the shapes on the interactive whiteboard positioned at the top of her classroom. The pupils worked in groups of three. I was impressed by Lisa's actions in encouraging the pupils e.g. "Have a bit of faith in yourself". "That's how it works, trial and error". "You're on the right track". Speaking of errors, Lisa made one herself when she presented an incorrect solution to the pupils. A transcript of Lisa's conversation with the pupils is enclosed as Appendix 10. Remarkably, Lisa turned this error to her advantage when she asked the pupils to help her. For me, this showed that Lisa did not want to be the 'sage on the stage', but could allow for pupils' opinions to come to the fore. I observed that pupils gaining ownership over their work should be one trait of a constructivist – led classroom as it promotes learning. Lisa referred to this in her pre-lesson interview on 11.10.2010 (PLIL) when asked how constructivism affected what pupils do in classrooms. She stated:

> It places them in a central role in the lesson as more active participants and in charge of their own learning. It gets them thinking for themselves and engaged in cooperative discussion, something which they find quite difficult across all curricular activities.

Lisa appeared to be travelling the constructivist path she had laid out for herself. At 17 minutes 27 seconds (17:27) into the lesson Lisa introduced the main activity. She asked the pupils how many squares there were in a 3x3 grid drawn on the interactive whiteboard as shown in Figure 11 which follows:

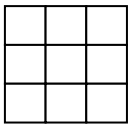
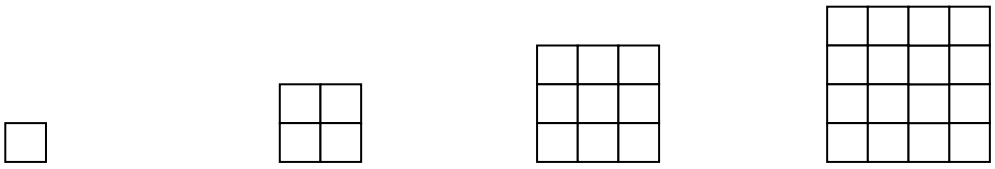


Figure 11: The count the squares activity

This activity was one I suggested to Lisa as it would introduce the idea of investigative mathematics to pupils. I had used this activity on previous mathematics in-service courses for teachers and found it very worthwhile in that the teachers had to put themselves in the role of pupils and work systematically in small groups to find a solution. Furthermore, the activity has broad appeal among pupils as it simply involves the operations of addition and multiplication. With this type of activity there is more likelihood of children becoming involved in creating their own mathematics as they proceed to discover the different types of squares involved. The pupils worked in small groups and began to see that there were 9 1x1 squares, 4 2x2 squares and 1 3x3 square totalling 14 squares in all. One pupil, Claire (pseudonym), gave an insight that the intersections between the lines were squares in themselves. It must be remembered that Claire was looking at a large grid on the interactive whiteboard, when she spotted these squares. Nevertheless, it was an example of a pupil creating her own mathematics. For me, it showed that the choice of activities is crucial for powerful mathematics to emerge. In a discussion I conducted with Dr. Hugh Gash on 25.02.2011, he suggested choosing the activities carefully for the teachers involved in the project. Seeing that Dr. Hugh Gash has written extensively on constructivism I took his advice on board. That is not to say I wanted to 'spoon feed' the teachers by providing them with all the activities they should try out with the children. I believed this would be anti-constructivist in that the teachers, as well as the pupils, should be encouraged to negotiate meaning for themselves. What I mean here is that they should be encouraged to source activities, which *they* believe will help pupils to engage in meaningful mathematics.

At 18:58 Lisa introduced a 4x4 grid on the interactive whiteboard and asked pupils to count the squares. Again she provided plenty of encouragement to the pupils to keep them motivated: “You need to work together as a group” (23:45); “Keep going now, you’re on the right track” (25:26). I believe that such encouragement is really essential for pupils to persevere at tasks, which are new and challenging for them. This shows that constructivism is not just about the cognitive domain, but is also influenced by the affective domain of pupil learning. One group was doing so well with the 4 x 4 activity grid, that the pupils in it were encouraged to try a 5 x 5 grid. Strategically, Lisa was astute in her actions in asking this group to report back to the other groups on how they solved the 4 x 4 activity (35:00). A sample of one group’s work, with Nora as recorder, is included as Appendix 11. It can be seen t

hat the work is recorded iconically and not symbolically. Therefore, I was interested when Lisa introduced the symbolic notation of indices by drawing the following diagrams on the whiteboard:



1 X 1 = 1	2 X 2 = 4	3 X 3 = 9	4 X 4 = 16
1^2	2^2	3^2	4^2

The pupils never spotted that the solution to how many squares there are in a 4 x 4 grid can be formulated as $4^2 + 3^2 + 2^2 + 1^2 = 30$. Yet they engaged well with the process of a task, which was challenging them at their zone of proximal development as evidenced by their level of engagement. The introduction of the index notation meant that Lisa was enculturating the pupils into the kingdom of mathematicians, where one cannot survive without knowing the symbols of the realm and the language, which accompanies them. Such symbols and their accompanying language become important cognitive tools which pupils can use to

enhance their mathematical learning. The pupils have to serve as squires or apprentices before they can become knights, fully conversant with the discourse of mathematics. In using the index notation Lisa was linking the two strands of number and shape. Lisa referred to this in her reflection on the lesson (LRLI), when she observed that she needed to link the mathematics more with the stated curriculum. This is a very practical way of dealing with the conundrum of covering curricular content, while at the same time introducing pupils to investigative problem solving. I will return to this point later in the appraisal of her activities. Lisa also pointed out that she needed to give more time to groups to explore concepts "without interrupting them and trying to show them the path to the solution"(LRL1). This requires great faith on the part of the teacher engaging in constructivist approaches as it means that the teacher has to hold back, 'bite her lip' and trust the pupils to solve the problem.

5.2.1 A constructivist analysis of Lisa's second lesson on 15.02.2011

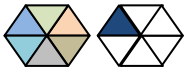
Lisa introduced this lesson with a loop card warm-up activity. Her rationale for her actions in using these warm-up games was that they provided "a focus and challenge" (LRL2). For example, a pupil might say, "Who has a shape with six sides?" The pupil with "hexagon" on her card reads an accompanying question for the next pupil to follow and so on. One advantage of this game was that the pupils had to visualise the shape in their minds before they could answer. Lisa followed this activity with another warm – up game called "Fraction Scrabble". She drew the following grid on the whiteboard:

½	$^{2}/_{4}$	¾	$^{4}/_{5}$
$^{6}/_{12}$	$^{4}/_{8}$	$^{8}/_{10}$	$^{7}/_{10}$
$^{8}/_{12}$	$^{3}/_{5}$	$^{6}/_{10}$	$^{11}/_{12}$

The objective was to get the pupils to make mathematical statements using the fractions above and the symbols >, <, =. Pupils gave answers such as 6/10 = 3/5, 8/10 > 7/10 and 4/5 > 1/2. Lisa displayed a fraction wall on the whiteboard, which the more visual learners could use as a comparative tool. At 14:02 Lisa strategically put the pupils in pairs. I presumed this was done on the basis of mixed ability. However, Lisa showed in her reflection on the lesson (LRL2) that her actions in organising groupwork had a higher level of sophistication than I had originally anticipated. She stated:

> I choose pairs according to maths ability but also according to motivation, determination and participation factors. I think this mix is necessary to allow children who find maths challenging, a supportive partner, to encourage them along. Working in pairs, rather than groups, allowed opportunity for a higher level of involvement with the lesson.

Although a pair is a group we can interpret what Lisa meant. Working in pairs ensured that the children stayed on task and worked at a high level of intensity. As her main activity Lisa gave the children a complex worksheet with which to work (see Appendix 12). For example, question 4 asked, "how many in a ?" However, Lisa made a useful link back to question 3 which had asked, "how many in a (semi hexagon)?" In doing so, she scaffolded the knowledge for the pupils, so that they could see that if the answer to question 3 was 3, then the answer to question 4 was 6. I realised that such scaffolding is essential for a teacher to adopt

a constructivist approach to their work. As the worksheet became more complex Lisa was faced with the constructivist's dilemma: whether or not to intervene. For example, pupils struggled with the following question "What fraction of a hexagon is this shape?" 

They thought the answer should be $^{7}/_{12}$ rather than $1^{1}/_{6}$. This is a common misdemeanour in that pupils misinterpret what the "unit" should be. In her reflection (LRL2) Lisa stated:

> They needed guidance with the 3rd part of the lesson especially. From circulation of the classroom, I noticed a lot of them had gone wrong, so I intervened and modelled an example to set them right. This worked and all groups then proceeded to get it right. From a constructivist point of view, I'm not sure if this was the right or wrong procedure. Perhaps I should have spent more time exploring why exactly it was wrong, allowing more time for exploration of how to arrive at the proper solution. I was a little caught up with the time constraints.

I believe this is a very rich piece of data. The first point I would like to make is that Lisa believed it was important for each group to "get it right". She was fortunate in that each group arrived at a correct solution. However, a constructivist approach has to allow for differentiation in tasks, as pupils will arrive at solutions as different times. It is not desirable for every pupil to arrive at a solution at the same time, as it implies that the teacher is aiming at a perceived mean range of ability, without allowing for individual difference.

The second point I wish to highlight is the dilemma Lisa faced in wanting to allow more time for exploration, but feeling frustrated by time constraints, such constraints becoming an emerging theme in the data. This is allied to the dilemma she faced earlier in wondering whether to intervene or not. Even if a teacher does not intervene she is then faced with the problem of how much time to give the pupils to allow

them to come up with a solution appropriate to their ability. The only advice I can offer here is that each teacher gets to know individual pupils' strengths gradually, which enables them to pitch the level of challenge and the appropriate time to be allowed. The reader will notice on the worksheet that the teacher used the label "pentagon" instead of "polygon". However, as the lesson emphasis was on fractions, rather than on naming shapes, no pupil spotted the error and it manifestly did not interfere with the thrust of the lesson.

In her reflection on the lesson (LRL2) Lisa showed that linkage was becoming a powerful conceptual tool for her. She stated:

> In the last session I stated I wanted the lessons to link more to the curriculum. I feel this was achieved to a great level, as it (the lesson) combined area, fractions, 2D shapes and spatial awareness. The fact that we had previously covered 2D shapes and fractions extensively was a great help and a nice means of revision and consolidation (Brackets added).

Linkage was a superb strategy for Lisa in connecting several areas of mathematics together. It seemed to allay her fears of being able to cover the prescribed curriculum, while at the same time exploring a problem solving approach. Lisa was taking ownership of the strategy of linkage and it was to prove very useful in her lessons. It is interesting that the Mathematics Curriculum for $5^{th}/6^{th}$ class (NCCA, 1999, p. 92) suggests constructing diagrams to illustrate simple square and rectangular numbers; yet it never makes the link between square numbers and calculation of area explicit. Indeed there is no suggestion for linkage on page 92. Therefore, teachers need to be open to linkage as such opportunities will open pupils' minds to perceive mathematics topics as being interrelated and not as

discrete blocks of knowledge. In turn, this will encourage pupils' creativity in mathematics and lead them to further discoveries. It could be that the pupils did not share their linkages with the wider class grouping as they only worked in pairs. The main activity also appeared closed in that fixed solutions were required and this meant that the pupils seemed to have very little opportunity to engage with one another in determining solution methods.

5.2.2. A constructivist analysis of Lisa's third lesson on 17.5.2011

I had witnessed how linkage had become embedded in Lisa's practice. For this lesson, occurring over three months on from Lisa's previous one, I encouraged her to experiment with more open-ended activities as this was one of the affordances offered by this research. The delay between the two lessons was down to the fact that teachers are busy people and it is difficult to schedule appointments when they have to try and 'step out of the glue which binds them' to their standard practice. The fact that the Easter holidays had intervened did not help either. Anyway, Lisa started the lesson with an open-ended computational activity. She asked the pupils to list possible sums where the answer is 11. It was interesting that the pupils did not interpret the word "sum" as meaning addition here, but gave it a broader meaning and came up with solutions such as 20 – 10 + 1 = 11, 12 – 1 = 11 and 10 + 3 – 2 = 11. Indeed, this interpretation gave a wider display of answers than if the pupils had interpreted "sum" as addition only. Next, the pupils were asked to do the same for 25. Solutions given included 10 x 2 + 5, 75 – 25 – 25 and (11 x 2) + 3. Next, the computation became slightly more complex in that the pupils were asked to find "sums" for 8.3. Answers given included (4.0 x 2) + 0.3, 8.1 + 8.2 – 8 and 16.0 – 8.0 + 0.3. I believe increasing the complexity ever so slightly in activities, as Lisa did, is an important tenet of constructivism, as it links well with Vygotsky's notion of the

“zo-ped” i.e. that pupils are required to work at the very edge of their current conceptualisations.

At 9:34 Lisa moved on to another warm-up computational activity. She placed the following sum on the whiteboard and asked the pupils to come up with possible answers:

$$\begin{array}{r} 2\,?\,? \\ \underline{?\,7\,?} \\ 4\,2\,2 \end{array}$$

The pupils rose to the challenge and answers offered included:

$$\begin{array}{rrrrr} 250 & 252 & 246 & 248 & 251 \\ +\underline{172} & +\underline{170} & +\underline{176} & +\underline{174} & +\underline{171} \end{array}$$

Lisa’s actions provided for plenty of encouragement; “There’s more than one answer, you could have something else as well” (10:53) and “That’s three different solutions. Did anyone get anything different?” (12:53). It has to be stated that from my observations, the action of providing encouragement was very much a classroom norm for Lisa and this seemed to motivate pupils to engage with whatever challenge she provided. Although Lisa did not explicitly state the strategies one might use in solving such activities the pupils arrived at their solutions intuitively.At 13:23 Lisa placed the following sum on the whiteboard:

$$\begin{array}{r} ?\,4\,3\,? \\ \underline{3\,9\,?\,3} \\ 7\,3\,6\,0 \end{array}$$

I was slightly puzzled by this choice of activity as, unlike the previous ones there is only one possible solution, which the pupils quickly realised:

$$\begin{array}{r} 3\,4\,3\,7 \\ \underline{+3\,9\,2\,3} \\ 7\,3\,6\,0 \end{array}$$

Perhaps Lisa missed an opportunity here to create a discussion with the pupils on what constitutes an open or closed activity. However, it has to be conceded that this type of "reflection-in-action" (Schon 1983) is difficult for practitioners to engage in, as they are caught up in the net of immediacy of the task at hand. However, Lisa's next activity showed that with planning she was very capable of challenging the pupils with an open-ended task. Lisa presented the pupils with the following problem, which she had sourced on a self-discovered site named www.figurethis.org.

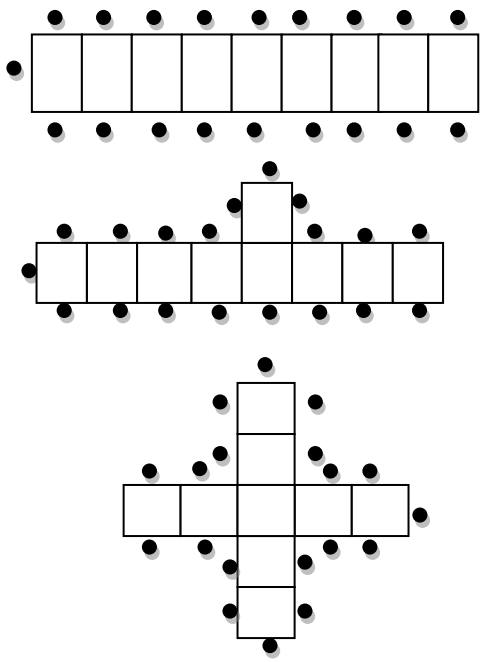
5.2.2.1. Polygon's restaurant

Polygon's restaurant has square tables that seat one person on each side. To seat larger parties two or more tables are pushed together. What is the least number of tables needed to seat a party of 19 people who want to sit together?

She revised a problem-solving strategy with the mnemonic RUDE for the pupils. RUDE is an acronym for Read-Underline-Draw-Estimate. The pupils were excellent at writing down key words in the problem e.g. "one person on each side", "19 people", "pushed together", "square" and "least number of tables". The pupils had obviously practised the RUDE approach on previous occasions. Lisa also gave the problem an authenticity, when she stated that finding patterns and arranging geometric shapes are strategies used by architects, landscapers, quilt makers and carpet layers in their work. In section 2.18 I discussed the importance of giving pupils authentic problems i.e. problems encountered by people who use mathematics

in their professional lives. Ross et al. (2002) suggest that such student tasks should be complex, open-ended problems; embedded in real-life contexts and possibly not affording a single solution. Lisa's self-chosen task would fit into that category.

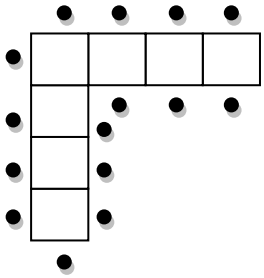
As before, Lisa's actions included the provision of encouragement by asking the pupils to draw the shapes as they went along (22:13). At 27:28 she further elaborated on this point when she prompted, "Draw it out and you'll get to your solution. You'll get there". This reminded me of the relevance of Bruner's iconic stage of representation in mathematics, as Lisa was encouraging the pupils to draw a diagram to assist them in finding a solution. Below are some diagrams, which pupils drew to illustrate their solutions:



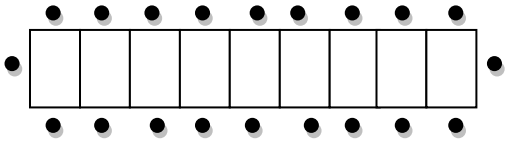
Again Lisa provided scaffolding for the pupils. She asked them to consider how many pupils would sit at 2 or 3 tables. She stated that tables would have to be joined together at some point because "space is money". To be fair Lisa did not shepherd or coral the pupils into finding one particular solution to the problem but instead wanted them to experiment with their drawings. At 30:31 Lisa started taking

feedback from pupils by allowing them to come to the whiteboard and represent their solutions. It was interesting that one of the pupils, Karen, who was considered to be a low achiever in mathematics, gave the following insight: “I did 2 into 19 and I got 9 remainder 1”. This shows that more open-ended approaches to mathematics can help teachers to reassess their opinions of pupils’ ability.

Lisa’s next move in the lesson surprised me. She asked the pupils to find the least number of tables needed to seat a party of 15 people. Nora provided the following solution:



I was surprised by this activity choice, as Lisa had lowered the level of challenge. This has the motivational benefit of allowing pupils practise their problem solving strategy at a lower level, but may also disincentivise pupils of higher ability who require challenge in their work. Lisa must have realised this because she asked the pupils to find the least number of tables for 20, 23 and 26 people. Then, the introduction of an even number of people (20) provided a new impetus for the lesson. Kiely gave the following solution:



At 57:00 Lisa asked the class if they had noticed any pattern emerging between odd and even numbers. Nuala gave a valuable insight when she stated that with an even number there are no empty spaces. Lisa asked the pupils to fill in the following

equation where ? represented the number of tables and the answer represented the number of people:

(? X 2) + 2 = $\boxed{}$

Lisa wanted the pupils to write the equation for 4,5,6 and 7 tables. She asked the pupils why 2 was added to make it work. Paula replied, "Because there's 2 on each end". This was a good reply which showed Paula had made the connection between the equation and its visual representation of tables with people at both ends. The use of the equation also provided the lesson with a fitting conclusion. In Lisa's own words, "it helped to tie the lesson together nicely at the end" (LRL3).

5.2.2.2. Lisa's view of her third lesson

Lisa was very pleased with how successfully the lesson had progressed (LRL3):

> It was very open-ended in nature, with lots of different methods for achieving the answer. This allowed for a lot of exploration, facilitated the use of mixed-ability groupings and catered for individual difference. I observed that the typically "weaker" children in maths were really "getting into" the problem solving, were giving great ideas and experiencing success in maths. Surprisingly, some of the more "able" students did not make the connections initially and needed guidance.

As a researcher, I too was interested to see how children, perceived as being of lower ability in mathematics, can surpass teachers' expectations of them given the right environment. It helps if one sees ability as not being static but elastic. Yet again, Lisa referred to linkage. She stated that she liked the way this open-ended problem linked different areas of curriculum such as logical reasoning, spatial awareness, geometric patterns, sequencing and algebraic equations. Linkage was to become the 'bedrock' for Lisa in adopting a constructivist, open-ended approach to her work. It gave her the freedom to experiment with various approaches while at

the same time it gave her the security of knowing she was covering different areas of the prescribed mathematics curriculum. This sense of security helped her overcome the guilt that often comes with not covering the set curriculum. From my viewpoint, I believed I had witnessed pupils engaging in powerful mathematics in this lesson with pupils making strong connections between several mathematical areas such as geometry and algebra. In her reflection (LRL3) Lisa also wrote about the organisation of her groupwork; such organisational strategies becoming an emerging theme in the research:

> The children worked in groups of 3 mostly. I choose the groups according to mixed ability; along with motivation, determination and personality factors. I feel this is very important for successful groupwork in maths. It worked quite well today and everybody was motivated and engaged with the lesson.

It is interesting that Lisa wrote in the present tense in referring to her groupwork. It indicates that her choice of mixed ability groupings was becoming an ongoing part of her practice. She shows that grouping children according to mixed ability is not enough on its own; one must also consider the individual characteristics of each pupil so that a group can be formed, which will work well together.

Lisa also referred to the issue of progression in her lessons. This was an area which was to feature strongly with the other participants in this study. She stated that each stage of the lesson naturally progressed into another stage. She believed that the lesson gave the children plenty of opportunities to practise what they had learnt and also try out new and different combinations for seating people at tables. The structure of the lesson helped the children to begin with diagrams before extending the lesson to identify a number pattern and use algebraic equations. I have to agree

with Lisa that, apart from one incident, the lesson showed a logical progression. It occurred to me that the issue of progression in a lesson is linked to Vygotsky's notion of the zone of proximal development. What I mean here is that one has to have an increasing level of difficulty in a lesson to challenge pupils at their frontier zone or zo-ped. A teacher may decide to pause and keep the level of challenge static in a lesson for a short period to maintain pupils' motivation and to revise key concepts but surely the teacher must soon return to increasing the challenge in an onwards and upwards direction. The sourcing of challenging material and pupils' associated level of engagement was becoming a dominant theme in the research.

I have previously mentioned how Lisa found linkage was a great means of covering several mathematical topics at once. In simultaneously covering such topics, Lisa found that time was a constraint but she also came up with a viable solution. She stated (LRL3):

> I personally learned that problem solving and a constructivist approach requires a lot of time. The lesson could have taken up a lot more time and maybe even be taught over two sessions. I had initially planned to cover a lot more in one session and it was an eye-opener to me as a teacher.

To cover several mathematical topics simultaneously Lisa had used linkage as a strategy. Now she found she had to take the pressure off herself in attempting to get everything done in one lesson. Her insight that a mathematical topic could range over several lessons was a major breakthrough in her thinking. She was beginning to realise that a constructivist approach did not just involve her doing the teaching, but also involved giving more ownership to the pupils to control their own learning, at a pace which suited them. This approach takes time. It occurs to me that a

constructivist approach implies that it is the pupils who set the pace and not the teacher. This does not mean that the teacher is redundant. Rather, it means that the teacher needs to be adept at assessing when pupils are ready for further challenge and when they need to be left alone to explore a topic by themselves. Such an approach requires sensitivity to pupils' needs, a category of Jaworski's (1994) Teaching Triad.

5.2.3. A constructivist analysis of Lisa's fourth lesson on 15.06.2011

The theme of Lisa's 4th lesson was probability. Lisa commenced the lesson with two warm up activities; a trademark of how she began lessons. During these activities the children worked in pairs. Each pair was given 5 dice and asked to roll the dice as individuals to make the greatest number possible. The children recorded the outcomes on a sheet as follows:

Round no.	Player 1	Player 2
1	44421	31426

The player with the greatest number won the round. The children were given approximately 5 minutes to complete the game. At the end of the game Lisa asked the children to name the highest number they had encountered. This turned out to be 66655. Then Lisa asked the children for the highest possible number they could get in the game. The answer given was 66666.

The second activity was a version of bingo. The children were asked to draw a number line from 1 – 15 as follows:

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15

Then they were asked to roll 2 dice, multiply the resulting digits and cross out the product on the number line. For example, 1 (die 1) and 3 (die 2) meant the pupil had to cross out 3 on the number line. The first pupil to cross out all their digits won the game. Again the children were given 5 minutes to complete the game. This game of "beat the clock" was described by Lisa in her reflection (LRL4), as being constructivist in nature, as the children discovered that the numbers 13, 14 could not be obtained using the two dice. It was interesting that Lisa was developing her notion of constructivism to include the construction of *new* knowledge by the children. As the children worked in pairs it could be said that there was not only construction but co-construction of knowledge involved. In my own journal notes on the lesson, I recorded that constructivist rooms are noisy rooms, where there is a lot of social interaction going on. It was evident that this type of co-construction of knowledge happens best when more and more control of learning is handed over to the pupils. At 16:20 Lisa called time on the activity. She had a brief discussion with the pupils on which numbers could *not* be obtained by multiplying the outcomes of rolling the two dice together. The pupils were quick to point out that it was 13 and 14 which could not be obtained.

At 18:20 Lisa decided to move on to the main part of her lesson. I had suggested to Lisa in a pre-lesson meeting on 13.06.2011 that she consider using an activity, which was explained on page 37 of the Teacher Guidelines in Mathematics. I suggested this activity as Lisa wanted to do something from the probability strand and was seeking advice. In this activity the pupils are asked to roll two dice, find the sum of the outcomes and decide whether even or odd totals occur more frequently. I had suggested this activity as it seemed to be age-appropriate and would enable the pupils to engage in investigative work. It would also assist the pupils in making links

among the curricular strands of number, algebra and data. Lisa liked using activities which involved linkage. Lisa decided to let the pupils work in pairs for this activity with one pupil recording the even totals and the other recording the odd totals. Initially, pupils thought it was luck decided. I think that working in pairs gave pupils an incorrect notion that the odd totals would occur 50% of the time and the evens likewise. Therefore, one has to be mindful, in doing probability activities with groups of pupils, that they may incorrectly conceive that the number of children in a group activity has some bearing on the probability of an independent event occurring e.g. if there are 4 pupils in a group this does not necessarily mean that there is a 25% chance of an event occurring. At 29:06 Lisa took feedback from each pair of pupils. Six pairs reported that "even" totals occurred more often. Two pairs reported that "odd" totals occurred more often and one pair reported the chances of even or odd totals occurring as being 50/50. Then Lisa strategically asked the pupils to list all the possible outcomes and she recorded pupils' answers on the whiteboard as follows:

Possible Odd Combinations	Possible Even Combinations
5 + 6 = 11	6 + 6 = 12
5 + 4 = 9	6 + 4 = 5 + 5 = 10
3 + 6 = 9	4 + 4 = 6 + 2 = 3 + 5 = 8
4 + 3 = 6 + 1 = 5 + 2 = 7	5 + 1 = 3 + 3 = 4 + 2 = 6
3 + 2 = 4 + 1 = 5	2 + 2 = 3 + 1 = 4
2 + 1 = 3	1 + 1 = 2

Pupils asked whether they could subtract or multiply to obtain the totals. Another pupil asked if she could go past 12. Lisa adeptly guided the pupils through the activity. She stressed that they must stop and think about the logic of the game. She asked them to name the highest possible odd outcome and the highest possible even

outcome (time 33:13). I was impressed that Lisa did not just list the outcomes, but asked the pupils to explain why there were more evens than odds. Hilda gave a good insight when she stated that doubles give you evens. Lisa elicited from the pupils that to make an odd total you needed to have an even and an odd number. She also elicited that to make an even total you could have an odd plus an odd, doubles or an even plus an even.

In eliciting this information Lisa helped the pupils to synthesise the work they had done while working at their zo-ped. Lisa reinforced this view when she stated in her reflection (LRL4) that initially she had thought the odd/even dice game would be too easy for the pupils, but "was surprised at how challenging some of them found the task". She also learned that process, and not just product, is important when adopting a constructivist approach. Windschitl (1999) has succinctly stated that pupils need feedback on the processes, as well as the products of their work. I referred to this in section 2.19 when I recommended a process orientation for mathematics lessons i.e. that the process of mathematics investigation can become the product of the lesson. Lisa commented that everyone experienced varying degrees of success in this activity but that "no pair got the exact correct ratio" of odds to evens i.e. 9:12. In other words, the children did not necessarily need to discover the correct ratio to realise that there were more even than odd totals. Their engagement with the process had helped them in this regard. Once again in her reflection (LRL4) Lisa promoted the use of linkage:

> I liked the way this lesson covered the topic of probability and odd/even numbers. It linked into my own scheme of work, which made it much more realistic to be able to incorporate into your weekly plans. Also it is a great lesson for follow-on opportunities.

It seems to me that Lisa is stating that linkage is an evolutionary process and teachers need to be alert to its power in helping pupils make connections between different areas of mathematics. Linkage is a social cognitive construction, but Lisa also highlighted the affective dimension of learning when she finished the lesson by asking the pupils if they enjoyed the lesson and, if so, what did they enjoy about it. Pupil comments were very positive and are summarised as follows:

> **Ciana**: It's fun working together. You don't work under as much pressure.
>
> **Nora**: I like the challenge.
>
> **Sarah**: I like working together. I like the competition aspect.
>
> **Klarisa**: I just like the way, love the way it was all fun and games.
>
> **Clara**: I like the way that maths can be fun.

Although one needs to be cautious that pupils are only being positive because the researcher is present, I believe that the comments were heartfelt. They suggest that an investigative approach helped to motivate the pupils to engage with mathematics.

5.2.3.1. Conclusion on Lisa's four lessons

In this section I have outlined how Lisa found the use of linkage of great support to her in reconciling the time needed to cover problem solving activities with the demands of curriculum coverage in mathematics. She was innovative in experimenting with mixed-ability groupings and highlighted the need for a sociocultural approach to such groupings, when she stated that she chose the constituents of the groups based on personality, motivation and determination factors. In chapter 2 I mentioned Graham Nuthall's Seven Principles for Effective Implementation of Social Constructivist Teaching. One of these is the need to train students in group interaction procedures. This highlights the need to see

constructivism as more than just a cognitive theory. Pupils' affective factors need consideration also. Lisa was both delighted and surprised to see children, perceived as being of low ability, giving answers which showed great insight. It suggests that if the teacher encourages creative thinking, children will rise to the challenge. It is one of the benefits of presenting pupils with frequent occasions to deal with complex, meaningful problem-based activities as Windschitl (1999) suggested in chapter 2. Windschitl (1999) claimed that such activities were one of the key features of constructivist classrooms. Lisa showed great initiative by increasing the complexity of the mathematics ever so slightly to keep pupils challenged. At times she lowered the challenge, but this may also be useful in helping to consolidate previous learning. It was evident to me that Lisa's third and fourth lessons showed that she had gained in confidence in giving children more investigative-based mathematics to work on and had become more adept at adopting groupwork approaches. What I learned during classroom observations was that a constructivist approach involves a teacher handing more control of learning over to the pupils and thereby allowing them to dictate the pace of the lesson as necessary. Indeed, Lisa realised that a mathematical topic may encompass several lessons and need not be neatly packaged into just one succinct lesson. I now wish to appraise Lisa's activities according to the six criteria outlined earlier in the chapter. In appraising the activities I am drawing attention to their potential as problem solving tasks. This will be done with each of the four participants. It is not my intention to appear to rate the teachers adopting these tasks. Indeed teachers could carry these tasks in many different directions, particularly if the teacher is willing to listen to and adopt the pupils' interpretations of such tasks.

5.2.3.2. Appraising the affordances and constraints of Lisa's problem solving activities

Lisa's first activity was one I had previously used on in-service courses with great success. The Count the Squares activity forces teachers to adopt the role of learners themselves and, therefore, assists teachers in realising what is required during the problem solving process. It satisfies five of the six criteria outlined earlier although it has to be stated that only two operations, addition and multiplication, are involved. It is not open-ended in that there is only one solution but there are many solution paths.

Lisa's second activity was based on a fraction worksheet which she herself compiled. It satisfies two of the six criteria outlined. The worksheet allowed for correct or incorrect answers to be inserted with no opportunity for pupils to discuss the results. It shows that many textbook activities do not afford room for discussion as they have the constraint of being closed tasks requiring only one answer. They do not fit into the reformist agenda of Ross et al. (2002) who recommend complex, open-ended tasks, embedded in real life contexts.

I have to compliment Lisa on the choice of her third activity. She had researched it on a website entitled www.figurethis.org. She had obviously benefited from any pre-lesson advice given during the design project. The Polygon's Restaurant activity satisfies all six criteria. In particular, it affords pupils the opportunity to discuss various solutions with one another.

Lisa wanted to choose from the area of probability for her fourth activity. In discussion with Lisa I suggested an activity from the Teacher Guidelines which required pupils to investigate the prevalence of odd and even totals when two dice

are thrown and the outcomes are added together. This activity satisfied three of the six criteria but it had the distinction of being the noisiest activity I witnessed with pupils hopping dice off their desks! This meant that there was great interest in the activity and the pupils were afforded the opportunity of discussing the results with one another as they worked in pairs. In my journal entry for the lesson I noted that constructivist classrooms should often be noisy classrooms. I was also glad that the activity came from the probability strand which has previously received little attention from teachers.

In summary, I just wish to state that although I had been influential in helping Lisa choose two of her four activities I was very pleased that one of the two activities she chose herself satisfied all six criteria and showed her growing into a constructivist-compatible approach to her work. The affordances and constraints of Lisa's problem solving activities are synopsised in Table 5 below.

Table 5: Affordances and constraints of Lisa's problem solving activities

Activity	Allows for Several Formats (Bruner)	Allows for a Smaller Case (Polya)	Open-ended problem	Involves a Variety of Mathematical Operations	Allows for Engaging Classroom Discussion	Involves the study of Patterns (Boaler)
Count the squares Activity	Yes, pictorial and symbolic	Yes	No, but various solution paths	Yes, primarily multiplication and addition	Yes, pupils do not see the solutions easily	Yes, 4x4 grid solution=$4^2+3^2+2^2+1^2$
Fraction Worksheet	Yes, pictorial format but symbolic assessment	No	No	Yes, primarily division with remainders by subtraction	No	No
Polygon's Restaurant	Yes, pictorial presentation including symbolic assessment	Yes	Yes, different seating arrangements possible	Yes, primarily addition and multiplication	Yes	Yes i.e. Tx2+2=□
Odd/even dice game	Yes, symbolic and pictorial	No	No, but various solution paths	No, addition only	Yes	Yes i.e. odd and even totals

5.3. A constructivist analysis of Anita's first lesson on 10.03.2011

In this section I will look at the path followed by the 2nd teacher participant, Anita, as she attempted to pursue a constructivist trail. It is worth mentioning that Anita came to the research late as she replaced Clarissa, who had obtained a teaching position in another school. This explains why her 1st observed lesson occurred so late into the school year.

Anita started the lesson with a card game. The pupils worked in groups of three. Two of the pupils paired up and placed a numeral card on their own forehead. No pupil could see her own card but she could see her partner's card. The third pupil called out the product of the two numbers on view. The object of the game was to see which of the two pupils would be quicker at the division calculation to determine their partner's number. The pupils enjoyed the game and it served as a stimulating way to revise division tables, which are difficult to teach in an interesting way.

At 9:33 Anita proceeded to establish classroom rules, which the pupils could adopt while engaging in groupwork. She had asked the groups of three to work together to devise some rules. Suggestions included listening to everyone in the group, not allowing one person do all the work and explaining something to someone who doesn't understand. One pupil summarised it well when she stated, ''There's no I in team.'' Afterwards, Anita showed a summary of the groupwork skills required on the Interactive White Board (IWB). It read as follows: Listen, respect opinions, take turns, include everyone and discuss. I think it would have been more democratic if Anita had placed the pupils' suggestions on the IWB and added in her own thoughts; although I have to concede that there was a huge overlap anyway.

Anita had a list of general classroom rules on her wall, which I wrote into my journal. These were:

1. Please put your hand up when you want to say something.
2. Everyone needs permission from the teacher before you move out from the desk.
3. The class listens carefully when teacher speaks.
4. Everyone respects each another and each other's property.

It can be deduced that, apart from rule 4, these rules are very teacher-oriented. Compare these rules with the ones for engaging in groupwork, which Anita placed on the IWB and it is evident that there the emphasis is pupil-centred. I believe this was an enormous leap for Anita to make in her practice as she was endeavouring to change current classroom norms for more democratic rules of engagement, which she herself hoped to establish. The development of such classroom norms in the area of groupwork was an emerging theme in the research. Like Lisa earlier, she was attempting to give more control to the pupils over their own learning and, for me; this was an emerging tenet of a constructivist approach.

I now turn to the cognitive aspects of the lesson. Anita's main activity was to ask pupils to compile addition sums, which would have an answer of 17.5. Anita gave one example: 17.0 + 00.5. The pupils worked in groups of 3 to 4 pupils. Anita used a novel idea of allowing one pupil from each group go to another group to share strategies they had encountered in their original group. They spent approximately 5 minutes in their second group before returning to their original group. At 32:00 Anita started to get feedback from the groups on the strategies they had employed. A spokesperson from each group came to the whiteboard and wrote three strategies .For example, groups 1 and 5 recorded as follows:

8 ½ + 9 =17.5; 12.2 + 5.3 = 17.5; 5.5 + 5.0 + 7.0 = 17.5

15% + 2.5% = 17.5%; 8.75 + 8.75 = 17.5; 16.0 + 01.5 = 17.5

To me the pace of the lesson appeared slow (journal entry10.03.11). It wasn't until 45:00 that Anita asked the pupils to use other operations instead of addition to help them with their answers. I believe the lesson lacked cognitive challenge for the pupils. This was a point conceded by Anita when she wrote in her reflection (ARL1):

> If I was to repeat this lesson again (I feel that the task was a little too easy) I would make it more challenging and possibly introduce ×, ÷ earlier in the lesson.

I agree with Anita's sentiments and believe that she was faced with a dilemma. Her pupils were obviously unfamiliar with groupwork in mathematics. She was trying to establish new classroom norms in this area and simultaneously maintain cognitive challenge for the pupils. I believe she 'sided' in favour of the former. It was understandable as she wanted to keep the pupils' interested in the lesson, but not alienate them through introducing too high a challenge. In terms of Jaworski's Teaching Triad, Anita sought to improve her classroom management skills in the area of groupwork, but had to make sacrifices in the area of mathematical challenge. In section 2.5 I mentioned that Potari and Jaworski (2002) define *harmony* as the extent to which the degree of challenge in a lesson is appropriate to the particular cohort of students involved. Harmony involves achieving a balance between sensitivity and challenge. I believe Anita was witnessing how difficult it is to achieve such harmony. The main activity allowed pupils to link various aspects of number together; such as fractions, decimals and percentages but ranked lowly in terms of cognitive conflict for the pupils. However, I anticipated that Anita's

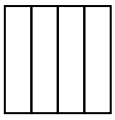
sensitivity to pupils' needs would heighten as she endeavoured to pitch the cognitive demands to pupils' abilities in future lessons.

5.3.1. A constructivist analysis of Anita's second lesson on 05.05.2011

Anita's 2nd observed lesson occurred two months after her first lesson. She had postponed the lesson, for personal reasons, until after Easter in a text sent to me on 13.04.11. Having revised her rules for groupwork, Anita commenced the lesson with a simple activity, which required pupils to work in pairs. Her rationale for using pairs was that it "allowed for more interaction and discussion"(ARL2). Interaction is one thing, it can lead to 'hands-on' activities. Discussion is another thing, it can lead to 'minds-on' activities. It is the blending of the two which can lead to activities which are both 'hands-on' and 'minds-on'. In my absence, Anita had obviously experimented with groupings and believed that "the pairs were working better than the groups of four in the previous lesson" (ARL2). For this activity, one child had to draw a regular or irregular 2-D shape and the second child had to name the shape. The children then swopped roles.

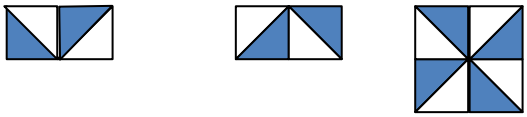
At 7:30 Anita drew a square on the IWB. She asked pupils for ways to divide it in half. The pupils indicated standard ways as drawn below:

At 26:43 she drew the following diagram of a square divided into quarters on the IWB:



She asked the children how it might be divided into eighths. The pupils gave her two suggestions as follows:

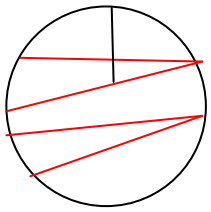
Anita conceded she hadn't thought of the first solution. This illustrates that when pupils take ownership of the mathematics, in line with a constructivist approach, they come up with unanticipated solutions. Admittedly, it was at a very simple level in this case. Afterwards, Anita requested the pupils to work in pairs to come up with other ways to divide the square using eighths as above. To prompt the pupils she stated that some groups were using squares, while other groups were using triangles to split up the square. Two groups came up with the following solutions:

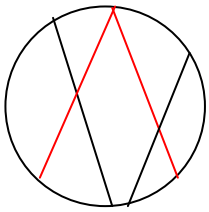


It occurred to me that perhaps Anita missed an opportunity to create linkage here between the strands of number and shape and space. She could have used the fractions of the square to identify the equivalence of 1/2 with 2/4 and 4/8. Teachers can often fail to respond to pupil questions, which may appear tangential, but which may actually lead to further learning. Here I am also saying that teachers need to remain alert to linkages within mathematics itself, which can lead to further learning. In my journal notes (05.05.11) I questioned whether there was enough progression in the lesson for 6th class pupils (11-12 years). I reasoned that maybe the issue was one of pacing. In other words, if the pace is strong in a lesson then progression is more likely to occur. After all, it wasn't until 40:24 that Anita decided to move on to an activity, which would really test pupils' ability. It came from a publication I suggested to her called 'Maths to Think About' by Claire Publications (2000). This

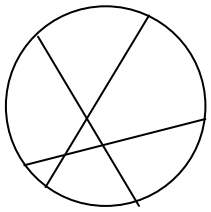
came about in conversation with Anita after her first lesson when she stated that her previous activity had been too easy for the pupils. I believed that this book contained activities which were designed to challenge pupils cognitively. This particular activity is called the Pie Activity.

Anita asked the pupils to cut a circular pie in 7 pieces using only 3 cuts. At 41:00 she gave the pupils great encouragement when she stated, "There isn't necessarily one answer to the question, so it's about you exploring. You're explorers, okay." The image of pupils exploring ''big ideas'' in mathematics, while going into unknown territory, is very powerful. In her reflection Anita remarked, "The children responded well to the lesson and they seemed to take ownership over their answers (ARL2)." It was interesting to see that the children did not interpret a cut to mean one line. For example, Lydia and Orla gave the following solutions respectively:





These solutions became a great discussion point with Anita seizing the opportunity to ask if these pupils had 'stuck to the rules'. These pupils seemed to take the view that a line was not complete until one lifted one's pen from the paper. They justifiably constructed their knowledge based on this interpretation. Other children like Yasmin came up with a more traditional solution such as:



When Anita saw that that some pupils were well able for the activity she asked them to explore how many pieces would be created using 10 cuts. Perhaps Anita could

have considered using 4, 5 and 6 cuts as an extension. I say this because the children came up with varied answers such as 23, 28, 32 and 35; which showed resilience but not necessarily a logical approach. The pupils did not record answers or solutions as they progressed through the lesson. Adopting an investigative approach implies the need to be systematic in recording observations or patterns; otherwise insights can be forgotten and conclusions difficult to derive. A diagram like the following may have been useful for recording answers:

Cuts	1	2	3	4
Pieces	2	4	7	?

In her reflection Anita stated that time ran out and she couldn't complete the entire lesson. In hindsight, she stated that she "was perhaps over ambitious (ARL2)." In conclusion, I think Anita was beginning to realise that open-ended problems can be extremely time-consuming and that adequate time needs to be allowed for a plenary discussion. These were points Anita was later to highlight in her exit interview (AEI). In terms of Jaworski's Teaching Triad my analysis of the Pie Activity showed me that Anita had become more aware of the cognitive challenge required in investigative mathematics. Although the main activity contained only one operation i.e. cutting the pie into sections, the cognitive challenge was constant for the pupils. In her management of learning she found that her actions of pairing pupils allowed for better interaction than larger groupings. Perhaps, she just needed to ensure that pupils adopted a logical approach to the activity by recording their findings as they progressed through the lesson. This recording would also enhance Anita's sensitivity to the pupils as she would be more aware of their level of progression.

5.3.2. A constructivist view of Anita's third lesson on 09.06.2011

In this lesson Anita attempted to open up her traditional teacher-led approach to mathematics. In the first minute she emphasised that there could possibly be more than one answer or more than one method. In this way, she was alerting pupils to the open-endedness of the task ahead. Her first action was to ask pupils to list possible sums with an answer of 11. At 2:40, she reminded pupils to share ideas with their partners, as they were going to work in pairs. She proceeded to use an approach which I will term "harvesting and sowing". As she circulated the room she orally gathered pupils' solution methods together and scattered them around the classroom for other pupils to use. She stated that one group had used brackets, another had used division, while yet another had used powers or indices. I thought this approach was inspirational and it was in line with a social constructivist approach to learning, which views knowledge as a social construction. At 7:30 pupils were asked to write their favourite solution and to share it with the class. Two pupils, Ciana and Lydia, gave their answers as follows:

Ciana (8+1)+(2x1)=11

Lydia (8+1+2) x 1=11

Anita asked the pupils to state if they had used Ciana's or Lydia's approach. Then she asked the pupils an interesting but ambiguous question: Which approach used the BOMDAS method? BOMDAS is a mnemonic for "Brackets, Of, Multiply, Divide, Add, Subtract". It helps pupils to prioritise operations within a sequence of operations. It could be argued that both pupils had used BOMDAS, but I think Anita wanted pupils to realise that multiplication takes priority over addition and the pupils accepted this without difficulty. At 18.35 Anita extended the previous task by asking pupils to list possible sums with an answer of 17·5 Anita entered into a

second cycle of “harvesting and sowing”, whereby she distilled pupils’ ideas and disseminated them among the class. She pointed out that some pupils were using whole numbers and decimals. She encouraged pupils to use percentages and fractions and also suggested using metric measures. Although it occurred off camera an interesting misconception of the task by teacher and pupil took place near me. Anita and a pupil, Nora, were working together. Nora suggested the following as a sum for 17·5:

Hrs.		Mins.
14	:	05
3	:	00
17	:	05

It can be seen that 5 minutes is not the same as 0·5 of an hour. Anita made an error by suggesting the following solution:

Hrs.		Mins.
14	:	50
3	:	00
17	:	50

Again, it can be seen that 50 minutes is not equivalent to 0·5 of an hour either. Such mistakes can occur in the melée of a busy lesson, but they highlight that we have units of measurement that do not rely on a base ten system. This needs to be pointed out to pupils as it forms the basis of modular arithmetic, which pupils encounter at second level.

At 32:11 Anita asked one pupil from each pair to join with another pupil to form a new pair. Her rationale was that ideas would be shared from one pairing to another.

In Anita's own words this seemed to work well and again I believe it was in line with an approach involving the social construction of knowledge. At 38:37 Anita placed the following calculation on the whiteboard for the pupils to solve: 0·75+1x10. Carmel offered the following solution: 1·00+0·75=1·75 and 1·75x10=1·75. Carol stated that the answer was 10·75. Anita probed further by asking how one obtains a solution of 10·75. Yasmin commented that 1·00x10=10 and 10·00+0·75=10·75. Anita astutely asked for a solution which coincided with the BOMDAS method. The pupils began to realise that multiplication preceded addition, so the correct solution belonged to Carol and Yasmin. Anita had not allowed pupil misconceptions to become embedded. Constructivist approaches do not imply that it's a case of anything goes. Phillips (1995) is graphic in describing such laissez-faire approaches as the *bad* in constructivism as mentioned in section 2.6. When pupils are in danger of assimilating incorrect subject knowledge the teacher needs to intervene to get pupils back on track. This is in line with a social constructivist viewpoint, which sees the teacher as an essential player in the knowledge construction process. Pupils are unable to work at their zo-ped if they make incorrect mathematical assumptions along the way. Anita concluded the lesson by asking pupils to find possible solutions for the following calculation:

$$\begin{array}{r} 2?? \\ \underline{?7?} \\ 422 \end{array}$$

As there wasn't a lot of time left in the lesson, the pupils did not get an opportunity to share their solutions.

5.3.2.1. Comment on Anita's third lesson

As part of my journal entry (09.06.11), I wrote that Anita was clearly relaxed during the lesson. In her reflection (ARL3) Anita commented that the pupils felt more at ease and confident doing "Team Maths", a name she gave to pupils working in a group (pairs in this case) during mathematics. She wrote that communication skills amongst the pairs had improved considerably and that children, who were working independently in previous lessons, were now sharing, discussing and problem solving with fellow classmates. From my perspective, I believe Anita had worked tirelessly to establish classroom norms whereby children felt relaxed whilst working cooperatively in groups. However, I believe this came at the cost of lowering the level of challenge she could introduce to the pupils. In other words, to maximise the level of engagement she minimised the level of challenge in the material presented, so that the pupils could experience satisfaction in their mathematics classes. At a minimum her activity generated a lot of discussion as pupils tried to interpret the BOMDAS rule. Pupils were also able to link the number and measures strands of the curriculum in coming up with sums for 17·5. For her fourth, and final lesson, Anita decided to use the Polygon's Restaurant activity, which Lisa had used in her third lesson; displaying cooperation between the two teachers. This activity introduced a higher level of mathematics challenge and the pupils rose to the occasion by suggesting some innovative solutions as the reader shall see.

5.3.2.2. A constructivist view of Anita's fourth lesson on 10.06.2011

This lesson took place just one day after Anita's 3rd lesson. There were two reasons for this. Firstly, in a pre-lesson conversation on 08.06.2011 Anita had wondered how long her third lesson should take. I wanted her to see that a lesson did not have to be completed in one hour and could range over several days if the topic was interesting

enough. Secondly, I was under pressure myself to complete the research before the end of the 2010/2011 school year. Anita's fourth lesson was therefore a continuation of her third lesson. Anita commenced the lesson with a 'fill in the missing numbers' problem. She wrote the following on the whiteboard:

$$\begin{array}{r} 4\ ?\ 6\ ? \\ \underline{?\ 2\ ?\ 3} \\ 1\,9\ 4\ 4 \end{array}$$

Pupils were encouraged to share their answers with their partners, so that they could learn from one another. Kate came up with the standard solution as below:

$$\begin{array}{r} 4\,1\,6\,7 \\ \underline{-2\,2\,2\,3} \\ 1\,9\,4\,4 \end{array}$$

Realising that there really was only one solution to the problem Anita introduced an open-ended element by asking the pupils to come up with answers close to 1944; if not 1944 itself. Nora and Rhona came up with the following solutions respectively:

$$\begin{array}{r} 4\,1\,6\,0 \\ \underline{-\,2\,2\,1\,3} \\ 1\,9\,4\,7 \end{array} \qquad\qquad \begin{array}{r} 4\,1\,6\,6 \\ \underline{-\,2\,2\,2\,3} \\ 1\,9\,4\,3 \end{array}$$

However, it was Carol who came up with a creative solution which showed a brilliant piece of independent thinking. She introduced a decimal point into the problem and offered the following solution:

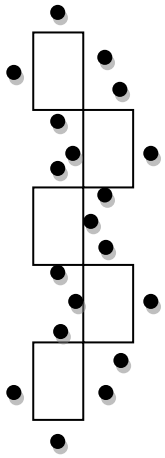
$$\begin{array}{rr} & 4\,1\,6\cdot 7 \\ + & \underline{1\,5\,2\,7\cdot 3} \\ & 1\,9\,4\,4\cdot 0 \end{array}$$

It is evident that by introducing a 1 into the thousands column she was also able to change the operation to addition. Over the four lessons observed, I believe that Anita

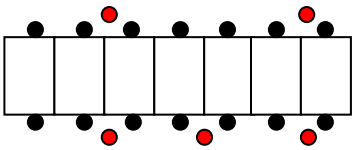
had helped establish a classroom climate wherein pupils felt free to experiment with their mathematics. Initially, I had wondered if Anita had pitched the mathematical challenge too low in her lessons. However, maybe this is a 'trade-off' that teachers need to do to give pupils confidence, so that they will engage with more difficult material at a later stage. At 23:30 Anita proceeded to revise the RUDE approach to solving a mathematics problem. RUDE is an acronym for Read the problem, Underline key words, Draw a diagram and Estimate what the answer could be. Anita asked the pupils if E could stand for anything else. One pupil suggested the word Evaluate. Anita agreed and then stated that the word evaluate meant to analyse, to focus in; to actually do the operation. I was puzzled by this as the word evaluate for me implied reflecting on one's solution when the problem was completed. I think the RUDE approach is best suited to helping pupils' get started on a problem as opposed to reflecting on the merits of their problem solving approach. On reflection, I believe Anita had confused RUDE with another acronym, ROSE which appears on the Primary Curriculum Support Programme website (www.pcsp.ie). I worked for PCSP for two years and was instrumental in placing this acronym on the website, as it was one I had found useful in delivering in-service courses to teachers. ROSE stands for Read the problem, Organise it into constituent steps, Solve the problem and Evaluate one's solution. It helps a pupil to work through the problem as opposed to the RUDE approach which is a pre-problem solving tactic.

Having revised the RUDE approach Anita decided to tackle the Polygon's Restaurant problem, which Lisa had introduced in her third lesson. This meant the two teachers were learning from one another. In a social constructivist approach it is envisaged that pupils will learn from one another. In adopting such an approach it makes sense that the teachers will also learn from one another. Such cooperation

helps greatly in the research design process as it enables the project to move forward smoothly. I now wish to refer to two innovative solutions, which two girls, Bria and Lea brought to the problem. It will be remembered that the problem required pupils to find the least number of tables needed to seat a party of 19 people who want to sit together. Lea drew the following solution:

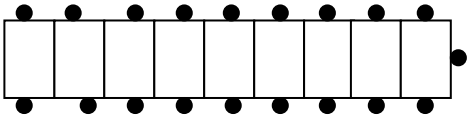


Lea used only 5 tables in her solution by pushing the corners of tables together as opposed to the sides. A minority of pupils thought Lea had broken the rules of the problem. Yet, it is this type of varied interpretation which can bring out creative solutions in pupils. Bria's solution involved interpreting the problem from a real world perspective. She drew the following solution:



In Bria's solution the black dots stand for adults at the tables whereas the red dots stand for babies' highchairs. What was interesting was that the majority of pupils accepted this as a valid solution to the problem. It has to be conceded that most restaurants allow for highchairs to be placed near tables. Bria had brought her real-life experience of witnessing babies in high chairs into the mathematics classroom. This gave the mathematics a cultural context for her and she responded by placing the mathematics into such a context. This gave Bria ownership of the mathematics

involved. Bria also gave another more predictable solution, presumably based on her experience of how children and/or adults usually sit around tables, as follows:



Many other pupils gave similar solutions. This type of solution led Anita to ask the pupils if they could predict the maximum amount of people, which could be seated at a particular number of tables. She stipulated that the tables had to be pushed together fully. The following grid emerged:

No. of tables	People	Pattern	Diagram
1	4	(1 X 2) + 2	
2	6	(2 X 2) + 2	

Anita asked the pupils to predict how many people could be seated at 7 tables and 10 tables. The pupils readily gave the required answers of 16 and 22 respectively. Anita then introduced an interesting twist to the problem by asking the pupils to find out how many tables would be needed if there were 26 people at a party. This meant that the pupils had to work backwards and manipulate an equation as follows:

$$26 = (12 \text{ X } 2) + 2$$

Answer: 12 tables

Overall, I rate the Polygon's Restaurant activity highly as a problem solving task as it combines several strands of the mathematics curriculum; number, algebra and shape and space. In this way, it is in line with the reformist agenda set by Ross et al.

(2002) which seeks to move mathematics outside the number strand. It generates a lot of discussion among pupils and allows for differentiation as pupils are able to interpret the activity in various ways. Pupils are also able to explore pattern in their quest to find solutions.

5.3.2.3. Conclusion on Anita's four lessons

In her reflection on her fourth lesson (ARL4), Anita commented that she underestimated the duration of the lesson and that her time management skills were affected when she did Team Maths. I can testify to this as I failed to capture her investigation of the pattern for the maximum number of people seated at tables, as the sixty minute film duration on the flip camera expired. Anita surmised that in future she would allocate plenty of time for her lessons, even suggesting that an entire morning would be an adequate period of time. I think this may have been an over-reaction, as it would be difficult to maintain pupil interest for such a long period of time. Nevertheless, Anita's adoption of the term Team Maths demonstrates that she worked hard to implement one of Windschitl's (1999) key features of constructivist classrooms quoted in section 2.14: pupils work collaboratively and are encouraged to participate in task-oriented dialogue. She used very novel strategies to assist pupils in sharing knowledge with one another. These included "harvesting and sowing" as explained earlier and allowing pupil representatives to move from one group to another; thereby sharing knowledge gained in a previous group. To paraphrase Windschitl (1999) she encouraged pupils to augment or restructure their existing knowledge.

It is necessary to state that I had been curious about the level of challenge Anita introduced in her earlier lessons. However, she coaxed her pupils along, slowly but

surely. In Jaworskian terms her actions meant she grew in sensitivity to her pupils' needs, raised the mathematical challenge by degrees and became more accustomed to a classroom management style, which gave pupils more ownership over their own learning through her Team Maths approach. Before I move on to the next section I wish to grade Anita's activities in terms of their problem solving characteristics.

5.3.2.4. Appraising the affordances and constraints of Anita's problem solving activities

Anita came to the project late when one of the participants obtained a teaching position in another school. As a result, she needed more guidance than the other participants in interpreting constructivist-compatible pedagogies. Anita attempted to choose something which was open-ended for her first activity. She asked the pupils to find sums with an answer of 17.5. The task was a simple open-ended one. However, apart from that, the task did not meet the other five appraisal aspects.

With more discussion Anita chose an activity for her second lesson which would satisfy five of the six criteria. This was the activity where pupils had to cut a pie in seven pieces using only three cuts. The activity requires a logical approach and gives pupils plenty of opportunities to discuss their findings with one another. It was an activity which two other participants, Aoife and Claire, found beneficial also. According to the criteria above it has a disadvantage in that it doesn't involve a variety of operations but this is to be expected in an activity which primarily involves drawing segments of a circle and investigating the related search for pattern.

Anita chose a task from the standard curriculum as her third activity; namely establishing the priority of number operations over one another. The task fulfils only

two of the six appraisal factors but it does have the advantage of initiating pupils into the mathematical convention of prioritising multiplication and division over addition and subtraction. This can involve a lot of classroom discussion as pupils grapple with the associated rationale. The task also highlighted the fact that the teacher participants had to come to grips with covering the standard curriculum while simultaneously engaging with more open-ended activities. This was not an easy alliance for them to uphold.

Anita's fourth activity was the Polygon's Restaurant task which her work colleague Lisa had researched on www.figurethis.org. This was a great activity for her to choose as it demonstrated all six appraisal factors and exposed Anita to an open-ended way of working. It also showed cooperation between the two teachers involved. Anita had chosen to finish the project with an activity which exposed her to constructivist-compatible pedagogies.

In summary, Anita's found the transition from textbook to investigative mathematics difficult. However, from my classroom observations I can certainly say that Anita had worked hard at establishing classroom norms which encouraged groupwork in her lessons. Having started with a teacher-dominated approach to classroom tasks she had progressed well along a constructivist-compatible continuum in giving more control of their learning to the pupils. Her progression was in line with one of the recommendations in the reform agenda set by Ross et al. (2002) who state that "instruction in reform classes focuses on the construction of mathematical ideas through students' talk rather than transmission through presentation, practice, feedback, and remediation" (p. 125). The affordances and constraints of her problem

solving activities are synopsised in Table 6 below. I now proceed to the activities of my 3rd teacher participant, Claire.

Table 6: Affordances and constraints of Anita's problem solving activities

Activity	Allows for Several Formats (Bruner)	Allows for a Smaller Case to be Considered (Polya)	Open-ended problem	Involves a Variety of Mathematical Operations	Allows for Engaging Classroom Discussion	Involves the study of Patterns (Boaler)
Sums with answer 17·5	No, symbolic only	No	Yes, various solutions possible	No, addition only in the way problem posed	No, pupils see the solutions easily	No
Cutting a pie in 7 pieces using only 3 cuts	Yes, pictorial presentation but symbolic assessment	Yes, if pupil adopts a logical approach	Yes, various solutions possible	No	Yes	Yes, if pupil adopts a systematic approach
Priority of Operations for 0·75+1X10	No, symbolic only	No	No	Yes, addition and multiplication	Yes, as pupils interpret task differently	No
Polygon's Restaurant	Yes, pictorial presentation but symbolic assessment	Yes		Yes, primarily addition and/or multiplication	Yes	Yes i.e. T X 2 + 2= P T=Tables P= People

5.4. A constructivist view of Claire's first lesson on 10.11.10

This lesson was a standard one based on tangrams. Claire stated in her lesson plan that she wanted to give a general background on the origin of tangrams and that she also wanted to allow the children to form various shapes using tangrams. In moving outside the number strand she was in line with one of the recommendations of the reformist agenda set by Ross et al. (2002). The first aim was easily realised as Claire introduced the lesson with a brief history of tangrams using the interactive whiteboard. I think Claire's second aim was fairly loose and, possibly, did not encourage progression in the lesson. I justify my stance through an analysis of the difficulty of the tasks Claire set for the pupils. Her first task was to ask the pupils to

put the 7 tangram pieces together to form a square as in Figure 12 below. The pupils did not have a template to help them.

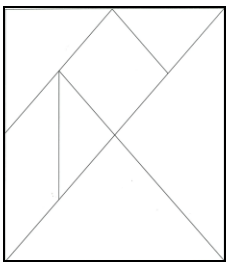


Figure 12: The square tangram activity

This was quite a difficult task for most pupils, especially as Claire's pupils were inexperienced with tangrams. This became obvious to Claire, so she gave the pupils a glimpse of a picture of a completed square, as in Figure 12 above, and later at 8:36 she gave them a hint by remarking that the two big triangles should make up half the square straight away. At 11:14 Alfred shouted, "I've got it." Claire gave a few minutes for other pupils to arrive at a solution.

At 15:54 Claire gave pupils a picture of a whale (see Figure 13) into which they had to correctly place the 7 pieces as in a jigsaw. If anything, this puzzle seemed somewhat easier to me than the square activity. Therefore, in my journal observations (10.11.10) I asked myself if there had been a progression from the simple to the more difficult in the choice of activities. It occurred to me that not only is the choice of activities important, as Hugh Gash had earlier advised, but the progression inherent in such activities is also vital. In her reflection on the lesson Claire later conceded that the pupils had found the first activity of making a square 'harder' than she had initially thought.

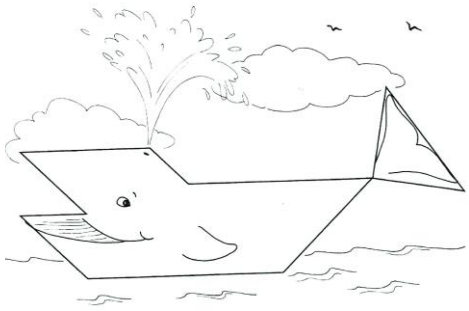
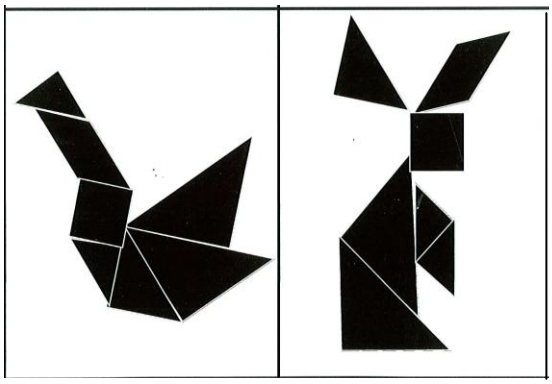


Figure 13: The whale activity

For Claire's third activity the pupils were requested to work individually to place the pieces onto pictures of animals or birds in which the delineation of the required shapes was clearly visible (see Figure 14). This appeared to be an even easier activity than the second one. Claire's logic seemed to be that the activity for the group had to be more difficult than the one for individual pupils. Such a view is interesting in that it emphasises the social construction of meaning at the expense of individual construction. I would like to hypothesise that ideally mathematics lessons should contain activities, which promote both individual and social acts of construction. Interestingly though, Claire she also gave the pupils a follow-on activity sheet on animals in which the pieces were not delineated. This activity required the pupils to think more and it certainly challenged the more able pupils.



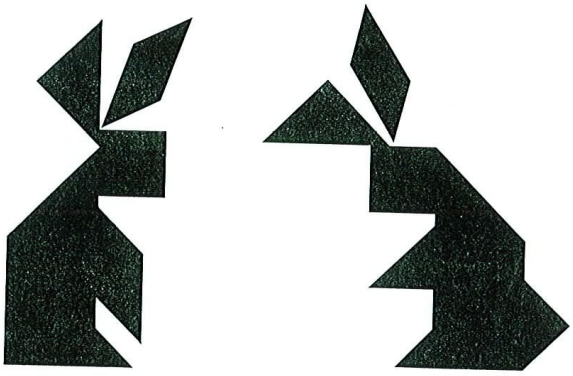


Figure 14: The animals and birds tangrams

The pupils were allowed work through the activities either as individuals or in a group situation. Claire remarked in her reflection that they "helped each other and encouraged each other but lots of individual work also (CRL1)." There was no time for a plenary session as the pupils were kept busy with the amount of activities.

5.4.1. Comment on Claire's first lesson

Claire was ambitious in this lesson in the amount of tangram activities she tried to cover. The noise level in the classroom was very high throughout the lesson, which clearly indicated that the children enjoyed the activities. I witnessed "lots of excited chatter and encouragement" within groups as Claire testified in her reflection (CRL1). As regards the level of progression in the lesson, Claire seemed to oscillate between higher and lower level cognitively-challenging activities. Perhaps a clearer progression from simple to more complex activities could enable the pupils to be more cognitively challenged. I discussed this with Claire, so that she could design future activities around this principle. I also discussed moving more towards group initiatives, as there was evidence of more individual work than group work in her lesson; a point conceded by Claire in her reflection (CRL1). In terms of Jaworski's Teaching Triad, Claire's actions concerning the management of learning appeared to show an awareness of how to organise groupwork. However, she did not appear to

display a great deal of sensitivity to the pupils as the activities oscillated between higher and lower order cognitive challenge.

5.4.2. A constructivist view of Claire's second lesson on 08.03.11

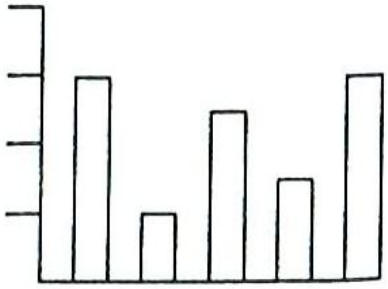
It can be seen that this lesson was observed almost four months after Claire's first lesson. At this stage the design of the project had changed in that I realised that the participants were struggling to create open-ended problem solving activities. To assist in this process I photocopied two pages from a book called Thinking Allowed by Sue Cunningham (2003) on how to create open-ended tasks in the mathematics classroom (see Appendix 13). Claire chose five tasks from those sheets to create her lesson.

Claire commenced the lesson with two warm-up activities in mental computation. In one of these activities pupils used loop cards to succeed one another when they had the correct answer to a given question. For example, when one pupil stated that she had 18, but asked who had 4 X 6, the pupil with 24 on her loop card replied and posed the next question. These loop card activities were very prevalent as an introduction to many of the lessons observed. They were justified as the children certainly appeared to enjoy them. At 9:49 Claire introduced the lesson proper by asking if any pupil knew what an open-ended problem was. Gerry replied that an open-ended problem was one where there was more than one answer. Afterwards, Claire randomly assigned each pupil a number from one to five. All the *ones* were to work in one group, the *twos* in another group and so on. Each group was then given a problem and told that its members had five minutes to complete the problem before moving clockwise around the classroom onto another problem. The five problems were as follows:

Problem 1: What numbers could replace the asterisks?

$$\begin{array}{r} 2\,{*}{*} \\ +\ \underline{{*}7{*}} \end{array}$$

Problem 2: What might this be a graph of? Label the axes appropriately. Describe what the graph tells us.



Problem 3: The answer is 17.5. What might the question be?

Problem 4: Zoo entry is €5.50 per adult, €3 per child and €3.50 concession. If I had €30 to pay for entry, what combinations of people could I afford to take based on these prices?

Problem 5: The West Gate Bridge has an average of 2.5 passengers per vehicle at any time. Draw what this might look like if there were 8 vehicles on the bridge.

The pupils proceeded to work in their groups for the designated five minutes on each problem. However, not all pupils stayed on task. At 21:50 two pupils played a 'clap hands' game while at 36:00 several pupils made hand gestures to the camera. Moreover, some pupils worked as individuals within groups instead of contributing to an overall group effort. This was evident in the previous lesson also and shows the difficulty teachers face in introducing groupwork to pupils accustomed to working as individuals in mathematics lessons. At 39.00 Claire introduced a plenary session

and asked for feedback from the groups. In Problem 1, pupils came up with several solutions but Claire did not ask the pupils for a general strategy to solve the problem. I believe this was a missed opportunity. No significant data emerged from Problems 2-4 but when it came to Problem 5 one group displayed an interesting misconception. They stated that there were 2 adults and 1 child in each car and interpreted this as 2½ adults in each car to justify a total of 20 people in 8 cars. Claire explained that their solution had 3 people in each of 8 cars and that their total was 24 and not 20.

5.4.3. Comment on Claire's second lesson

Claire was very ambitious in this lesson in attempting to cover five problems through a group-centred approach. Perhaps the pupils would have been better served had she chosen one or two problems on which to concentrate. Fewer problems would have given her the opportunity to strive for a deeper level of understanding. The pupils would have been afforded an opportunity to work at a more challenging level. Instead, I believe Claire engaged in what I shall call 'skimming' which I define as a teacher covering a lot of content but at a very superficial level. Skemp (1995) would have described this superficiality as instrumental understanding, as opposed to a deeper level called relational understanding. A metaphor may be a teacher skimming along the surface waters of mathematics without diving to a deeper level.

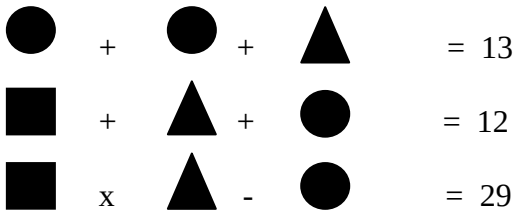
On the other hand I learned a lot about classroom management from Claire's organisation of her groups. She randomly assigned pupils numbers from 1 to 5 and asked the *ones* to come together, then the *twos* and so on. It was a very efficient method. Perhaps assigning pupils to groups by giving individuals a number could be

made into an even more powerful strategy, by ensuring pupils who work well together stay together. Therefore, the change I suggest is to strategically, rather than randomly, assign pupils numbers. Obviously, pupils would have to be rotated from time to time, as their attitudes and motivations change.

In her reflection, Claire mentioned that she could have dedicated more time at the end of the lesson to discuss pupils' outcomes (CRL2). As it was, she had only allowed 11 minutes for the 5 groups to provide their feedback. In view of the amount of material she attempted to cover in the lesson, I would agree with her appraisal. A design alternative would be to take fewer problems but go deeper with them as mentioned above. I also suggested the exploration of patterns and generalisations in her activities. Claire responded positively to these suggestions when I spoke to her post-lesson. She mentioned in her reflection that some pupils needed more encouragement to get involved in problem solving and to find more than one solution to the problems. I can testify to this as several pupils were distracted and made gestures to the camera during the lesson. However, this was understandable as the pupils were moving around the classroom to different problem solving stations and it was difficult for Claire to keep them all on task. In terms of Jaworski's Teaching Triad Claire's management of learning showed that she was becoming more adept at organising groupwork. As in her first lesson, cognitive challenge was an issue in that Claire appeared to cover too many problem solving tasks, which resulted in 'skimming' as outlined above. As a result, sensitivity to pupils' needs became an issue for me. I proceed to Claire's third lesson, in which some of the issues raised in this lesson re-emerge.

5.4.4. A constructivist view of Claire's third lesson on 18.05.11

Claire commenced the lesson with what she called three 'warm-up' activities. In the first one the children had to come up with solutions to these individual equations:



I was surprised at how fast the children produced solutions. Ian came up with the solution 6 + 5 + 1 = 12 and 6 x 5 – 1 = 29 which showed he had treated the last two equations as being simultaneous ones. Jim intervened to say the 5 and 6 were interchangeable. These two pupils certainly seemed to be cognitively engaged.

In the second activity the children were asked to discover how many ways there were to make 50c using 7 coins. The pupils readily gave solutions to this task. Claire introduced a second challenge to this activity when she asked the children to find the largest amount of money one can make with seven coins. Bob stated that €3.50 (7 x 50c) would be the highest amount, excluding €1 and €2 coins. At 8:50 Claire gave the third activity. Pupils were shown the following numbers on the interactive whiteboard and asked to discover the rule involved to find the missing numbers:

$4 \rightarrow 8 \qquad 7 \rightarrow ?$

$9 \rightarrow 13 \qquad 16 \rightarrow ?$

In this case the pupils were required to notice that the pairs of numbers given increase by 4 each time. Claire gave two other examples for the pupils to complete, which they readily did. At 11:21 Claire asked pupils to go into pre-arranged groups of four or five pupils. She gave them an activity which required them to put 12 crosses in a 6 x 6 grid but there could be no more than 2 crosses in any row across,

down or diagonally. The pupils worked on this activity for approximately 11 minutes. I wrote in my journal (journal entry 18.5.11) that there was genuine group work going on and that the pupils were not just working as isolated individuals. At 22:56, Frank gave feedback on his group's work by drawing the following diagram on the standard whiteboard:

X					X
		X	X		
	X			X	
	X			X	
		X	X		
X					X

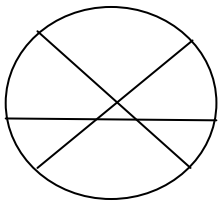
Then Claire drew another solution on the whiteboard as follows:

X			X		
X			X		
	X			X	
	X			X	
		X			X
		X			X

Claire did not generate a discussion on the tactics required to solve the puzzle. She chose instead at 25:25 to introduce the pie activity, which Anita had done with her class on 5.5.11. This was the activity where the pupils had to cut a circular pie into 7 pieces using only 3 cuts. I had suggested this activity to Claire and the fourth participant, Aoife, as it had worked well with Anita’s class. This was a design change for the project as I had initially planned for the teachers to devise all their

own activities. However, Claire and Aoife requested more guidance in choosing their activities. A possible explanation for this is they were not used to new initiatives, unlike Lisa and Anita, who taught in a disadvantaged setting where new initiatives in literacy and numeracy are commonplace. I gave them some activities to consider, but I did not give them a lot of pedagogical advice, as I wanted them to construct their own methodologies in partnership with one another and take ownership of them in line with a constructivist viewpoint.

At 25:45 Claire outlined that she wanted the children to figure out the largest number of pieces one could make with 10 cuts to the pie and investigate if there was a pattern. The children had difficulty with this aspect of the activity. At 37:35 one girl exclaimed, “I don’t get the number sequence.” Claire explained that it meant the relationship between the cuts and the pieces. The difficulty was that the children were trying to predict what would happen with 10 cuts to the pie instead of possibly starting with 2, 3 and 4 cuts and building up a sequence. At 39:00 Claire called time on the activity and I believe this was the right move, as some of the children were getting frustrated. Rose came to the whiteboard and gave a standard solution to the 3 cuts problem:



Claire asked the pupils to create a sequence or relationship between the cuts and the pieces. Greg stated, “It’s one for every two except when you get to three.” This sounded so profound that some pupils laughed in amazement. In the absence of other generalisations Claire developed the following sequence on the whiteboard with the help of the pupils:

Cuts	Pieces
1	2
2	4
3	7
4	11
5	15/16

It can be seen that pupils were not sure how many pieces pertained to 5 cuts. This could explain why the majority of pupils were unable to generalise up to 10 cuts. At 45:00 Claire concluded the lesson.

5.4.4.1. Comment on Claire's third lesson

When Claire began this lesson, my initial reaction was that I was partaking of a smorgasbord of simple mental activities, but that there was no main meal of intellectual substance. Certainly, Claire spent the first eleven minutes of the lesson dealing with an array of mental activities, which bore little relation to one another. O' Shea (2007) also noted the prevalence of mental mathematics at the beginning of Irish mathematics lessons. Perhaps some of this time could be used at the end of a lesson, in giving children adequate time to give their feedback in a plenary session.

I was fearful that Claire was yet again going for breadth of activities rather than depth as occurred in lesson two. However, Claire proved me wrong in the pie activity, as she tried very hard to get the pupils to generalise the ratio of pieces to cuts as outlined above. I believe she was over-ambitious in attempting to get the pupils to generalise the pattern up to 10 cuts, as they clearly ran into difficulty after 5 cuts. Nevertheless, this showed that she was becoming more conscious of getting the pupils to work at a challenging level by introducing the generalisation of patterns

into her lessons. In her reflection, Claire stated that she enjoyed watching the children "think outside the box with just a little guidance" (CRL3). Again, Claire was confronting the constructivist's dilemma of whether to intervene or not. To me it seemed that she needed to give the children a little more guidance when they clearly ran into difficulty. Such carefully-timed guidance can keep children motivated and lead them on to further discoveries. Claire also remarked in her reflection that the children worked well in groups, encouraged each other and were "edged on by one group finding the answer" (CRL3). This illustrates that success breeds success, and that when some children work at a challenging level, others may be motivated to do the same. I rate the Pie Activity highly as a mathematical endeavour as it serves to encourage pupils to discuss their solutions with one another and, if used properly, it also helps them to generalise the pattern generated by the ratio of cuts to pieces. In terms of Jaworski's Teaching Triad, Claire's actions in the management of learning demonstrated there was genuine groupwork going on, with the pupils encouraging one another. They were engaged in a challenging activity but Claire could be more sensitive to their abilities, particularly in asking the pupils to generalise the pattern up to 10 cuts without adequately considering a number of smaller cases. I now move on to Claire's fourth and final lesson.

5.4.5. A constructivist view of Claire's fourth lesson on 25.05.11

Claire's 4th lesson plan was vague on details (see Appendix 14) so I was somewhat unsure what to expect. Claire commenced the lesson with a puzzle:

Pick a number between 50 and 100. Add 73. Remove the hundreds digit and add it to the other 2 digits. Subtract from the number you began with.

Claire did an example on the whiteboard. A pupil had picked 99 as the starting number: 99 + 73 = 172. 72 +1 = 73. 99 – 73 = 26. Claire did two other examples and

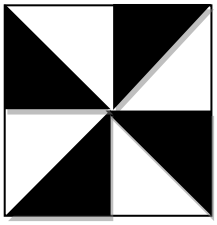
then at 4:57 asked, “How’s everyone getting 26 all the time?” She didn’t get any satisfactory answers, so at 7:11 she gave the following explanation: “If you add the 73 and the 26 you get up to 99 and then the 99 is why everybody ends up with the same relationship.” Personally, the explanation was not very clear, and I doubt if it was to the pupils either. Also Claire did not hear a valuable interruption (6:20) when a pupil asked, “Would it work if it wasn’t between 50 and 100?” The answer is ‘sometimes’ but it would have provided the pupils with a very interesting investigation to explore the number ranges within which the puzzle works and thereby construct their own knowledge. Unfortunately, the moment was lost in the midst of mathematical time. At 7:31 Claire introduced a second puzzle to the pupils. She asked them to use 6 sixes to make zero. The pupils worked individually on this one and came up with some interesting answers:

6 + 6 – 6 – 6 – 6 + 6= 0; 666-666=0 and (6 x 6) – (6 x 6) + 6 – 6=0.

At 10:36, Claire introduced a third puzzle. She asked the pupils to split the numbers 2, 4, 6, 8, 10, 14 into two groups, so that each group added up to the same number. Ian suggested the following solution: 8 + 14 = 2 + 4 + 6 + 10 =22 and Brian followed with 14 + 2 + 6 = 10 + 8 + 4 = 22. At 14:07 the smorgasbord of activities continued with Claire introducing an activity, where pupils had to find seven more ways of dividing a square in half in addition to three ways already given on an activity sheet entitled Challenge for the Week (see Appendix 15). This activity was taken from a book called ‘Maths to Think About’ which I had given to Claire and Aoife, her work colleague and fourth participant in the project. Introducing the book was a design change, that I believed was necessary to assist them in choosing mathematical activities that would bring depth and cater for ‘‘big ideas’’. As mentioned earlier, I did not choose the actual lesson activities from the book for the

teachers, as I thought this would be too prescriptive and run counter to the constructivist idea of teachers negotiating pedagogical knowledge for themselves.

Claire left the pupils work on the activity, for seven minutes approximately, in their groups of four or five pupils. Then she took feedback. Individual pupils came to the whiteboard to give solutions from their groups. Seven such solutions were as follows:



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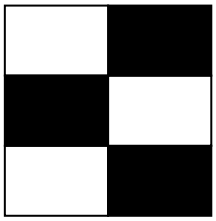

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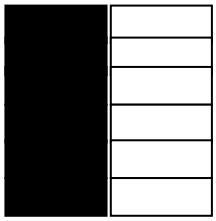

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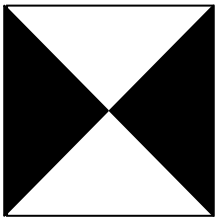

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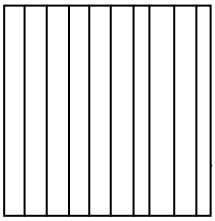

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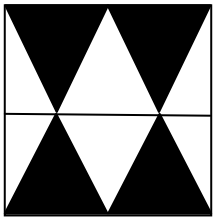

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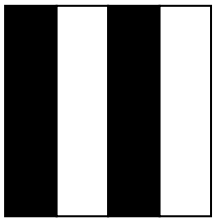

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There were several points I noted on the construction of the pupils' solutions. At 21:30 Claire remarked as regards solution 4 above, "They (the group involved) had divided it into 2 and then they divided it into 4 and then coloured 2 out of the 4 parts; which is relating back to your *what* fractions?" Greg replied, "Equivalent fractions." I was glad to see Claire help the pupils make the link between the fraction

(1/2) and the equivalent parts of the 2-D shape. This linking of the symbolic and the pictorial modes helps to open up mathematics for the pupils, so that they do not perceive the different branches of the subject as being discrete from one another. I stated earlier that linkage was an important construct for another teacher participant, Lisa, as it helped her introduce open-ended problems, while dealing with the constraints of covering a prescribed syllabus. She was far more willing to introduce such problems when she believed she could cover several aspects of the maths curriculum simultaneously. I was beginning to see that linkage could be a way forward for Claire also. Claire reinforced my view in her post-lesson interview (CPLI4), when she stated that not only had she linked equivalent fractions into the lesson, but that she had also brought in number sentences, which she had covered earlier in the week. From the examples above Claire pointed out that ½ is equivalent to other fractions also such as 3/6, 4/8, 6/12 and 10/20. In example six, George astutely observed that it was not essential to use the horizontal line to show half of the shape. Although he didn't verbalise it, I believe George had realised that the square, as drawn, had both vertical and horizontal line symmetry.

At 27:45 Claire introduced a final challenge to the pupils. She asked the pupils to fill in the numerals 1-9 on a 3 x 3 magic square. The pupils worked on this activity in their groups for approximately seven minutes, while Claire circulated the room. At 34:58 she strategically intervened when she proclaimed, "I'll give ye one clue; I'll do the middle box." This was designed to assist pupils who were struggling with the activity. At 36:55 there was genuine excitement emanating from George's group at the back of the room as one pupil shouted, "We have it." At 38:12, Claire asked George to provide the top row for the other pupils in the class. As soon as this

occurred the pupils were quick to fill in the remaining numbers. George then wrote the complete solution on the whiteboard as in Figure 15 below:

4	9	2
3	5	7
8	1	6

Figure 15: The magic square activity

In dealing with the magic square activity the children had engaged in a very basic form of discovery learning, but with the teacher's help. The 1971 curriculum stated: "Since the child needs to have a reasonable amount of success, the understanding teacher will guide the pupils along the paths where useful discovery is most likely to occur" (p. 126). However, constructivists assert there is no 'discovery learning' in the sense of a revealing or unmasking of any kind of external structures (Bauersfeld, 1995). Pupils arrive at new descriptions and regularities depending on the creative combination of elements already available to the subject; accommodation in Piagetian terms. Therefore, the crucial constructive processes are strictly subjective and developed through social interaction, rather than something like reading a book of nature or discovery. Von Glaserfeld (1987) comments that to say "we can discover aspects of an objective reality, because we are able to experiment or modify our environment, is merely to extend the realist illusion" (p.108). Even Bruner (1986), an early advocate of discovery learning, has renamed it as 'learning by invention'. He states that he has come increasingly to recognise that most learning in most settings is a communal activity, a sharing of the culture, so to speak. "It is this that leads me to emphasise not only discovery and invention but the importance of

negotiating and sharing- in a word, of joint culture creating" (p. 127). It can be seen that there is a large role here for teacher-pupil dialogue.

It was here that the discovery learning ended in Claire's lesson. Instead of investigating the possibility of further solutions to the magic square, Claire asked the pupils to go back to a revision exercise in their mathematics textbook for the remaining five minutes of the lesson. It seemed to me that the intellectual gourmet meals had come to an end and it was now time for Claire to go back to the staple diet of the 'mathematics cookbook'. Of course, this reflects the constraints under which teachers work; following a textbook, which mediates the curriculum for them, but often stifles their own creativity.

5.4.5.1. Comment on Claire's fourth lesson

As in lessons two and three, I believe Claire tried to cover far too much material in her fourth lesson. This meant that any challenge introduced into the lesson had to be fleeting and could not be sustained due to time constraints. In her reflection (CRL4), Claire stated that she could have explored some avenues further; like the relationship between the number in the middle of the square and the magic number. She commented that the magic square activity took longer than expected. To help the pupils along, she stated that after sufficient time she gave the pupils a clue; presumably referring to when she told the pupils the middle number. She followed the clue by showing them the top row. Therefore, her scaffolding was praiseworthy but, perhaps, the challenge needed to be more sustained to keep pupils working at a high intellectual level. Hopefully, this would become easier for Claire as she gained experience in becoming more aware of pupils' problem solving capabilities when they worked in groups. Claire stated that the groups were interested in the problems

she set for them and that she could see ‘their brains working’ but she also conceded that one or two pupils left all the work to the others and needed encouragement to engage. To summarise, in Jaworskian terms, Claire’s actions in the management of learning was exemplary in terms of mixed ability group organisation. However, the dearth and sporadic nature of mathematical challenge in the lesson for the more able pupils meant that perhaps Claire’s sensitivity to such pupils’ mathematical needs needed to be raised. Her main activity consisted of finding seven ways to divide a square in half. I would not rate this activity highly in terms of mathematical challenge but it had the advantage of linking the number strand (fractions) with the shape and space strand. Furthermore, the iconic nature of the activity encouraged the pupils to share and discuss their solutions with one another. Before I move on to the next teacher participant I wish to appraise the affordances and constraints of Claire’s main problem solving active

ities.

5.4.5.2. Appraising the affordances and constraints of Claire’s problem solving activities

Claire’s first activity involved assembling tangram pieces. The pupils were physically active and engaged in quite a bit of discussion with one another. The activity fulfilled three of the six criteria but was somewhat closed as there was very little scope for the pupils to be creative in their solutions. It demonstrated that being physically active does not necessarily mean being cognitively challenged. This reminds me of Fosnot’s (2005) comment that reform-based pedagogical strategies can be used without the desired learning necessarily resulting.

Perceiving that Claire might need some help in choosing more open-ended activities I recommended some from Sue Cunningham’s (2003) book ‘Thinking Allowed’ for

Claire's second lesson. This meant that the activities chosen fulfilled four of the six criteria but I would hold that perhaps Claire tried to cover too many tasks in the one lesson with the result that too much 'skimming' occurred and, yet again, pupils were not sufficiently cognitively challenged. In my opinion, Claire could concentrate on one or two of the tasks so that she could get the pupils to probe them at a deeper level. This would necessitate a more active role for Claire in helping the pupils to scaffold their thinking. Initially, this scenario involves a lot of questioning by the teacher as she encourages the pupils to join her in solving the task. Then, as the learners become more competent, the teacher steadily withdraws the scaffolding so that the learners can perform independently (Cowan 2004). In construction parlance, it evokes for me an image of a ladder being removed from a structure once it is near completion. After all, as Bruner (1986) remarks, it is the loan of the adult's consciousness that gets the pupil through the zo-ped.

With more guidance Claire chose an activity for her third lesson which would satisfy five of the six criteria. This was the activity where pupils had to cut a pie in seven pieces using only three cuts. The activity requires a systematic approach and gives pupils lots of opportunities to discuss their findings with one another. It was an activity which two other participants, Aoife and Anita, found beneficial also in opening up discussion.. It is an activity which makes it easy to follow Graham Nuthall's advice as quoted in Brophy (2006) i.e. set up a common experience, preferably a small group cooperative activity, that produces the data or knowledge that will be the focus of the discussion (see section 3.13). According to the criteria outlined earlier, the activity has a disadvantage in that it doesn't involve a variety of operations but this is to be expected in an activity which primarily involves drawing segments of a circle with a related search for pattern.

For her fourth lesson Claire used an activity from a book I introduced to the participants called ‘Maths to Think About’ by Claire Publications. The children had to devise seven different ways to divide a square in half. As the activity is primarily non-numerical it only satisfies three of the criteria but this probably underestimates its worth as it is open-ended and generates a lot of classroom discussion. It also exposes pupils to the difference between the concept of halving a shape and checking it for axial symmetry. Instead of exploring such symmetry with the pupils Claire introduced a further task whereby pupils had to solve a magic square. Again, perhaps this meant that depth of knowledge had been sacrificed at the expense of breadth of material covered.

In summary, choosing less tasks, but covering them in more depth, could be useful for Claire’s pedagogy. It also occurred to me that Claire did not want to stray too far from the standard curriculum in her choice of activities. Therefore, the linking of mathematical strands with one another (linkage) could be another way forward for her in choosing her mathematical activities so that she could cover prescribed curriculum content while simultaneously introducing a more open-ended approach to her work. The affordances and constraints of her activities are outlined in Table 7 which follows. I then proceed to apply a constructivist lens to Aoife’s lessons, the fourth and final teacher participant.

Table 7: Affordances and constraints of Claire's problem solving activities

Activity	Allows for Several Formats (Bruner)	Allows for a Smaller Case to be Considered (Polya)	Open-ended problem	Involves a Variety of Mathematical Operations	Allows for Engaging Classroom Discussion	Involves the study of Patterns (Boaler)
Tangram Activities	Yes, enactive and pictorial	No	Yes, various solutions possible	No, activity involves spatial awareness.	Yes, as pupils manipulate shapes.	No
Sue Cunningham Activities	Yes, pictorial and symbolic	No	Yes, designed as such	Yes, all four operations could feasibly be involved.	Yes, as group work Involved.	No
Cutting a pie in 7 pieces using only 3 cuts	Yes, pictorial presentation but symbolic assessment	Yes, if pupil adopts a logical approach	Yes, various solutions possible	No	Yes	Yes, if pupil adopts a systematic approach
Seven ways to divide a square in half	No, pictorial only	No	Yes, various solutions possible	No	Yes	Yes, involving one-to-one mapping

5.5. A constructivist view of Aoife's first lesson on 09.11.10

Aoife commenced this lesson in an unusual way in that she asked the pupils to do some physical warm-up exercises at their desks. I guess she believed in the old Roman adage of a healthy mind in a healthy body (mens sana in corpore sano). Afterwards she asked individual pupils to recite the square tables i.e. $1^2 = 1$, $2^2 = 4$, $3^2 = 9$ etc. Her actions in revising this prior knowledge was to prove instrumental in how some pupils later approached the main lesson activity. Aoife wrote the word 'inverse' on the IWB, elicited that it meant 'reverse' and asked the pupils the value of $\sqrt{25}$, $\sqrt{49}$ and $\sqrt{100}$ respectively. It was clear that Aoife was establishing links for the pupils between the square of a number and the inverse of the square being the square root.

At 3:55 Aoife wrote the equation 5 + 5 + 5 = 550 on the IWB and asked the pupils to work in pairs to make one adjustment to it to make the equation true. The pupils were finding it difficult to come up with solutions and at 7:05 Aoife gave a hint to the pupils when she asked them to 'think back to lines and angles.' As the pupils were clearly struggling with the problem, I believe Aoife was correct in giving a hint, which might help pupils to exceed their current mental grasp of the problem and work at their zo-ped. The pupils worked on the problem for another five minutes and then Aoife stated that she wanted to talk about Emma and Paul's solution. Their solution was as follows:

5 + 5 + 5 = 550 **X**

It can be seen that the two pupils had made the equation balance by placing an **X** at the end of it, to show that the initial equation was incorrect. It could also be said that they had employed Aoife's hint of using lines and angles. Aoife proceeded to give the pupils two other possible solutions i.e. 5 **4** 5 + 5 = 550 and 5 + 5 + 5 ≠ 550. It was interesting that the pupils had come up with a solution which Aoife had not anticipated and she praised them accordingly. Next, Aoife moved from pair work to four-pupil groupwork in a most effective and enlightening fashion. She gave an envelope to groups of four pupils, which contained the names of who would act in the roles of recorder, reporter, timekeeper and chairperson. Aoife gave the pupils a rectangular sheet of centimetre-squared paper measuring 28 by 19 centimetres. The dimensions seemed to vary slightly depending on how the sheet had been photocopied. They were then given fifteen minutes to tackle the very open task of discovering how many squares were on the page. My initial reaction was that the task was too open and that the pupils might struggle to interpret it. Aoife's first lesson plan (see Appendix 16) did not provide any further elaboration. At 18:20, Aoife reinforced her terms for the task when she reiterated that she wanted to know

how many squares there were. At 31:20, Aoife asked the groups to put the finishing touches to the task, so that the reporter could give feedback. As this was a 50 minute lesson Aoife had allowed adequate plenary time; something which had not always happened in previous participants' lessons. The following conversation occurred between Aoife and James, who was asked to give feedback for his group. By way of background, James, a high achiever in mathematics, had been somewhat aggrieved that, in his opinion, his group had not given him a fair hearing.

James: I multiplied 19 by 29 and then I got 551.Then I divided by…

Aoife (interrupts): I'm just going to pause there. Would that be a perfect square if you multiplied by two different numbers? Would ye get a perfect square on the sheet? So you had the right idea, you were on the right track, but maybe using the wrong numbers.

James (elaborating): That was the small squares, the small ones that were on the sheet. Then I divided it by 4, because if you make 4 squares that equals a square, I got 137. Then I divided it by 9, I got 61. I divided it by 16, I got 34. I divided it by 25, I got 22. I divided it by 36, I got 15. What was the next one (pauses)? I divided it by 49, I got 11. I divided it by 64, I got 8. Then I divided it by 81, I got 6. Then I divided it by 100, I got 5. And then I added them all together.

Aoife: And tell people now who mightn't know, why you divided by, we'll say 49 and 25.Where did you get these numbers from?

James: You just square them like that (draws a square in the air). If you get one square and then you have 5 down and 5 across you get 6 to make a square.

Aoife: Very good. I like the way you think.

James: And then I got the answer.

Aoife (addressing the other group members): You should have listened to him a little bit better.

I believe this conversation vignette illustrates two points. Firstly, there seemed to be a disparity between where James wanted to go with his investigation and where Aoife wanted him to go. Perhaps Aoife didn't give him due credit for discovering that there were 551 small squares on the sheet with dimensions one centimetre by one centimetre. Aoife seemed to want to funnel James into using other perfect squares to divide up the page. James had come to this method of proceeding by himself anyway. When Aoife realised what James was doing she acknowledged his contribution. Secondly, as James was such a high-achieving pupil, he was operating at a cognitive pace well beyond the other members of his group; so much so that they weren't listening to him. This illustrates that sometimes pupils, like James, need to work as individuals within groups to construct their own knowledge and create their own 'eureka' moments. There are other times when the group dynamic will determine the type of knowledge generated. My point is that one cannot sit solely in the camp favouring the individual or the social construction of knowledge; praxis requires both.

Feedback from other groups also suggested that they had multiplied the length by the width of the page to find the number of individual 1cm x 1cm squares. Then they had divided the area by 4 (2x2) or 9 (3x3) etc. at a numerical level to discover how many squares there were of such dimensions. Other groups like Jeremy's found the numerical sum of square numbers, but had to stop at 19^2 as they were constrained by the width of the page. Appendix 17 of Jeremy's work illustrates this approach. Jeremy was working at a symbolic level whereas Emma's group produced similar findings, but at a pictorial level. Appendix 18 of Emma's work shows that both the pictorial and symbolic modes of Bruner's representations were at play in the classroom. Emma's work gave Aoife the idea for an inspired conclusion to the

lesson; one not anticipated in her lesson plan. At 42:18 Aoife asked the pupils to use highlight markers to shade in the square numbers on their sheets in much the same way as Emma had done. This showed that Aoife had the ability to think and act spontaneously in using pupils' contributions; a useful trait when one is adopting constructivist-compatible pedagogies.

5.5.1. Comment on Aoife's first lesson

Although Aoife did not write written reflections on her lessons her immediate post-lesson interviews were rich sources of data, as Aoife spoke very freely for them. She raised three intertwined issues of control, telling and linkage in her first post-lesson interview (AOPLI1). As regards control, Aoife stated that the hardest thing for her, was to try *not* to tell the pupils what to do, while she was circulating the groups. Giving more control to the groups was problematic for certain children as well as for the teacher. "Some people were like trying to take over and other people were quite happy to sit back and just listen to what everyone else had to say. And I don't know if that is part of constructivism or not (AOPLI1)." In handing more control over to the pupils Aoife was faced with the typical constructivist's dilemma of whether or not to tell pupils what to do. She used a graphic metaphor to describe the situation she faced: "It's like the monkey and the banana and the box. Give them the box but don't tell the monkey how to get the banana down, just see if he gets the banana down by himself, you know (AOPLI1)." On the other hand, she realised that she was not on some 'kind of secret mission' and needed to make certain matters more explicit. A good example of this was when the pupils recited their square tables (1,4,9,16…) but did not make the connection later between these numbers and the square shapes (1x1, 2x2, 3x3, 4x4…) that these numbers generate when drawn on paper. Aoife was disappointed that the pupils had not spotted the required linkage.

Yet, Aoife herself had not planned for the linkage in her lesson plan. However, she may have been too hard on herself as, clearly, high ability pupils like James and Emma had made the necessary link. Nevertheless, Aoife gained insight into how to proceed in future lessons: "So that was actually something that I learned today as a teacher; that I should be just very explicit at the start of my lessons and saying 'this is what we're doing, this is where we're going to get to and this is *how* I hope we're going to get there '(AOPLI1)." I'm not sure if Aoife was aware of it but such delineation of the phases of a lesson is an important part of Assessment for Learning (AfL). I think the important word here is *how* in that Aoife was beginning to realise that she needed to give thought to the teaching methodologies (like linkage), and not just the content she needed to employ; so as to maximise the learning opportunities for the pupils. In this lesson, I believe that Aoife's instructions were possibly too loose in outlining the square numbers' task for the pupils. I hoped that Aoife's future lessons would provide more guidance for the pupils but not constrain them too much in their inquiries. In terms of Jaworski's Teaching Triad Aoife's management of learning was exemplary in that she switched from using pairs to groups of four pupils with ease. There certainly was mathematical challenge in the lesson but perhaps Aoife needed to be more sensitive to higher ability pupils, like James, who wished to follow their own path and not be funnelled into following the teacher's thinking flow. The Counting the Squares activity contains linkage (number with shape and space strand) and allows for differentiation of abilities as pupils can consider smaller cases of the problem. A corollary is that pupils can investigate the sequence of square numbers. Normally, I limit the investigation to a 4 X 4 grid for primary school pupils. Aoife was somewhat ambitious in tackling a 28 X 19 grid but, in my opinion, her endeavours resulted in worthwhile pupil learning. I now move on to Aoife's second lesson.

5.5.2. A constructivist view of Aoife's second lesson on 09.03.11

Again, Aoife commenced the lesson with a physical warm-up activity and moved quickly on to a challenge in mental computation whereby pupils had one minute to write down as many multiplication facts as they could from the 5 to 9 times tables. The objective was to beat their previous 'personal best' amount of tables written. The pupils clearly enjoyed this activity. At 5:33 Aoife both elicited and provided information about the roles that pupils would adopt in the group allocation to come. She stated that the job of the recorder was to write down the group's explanation of the problem. She wanted the timekeeper to also take on the role of motivator by praising other pupils. She explained that the reporter was there to feed back all the ideas of the group. Therefore, the roles of the reporter and recorder may overlap, but the former had an oral role, whereas the latter had a transcribing one. The final role fell to the chairperson/manager whose role it was to make sure everyone got a fair opportunity to contribute to the group discussion; be it a right or wrong answer. Furthermore, Aoife reinforced at 7:10 that "sometimes we have to do things wrong to get to the right answer." I think this is a trait that teachers adopting constructivist-compatible pedagogies need to employ, as there will be many occasions where a trial and error approach will be required. Aoife then proceeded to revise a problem solving acronym which the pupils had on a bookmark. The acronym was RAVECCC which stood for Read the problem, Attend to the key words, Visualise the problem, Estimate, Choose numbers, Calculate and Check. This acronym is available on the Professional Development Service for Teachers website (www.pdst.ie) and is similar to the RUDE acronym adopted by Lisa in her third lesson. Although such acronyms are useful, they rarely encompass the entire range of strategies that are needed in a problem solving situation. Therefore, they can stifle pupils' intuitive and creative reactions to a problem. Furthermore, they are very

much oriented around the number strand. Ross et al. (2002) state that one of the reform characteristics of mathematics education should be broader scope e.g. "multiple math strands with increased attention on those less commonly taught such as probability rather than an exclusive focus on numeration and operations" (p. 125). The 1999 curriculum has encompassed this agenda as it contains five strands (number, algebra, shape and space, measures and data) as opposed to the heavy emphasis on number in the 1971 version. Therefore, such acronyms as RUDE and RAVECCC can help to put structure on problems in the number strand but lack the flexibility to be of any significant use in other strands whenever pupils encounter ill-structured investigative problems.

At 10:25 Aoife gave each group of pupils a pack of three sheets with an open-ended problem on each sheet. Although the pupils had a choice of three problems the groups concentrated on the following two:

Problem 1: Zoo entry is €5·50 per adult, €3 per child and €3·50 concession. If I had €30 to pay for entry, what combinations of people could I afford to take based on these prices?

Problem 2: The West Gate Bridge has an average of 2·5 passengers per vehicle at any time. Draw what this might look like if there were 8 vehicles on the bridge.

It can be quickly realised that acronyms are of little use with these problems as they require more creative thinking than just following a set procedure. It can also be seen that these are two of the problems that Claire used in her second lesson. This was a design change in that Claire and Aoife were not solely working as individuals anymore, but were also sharing ideas as I had encouraged them to do. I think such

collaboration is important for any teachers planning to adopt a constructivist approach to teaching mathematics. Indeed, such cooperation is essential if teachers are seeking to sustain a long-term view which involves the *fluidity of content* approach I wrote about in section 2.17. The groups were to spend ten minutes on the first problem before switching to the other one. The pupils did not seem to have any difficulty with the first problem, but the second one raised some issues from a constructivist viewpoint which I will deal with in the comment section below. During the lesson the group of pupils nearest camera seemed to have difficulty with the idea of 2·5 passengers per vehicle. At 25:43, Aoife approached the group and the following dialogue ensued:

> **Aoife**: What would you do, Sonya, you can't draw half a person?
>
> **Sonya**: Draw a child!
>
> **Aoife** (repeats): Draw a child. What else could you do, maybe?
>
> **Florence**: A dog.
>
> **Aoife**: A dog isn't a person, no (laughs). I suppose it does say passengers; it doesn't say person, but that was a good idea to throw out (Aoife kneels on the ground to be at pupils' work level). So she said, right, a child; could be half a person or would it be like, get a chainsaw, cut someone down and throw one half into one car and another half into some…(pauses).That's called murder. You probably get a bit of time for that. So what else could you do? Think!
>
> **John**: Maybe you could put three people in one car.

At this point Aoife seems satisfied. She encourages the group to do some doodling of the cars, look at averages again and come up with an equation. At 27:52, she leaves to attend to another group. At 30:48, Aoife returns again to engage with the first group. She reminds the group of the practical suggestion that somebody has to be in a car, as it cannot drive itself. She looks at Sonya's doodles of two people in a

car and encourages the pupils to try different combinations of people in each car. She repeats her demand that the work should be recorded or drawn or written as an equation. At 32:27, Aoife again leaves to attend to another group. The group works on but does not reach a resolution of the problem. Surprisingly, Aoife does not take feedback from the groups at this point. Instead, at 35:52, she asks pupils to tackle the third task on their sheets. At 42:00 the lesson concludes, as it is the children's lunchtime. After lunch, Aoife decided to take feedback from the groups for another seventeen minutes but, from my perspective, no significant data emerged.

5.5.2.1. Comment on Aoife's second lesson

What intrigued me, from a constructivist viewpoint, was the difficulty pupils were having with the idea of 2·5 people on average in each car. Aoife worked very hard with the group nearest camera, and other groups, to try and elucidate the concept for the pupils but to little avail. The pupils seemed to interpret an average of 2·5 people as literally meaning 2 adults and a child; not realising that this interpretation meant there were 3 people per car. Considering that a child is generally regarded as being half an adult's height at two years of age, this was an interesting sociocultural interpretation. In two visits Aoife spent an aggregate of 3 minutes and 48 seconds with the group. (Incidentally, throughout this research two minutes seemed to be the average time spent by each of the teacher participants with a group of pupils at any one time). In the post-lesson interview (AOPLI2) I raised the issue of direct teaching on the meaning of an average of 2·5 people in a vehicle. (As a lot of relevant data is quoted from Aoife's second post-lesson interview I enclose a copy of it, with relevant sections highlighted, as Appendix 19). Aoife seemed to be under the impression that no structured teaching could take place in an investigative setting. Yet, the pupils were clearly struggling with the concept of an average of 2·5 people.

It will be remembered that one group of pupils in Claire's class had a similar difficulty during her first lesson. I would contend that where a misconception is widely evident in a class, it is better for the teacher to try and explain the concept to the class as a whole than to let them struggle on unaided. Such an approach is more expedient and would help to spare constructivist approaches from Schoenfeld's (2006) criticism that they are too laissez faire and lack structure. Having gained some elucidation of a difficult concept the pupils can then proceed more efficiently with the investigation.

In the interview Aoife repeated her concern from lesson one that she found it difficult *not* to tell the pupils what to do. In contrast, she believed the pupils enjoyed taking ownership of the material taught and coming up with their own ideas. She remarked, "I think they're taking to the whole constructivism (thing) faster than I am" (brackets added). She said this in the context of the pupils not asking her as many dependency questions as before and being more willing to try to figure out their own solutions. In lesson one, perhaps Aoife had been too loose in her choice and explanation of activity. In this lesson maybe Aoife, like Claire in her second lesson, had tried to cover too much material. This resulted in Aoife being unable to take feedback from the pupils before lunch break. Aoife conceded that feedback needed more priority when she stated:

> If I was to change anything the next time it would be the time management on my own part; just because the feedback needs more, nearly as much time as the group work does, because usually when we do Maths from 'Action Maths' there is a kind of given solution and you know what's the answer; it's either going to be the answer or not (AOPLI2).

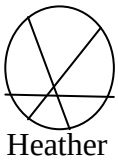
In attempting to cover three problems in the lesson Aoife conceded that lack of time for feedback was not the only issue which arose:

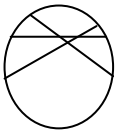
> Because I had three activities today I just like, because I wanted to hit the three of them I just kind of, like, skimmed them and I didn't give them (the pupils) the opportunity to discuss or to argue , you know to kind of say, 'Oh why did ye do that?' (AOPLI2).

As 'skimmed' was a word I had used earlier, I found it interesting that Aoife employed the same word to describe her treatment of the three problems and the resultant lack of attention given to discussion and argument. Aoife was a joy to interview as she gave freely of her opinions. I now attempt to summarise her developing views from this lesson in terms of Jaworski's Teaching Triad. She appreciated that introducing a lot of problems meant that such problems were skimmed superficially rather than dealt with in depth and that there was little or no time left for feedback. This meant that mathematical challenge had been diminished. However, her actions during the activities showed that Aoife was sensitive to pupils' cognitive needs and was willing to adapt her management of learning style accordingly. The Sue Cunningham activities are designed to introduce teachers to a more open-ended way of working, which in turn generates pupil discussion. The ones Aoife chose also displayed linkage in that pupils were exposed to the strand units of money, decimals and averages. I found myself looking forward to Aoife's third lesson, not only because I admired her teaching, but also because I valued her evolving perspective as a learner of constructivist-compatible approaches.

5.5.3. A constructivist analysis of Aoife's third lesson on 19.05.11

Aoife commenced the lesson with a revision of the parts of a circle i.e. circumference, radius, diameter, chord, segment and arc. She used the IWB to illustrate these parts. The pupils practised drawing a circle with a compass and naming the parts thereof. At 6:37 Aoife introduced the main activity for the lesson. She chose the same activity which her colleague, Claire, had opted for on the previous day; demonstrating cooperation between these two teachers. Again the task was to cut a pie into 7 pieces using only 3 cuts. The Pie Activity is one I like as an investigative task. One advantage is that it links the number strand (fractions) with the shape and space strand (parts of a circle). However, the activity is heavily dependent for its success on the teacher encouraging the pupils to explore the pattern for the ratio of cuts to pieces. That is the aspect which promotes pupil discussion. Aoife commenced the activity by prompting the pupils to investigate freely. At 9:27 she encouraged them as follows; "There's lots of different ways of getting there." She also urged them to peer tutor; "If there's someone next to you and they're stuck, you might give them a hand." It can be seen that Aoife did not opt to put pupils working in groups of four in this lesson. Instead she elected for pupils to work as individuals or in pairs. Her rationale was that some of the group leaders who were high achievers had become too dominant and the rest of the pupils "would nod in agreement so I wanted them to have a go at it themselves today"(AOPLI3).The pupils worked on task for over eight minutes and at 14:43 Aoife asked for pupils to come to the IWB to share some solutions with the rest of the class. Heather, Jim and Jeremy gave the following solutions respectively:


Heather


Jim

Jeremy

At 19:05 Aoife made a surprise move by introducing a new activity. I had expected her to carry on with the pie activity and explore the ratio of cuts to pieces. In her post-lesson interview (AOPLI3) she later conceded that she ‘forgot to do the patterns’ and therefore introduced a new extension activity instead. This activity was similar to Claire’s one the previous day, except that the grids were 12 x 12 instead of 6 x 6. The pupils were requested to put 12 crosses on the grid, but there could be no more than 2 crosses on any horizontal, vertical or diagonal row. Aoife provided one example on the IWB before pupils set to work. Again Aoife encouraged the pupils to work cooperatively in pairs: “You might check your partner’s (work) as well” (25:15, brackets added). As Aoife circulated the room she saw where pupils were having difficulty and prompted them accordingly: “Check your diagonal ones; that’s where people are getting caught” (30:00). At 35:20 Liam and Emma came to the IWB and drew the following two solutions respectively:

X											
	X										
	X										
		X									
		X									
			X								
			X								
				X							
				X							
					X						
					X						
						X					

Liam’s solution

		X	X								
											X
				X	X						
			X								
						X	X				
									X	X	
X	X										

Emma's solution

At 39:30 Aoife asked the pupils to return to the pie activity and try to find out how many pieces they would get with 4 and then 5 cuts. She urged the pupils to use different line colours to represent each cut so that she could assess how pupils were progressing. This was a very practical and visual assessment strategy as it can be difficult to gauge how pupils are constructing knowledge during investigative mathematics. This is because assessment of student learning in constructivist classrooms is interwoven with teaching, according to Brooks and Brooks (1999). This also ties in with the type of formative assessment envisaged in Assessment for Learning (AfL). Fortunately, Aoife advised pupils to record the ratio of cuts to pieces as they progressed through the activity. When it came to recording this ratio on the IWB she wrote the following:

(Cuts	Pieces)
3	7
4	10 or 11
5	15 or 16
6	20

Aoife did not record the pattern for the 1st and 2nd cuts to assist pupils in generalising the pattern. At 53:08 Donncha asked if chords were allowed bend. It seems that Donncha believed that as long as he didn't lift his pencil from the page he was still drawing a line even if it changed direction. It will be remembered that Lydia and Orla had the same misconception in Anita's classroom during her second lesson. It has been informative for me to witness such misconceptions across several different classrooms. It shows that a certain amount of 'telling' may need to take place to define a line or other mathematical construct for the purposes of a particular investigation. At 55:25 Aoife attempted to reach a consensus with the pupils on what the pattern was for the ratio of cuts to pieces. This proved difficult as pupils had varying answers as shown by the recording on the IWB. All Aoife could do was encourage the children to continue investigating for the remainder of the lesson by repeating her earlier instructions to use coloured lines as chords and to keep a written record of the ratio of chords to pieces.

5.5.3.1. Comment on Aoife's third lesson

In our post-lesson conversation Aoife told me that she didn't work out the pattern for the ratio of cuts to pieces before the lesson. There were advantages and disadvantages to this modus operandi. An advantage was that it forced Aoife to work with the pupils as a peer in a common struggle to find the solution. A disadvantage was that Aoife was unsure where the pupils were heading, as she had not worked out

the pattern for herself. Aoife stated that in English lessons she often gave riddles to which she did not know the answer and was happy to proceed in that way. However, in this mathematics lesson Aoife reflected; “I think that I should have, maybe, in hindsight sat down and made sure I had a solution myself. But I kind of thought, ‘ah sure there’ll be so many solutions anyway’”(AOPLI3). Furthermore, she wondered, as she had done in her 2nd lesson, “If I tell them too much am I telling them the answers?”(AOPLI3). From my observations, I would hold that Aoife was a teacher who liked to be in control and give pupils information. She wanted to be the captain steering the ship. In this lesson she sailed into a mathematical storm and had to row with the other members of the crew. Some teachers learn to be comfortable in such situations, but it takes experience as Schifter and Fosnot (1993) found in their research. Perhaps Aoife could plan for where she thought an investigation might lead in a lesson. In that way she could make more informed decisions about whether or not to intervene in an investigation. Pupils will often lead an investigation down unanticipated paths. Indeed, this is one of the appeals of constructivist-compatible approaches. However, this is not an excuse for teachers to be ill-prepared and adopt a ‘sit back and see’ approach to all lessons. As mentioned previously, such an approach encourages critics of constructivism to condemn it as lacking structure and accountability.

On the issue of experience, I asked Aoife if she had undertaken other investigative lessons similar to the one I had witnessed. In her post-lesson interview she replied as follows:

> Ahm, I try to but I obviously give them… I obviously tell them what to do. Because it was just, I only realised it yesterday when I said that you were coming in and they were like ‘Yes! The maths man is coming!’ and I was just like ‘but we’ve being

> doing this you know'. They were just like 'oh no, okay.' But like so I mustn't be… I must be doing it differently unknownst to myself. A good thing would be to video me or someone else over a period of time. You know, when you're not in the room because I obviously must be acting or dealing with it differently when you're in the room to when you're not in the room (AOPLI3).

I believe this is a rich quotation on two fronts. Firstly, Aoife may have thought that through her actions she wasn't telling pupils what to do and that she was adopting an investigative approach, but her pupils obviously thought differently. Secondly, her behaviour was to give me a clue as to the long-term effects of this study, which will be dealt with in the next chapter. In terms of Jaworski's Teaching Triad I believe Aoife's management of learning was praiseworthy as she had placed pupils in pairs because certain pupils had become too dominant in larger groupings. The Pie Activity is inherently a challenging task for pupils. Perhaps Aoife could have been more sensitive to pupils' needs by commencing the pattern (for the ratio of cuts to pieces) with one cut and not three cuts as shown on the IWB. This could have assisted the pupils in generalising the pattern. On a final note, I was also amused and flattered that the pupils had referred to my arrival as 'the maths man is coming'. I liked the phrase so much that I used it to name this chapter, as it conveyed to me a deep sense of mission in coming to Aoife's classroom to gather data on investigative approaches. I now continue that mission as I move on to Aoife's fourth and final observed lesson.

5.5.4. A constructivist analysis of Aoife's fourth lesson on 26.05.11

Aoife commenced the lesson with a revision of the properties of 2-D shapes on the IWB. She dealt with the square, rectangle, circle and the three different types of triangle; equilateral, scalene and isosceles. She highlighted the differences between squares and rectangles. One pupil commented that a rectangle was like two squares put together, which reminded me of the traditional way a rectangle is presented in textbooks. At 9:07, Aoife asked the pupils to keep the material she had just covered in their memory, as it might help them with the main activity of the lesson. I consider such advice to be very constructive, as it focuses pupils' attentions on particular mathematical facts, which may save time when they begin to struggle with an investigation. Therefore, I was surprised when Aoife's next move was to revise that 0·25 = 1/4 = 25/100 =25% and 0·5 = 1/2 = 50/100 = 50%. It is evident from this thesis that I am a huge enthusiast of linkage within the curriculum. However, on this occasion, I believe it was somewhat contrived and unnecessary, as will be evident from the outline of Aoife's main activity. Perhaps Aoife's intention was to revise some number facts from a previous lesson.

At 10:35, Aoife introduced the main activity, which was the same one Claire had employed in her fourth lesson. Again, it was interesting to witness the cooperation between these two teachers. Aoife asked the pupils to find seven or more ways of dividing a square in half. As mentioned earlier, I believe this activity lacks mathematical challenge but it does have various solutions which can help to generate worthwhile pupil discussion. Aoife drew one example on the IWB where she coloured in the bottom half of a square. Then the pupils proceeded to work on the task as individuals; although, at 11:40, Aoife encouraged the pupils to cooperate by saying, 'work together if you need to.' Her actions included giving the pupils

various prompts throughout the lesson. She encouraged the pupils to use symmetry but later began to realise that introducing symmetrical designs into a square does not necessarily guarantee that it is split into two equal parts. One pupil, George, mentioned using countries' flags to create halves and Aoife encouraged him to pursue this line of thought. Although it was mentioned in her lesson plan, attached as Appendix 20, I found it interesting that Aoife did not allow time for a plenary session. Instead, at 41:46, she asked the pupils to share their work with their partner and justify, where necessary, how the halves were obtained. I thought this was an interesting method of assessment, as it forced the pupils to reveal their solution methods and associated thinking to one another. Sensing that some pupils had done enough work with squares, Aoife introduced an interesting final lesson activity (43:28) by asking pupils to come up with different ways to split a rectangle in half. Unfortunately, the pupils did not have long to work on this activity as the lesson concluded at 45:30.

5.5.4.1. Comment on Aoife's fourth lesson

The absence of a plenary session meant that it was difficult for me to comment on pupils' work as a whole, as it was not shared with the whole class. However, I collected individual samples of children's work and I now take the opportunity to comment on one child's work, namely Naomi's, as it illustrates how the lesson proceeded and how a misconception passed without notice. Naomi's initial drawing of the squares (see Appendix 21) suggests she understood how to split the square in half and that 1/2 is equivalent to 4/8. Aoife praised Naomi for the 'board game effect' of her second drawings (shown as Appendix 22). Naomi had coloured in 66 out of 132 squares. However, neither teacher nor pupil seemed to realise that the 132 squares derived from an 11 X 12 grid representing a rectangle and not a square.

Naomi had the right answer but to the different question of finding half a rectangle. However, Naomi's earlier work suggests that she understood how to draw half a square and that her second drawing merely reflected a calculation error on her behalf. Therefore, the incident illustrates that when pupils are constructing their own knowledge in open mathematical activities, teachers have to very alert to the possibility of pupils forming misconceptions along the way. That is why Jaworski's 'sensitivity to students' category appealed to me as a way of analysing teachers' classroom interactions. In this lesson there was possibly a lack of mathematical challenge in the main activity of dividing a square in half. Perhaps, in terms of management, the lesson needed a plenary session so that the pupils could share their solutions with one another.

5.5.4.2. Comment on Aoife's four lessons as a whole

I now wish to summarise Aoife's four lessons in terms of Jaworski's Teaching Triad. As regards classroom management, Aoife conceded that she found it difficult to hand control of learning over to the pupils themselves. Nevertheless Aoife showed a great willingness to let pupils work individually, in pairs and in groups of four or more. She endeavoured to take Windschitl's (1999) advice, quoted in section 2.14, that teachers should design learning situations that encourage pupils to augment or restructure their current knowledge. In so doing, she had to relinquish her own desire to be the dominant speaker in the classroom. It stands to reason that if pupils are expected to work collaboratively then teachers should do so also. I admired her cooperation with her colleague, Claire. Such cooperation certainly gave the project a synergy, which might otherwise have been absent. As regards mathematical challenge there were times when Aoife introduced several problems at once into a lesson (see lesson two) with the result that the pupils were *skimming*

rather than learning in great depth; a point which Aoife acknowledged. In lesson three Aoife discovered that it is pedagogically advisable for novice practitioners of constructivist approaches to work out a mathematical problem in advance of a lesson, than to attempt to struggle through it afterwards with the pupils in an ad hoc way. In the category of sensitivity to students Aoife found that there were times when it was necessary to *tell* pupils information to enable them to proceed with an investigation. In ill-structured mathematical problem solving situations it would be particularly important for the teacher to intervene if pupils began to exhibit misconceptions while grappling with a concept. Lack of intervention would be pedagogically unsound.

5.5.4.3. Appraising the affordances and constraints of Aoife's problem solving activities

Aoife's first activity was concerned with finding the squares on a 28cm X 19cm grid. Although the activity satisfies all six criteria this does not tell the full story. The activity is based on one I had shown the participants whereby the squares on a 4cm X 4cm grid had to be counted. Aoife adapted this activity to suit her lesson. The activity is open-ended and generates a lot of discussion. I think Aoife wanted the pupils to divide the grid into square shapes (1X1, 2X2, 3X3...) but the looseness of the task meant that many pupils did not interpret the task in that way at all.

Aoife's second task was based on two problems taken from Sue Cunningham's (2003) book 'Thinking Allowed'. I had introduced this book to the participants to give them some guidance on the framing of open-ended problems as they were clearly struggling in this area. The problems satisfy four of the aspects but do not allow for a smaller case to be considered or for pattern to be investigated. Aoife and Claire found it more difficult than the other participants to source more open-ended

activities. Perhaps they were not used to new number initiatives whereas the other two participants were used to such initiatives as they taught in a DEIS school where they are commonplace. However, Aoife and Claire cooperated in framing this activity and the following two also.

Aoife chose an activity for her third lesson which would satisfy five of the six criteria. This was the activity where pupils had to cut a pie in seven pieces using only three cuts. The activity requires a logical approach and gives pupils plenty of opportunities to discuss their findings with one another. It was an activity which two other participants, Anita and Claire, also found beneficial in generating discussion. According to the criteria outlined earlier, it has a disadvantage in that it doesn't involve a variety of operations but this is to be expected in an activity which primarily involves drawing segments of a circle with a related search for pattern. In the lesson proper, Aoife came to the conclusion that she should have completed the activity herself prior to the lesson so that she could anticipate the problems the pupils would encounter as they tried to derive said pattern for the ratio of cuts to pieces. In planning for an open-ended activity the teacher needs to anticipate where the pupils might reasonably go with the activity and update their own subject knowledge accordingly. There will be occasions when pupils will take an activity into unexpected territory and this is to be welcomed also so that those pupils will be challenged and require scaffolding.

For her fourth lesson Aoife used an activity from a book I introduced to the participants called 'Maths to Think About' by Claire Publications (2000). The children had to devise seven different ways to divide a square in half. As the activity is primarily non-numerical it only satisfies three of the criteria but this probably

underestimates its worth as it is open-ended and generates a lot of classroom discussion. It is in line with Windschitl's (1999) advice, quoted in section 2.14, that pupils should work collaboratively and be encouraged to engage in task-oriented dialogue with one another. The activity has variety in that it also exposes pupils to the difference between the concept of halving a shape and checking it for axial symmetry. Towards the end of the lesson Aoife introduced an interesting extension activity by asking the pupils to find different ways of dividing a rectangle in half.

In summary, Aoife sought guidance in generating open-ended activities. She cooperated well with her colleague Claire in this regard. What isn't obvious from the appraisal of the activities is the struggle Aoife had within herself to give more control of learning to the pupils themselves. She was very frank in her endeavour to tackle this issue. The affordances and constraints of her problem solving activities are outlined in Table 8 which follows.

Table 8: Affordances and constraints of Aoife's problem solving activities

Activity	Allows for Several Formats (Bruner)	Allows for a Smaller Case to be Considered (Polya)	Open-ended problem	Involves a Variety of Mathematical Operations	Allows for Engaging Classroom Discussion	Involves the study of Patterns (Boaler)
Counting the Squares on a 28 X 19 Grid	Yes, pictorial and symbolic	Yes, by considering a smaller grid.	Yes, allows for various interpretations	Yes, primarily multiplication but addition also.	Yes, as task can be interpreted in different ways.	Yes, patterns generated by square numbers.
Sue Cunningham Activities i.e. combination of fares problem and vehicles on bridge problem	Yes, pictorial and symbolic	No	Yes, allows for various interpretations	Yes, all four operations could feasibly be involved.	Yes, as group work involved and solutions can be shown in different ways.	No
Cutting a pie in 7 pieces using only 3 cuts	Yes, pictorial presentation but symbolic assessment	Yes, if pupil adopts a logical approach	Yes, allows for various interpretations	No	Yes, as solutions can be shown in different ways.	Yes, if pupil adopts a systematic approach with teacher's help.
Seven ways to divide a square in half	No, pictorial only	No	Yes, allows for various interpretations	No	Yes, as solutions can be shown in different ways.	Yes, involving one-to-one mapping.

5.6. Summary

In this chapter I detailed how Lisa used linkage to engage with investigative mathematics without feeling she was neglecting the prescribed curriculum. Indeed, I would say that linkage helped Lisa to teach to the ''big ideas'' in mathematics as discussed in chapter two. In Anita's classroom I outlined the balance she sustained between increasing cognitive challenge on the one hand and maintaining motivation on the other. Her development of the teaching methodology I called 'harvesting and sowing' was interesting. The issue of cognitive challenge also arose in Claire's

classroom. I reflected on how introducing too many activities only leads to what I term 'skimming'. In Aoife's classroom I described how she found it difficult to hand control of learning over to the pupils. Yet, when she handed over control she learned to reintervene with groups to ensure pupils were working at a challenging level. Indeed, Vygotsky's construct of the zo-ped was one which was central to this chapter. For me the more successful activities were the ones which had cognitive challenge but at different levels to allow for pupils of varying abilities. That the activity contained an opportunity for children to explore mathematical pattern was another crucial factor as it led to fruitful and engaging discussion. Therefore, from the sixteen lessons observed, four activities stand out for me as being worthy of replication in other classrooms. These are:

1. The Count the Squares Activity
2. Polygon's Restaurant
3. Cutting a Pie in Seven Pieces Using Only Three Cuts
4. Rolling Two Dice, Finding the Sum of the Outcomes and Deciding Whether Even or Odd Totals Occur More Frequently

I now proceed to the penultimate chapter in which I analyse questionnaires and interviews using Jaworski's Teaching Triad as the analytical tool.

Chapter 6: Using the Teaching Triad to appraise the short and long term impact of the study

6.1 Introduction

In this chapter I will draw on six sources of data and comment on each teacher participant's constructivist trajectory. Three of these sources are questionnaires which I gave to the four teacher participants. The first one was given to the participants at the beginning of the study in October/November 2010 and was used to gauge their understanding of constructivism at the time. It is called Pre-lesson Interview (PLI) and is included as Appendix 23. To prepare for this interview the teacher participants were asked to read pages 3-4 of the Mathematics Teacher Guidelines and page 5 of the Mathematics Curriculum as these gave a basic overview of constructivism. The second questionnaire was given at the end of the project in June 2011. The project was undertaken during the school year 2010-2011. The purpose of this second questionnaire was to look at the impact of the project on the participants' teaching and the children's learning. It is entitled Exit Interview and is enclosed as Appendix 24. The third source of data was an open-ended focus group interview (GI) I conducted with the participating teachers in June 2011, which gave them an opportunity to interact with one another and give their views on how the project had been perceived by them as individuals. The Exit Interview questions were a crutch for the participants when I conducted the group interview on 16th June 2011, but I found the participants strayed from the questions and spoke freely, which was very positive for data collection. At that stage, the teacher participants were more familiar with me and were relaxed now that the project was coming to an end. The transcript of the Group Interview is enclosed as Appendix 25. I mentioned

earlier in chapter 4 that I had to take a year out of the PhD programme during the school year 2011-2012, due to the illness of a family member who needed full-time care. However, in conversation with my supervisor, I realised that I had been afforded an opportunity to look at the possible long term views of the participants, one year on, in May 2012. To this end, I gave the participants a survey entitled Long Term Impact Questionnaire (LTIQ) and this is enclosed as Appendix 26. Again, these questions acted as a prompt for the Long Term Impact Interview itself, which I conducted on May 2012. The transcript of this interview appears as Appendix 27. Obviously, such questionnaires only give a fleeting snapshot of the participants' views at a particular point in time. A more comprehensive view would require a further longitudinal study beyond the time span of this thesis. The sixth source of data was a short interview I conducted with twelve children of mixed ability (three from each class) at the conclusion of the research in May/June 2011, in which I asked them for their views on the project. Such views are given as Appendix 28. I believe it is best to categorise the data for each of the teacher participants using Jaworski's Teaching Triad i.e. Management of Learning, Mathematical Challenge and Sensitivity to Students. Each participant's views are paraphrased from their questionnaire responses.

6.2 Conclusions on Lisa's data

In this section I will deal with the conclusions arising from Lisa's data.

6.2.1 Management of learning

In the initial pre-lesson interview questionnaire, Lisa believed that constructivism placed the children in a central role in lessons as more active participants and in charge of their own learning. In her exit interview, Lisa elaborated on this view by stating that what impressed her the most about the project was the effect that

working in pairs/groups with open-ended problems had on the children's confidence and attitude towards mathematics. This is in line with Ross et al's. (2002) summary comment that children exposed to reform mathematics had more positive attitudes toward the subject. She believed the open-endedness helped children experience different levels of success. As regards long-term impact, she commented that the project had encouraged her to do a daily word problem or puzzle at the end of each maths class and to split the children into pairs to solve such problems. Splitting children into pairs was her way of offering support to pupils of lower ability in solving the problems; presumably through peer tutoring. It is interesting that Lisa tackled the problems at the end of her lessons. This suggests that in the longer term she reverted to teaching maths using a transmission model, but kept a 'pedagogic nugget' from the project in retaining the usage of pair work.

6.2.2 Mathematical Challenge

In her pre-lesson interview questionnaire Lisa expressed the view that children are often afraid and overwhelmed when it comes to solving word problems. In her exit interview Lisa stated that working on open-ended problems promoted higher-order thinking, confidence in maths, mathematical language and a positive attitude to maths. She believed that working on open-ended problems created a knock-on effect to solving word/text book problems, as a result of the skills, strategies and confidence children had gained along the way. This suggests that the process of working on open-ended problems had a positive attitudinal impression on children's engagement with text book problems. This is a very interesting finding from an affective point of view. As regards long term impact Lisa commented that the project had encouraged her to use more open-ended problems. As a mathematics educator, As a researcher, I found it interesting that Lisa was willing to incorporate

more open-ended problems into her work. In her exit interview, Lisa had aspired to incorporating an open-ended investigation into her lessons once a week or fortnight and to link it into the topic she was covering. I suspect that this initial enthusiasm had waned somewhat in the intervening year, but it was interesting that the willingness was still present.

6.2.3 Sensitivity to students

This section is derived from the participants' interpretations of the constructivist term 'scaffolding' in the questionnaires. My logic here is that scaffolding during a task requires a teacher to be sensitive to pupils' cognitive needs. In the pre-lesson questionnaire Lisa interpreted scaffolding as supporting children's learning with the aim being to release support and foster independence. In her exit questionnaire Lisa's interpretation had evolved spectacularly. She wrote that scaffolding meant guiding the children through discussion, open-ended investigations and the support of a peer/group network to construct their own learning in a meaningful way. She elaborated by stating that it meant circulating to give help and re-teach concepts where necessary, sometimes using direct teaching. I am inclined to agree. I think too much has been made of direct teaching being anathema to constructivist instruction. For example, Tobias and Duffy (2009) have edited a volume debating the subject. In that volume Klahr (2009, p. 291) declares that "even the most zealous constructivist would acknowledge that there exist combinations of time, place, topic, learner and context, when it is optimal to simply tell students something, or to show them something, or to give them explicit instruction about something." The task then becomes the identification and characterisation of such instances. As regards long term impact, Lisa believed that the project had encouraged her not only to use more open-ended problems, but also to allow time for exploring different strategies for

solving a particular problem so that the children could see there was often more than one way to solve a particular problem. The National Centre for Education Evaluation and Regional Assistance in America (2012) found that when regularly exposed to problems that require different strategies, students learn different ways to solve problems. As a result, they "become more efficient in selecting appropriate ways to solve problems and can approach and solve math problems with greater ease and flexibility" (p. 32). As a researcher, it was interesting that the willingness to let children explore various solutions had had a lasting effect on Lisa's consciousness.

6.2.4 The genesis of a teaching quadriad arising out of Lisa's data

Although I have found Jaworski's Teaching Triad invaluable in categorizing what teachers do in attempting to adopt constructivist-compatible methodologies, it does not encapsulate the policy constraints under which teachers work. Jaworski (1994) used a (John) Venn diagram to represent the Triad. I believe this could be amended by placing the Triad within a Universal Set entitled Policy Constraints as shown in Figure 16 below. Such policy constraints include the pressure to cover a prescribed curriculum within a set period without a proper emphasis being placed on the resourcing of a problem solving approach to mathematics. The set of policy constraints is similar to what Chevallard (1985) calls the 'noosphere' as discussed in section 2.4. The reader will recall that the 'noosphere' is the bureaucratic universe that shapes schooling, which influences what happens in classrooms.

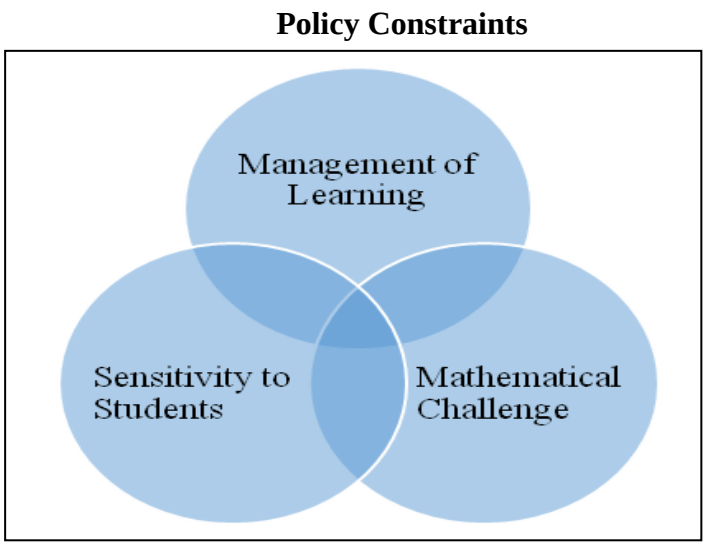


Figure 16: Jaworski's Teaching Triad within a universal set of policy constraints

In the long term impact questionnaire Lisa pointed out that time constraints remained an issue in adopting a constructivist approach. This was also borne out by the other participants in their long term impact questionnaires. For instance, Aoife commented that because teachers have to deal with a lot of content and testing they find it easier to use direct instruction. Claire stated that 'time restraints' (her words) and sourcing materials (presumably suitably open-ended problems) made it difficult to do groupwork on a regular basis. Anita remarked that there are so many demands placed on the classroom teacher nowadays that it is impossible to adopt a constructivist approach on a daily basis. In particular, she mentioned the vastness of the mathematics curriculum as a factor militating against the adoption of a constructivist approach. Therefore, what can teachers do? In the long term impact questionnaire, I also asked the participants for any advice they would give to another teacher attempting to adopt a constructivist approach. Lisa's advice was to integrate such an approach into existing maths lessons. She elaborated by suggesting the use of linkage within the maths curriculum to enhance lessons, rather than seeing constructivist instruction as an isolated approach to teaching. Lisa suggested that

teachers needed to plan how to incorporate such an approach into their work. I believe her advice is sound. She has acknowledged the time constraints under which teachers work but, instead of despairing, she has made a very practical suggestion in the form of linkage. Linkage could probably be categorised in the Teaching Triad under Management of Learning, although this is arbitrary as the categories are interrelated. Instead I prefer to expand Jaworski's Triad into a Quadriad by forming a new category called Connectivity. For me Connectivity could have two stages:

1. A Planning Stage: This is where a teacher would plan for linkage within the maths curriculum while preparing their weekly or fortnightly plans as is the custom in Ireland. This would assist the teacher in anticipating connections which the pupils might make from one strand of the curriculum to another.
2. A Spontaneous Stage: As children are constructors and co-constructors of their own meanings it is impossible to anticipate all the connections, which pupils might make before a lesson commences. Therefore, the teacher needs to be alert to the connections pupils *are* making in the throes of a lesson. This is facilitated through teacher-questioning, observation of pupils' work and constant feedback as advocated in the AfL initiative referred to in section 3.11. Then the teacher's responsibility becomes the facilitation of such connectivity by introducing mathematical knowledge, which may not be part of pupils' prior knowledge, but which may be useful to the investigation in hand. The main criterion for introducing the new material is that it must assist the pupils in furthering the investigation. An example from this research in Lisa's classroom might be introducing index notation into the activity where pupils had to count the squares in a 4 x 4 grid. Pupils unfamiliar with index or exponential notation would be faced with a cumbersome way of recording their findings when a simple way of recording the number of squares in a 4 x 4 grid can be given as $4^2 + 3^2 + 2^2 + 1^2$.

Jaworski's initial Teaching Triad diagram could be amended as shown in Figure 17 below:

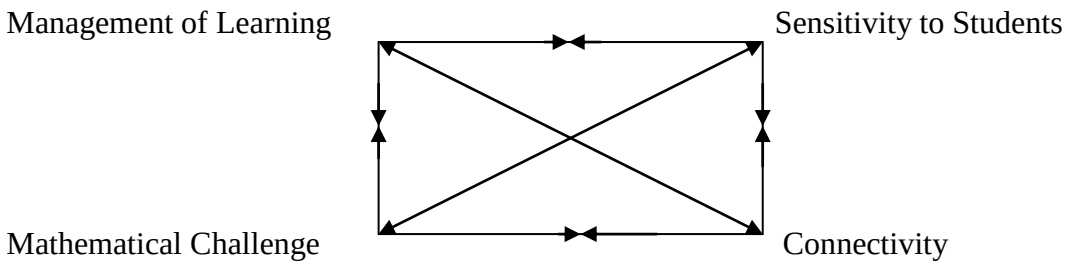


Figure 17: The Teaching Quadriad

Such connectivity should seek to link mathematics with students' real life experiences as for instance, in the case of RME and promoted in the reformist agenda of Ross et al. (2002). This requires sensitivity to students in the form of awareness of their mathematical interests. Connectivity should also seek to link one student's evolving knowledge of a mathematical concept with other pupils' evolving concepts. Obviously, this requires a high degree of awareness from teachers as to where a pupil is at and where she may reasonably be expected to go to work at her zo-ped. It necessitates skills in the management of learning, be it peer or group work, and in pitching an appropriate level of cognitive challenge in activities chosen. I am reminded of the useful Team Maths strategy, as witnessed in Anita's classroom, which allowed pupil representatives to move from one group to another; thereby sharing knowledge gained in a previous group while possibly introducing a fresh challenge to a new group. Connectivity could be like the 'glue' which binds the other elements of the Teaching Triad together. Potari and Jaworski (2002) linked two elements of the Triad together with the concept of *harmony* which they defined as the extent to which the degree of challenge in a lesson is appropriate to the particular sensitivities of students. Connectivity is broader in that it seeks to link the three elements together. In conclusion, it can be seen that in response to the

participants bemoaning the time constraints under which they worked, Lisa's use of linkage helped me theorise on Jaworski's Teaching Triad, which in turn led me to the genesis of a Teaching Quadriad. I now move on to my conclusions emanating from the data in Anita's questionnaires.

6.3 Conclusions on Anita's data

In this section I will deal with the conclusions arising from Anita's data.

6.3.1 Management of learning

In Anita's initial interview she defined constructivism as the promotion of groupwork. In such groupwork children would ideally share their solutions in a whole class setting and every child would be active. However, as I highlighted in section 2.6, children being active does not necessarily equate to the construction of meaning at their zo-ped. In her exit interview Anita articulated the difficulties with implementing a group approach to mathematics; as opposed to the whole class approach so prevalent in Ireland. She gave a very honest appraisal when she stated that it wasn't until the fourth lesson that children became comfortable sharing and discussing problems amongst their peers. Anita developed her own language for a group approach to mathematics, calling it 'team maths'. She also developed a very worthwhile strategy called 'Move About' for children to share solutions in other groups. This strategy involved a child moving from one group to another to share strategies devised in the previous group. Acknowledging that working on open-ended problems can be extremely time-consuming, Anita aspired to adopting such an approach once a fortnight for the coming year. In her long term impact interview her enthusiasm had obviously waned somewhat as Anita surmised that she had become more aware of constructivism and used it to a small degree in her classroom. She boldly stated that it is impossible to adopt a constructivist approach on a daily

basis, as there are so many demands placed on a classroom teacher. In particular, she blamed the vastness of the mathematics curriculum as a major constraint. Nevertheless, Anita remarked that, one year on, she continued to encourage more pair and group work during problem solving sessions and that she allowed children time to explain their findings to their peers. This is partially in line with Ross et al.'s (2002) recommendation that instruction in reform classes should focus on the construction of mathematical ideas through students' talk rather than transmission through presentation, practice, feedback and remediation.

6.3.2 Mathematical challenge

In Anita's initial interview she aspired to promoting more open-ended problem solving in her classroom. She hoped that such a constructivist approach would have the knock-on effect of helping the children develop better problem solving skills in dealing with written problems. She acknowledged that it was difficult to dedicate time for open discussion, as her lessons typically started with a maths game, followed by the standard operations such as multiplication and division. This type of lesson format illustrates the difficulty teachers have in adopting Ross et al's (2002) recommendation above concerning the use of students' talk. In her exit interview, Anita confided that locating suitable open-ended problems had been difficult. She was not alone in this view and that was why I introduced the participants to strategies from Sue Cunningham's book Thinking Allowed (2003). Such strategies illustrated how to make tasks more open-ended. Furthermore, it highlighted to me that it was not enough to adopt a 'see what they come up with' approach to the research. Promoting constructivism required active facilitation and several design changes were implemented on my part. These usually involved my introduction of problem solving material to the participants or encouraging them to share suitable material with one another. Anita conceded that adopting a constructivist approach

had allowed the children to take ownership of their own solutions and placed them in a ‘teacher role’ when explaining their answers. Interestingly, she believed that the children of perceived ‘lower ability’ felt more comfortable in a group situation; presumably there was less anxiety present than when they were asked a question on an individual basis. In her long term impact questionnaire, Anita had stated that the project had encouraged her to use more pair and group work during problem solving and to allow children to explain their findings to one another. It was interesting that the impact on Anita’s practice in the longer term indicated a tentative leaning towards a constructivist-compatible view of learning.

6.3.3 Sensitivity to students

Initially, Anita had a rather paradoxical view of what scaffolding entailed. She believed it meant supporting the pupils until they could work out a problem independently. At the same time she believed it meant giving the children mathematical rules, such as putting in an ‘automatic zero’ in long multiplication. Such a view would mean that pupils would be dependent on such rules without understanding them properly. This type of bedrock would not be a strong foundation on which to build further learning. In her exit interview, Anita gave a vague definition of scaffolding. She defined it as a platform or framework, which assisted teacher and pupils in the teaching and understanding of maths problems. It was unclear what she meant but she gave me a better indication of her developing sensitivity to students when she remarked that she took more time to listen to their methods. This indicated that she was no longer relying on rules of thumb alone to teach mathematics. In her long term impact questionnaire, I had further cause for optimism when Anita remarked that she was more open to exploring alternative methods. She stated that if a pupil came up with a different method she would

highlight it more than heretofore. I will now look at Anita's data under the heading of connectivity; the newest element of the quadriad.

6.3.4 Connectivity

I induced the category of connectivity from Lisa's data. Now I attempt to look at Anita's questionnaires, and with the benefit of hindsight, apply the connectivity categorisation to her data. In her initial interview, Anita mentioned that pupils experience a real sense of achievement when they can come up with their own theory. In her exit interview, Anita commented that a teacher working on open-ended problems needed to prepare for numerous possibilities; particularly those that may not have occurred to the teacher in advance of the lesson. In her long term impact interview, Anita stated that she had adopted more group and pair work in her problem solving maths lessons. She quoted the old counselling adage; a problem shared is a problem halved. Allowing pupils to theorise, being aware of different mathematical possibilities and permitting pupils to share their opinions are all indicators for me of Anita's awareness of the need to help pupils make connections in the mathematical domain. As I did not ask questions on connectivity, I can only surmise on Anita's awareness of it, but I believe the construct of linkage would be of great benefit to Anita in planning for connectedness in her lessons. I now move on to Claire's data.

6.4 Conclusions on Claire's data

In this section I will deal with the conclusions arising from Claire's data.

6.4.1 Management of learning

In her initial interview, Claire thought that by adopting a constructivist approach a teacher would benefit greatly from seeing children work in pairs or larger groups

and applying their knowledge together to solve problems. In her exit interview, she seemed to have gained such a benefit as she stated that she had become more aware of listening to pupils' suggestions and 'following them up' if time allowed. This showed an awareness of the constructivist principle that children should be given opportunities to construct meaning for themselves i.e. knowledge is actively built up by the cognising subject (von Glaserfeld, 1990). Claire hoped to use open-ended problem solving once a week into the future. In her long term impact interview, Claire stated that she had become more aware of the fact that individual pupils learn and see things differently. She reiterated that when time allowed she tried to hear all their explanations of a problem. Although I didn't see evidence that Claire had adopted a regular open-ended problem solving approach, I could say that she had become more willing to allow for a variety of pupil solutions. That willingness should prove useful to her into the future, even in dealing with standard textbook problems.

6.4.2 Sensitivity to students

Claire's initial definition of scaffolding was that it meant supporting the children who needed it in a discreet manner, while still encouraging and challenging them. It was an interesting definition in that it did not acknowledge that children of higher ability would need support in working at their frontier zone. In her exit interview, Claire realised that the support provided depended on the type of mathematical problem presented to the children. She stated that there were ways of guiding and helping the children along the right path. She was conscious that children clearly helped one another while solving such problems. In her long term impact interview, Claire showed that her awareness of constructivist problem solving approaches had not really influenced her day-to-day mathematical practice. She commented that her

need to differentiate for a wide range of abilities in her class meant that she tended to stick with handouts and textbooks. By staying with such textbooks, she was adhering to problems where there was a 'right path' to their solution. Her comments implied that she preferred textbook problems where there was one right answer, rather than the less predictable open-ended problems, which could lead pupils down different routes. Paradoxically, I would hold that it is often the open-ended problems which allow pupils of different achievement levels to contribute more in class.

6.4.3 Mathematical challenge

Claire's initial interview indicated that she believed that discussion amongst pupils during problem solving was beneficial to them across all curricular areas. However, her proviso was that it was important for pupils to achieve a single solution, although different methods could be used to arrive at such a solution. In her exit interview Claire stated that 'brighter children' benefit from being challenged more in the classroom and from figuring things out on their own. However, she believed that 'weaker children' struggle in such situations. She commented that some children lose interest when they are struggling to find a solution. As stated above, Claire's view of scaffolding seemed to involve supporting the pupils of lower ability, while letting the pupils of higher ability to their own devices. Her comments coincide with studies by King (1993) and by Ross (1995) who found evidence in mixed-ability groups of passivity on the part of less able students in mathematical discussions and dysfunctional responses to their learning needs on the part of higher-ability students. In Claire's long term impact interview she was asked for the advice she would give to another teacher adopting a constructivist approach to their work. She advised having one day a week dedicated to groupwork and problem solving. She expressed the view that children enjoy such groupwork and don't even realise they are doing

mathematics. Her opinion was commendable, but her previous comments on adherence to textbook teaching indicate that such group work was unlikely to become a permanent feature of her own classroom practice.

6.4.4 Connectivity

For her preliminary interview Claire had been asked to read pages 3-4 of the Teacher Guidelines in Mathematics. She commented that pupils need to use existing ideas to make sense of new ones. This resonated with a Piagetian view of learning. It had also occurred to her that she should try to relate topics to one another. In her exit interview, Claire stated that it was hard to find open-ended problems to tie in with the topic being taught on a particular week. She declared quite honestly that she needed help in sourcing suitable problems. In her long term impact interview Claire reiterated that the sourcing of materials and 'time restraints' (her phrase) made it difficult to do groupwork on a regular basis. Nevertheless, as mentioned above she proposed dedicating one day a week to groupwork and problem solving. However, it seemed to me that Claire's difficulty in finding suitable problems meant that her own practice would fall short of the aspirations she held for others. Clearly, it is hard to foster constructivist-compatible approaches if the problems chosen do not link with the material or topic being taught. It appeared to me that Claire was aware of the need to source problems, which would encourage connection-making among the pupils, but she lacked the conviction required to pursue this awareness. Therefore, there was a lack of connectivity prevalent in the analysis of Claire's data. I now proceed to Aoife's data.

6.5 Conclusions on Aoife's data

In this section I will deal with the conclusions arising from Aoife's data.

6.5.1 Management of learning

In her initial interview Aoife derived a succinct definition of constructivism from her reading of the Mathematics Curriculum and Teacher Guidelines. She stated that it seemed to be a very democratic system, whereby the control was taken from the teacher and that it was more about the children's learning than the way a teacher teaches. She seemed to have grasped the essence of constructivism. In her exit interview she qualified this somewhat when she surmised that the project on constructivist-compatible approaches seemed to benefit the more mathematically-able pupils who would naturally look for different solutions. Interestingly enough, Aoife mentioned how there had been a benefit to her teaching of Science in that she now allowed children to try other methods besides her own ones. As regards the long term impact of the study on mathematics teaching, Aoife commented that she allowed children who used a different method to present it to the class. In offering advice to other teachers, Aoife promoted the release of control. She stated that constructivist approaches were more about facilitation than direct teaching. This coincides with Ross et al.'s (2002) statement that the teacher's role in reform settings is that of co-learner and creator of a mathematical community as opposed to sole knowledge expert. Aoife's awareness of constructivist approaches seemed to have heightened throughout the project.

6.5.2 Sensitivity to students

In her initial interview Aoife hoped that a constructivist-compatible approach would aid differentiation i.e. groups learning at their own pace and style. This contrasted somewhat with her view of scaffolding, which she defined as creating and maintaining the momentum of a lesson either through questioning, prompting or commenting to keep children on task. Her definition of scaffolding was very much

teacher-centred, in contrast to her aspirations for groups learning at their own pace, which demonstrated a child-centred approach. Throughout the project Aoife struggled with the relinquishment of control to the pupils. Although she promoted such relinquishment she found it difficult to implement.

In her exit interview Aoife redefined her definition of scaffolding to incorporate utilising what was learned as a stepping stone to a new but similar (if not a little harder) challenge. I think her definition changed in line with the more open-ended tasks she had presented to the children. Such tasks required more input from the pupils and less from the teacher and this is reflected in her definition. In her long term impact questionnaire, Aoife stated that she had become more aware that children are able to figure out methods by themselves through trial and error. She commented that she allowed for pupils' errors; more as a way of eliminating a wrong solution, so as to get to a correct one. It occurred to me that Aoife had become a partial advocate of constructivism as she had blended in aspects which gave pupils more ownership of their learning, while still steering a ship in which she was very much the captain; albeit a more democratic one over time.

6.5.3 Mathematical challenge

In her initial interview, Aoife had anticipated that finding different methods and skills would benefit 'weaker pupils' who don't understand the concept from a 'modh díreach' (direct transmission) method. However, in her exit interview, Aoife contradicted this by stating that the children of 'weaker ability' preferred more direct instruction and direction during the project. Moreover, she had found that it was the 'mathematically able' children who had been challenged and who had thrived. She commented that the project was like differentiation for 'stronger maths pupils'. In

her long term impact questionnaire this view seemed to have become embedded. Here, Aoife declares that constructivism only seems to suit higher achievers who have the ability to think ‘outside the box’. She believes that ‘weaker pupils’ need formulas and methods to follow. She points out that it is difficult to facilitate group work in mixed ability settings, as the group tends to rely on the input of one or two pupils. She comments that where pupils are in similar ability groupings, there are problems also as the ‘weaker pupils’ find problem solving very difficult and need constant assistance. One possible solution for Aoife would be to source different problems for the various ability groupings within the class. I say this because some studies that have focused on grouping strategies in mathematics have found that homogeneous ability grouping is preferable for complex problems (Fuchs, Fuchs, Hamlett & Karns, 1998), but only if students have different bodies of knowledge to draw on to solve problems (Mevarech & Kremarski, 1997). This is interesting in that it implies that even within a homogeneous grouping pupils need to bring their individual problem solving skills to bear.

6.5.4 Connectivity

In her initial interview, Aoife stated that she encouraged group work and pair work in all curricular areas, but not in every lesson. In her exit interview, Aoife thought that working on open ended problems was a beneficial exercise, as children learn to think laterally or ‘outside the box’. She believed that children needed to be given the tools to tackle problems. She quoted the example of the children needing to know what a sector was before tackling the problem of the maximum number of pieces generated when a pie is divided using three cuts. In terms of connectivity, she was conscious of relating one area of mathematics to another. As regards the long term impact on her practice, Aoife stated that she had become aware that individual

children can come to a solution by themselves and in different ways. She believed that teachers in general practised constructivist approaches to a degree, but that the amount of content to be taught and the associated testing meant that it was easier to teach children using direct methods. In summary, this project had impacted on Aoife's awareness of group work approaches in mathematics. She had previously connected group work with other subjects. However, the complexities of implementing group work whilst teaching in excess of twenty pupils meant that she had reverted to whole class teaching in mathematics. Aoife believed that constructivist approaches favoured the children of higher ability. However, her lack of group work meant that she would always be more likely to witness the individual, more than the social, acts of meaning-construction.

I now wish to turn to the group interview I conducted with the four participants on 16th June 2011 at the conclusion of the project. My supervisor, Dr. Paul Conway and Dr. Hugh Gash had advised me to bring the group together to give the project synergy. Unfortunately, the participants were so busy that it didn't materialise until the project was nearing its end. Nevertheless, the data collected proved very useful in highlighting issues the participants had perceived throughout the project and also in synopsising my findings.

6.6 The Group Interview

The group interview took place after school in Claire's classroom and lasted twenty six minutes. The proceedings were videotaped and as mentioned earlier, the transcribed conversation is included as Appendix 25. I will deal with the issues which each individual participant raised.

6.6.1 Claire's issues

One issue for Claire was sourcing suitable material for the lessons. Claire would have preferred if I had given her material to teach. I did not want to do this, as I wanted her to come up with her own material and not prejudice her with my views of what constituted suitable material. Nevertheless, I put forward Counting the Squares in a 4 x 4 Grid, as an example of a suitable activity to kick-start the project. Claire thought that the 'heavy curriculum' to be covered in 5th class was a constraint on her ability to find and link suitable activities with the content she was covering. She stated that she liked to finish a lesson at a certain point each day. In other words, she liked to cover a certain amount of content in each lesson. It is unsettling for teachers, such as Claire, to attempt ill-structured problem solving investigations, as there is no guarantee that a set amount of content will be covered.

As regards grouping Claire thought the project benefited the pupils of 'weaker ability' as they became involved in the problem solving activities. Heretofore, such pupils were confined to working with a textbook, lower in level to the class textbook. Her point was that the pupils felt more included in the group situations, as there was support from other group members if needed. She put in the proviso that care was needed in pairing off pupils, as they are very conscious of their own ability and don't like being paired with another pupil of substantially lower ability. According to social constructivist theory, pupils should be placed in situations where they can learn from one another. However, there is an egalitarian issue also in that pupils must be willing to partake in such situations. This requires great skill and tact on behalf of teachers in devising such social encounters.

6.6.2 Lisa's issues

Lisa viewed the written word problems in the standardised Sigma-T test (2011) as a constraint in that the test dictated the content to be covered in class, which left very little time for investigative problem solving explorations. However, Lisa had some useful ideas on how to improve the situation. She suggested linking a new problem solving situation with content, which had been previously covered in class, as this would aid revision. She was in favour of a mathematics publication, which would contain problem solving exercises arranged according to the curriculum strands i.e. number, algebra, shape and space, measures and data. She acknowledged that problem solving was an important skill but that teachers felt guilty when they had little content to report as covered when writing up their monthly summary. As previously stated, I worked with the Primary Curriculum Support Programme (renamed since as the Primary Professional Development Service and currently as the Professional Development Service for Teachers) for two years. Therefore, this comment interested me, as tutors like myself, were encouraged to get teachers to view 'problem solving skills as content' but obviously, not all teachers had internalised that message. Lisa advised that teachers would need in-service, where support personnel from outside (cuiditheoirí) would come in to the classroom to model problem solving lessons for teachers. The Professional Development Service for Teachers (PDST) currently provides this service on request. The difficulty is that personnel numbers have decreased in the PDST due to government cutbacks. The provision of modelling was a dilemma for me during the project. Tempted as I was, I did not provide modelling, as I wanted to see how the teachers would construct and evolve their own pedagogies without undue influence from me. Seeing that the vast majority of teachers have no ready access to modelling, I thought that it was best to

proceed without its influence, in order to gain a truer picture of life in Irish primary mathematics classrooms.

As regards grouping pupils, Lisa thought that working in groups had improved the children's confidence and attitude towards mathematics, especially the children perceived as being of lower ability. In Lisa's classroom these children had experienced success at mathematics. She believed that there was a positive knock-on effect to children's confidence in tackling textbook problems. When it came to pairing pupils, she had some practical advice. She suggested pairing children of 'good to very good ability' with children of 'average ability', but also children of 'average ability' with children perceived as being of 'lower ability'. However, ability was not the only criterion she considered, as she stated that a pupil's attention span and personality type (dominant versus laid back) needed consideration. Her advice was practical and showed that she had not only considered cognitive factors, but affective ones as well. Lisa had experimented with 'mixed ability' groupings and knew which pupils worked well together. It is worth looking at the converse situation. Does that mean that pupils who work well together will always come from mixed ability groupings? It seems not, as Aoife, the next participant teacher illustrates.

6.6.3 Aoife's issues

Aoife stated that at the start of the project I hadn't stated what constructivism specifically meant and as a result she felt frustrated because she wasn't sure whether her lessons were completely 'off the mark' or not. As already stated, I asked the participants to read the scant amount of advice provided in the Mathematics Curriculum and Teacher Guidelines, but obviously she believed more detail could

have been provided. The difficulty I had with providing such detail was that I wanted to situate this research within a constructivist paradigm which, according to Mertens (2005), acknowledges multiple, socially constructed realities. I did not want Aoife's creation of constructivist pedagogies to solely mirror my own views.

What was very interesting was that Aoife differed from Lisa in that she believed it was the children of 'higher ability' who had gained the most from the project. She described these children as the ones who could 'think outside the box'. Aoife thought that the children perceived as being of lower ability depended on her for assistance and had depended on teachers in previous years also. However, Aoife had earlier conceded that she was a teacher, who found it difficult to hand control over to the pupils. Interestingly, she compared her situation to that of a psychiatrist who is expected to provide the right answers to a patient. I think Aoife was happier in a class where she was the master of ceremonies. She described herself as regimental and quoted one parent who had described her as 'a benevolent dictator'; a mixed compliment indeed.

Another factor which would militate against Aoife's embracement of constructivist pedagogies was her desire, and Claire's, to have ready-made teaching material at their disposal. Aoife stated that she had 'exhausted the internet' in her search for age-appropriate problems, which she herself could solve. I have to say I found little evidence of such endeavours and that was why I provided resource material, such as Maths to Think About by Claire Publications (2000).

As regards grouping, Aoife tried mixed ability groups as well as pairs of similar ability levels. From my observations, her groups were organised efficiently in that

pupils had well-defined roles; such as recorder, reporter, encourager, timekeeper and chairperson. However, Aoife reported that the children of 'higher ability' tended to dictate to the rest of the group, who just followed suit, writing down whatever the children of 'higher ability' suggested. It was revealing that Aoife described the children of 'lower ability' as being passive in their groups. She compared their passivity level to that witnessed when she herself was teaching the class as a whole. In other words, being in smaller groups did not improve their level of engagement. Aoife believed she had better success with pairings of children of 'higher ability'. She stated that such children 'drove on' because they didn't have to stop to explain material to other children. It reminded me of Lisa's point that personality type (dominant versus laid back) needed to be taken into consideration when forming groups. As suggested earlier, it may also be necessary to provide different activities for groups, depending on their ability levels. If activities are pitched too high pupils may become disinterested in their groups and passivity levels may increase.

6.6.4 Anita's issues

Anita recalled that at the start of the project some of the 'higher ability' children were unwilling to share information with the other pupils, as they were used to working independently. She stated that it took a while for discussion to become established in the classroom. It was interesting that unlike Aoife, she found that the children of 'lower ability' were relaxed during the project, with no mention of passivity, as there was always someone in the group who could give them guidance or a bit of support.

Anita was glad that children often came up with answers that she hadn't anticipated. However, she was used to containing mathematics to a forty five minute lesson, and

now realised that investigative lessons could easily last for an hour and a half if pupils were given freedom to explore solution pathways. This created management problems for her, as she still had to cover other subjects also and do them justice in her monthly report (cuntas míosúil). Like the other participants, Anita believed that sourcing appropriate material was a major impediment to her adoption of a constructivist-compatible approach. She was very honest in stating that if she had to go scanning the internet or visiting the library or go looking at other maths books it would 'turn her off the idea'. She wanted readymade material in front of her. This indicates the appeal of a standard textbook as it is readily available to teachers. My visits to the schools made me realise how fortunate I was to witness how four conscientious teachers worked in their classrooms under the constraint of having to cover a prescribed curriculum in a set period of time. I will always be grateful for the privilege they bestowed on me.

6.7 A summary of the participants' issues

All of the teacher participants expressed a view that sourcing appropriate investigative problem solving activities was difficult for them. In particular, this was a major problem for Claire. Lisa stated that she would welcome more in-service on constructivist approaches, especially in the area of modelling of lessons. The Standards of Practice (NCTM, 1989) expected that teachers would be able to develop materials and practice to enact reform vision with little support. Coinciding with Lisa's request, Bitter & Hatfield (1994) state that experience since then shows that it is essential to provide ongoing professional development, with a particular focus on providing teachers with examples of constructivist teaching. In Aoife's case the main issue was the relinquishment of control. She stated that she found it difficult to hand control of their learning over to the pupils. What was also

informative was that she thought it was the children of 'higher ability' who gained most from the groupwork in mathematics, as she believed the children of 'lower ability' remained too passive in their groups. This would be contrary to what Lisa had found as she believed that the children perceived as being of lower ability had gained in confidence and had developed a positive attitude towards mathematics through the use of groupwork. For Anita the main issue seemed to be one of time management. She welcomed the fact that children could come up with unanticipated solutions, but stated that exploring such solutions took time and she was conscious of giving other subjects their due allocation of time also, especially with a view to completing her monthly report. Indeed, the constraint of needing to cover a prescribed curriculum in a set period of time proved to be an ongoing issue in data collection for all of the participants. I now wish to proceed to my final source of data which comprises the children's views of the project.

6.8 The children's views of the project

In June 2011, I interviewed three children from each of the four participating classes to ascertain their views on the project. The interviews were open in that the only set question was to ask children what they thought of the project. However, this question led to others through which children gave interesting insights into their teachers' teaching methods during the project. Parental permission was sought in advance and the children's names have been changed to protect their anonymity. The children were chosen by the class teachers. I asked them to choose children according to their perception of varying abilities as I wanted a cross-section of children's views. As I had only been in each classroom for four lessons I did not know individual children very well. However, during the research I had been surprised at how children, perceived as being of lower ability by their class teachers,

had answered questions with great insight. I wanted to see if children's views of the project differed, depending on teachers' perceptions of their ability. The full transcript of children's views appears as Appendix 28. I start by looking at a snippet from Greg, a pupil of 'high ability' in Claire's class.

6.8.1 Claire's children

Greg: And it was really fun, like, working in groups because with normal maths you're doing it by yourself.

Interviewer: Mmm.

Greg: But like in a group if you get something wrong some of the others might have gotten it right…

Interviewer: Okay…

Greg: … and then you can correct yourself.

Interviewer: All right. So did it take the pressure off you a little bit to perform on your own?

Greg: Yeah.

I thought Greg's phrase of 'normal maths' was interesting in that it captured what goes on in most mathematics classrooms in Ireland i.e. children working individually on textbook assignments. His phrase reminded me of the Evaluation of Curriculum Implementation Report (2005) which found that there was an over-reliance on textbook problems in classroom work. Greg enjoyed the camaraderie involved in group work as he could learn from other members. The difficulty of course is that pupils may decide to take a back seat and let others do the work; a point to which Aoife alluded earlier. However, Greg did not seem to fit into that category.

I asked the pupils about Claire's behaviour during the lessons, as I wanted to ascertain if the adoption of constructivist-compatible pedagogies had any discernible features for the pupils. Denis remarked that the teacher 'was explaining more stuff' and helped the pupils a lot compared to standard lessons, where she let the pupils do most of the work themselves. Ian reinforced this view of Claire when he stated that ''for normal maths she wouldn't explain the questions, but for this she would'' so that the pupils would know what to do. Greg suggested that Claire had to explain the maths more as 'some parts of it were a bit more difficult'. It seems that adopting the newer pedagogies meant more work than normal for the teacher in explaining the activities and in endeavouring to make sure pupils stayed on task.

6.8.2 Aoife's children

I wish to consider the following snippet from two of Aoife's pupils, Kay and John.

> **Interviewer**: Okay. Kay, could I start with you? What did you think of the maths project and working…?
>
> **Kay**: It was em, easy like… in most bits and like… it was fun.
>
> **Interviewer**: Okay. What did you find fun about it, we'll say?
>
> **Kay**: Em….because it was like different activities that we wouldn't normally do.
>
> **Interviewer**: Okay, okay. How about you James? How did you find it?
>
> **John**: It was great fun because it was like a challenge rather than like having to do sums constantly. And it was like trying to get at it from a different angle rather than just doing it the way we're supposed to, just kind of a different approach.
>
> **Interviewer**: Okay. So did you feel there were different ways of doing the sums, is it? Or try out different approaches?
>
> **John**: Yeah. Like rather than….you were free to try whatever way you wanted rather than like, you know, in the subjects you would have to do it one way; what the teacher says.

In a similar fashion to Claire's children Kay and John confirm that Aoife taught the children material, which they wouldn't normally cover and allowed them to try out different approaches. In particular, John, a high-achieving pupil seemed to relish being given the freedom to construct his own solutions. Kay later stated: 'You had to kind of use your mind a lot. You had to think about it before you'd kind of do it.' This shows that there was mathematical challenge for her in the activities presented. I asked the children if Aoife kept up these activities when I wasn't present. Kay and the third pupil, Gary, replied as follows:

> **Kay**: We kind of went back to normal like em… multiplying, dividing.
>
> **Gary**: Well, ahm...for the first one or two days she kept doing the different like sums and different approaches. We went back to the normal adding and subtracting. And now and again we just do the 'different thinking', like eh… the ways you would think to do different sums.

This would tie in with my long-term synopsis of Aoife's work as outlined earlier. Aoife probably regressed to whole class teaching when the camera wasn't there. The open-ended activities probably disappeared also. However, she did allow pupils to suggest different methods for solving text book problems, which, for me, was very positive. It was also informative when the pupils suggested that Aoife somehow behaved differently for the camera. Consider the following interview snippet:

> **Interviewer**: Okay. And did ye feel that the teacher behaved in a different way during… during these lessons? You're smiling, John.
>
> **John**: Yeah (laughs). I think she did because like… normally like… as soon as em… the camera's turned on she's started like… like acting better like… trying to be the best teacher she could.
>
> **Interviewer**: (laughs) In what way, John?

John: She was like, do you know the way she'd say, em… like the 'one, two, three' thing and you'd have to reply and stuff.

Interviewer: Yes.

John: She'd never do that normally. She just does it….

Gary: Showing off.

John: Yeah. Showing off.

Interviewer: Okay. Very good. What did you think, Kay, of the teacher behaving differently?

Kay: Yeah. She definitely behaved a bit differently, because like she wouldn't normally go like 'class, class, class' and like she was doing different activities that we, like normally, wouldn't do.

John and Kay were referring to drills, which Aoife used to get pupils' attention. For instance, when Aoife said 'one, two, three' the pupils were meant to reply with 'eyes on me' and similarly, when she said 'class, class, class' the pupils were meant to respond with 'yes, yes, yes'. From my observations these drills helped Aoife in gaining the pupils' attention. Furthermore, a certain amount of showmanship is understandable, as each teacher wants to come across as competently as possible on film; especially, when they know the tapes are reviewed after the lesson.

6.8.3 Anita's children

Consider the following snippet from Brianne and Isult taken from the interview with Anita's children:

Brianne: I liked it because it was different to all the other maths we did. Like… in pairs like… it was better because we're usually working alone by yourself and then, like, you get to talk about the way other people can figure out answers… like how different they think about maths than we do.

Interviewer: And do you feel you learned different ways from the other girls in your group?

Brianne: Yeah.

Interviewer: Were there times when you would have said 'ah yeah, yeah, I wouldn't have done it that way?'

Brianne: Yeah.

Interviewer: Yeah? Okay. Okay. Right. What did you think, Isult of the different ways of doing things?

Isult: Em… I thought it was good because then like… one time I didn't know like what to… and then my partner knew what to do and I was like 'I wouldn't have done it like that'.

Interviewer: Yeah?

Isult: And then that was like a better way to do it.

In the dialogue above, Brianne and Isult appear to have a strong awareness of how other pupils learn. They realise that other pupils have alternative views to theirs on how to do mathematics and that such strategies can be 'a better way to do it' to quote Isult. I was impressed by the girls' open-mindedness in assessing other pupils' viewpoints. It corresponded with data from the long term impact interview which indicated that Isult's teacher, Anita, had also become aware of pupils' alternative strategies and allowed time for them in her lessons.

Brianne also indicates that the pupils usually worked at mathematics as solitary individuals. Later in the interview, she went on to state that she liked to be 'allowed experiment' to come up with different answers. Such freedom to experiment emanates from the teacher's view of learning. When asked about any noticeable teacher behaviours, Isult described it succinctly as follows: 'She (the teacher) was

asking us to work together and usually she asks us to work independently.' Another pupil, Sharon, elaborated further when she commented that the pupils went in groups and actually discussed mathematics; but that when they worked by themselves they didn't discuss it. Brianne alluded to the pupils' standard modus operandi as doing written work based on the mathematics textbook. However, in the project the pupils had been allowed use the interactive whiteboard and did 'all different type of sums', not just 'division or fractions'. Interestingly, in contrast to the data from Claire's children, Isult and Sharon thought that the teacher had to give less explanation during the project activities as the teacher gave them 'a small bit of information' and they would have to 'go and figure out the rest of it' themselves. Furthermore, Isult and Sharon conceded that working in groups took the pressure off them, as they weren't 'put on the spot to give a particular answer', to quote Isult.

In summary, Anita's pupils appeared to enjoy the project as it freed them from the mundanity of the mathematics textbook, gave them freedom to experiment with varying forms of mathematical activities and took the onus off them to provide individually correct answers. I now move on to the final interview with three pupils from Lisa's classroom; Lauren, Karen and Kay.

6.8.4 Lisa's children

I would like to consider the following extract from the interview with the children:

> **Interviewer**: Okay girls… I'll start with you there, Lauren. What did you think of the maths project?
>
> **Lauren**: Em… I thought that it was easier than the usual maths, eh, because Miss Fogarty explained it more precise.
>
> **Interviewer**: Right. Okay. And what did you think, Karen?

Karen: I thought it was like different because like teamwork and… like all the different problems and like (inaudible)…

Interviewer: Right, right. And yourself, Kay?

Kay: I thought it was like, it was really like, fun and like she made it like… really… like easier.

Interviewer: Okay. How…why would you say that, Kay, 'she made it easier'? How do you think she made it easier?

Kay: She like put us into pairs and em… groups and like… she writ like stuff on the sheet.

Interviewer: Okay. Okay. What would you think of that Karen? Did she make it easier or harder for you or…?

Karen: Em, I thought it was easier.

Interviewer: Okay. Why would you say that?

Karen: Because like if… in the usual maths it's like all… em like … different like, er, the problems and stuff.

Interviewer: Okay.

Karen: But this was different.

Interviewer: And what was different about it?

Karen: Like we've never done these things before…

Interviewer: Okay.

Karen: … and like teamwork.

Interviewer: Okay. So the teamwork was different, was it?

Karen: Yeah.

Interviewer: Okay. What did you find about it, Lauren?

Lauren: I thought some of the questions were harder, but it made it kind of easier working in pairs and stuff.

I thought it was interesting when Lauren mentioned that she found the mathematics easier because the teacher had explained it more, as also occurred in Claire's classroom. Greg had suggested that Claire had to explain the maths more as 'some parts of it were a bit more difficult'. This contrasts with the view expressed by Isult and Sharon from Anita's classroom who thought that the teacher had less explanation to do. A possible reason for this is that the pupils in Anita's classroom were higher achievers and as a result needed less explanation.

I can surmise that exploring investigative mathematics was new to the pupils. Later in the interview Kay described the teacher's behaviour during the typical mathematics lesson as the teacher giving out worksheets with the pupils having to put their 'heads down and work'. She used the old adage 'time flies when you are having fun' to describe how quickly time passed for her while she was engaging with activities during the project. As regards new material presented, Lauren thought that some of the problems encountered were more difficult than normal but that working in pairs made it easier to tackle them. Lauren stated that she enjoyed the mental warm-up activities, which the teacher had created, and for which I can claim no credit. She used the word 'funner' as an adjective to describe her experience when there was a warm-up activity. She mentioned that the warm-up 'can give you an idea of what you did before and that would pop into your head in some of the questions'. Making mental connections was obviously beneficial for Lauren in dealing with mathematics problems. It reminded me of Piaget's theory of assimilation and accommodation where previously assimilated older knowledge is altered to accommodate newer knowledge. Lauren's comment resonated with me, as it was in Lisa's classroom that linkage became a significant catalyst for organising

lesson material in a coherent fashion. Indeed, such linkage led to the formulation of Connectivity as a fourth element for Jaworski's Teaching Triad.

6.8.5 Summary of children's views

This research took place in a disadvantaged girls' school and a middle-class mixed gender school. However, I found no discernible differences in the children's views based on either socio-economic status or gender or perceived ability. In general, the children appeared to enjoy working in groups on investigative mathematics. Kay (Aoife's class) mentioned that these were activities the pupils wouldn't normally do. Greg (Claire's class) stated it was fun as in 'normal maths' one works alone. He commented that the teacher had to work harder at explaining material. Lauren (Lisa's class) mentioned that she found the maths easier as the teacher explained it more. However, this contrasted with the view expressed by Isult and Sharon (Anita's class) who thought that the teacher had to work 'less hard' as the pupils primarily figured out solutions by themselves. I found it revealing when Kay and Gary in Aoife's class stated that Aoife reverted to teaching standard algorithms when the camera wasn't there. This highlights the difficulties involved in following a constructivist-compatible approach, while trying to teach to a prescribed curriculum. Barbara Jaworski had alluded to such difficulties when I spoke to her as part of my research. Indeed, Senger (1998), comments that even teachers chosen as exemplars of reform teaching regress from the ideal, displaying the height of reform one day, but reverting to traditional methods the next. This is understandable given that this research shows how difficult it is for teachers to elevate themselves out of the gravitational pull of a prescribed curriculum.

Having summarised the data, I proceed in the final chapter to draw my conclusions for the project and discuss its implications for (a) classroom practice (b) policy on continuing professional development and (c) constructivist theory.

Chapter 7 Conclusions and recommendations

7.1 Introduction

In this chapter I will summarise the research findings. This research involved the videotaping of sixteen mathematics lessons in which teachers attempted to adopt constructivist-compatible approaches to their work. The reader will recall that the teachers were interviewed individually at the start of the project and in a group situation at the end of the project. The primary purpose of these interviews was to ascertain if teachers' views and adoption of constructivist-compatible approaches had evolved. Children's views on the project were also sought and the teachers were also re-interviewed almost a year after the project had ended to determine any long term impact of the study. Jaworski's Teaching Triad was crucial as a developmental and analytical tool throughout the project. In synthesising the data I write the conclusions and recommendations arising from the research, insofar as they apply to implications for current classroom practice, policy on continuing professional development and constructivist theory. I conclude by questioning current values of what is deemed important in the teaching of mathematics in Ireland.

7.2 Implications for classroom practice

These implications for practice are written in the form of advice for an audience of teachers interested in adopting a constructivist-compatible approach in their mathematics classrooms. They are not written in any particular hierarchy of merit. One of the limitations of the research is that only four classrooms were involved and therefore it is difficult to generalise the findings to cover a wide variety of contexts.

Nevertheless, I believe teachers who wish to move the teaching of mathematics beyond the routine coverage of a textbook will find the following advice valuable:

1) **Pupils' attitudes and cognitive challenge:** When teachers adopt an investigative group-centred approach in their mathematics classrooms, pupils' attitudes towards mathematics seem to improve. Pupils describe the resultant mathematics experience as being more fun. One proviso I would include is that the teacher needs to be vigilant that pupils are working at a challenging level and not just 'coasting' in their groups, waiting for the higher-achieving pupils to come up with all the correct procedures. Stigler and Hiebert (1999) quote an unnamed mathematics education professor in their video study of 81 U.S. classrooms. The professor states, "In U.S. lessons, there are the students and there is the teacher. I have trouble finding the mathematics; I just see interactions between students and teachers" (p. 26). The point is that the interactions are not enough; there must also be cognitive challenge for the pupils. This thesis has stressed the need for cognitive challenge in activities presented to pupils; going so far as to use Jaworski's categorisation of mathematical challenge as one of the developmental and analytical tools throughout the study.

2) **Organising group work:** The organisation of group work was a major issue for the participant teachers. From a constructivist viewpoint a teacher hopes that splitting a class into groups will ensure that every group and every individual within a group gains an important amount of sustained attention of the kind that can produce the 'higher-order cognitive interactions' that Galton (1980) and Mortimore (1988) regarded as essential for purposeful learning. In general, the teacher participants thought that grouping suited the higher-achieving pupils more than the lower achieving ones. For instance, Aoife thought there were times when placing the

higher achievers together meant that they could ‘expand more on an activity’. However, Lisa thought that lower achieving pupils benefited more in groups as they gained assistance from higher achieving ones. The point I wish to make here is that it is difficult to generalise as to which groupings of pupils work best; seeing that affective as well as cognitive factors have to be considered in varying contexts. Teachers need to experiment with different groupings of pupils and adapt accordingly when pupils are not working at their optimum level. I believe there is a great need for representative pupils to share what they have learnt in groups with other members of the class. In other words, I am advocating plenary sessions where knowledge is shared in line with an emergent perspective. The reader will recall Anita’s strategy of ‘harvesting and sowing’ whereby the useful ideas from one group were gathered and disseminated to other groups by the teacher. She also dispersed ideas through the strategy called ‘Move About’ which allowed pupils to move from one group to another. These strategies counteract the traditional modus operandi of children completing individual, humdrum written tasks at their desks during lessons.

3) **Teaching as telling:** Teachers in Ireland work with the second largest class sizes in Europe. The pressure under which teachers put themselves to cover a set curriculum in such large classes was palpable and readily perceptible to this researcher. The DES study entitled An Evaluation of Curriculum Implementation in Primary Schools (2005) found that teacher-talk dominates as a methodology. Purists may argue that this is incompatible with a constructivist approach to learning. However, I have seen that there are times, even in investigative problem-based mathematics, that it is an expedient and efficient use of precious time to tell pupils information, which may help them in an investigation. The dilemma for the teacher remains *what* to tell and *when* to tell it.

4) **Teacher change:** Although I have to report that once the project ended, I do not believe the participant teachers adopted a constructivist-compatible approach to their work for the longer term, I believe that their knowledge of what such an approach entailed had increased. From what the teachers stated in the long term interviews, their awareness of pupils' varying learning styles had been heightened and a lasting effect was that they had become more open to letting pupils derive and describe their own individual methods. This showed willingness on the part of the participant teachers to relinquish some control and allow pupils construct their own problem solving meanings. The teachers could be described as 'part time' advocates of a constructivist approach. In similar findings to O'Shea and Leavy (2013) I found that the teacher participants placed a significant focus on computation and recall of basic mathematical facts. Such a focus remains deeply embedded in the teaching practices of teachers. Teacher change is a notoriously slow process. Ross et al. (2002) are graphic when they comment that progress towards "implementing reform ideals will be incremental, with advances occurring on a broken front with many backward steps" (p. 131). I agree with O'Shea and Leavy (2013) who comment, from their research in Irish primary classrooms, that although teachers were inspired by learning from a constructivist perspective, it was evident that methodologies that reflect constructivist principles would not usurp the traditional methodologies used by such teachers. There are many reasons for this finding. In this research, as outlined in Appendix 25, Claire blamed the heavy workload in 5th class and the unavailability of suitable problem solving activities. Aoife stated that she had to teach formulas and problem solving methods before she could consider taking on any other type of mathematical activities. On a similar note, Lisa thought that teaching problem solving methods to enable pupils to cope with standardised tests like the Sigma-T dominated her pedagogy. Anita thought that there were too many

subjects to be covered in the curriculum and this militated against her finding the time to scan the internet to find suitable problem solving activities. It seems that teachers' perceptions of what mathematics entails and the time available to teach the subject, according to those perceptions, acted as a barrier to their adoption of constructivist-compatible approaches.

4) **Choosing appropriate activities:** One of the difficulties the teachers experienced during the project was the sourcing of appropriate material for the problem solving activities. They thought that it would be extremely difficult to adopt a constructivist-compatible approach if they had to spend time searching the internet and looking up other textbooks to find suitable activities. They wanted readymade material to hand. Such views were also voiced by teachers at a problem solving workshop I attended during the Irish National Teachers' Organisation Education Conference held on 15/16 November 2013. The theme of the conference was Numeracy in the Primary School and the workshop was facilitated by Dr. John O'Shea, a lecturer in mathematics education and teaching methodology at Mary Immaculate College, Limerick and whose work I have quoted in this thesis. I believe those teachers' views reflect an over-reliance on standard textbooks and a lack of value on the merits of engaging in open-ended problem solving activity. Such a reaction is understandable from teachers who are already very busy teaching routine algorithms and standard textbook problem solving methods. The teacher participants believed they were under pressure to cover such prescribed content and report on the amount of material covered in their monthly reports (cuntaisí míosúla). As stated in chapter two there is very little emphasis in the curriculum on open-ended problem solving. If such problem solving was valued more and emphasised more in curriculum documents and in-service courses, I believe it would get more attention from teachers. Teachers could then be expected to draw on their professionalism in

sourcing additional material. If investigative mathematics is valued by teachers it will be covered by teachers. Both the Department of Education and Skills and the National Council for Curriculum and Assessment have a role to play in the promotion of investigative mathematics. From the American perspective Schoenfeld (2001) called for greater alignment of curriculum objectives with reform standards. As Alexander (1995, p.306) remarks, "Curriculum balance, then, is a product of decisions taken across the system as a whole, not merely within the school and classroom. It is a matter for policy-makers as well as teachers."

5) **The importance of linkage and connectivity:** A possible means of achieving a balance between covering a set curriculum and engaging in investigative mathematics is for teachers to use the methodology of linkage more often. In that way teachers can link a problem solving activity with prescribed curricular objectives and ease their conscience somewhat when it comes to writing up monthly reports. In this thesis, I have elaborated on the issue of linkage and broadened it into a concept called Connectivity which would expand Jaworski's Teaching Triad into a quadriad. For Connectivity to work a teacher must not only link investigative mathematics with the prescribed curriculum, as in traditional linkage, but must also link it with pupils' real world experiences of mathematical problem solving and with mathematical concepts emanating from other curricular subjects (integration). Such connection-making can be pre-planned but more of it should occur in the throes of a lesson when, for instance, a teacher spots a suitable opportunity to pursue a pupil's query.

7.3 Implications for policy on continuing professional development

O'Shea (2009) stated that the primary curriculum reflects the principles of the emergent perspective on constructivism, but that from a reader's perspective little

background is offered to place its centrality to the curriculum in focus. Moreover, he argues that the presentation of the curriculum's content in clearly defined units places significant restrictions on teachers engaging students in learning from an emergent perspective. I would agree with O'Shea on both points. The issue then becomes a question of what can be done to improve matters. I have highlighted above how traditional linkage needs to be broadened into the concept of Connectivity, whereby teachers constantly seek out material which encourages pupils to make connections between mathematics and their own lives. How best to provide in-service to teachers on constructivist-compatible approaches also becomes relevant. When I was a tutor on the Primary Curriculum Support Programme during 2001/2002 teachers received two days in-service on the revised mathematics programme. As this in-service was delivered to large groups of teachers in hotel rooms, it was not compatible with showing teachers how to divide a class into groups and engage the pupils in a problem solving activity. No wonder then that McCoy et al found in the Growing Up in Ireland Study (2012, p. 35) that "teachers of large classes are more likely to take more traditional approaches, perhaps reflecting greater logistical constraints and space constraints". O'Shea (2009) comments that successful in-service needs to be classroom-based with particular emphasis placed on prolonged periods of classroom support, which is consistent with current literature (Loucks-Horsely, Hewson, Love and Stiles, 1998). Ross et al. also (2002) state that the most powerful method for increasing implementation of reform mathematics is in-service. They add that it is essential to provide ongoing professional development, particularly focused on providing teachers with examples of constructivist teaching (Bitter & Hatfield, 1994) and explicitly addressing their beliefs about mathematics as a teachable subject (Grant, Peterson, & Shojgreen-Downer, 1996). O' Shea (2009) quotes Snyder, Lippincott and Bower (1998) who

suggest that the most effective method employed in the professional development of beginning teachers is a practice-oriented model; where participants devise plans, implement them and reflect upon what happens. There is a strong emphasis in the Irish system on newly qualified teachers providing copious quantities of written notes in order to be certified as probated when a Department of Education and Skills (D.E.S.) inspector visits. Thankfully, D.E.S. Circular 39/12 heralded a more reflective approach with school staffs encouraged to identify areas in need of development and to decide on actions that should be taken to bring about improvements in those areas as part of their three-year literacy and numeracy plans. This means that schools can identify areas like problem solving in mathematics as being in need of attention and give them full priority. Under the Haddington Road pay agreement (2012) teachers have to work an extra hour after school once every week. This means that teachers could prioritise the teaching of investigative problem solving, if the will to do so is present. With a shortage of support personnel on the ground from the main in-service provider, the Professional Development Support Service (P.D.S.T.), schools are forced to bring in outside expertise to help. Indeed, I have been personally asked to give in-service to several staffs in the area of mathematics problem solving. Hopefully, school staffs will use such in-service to gain insights on how to move away from a textbook-led approach towards a more investigative stance to mathematics problem solving. Such in-service is required, not only at primary level, but at second level also, where teachers need to explore investigative mathematics as envisaged in the Project Maths curriculum, as referred to in chapter three. The Project Maths curriculum was designed to follow on from the primary mathematics curriculum. The Dutch Realistic Maths Education (RME) movement has merit as a model for the teaching of investigative problem solving. That is not to say that such a change in teaching methods can occur quickly. Indeed,

Stigler and Hiebert (2009, p. 87) state that cultural activities like teaching "evolve over long periods of time in ways that are consistent with the stable web of beliefs and assumptions that are part of the culture". Delaney (2012) adds that because teaching is a cultural activity, it is difficult to change through teacher education or professional development for teachers. He proffers that teaching is an activity that is absorbed from culture through family conversations over meals, through watching television and listening to radio, and of course from spending 13/14 years as a pupil in various classrooms observing teachers teach. I would add that, unlike other professions, everyone feels qualified to give an opinion on teaching as everyone has been through the education system. Delaney (2012) suggests, and I agree, that the way forward is for teachers to make small changes to their practice over time. For instance, he recommends the development of new habits in mathematics classes; such as asking children to explain how they got their answers, or replacing textbook problems with open-ended problems from a site such as NRICH (or as participant teacher Lisa suggested; www.figurethis.org), or start referring to children as 'low achieving' at maths rather than 'weak' and 'high achieving' rather than 'strong'. His suggestions regarding the labelling of pupils became apparent in this research also. This research certainly heightened the teacher participants' awareness of allowing pupils to create various solution methods to problems and share them with their peers. Delaney (2012) comments that without changing our habits, we won't change a cultural activity like teaching. In chapter three I mentioned an inspirational programme for teachers called the Educational Leaders in Mathematics (ELM) Project as reported by Schifter and Fosnot (1993), which is a two-level programme. At the initial level, teachers who attend a two-week introductory summer institute receive weekly clinical supervision during the following academic year. Many of these teachers then proceed to an advanced level comprising a second institute and

an apprenticeship programme in which they learn to conduct workshops for their colleagues. Such a system is possible in Ireland also. Primary teachers already receive an incentive of four Extra Personal Vacation (EPV) days if they partake of two weeks summer in-service. If the summer course could be followed by a year-long course in mathematics education, with school-based assignments leading to a diploma from a recognised university, then I believe teachers might be motivated to experiment with constructivist-compatible approaches. As I write this conclusion the INTO has advertised a summer in-service course entitled " Maths Problem solving: Process, Not Product". This type of course echoes the sentiment of this thesis and is to be welcomed. Another movement to be welcomed is Assessment for Learning (AfL) as outlined on the NCCA website. This movement promotes the integration of learning with assessment in a formative way, while still allowing for summative assessment at intervals. This type of assessment is in line with a constructivist approach which sees learning and assessment intertwined. Ross et al. (2002) use the word 'integrated' to describe such assessment and see it as being "in contrast with end-of-week and unit tests of near transfer that characterise assessment in traditional programmes" (p. 125).

7.4 Implications for constructivist theory

The Growing Up in Ireland Study (2012, p.23) gives a concise summary of research into constructivist-led classrooms:

> Many of the studies show positive effects on student learning-including research in Korean classrooms (Kim, 2005), a Dutch study on primary students (De Jager, 2002) and the Maths Wings project in the US (Madden at al.,1999) – as well as on other outcomes like students' writing (Au and Carroll, 1997) and student motivation (Koebley and Soled, 1998). However, Muijs and Reynolds (2011) also point to

> research showing that pupils taught by teachers using a direct instruction approach have higher achievement levels than students taught by teachers with constructivist beliefs (Gales and Yan, 2001; Klahr and Nigam, 2004). Further, they note that good classroom management and a positive climate are essential to making constructivism work in the classroom. Kirschner et al. (2006) argue that much of the empirical evidence indicates that constructivist-based minimally guided instruction is less effective and less efficient than instructional approaches that place a strong emphasis on guidance of the student learning process. However, Spiro and DeSchryver (2009) note that many of the studies finding that direct instruction approaches have more positive learning outcomes than constructivist approaches are typically focused on well-structured domains like Mathematics and Science.

From the above quotation, it may appear that direct instruction is deemed to be more effective than constructivist approaches in most empirical studies. However, Mujis and Reynolds (2011) also cite earlier research by Good and Brophy (1986) which states that direct or teacher-centred instruction has been found to be most effective in teaching rules, procedures or basic skills, especially to younger pupils. This reminds me of the teaching of acronyms like RUDE, ROSE or RAVECCC, as discussed earlier in this thesis, to assist pupils in dealing with textbook problems. There is no mention of the development of higher-order thinking skills which constructivist approaches seek to inculcate. In this regard, D'agostino (2000) found that by fourth grade (9-10 years) pupils need to be provided critical thinking opportunities and they need to have occasions where they can direct their own learning. Moreover, Veenman et al. (2005) found a significant relationship between providing explanations in small groups and students' mathematics achievement. Therefore, evidence in favour of either direct teaching or constructivist approaches is not clear-

cut and comes with many caveats depending on the age and context of the pupils involved.

From this research, I would have to agree with Muijs and Reynolds (2011) comment that good classroom management and a positive climate are essential to making constructivist approaches work in the classroom. Indeed they remind me of two elements of Jaworski's Teaching Triad; management of learning and sensitivity to students. The participant teachers were efficient at organising their classes into groupings; experimenting with different arrangements of pupils. The pupils enjoyed working in such groupings so it seems that not only cognitive influences but affective ones also determine learning outcomes in constructivist-led classrooms. It can be deduced from the quotation above that much of the debate on constructivist approaches revolves around how much guidance to give children, particularly those of primary school age. For instance, in this thesis, I have advocated intervention where children need familiarity with standard procedural or mathematical conventions or where an investigation is leading pupils down a blind alley. I have even stated that for a teacher not to intervene can, at times, be pedagogically unsound.

Under Implications for Practice above (Point 2) I mentioned the need for plenary sessions where pupils come together and share their solutions. O'Shea (2009, p. 246) wrote that teachers that engaged students in explaining their solutions to their classmates found that such experience reinforced all students' understanding of the method utilised to solve the problem in question. I would agree with O'Shea's conclusion and would welcome more pupil discussion of solution methods in primary classrooms. Indeed, in America in 2012 the National Centre for Education

Evaluation and Regional assistance issued a report entitled Improving Mathematical Problem Solving in Grades 4 through 8 as outlined in section 3.2. This report recommended exposing pupils to multiple problem solving strategies. It stated that pupils who are exposed to such multiple strategies become more efficient in selecting appropriate ways to solve problems and can approach and solve such mathematics problems with greater ease and flexibility. Moreover, there are times when individual pupils construct idiosyncratic solutions to problems. I witnessed several during this research as outlined in the classroom episodes. One early example (see section 5.2) was when Claire in Lisa's first observed lesson stated that the intersection of two lines could be viewed as a square. It must be remembered that Claire was looking at a very large grid on an interactive white board. Sharing such solutions with the other members of the class means that there is reconciliation between the individual and the social construction of meaning, which is in line with the emergent perspective on constructivism.

O' Shea (2009) also concludes that successful efforts at facilitating learning from a constructivist perspective include the careful selection of suitable mathematical problems. This conclusion resonates with similar advice given to me early on in my own research by Dr. Hugh Gash. I would have to agree with O'Shea (2009, p.247) who surmises that good questions are those that promote debate and discussion and allow the pupil an appropriate amount of choice in terms of strategies to be employed in their attempts to solve them. From this research I would add that good problem solving questions allow for pupils of varying abilities to become involved in their solution and therefore require an appropriate level of cognitive challenge. Too little challenge means pupils will be bored, whereas too much challenge means pupils will lose interest. A type of 'easy difficulty' is required. Vygotsky's notion of

the zo-ped has been a central construct in this thesis. In chapter 5 I created a matrix for appraising problem solving activities which could be useful to teachers. Obviously, it is important that teachers exercise their professional judgement in choosing such activities. In this research, the teacher participants stated that they would welcome a textbook containing such investigative problems and while I concede there is a gap in the market for such a text I believe teachers should use the internet more to source problems, which would be suitable to their own circumstances. No one text is likely to suit all contexts. A textbook should only be a springboard for a teacher into other problem solving activities; either devised by the teacher or instigated by pupils' interests and suggestions.

This study did not just look at investigative problem solving in classrooms; it also looked at the recitation of number facts and the practising of number games at the commencement of lessons. While such activities could be deemed to be engaging pupils at their zone of *minimal* development, they nevertheless provided ample stimulation for pupils to later engage with the more challenging activities in the observed lessons. The motivational power of such early activities should not be underestimated. Indeed, Ross et al. (2002) comment that teachers in reform settings make the development of student self-confidence in mathematics as important as achievement. I now wish to make some suggestions for further research on a constructivist-compatible approach and, thereafter, draw this thesis to a close.

7.5 Further research: Using the appropriate lens and methods

Investigating constructivist learning environments is a complex process. Further research needs to be undertaken on methods which suit the investigation of such environments. Personally, I found Jaworski's Teaching Triad (management of

learning, sensitivity to students and mathematical challenge) invaluable as a lens for looking at teachers' endeavours to adopt a constructivist-compatible approach to their classroom practice. This lens was useful in a design-based approach which sought to improve teachers' investigative problem solving practices. Other researchers may well construct other lenses to look at such work and may cite 'Connectivity' as one such lens. For instance, Simon and Schifter (1991) used a different approach to mine in writing about the Educational Leaders in Mathematics Project (ELM) which involved the creation of an innovative in-service programme for precollege teachers of mathematics based on research and theoretical work. The programme ran over one year. It drew on two sources of data: teachers' writings and interviews with teachers. To assess the use of a constructivist epistemology, ELM developed the Assessment of Constructivism in Mathematics Instruction Instrument (ACMI). It ranged from Level 0 (teacher does not have a constructivist epistemology) to Level 5 (teacher assists or collaborates with colleagues to implement instruction based on a constructivist view). As mentioned earlier, the ACMI instrument appears as Appendix 3. The difficulty I had with such a hierarchy was that teaching is a very complex process and I did not believe teachers could be consistently categorised as falling neatly into one category or another. Rather, they would cross over from one category to another, depending on whether they adopted a transmission or investigative approach to their individual lessons. I found Jaworski's categories to be broad enough to encompass such shifts. Another approach to research in constructivist classrooms, again over a sustained period, is the tracing of sociomathematical norms as promulgated by Yackel and Cobb (1996). These are defined as normative understandings of what counts as mathematically different, mathematically sophisticated, mathematically efficient, and mathematically elegant in classroom activity. The development of such

sociomathematical norms during problem solving sessions provides ample scope for further research in Irish classrooms.

As stated earlier, the teachers in this study worked under the usual constraint of teaching to a prescribed curriculum within a set period of time. Yet, they gave some of their valuable time to explore the application of constructivist-compatible pedagogies in their classrooms. Such pedagogies imply the transfer of ownership of learning to the pupils. According to the teacher participants, this necessitated significant changes of practice for them. Tracking such change involved the adoption of Borko's Phase 1 research design. In this respect, the area of design-based research as detailed in chapter 4, which willingly encompasses and embraces such change, offers great hope for further classroom research on investigative mathematics.

7.6 Conclusion

In the conclusion to his own PhD thesis O'Shea (2009, p. 252) states that much research needs to be conducted concerning the employment of constructivist methodology with the entire mathematics curriculum (not just problem solving) and with other curricular areas. He suggests that all mathematical strands and strand units should be explored from a constructivist perspective to determine the optimum starting point for classroom teaching and learning. I strongly believe that the Realistic Maths Education (RME) movement, initiated in the Netherlands, offers the greatest hope of success for teachers adopting a constructivist-compatible approach. Although not explicitly constructivist, RME uses pupils' real world interests as a motivating springboard for learning. This is in line with Ross et al's. (2002) reform recommendation that student tasks should be complex, open-ended problems embedded in real life contexts, with many of these problems not affording a single

solution. RME also tries to represent concepts in a problem solving format in line with a constructivist viewpoint but does not deny the role of explicit instruction either. Therefore, I advocate using RME in Irish primary classrooms as it blends the all too familiar direct instruction approach with a more constructivist-compatible model. Teaching pupils how to solve textbook problems may well involve direct instruction. However, from this research it seems that dealing with more open-ended problems involves less direct instruction from the teacher and more pupil input and engagement. Other researchers will find ample scope for investigating the applicability of RME to Irish classrooms.

7.7 Final statement

Jean Schmittau (2006) states that constructivism still continues to maintain its pedagogical hegemony. This thesis has looked at the adoption of constructivist-compatible pedagogies in the senior primary classroom. It has followed four teachers over a one year period as they endeavoured to adopt such reform pedagogy; despite having to cover a prescribed mathematics syllabus with large pupil numbers. The teachers explored the implications of constructivism for primary mathematics instruction. They were able to reflect on their lessons, as was I, through the use of digital film technology. Whereas I do not believe the teachers were in a position to adopt a constructivist-compatible approach for the longer term, I believe that they became more aware of what such an approach entailed and were more willing to provide for pupils' insights and idiosyncratic methods during mathematics lessons. Like O'Shea (2009, p. 253) I found that this research "revealed the value in having primary students of mathematics debate, experiment with, and select a variety of problem solving strategies in collaboration with one another". The pupils certainly seemed to enjoy the experience. The adoption of constructivist-compatible

approaches will only prevail in the future if such approaches are given the same prominence and value as the teaching of a prescribed mathematics syllabus currently enjoys. If such flexibility ever gains prominence it is hoped that this thesis will provide valuable advice to those who wish to advance the cause of constructivism. As I conclude this thesis the Educational Research Centre has released a report on performance outcomes for the 2014 National Assessments of English reading and mathematics. The report shows that the percentage of pupils performing at or below proficiency level 1 (the lowest level) has decreased by five percentage points, at both second and sixth class, in both areas. The report also states that there was an increase of five percentage points in proficiency levels 3-4 (the highest levels) in both classes in both areas. The reader will remember from chapter three that these were targets set for the National Strategy to improve Literacy and Numeracy among Children and Young People 2011-2020. Although the targets have been met ahead of time for both English reading and mathematics the report cautions that there is scope for pupils in second and sixth class to improve further on higher level mathematical processes such as the ability to apply and problem solve. A focus on such processes has been at the heart of this thesis and I can only hope that my account of the experiences of the teacher participants and their reflections thereon will be of interest to both practitioners and policymakers alike.

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Table of Appendices

Appendix 1:

Graham Nuthall's Seven Principles for Effective Implementation of Social Constructivist Teaching

1. Develop an activity framework- a sequence of activities that form a coherent learning system.
2. Establish an accountability system.
3. Develop monitoring procedures.
4. Set up a common experience, preferably a small group cooperative activity that produces the data or knowledge that will be the focus of the discussion.
5. Ensure frequent repetition to routinize procedural aspects so that most time is spent on constructive discussion.
6. Repeat critical content and revisit main ideas frequently.
7. Train students in group interaction procedures.

Appendix 2:

Gagnon Jr. and Collay's Six Part Constructivist Learning Design

1. The situation frames the agenda for student engagement by delineating the goals, tasks and forms of the learning episode.
2. Groupings are the social structures and group interactions that will bring students together.
3. Bridge refers to the surfacing of student' prior knowledge before introducing them to the new subject matter.
4. Questions aim to instigate, inspire and integrate students' thinking and sharing of information.
5. An exhibit asks students to present publicly what they have learned.
6. Reflections offer students and teachers opportunities to think and speak critically about their personal and collective learning.

Appendix 3:

The Assessment of Constructivism in Mathematics Education (ACMI) Instrument

Level O: does not have/use a constructivist epistemology.

Level III:5 attempts to modify instruction based on a general view that instruction should involve students in active construction; struggles with how to integrate this view with teaching style and curriculum.

Level IVA: has modified teaching style to include regular activities to foster construction by students; focuses primarily on teaching behaviours.

Level IVB: focuses on student learning rather than teaching behaviours to shape instruction from a constructivist perspective.

Level V: assists or collaborates with colleagues to implement instruction based on a constructivist view.

Appendix 4: Levels of Understanding (LoU) Strategies

LoU ratings were based on the following strategies which were modelled during ELM instruction:

1. Using non-routing problems
2. Exploring alternative solutions
3. Asking non-leading questions
4. Using manipulatives, diagrams, and alternative representations
5. Having students work in groups and pairs
6. Pursuing thought processes on both “right” and “wrong” answers
7. Working with Logo
8. Employing wait time
9. Encouraging student paraphrase of ideas expressed in class

Appendix 5:

Child's Permission Slip

- I am happy to volunteer to be part of the mathematics project.
- I understand that I may appear in videotapes of class lessons but that these tapes will only be seen by my class teacher, the researcher Joseph McCarthy and possibly his supervisor, Dr. Paul Conway. The tapes will be kept in a secure and safe place.
- I am happy that samples of my written work may be collected by the researcher. I know I may be asked at a later date to be part of a 15 minute interview with 2 or 3 pupils from my class which will be audio taped. My real name will not be used when the researcher writes about his study.
- The project has been explained to me in class and I have been invited to ask the researcher questions if I want anything explained. I can ask the researcher or my teacher questions on the project at any time.
- The results of the project will be explained to me, if I so wish.
- I understand that I can stop being a part of the project at any time whatsoever.

Signature: ……

Date: ……..

Class teacher's name ……..

Appendix 6:

Parent's Consent Form

St. Patrick's B.N.S.

Gardiner's Hill,

Cork

Date: ……

Dear Parent(s)/Guardian(s),

My name is Joseph McCarthy and I am researching the teaching of mathematics as part of my PhD thesis. I wish to conduct part of this study in your child's class. It will consist of a series of mathematical lessons centred on how children and teachers approach mathematics as envisioned by the curriculum. Samples of your child's written work may be collected by the researcher and your child may be asked at a later date to take part in a focus group interview which will be audio taped. To preserve anonymity names will be changed in the write-up of these interviews. The class teacher will be teaching these lessons and I will be in attendance in early 2011 to videotape 4 to 6 50 minute lessons for observation purposes. Only the class teacher, my supervisor and myself will have access to the tapes which will be kept in a secure location. I would be grateful if you could complete the permission slip below and return it to your child's teacher indicating whether or not you would like your child to be involved in the research. Your child will be invited to complete a separate permission slip after I have explained the project in class and responded to any questions your child may have. Your child can decide to withdraw from the project at any time. If you have any questions yourself, I can be contacted during school hours on 021-4502024.

Yours sincerely,

_____

Joseph McCarthy

-

Parental Consent Form for Joseph McCarthy's PhD study

Parent

Please delete as appropriate:

I do/do not give permission for my child _____ to be part of this project.

Signed: _____Date: _____

Appendix 7:

Letter of Invitation to Teachers

xxxxx,

xxxxx,

xxxxx,

05.10.2010

Dear ……,

My name is Joe McCarthy and I am currently engaged in a PhD dissertation with the Education Department in University College Cork. I hope to research the teaching of mathematics in two schools over the next number of months.

To this end, I will be asking participants to partake in a professional development initiative in suitable locations designed around the teaching of mathematics from a constructivist perspective. Participants will be asked to teach a number of mathematics lessons in their own classrooms with their own students and engage in dialogue with the researcher over this period of time. These lessons will be videotaped by the researcher.

I would be very grateful if you would give me an opportunity to meet you in person about this project to discuss it further or answer any queries you may have. I am available to visit your school at a time of your choice for this. Alternatively, I can be contacted on 087-7987078 or by email at jmcpcsp@eircom.net.

Looking forward to your reply,

Yours sincerely,

_____

Joe McCarthy

Appendix 8:

Teacher's Consent Letter

PhD Research into the Teaching of Mathematics from a Constructivist Perspective

I _____, am a primary teacher. I am willing to take part in research being carried out by Joseph McCarthy in accordance with the requirements for his PhD. I have read and retained a copy of his invitation letter dated _____. I understand the conditions under which I am taking part in this research. I agree to be videotaped teaching 4-6 mathematics lessons and to discuss these lessons with Joseph McCarthy. I agree to be interviewed prior to the commencement of the videotaping and also when the 4-6 lessons have been completed. I understand that my permission will be sought if any of the videotaping is to be shown to other teachers/researchers as exemplifying best practice. I also understand that I will not be identified in any publications following on from this project. I am undertaking participation of my own free will. No pressure has been placed on me to take part in this research. I understand that I am free to withdraw from the research at any time without any repercussions.

Signed : ……. Date: ……

Appendix 9:

Social Research Ethics Committee Approval

UCC

Coláiste na hOllscoile Corcaigh, Éire
University College Cork, Ireland

Mr. Joseph McCarthy,
Education Department,
c/o Carthain,
Lovers' Walk,
Cork.

Oifig an Leas - Uachtaráin Taighde agus Nuálaíochta
Office of the Vice President for Research and Innovation

Urlár 4, Bloc E,
Áras na hEolaíochta Bia,
Coláiste na hOllscoile Corcaigh,
Bóthar an Choláiste,
Corcaigh, Éire.

4th Floor, Block E,
Food Science Building,
University College Cork,
College Road, Cork, Ireland.

T +353 (0)21 4903500
F +353 (0)21 4903506
E vpresearch@ucc.ie
www.ucc.ie

28th February 2011

Dear Mr. McCarthy,

Thank you for submitting your revised research (project entitled ***An Analysis of Teachers' Implementation of a Constructivist Approach to the Teaching of the 5th/6th Class Mathematics Programme*** #48r) to SREC for ethical perusal. I am pleased to say you have addressed all concerns raised in relation to your original submission and we are now happy to grant approval.

We wish you every success in your research.

Yours sincerely,

S Hammond

pp Mairéad Mooney

Sean Hammond

Chair of Social Research Ethics Committee

Appendix 10:

Lisa's conversation with the pupils on her error

Lisa: And when you were doing that puzzle, that's what it was, you had to use a lot of listening skills in group work; and it's actually a lot trickier than it looks, isn't it?

Children: Yeah! (chorus)

Lisa: Some people were making a lot of mistakes, that's how it works; trial and error. Hope you realise 'Oh I'm wrong' but you go again. So this is the solution to the problem. Now if you got it right, don't shout it out and don't scream the house down. Put up your hand and even if you didn't get it all right, if you got, let's say, 4 out of it, hang on.

You're encouraged already and you haven't seen the solution. (Directed towards children with hand up) So this is the solution to the problem (Teacher presents solution A on interactive white board) and let's have a quick discussion with it. On the top a lot of people made a mistake between right and left. Blue is on the right of pink. I had so many groups that put the blue there (points to the left) and the pink there (points to the right). Our left and our right. So on the top!

Pupil 1: It says blue is between white and grey.

Pupil 2: That's wrong!

Lisa: That is wrong! Blue is in between white and grey? Hang on now.

Pupil 3: The answer is wrong.

Lisa: Now red is not next to grey- that's fine. Green is not a square – that's fine. Blue is on the right of pink – that is grand. Blue is in between white and grey, so maybe that is a mistake, you're right. They should be swopped. That doesn't make any sense because here it's not in between them at all. So who's got the solution to the problem there? Kay? You're not Kay. I'm asking Kay. (Another pupil interrupts saying she has the correct answer but teacher persists with Kay).

Kay: Red is on the right- hand side.

Lisa: Will you come up to the board?

Kay: The red is up there (points to the top left). The green is there (points to the top right). (This is an incorrect solution also).

Pupil 4: Red is the white.

Lisa: And then if we had, well, the green has to be here (points to the bottom left). So we've got green, we've got red. Now we have to have blue here (centre right) and pink here (centre left). They're right. So how can we solve the problem? How do we do the rest? What do you think, Linda?

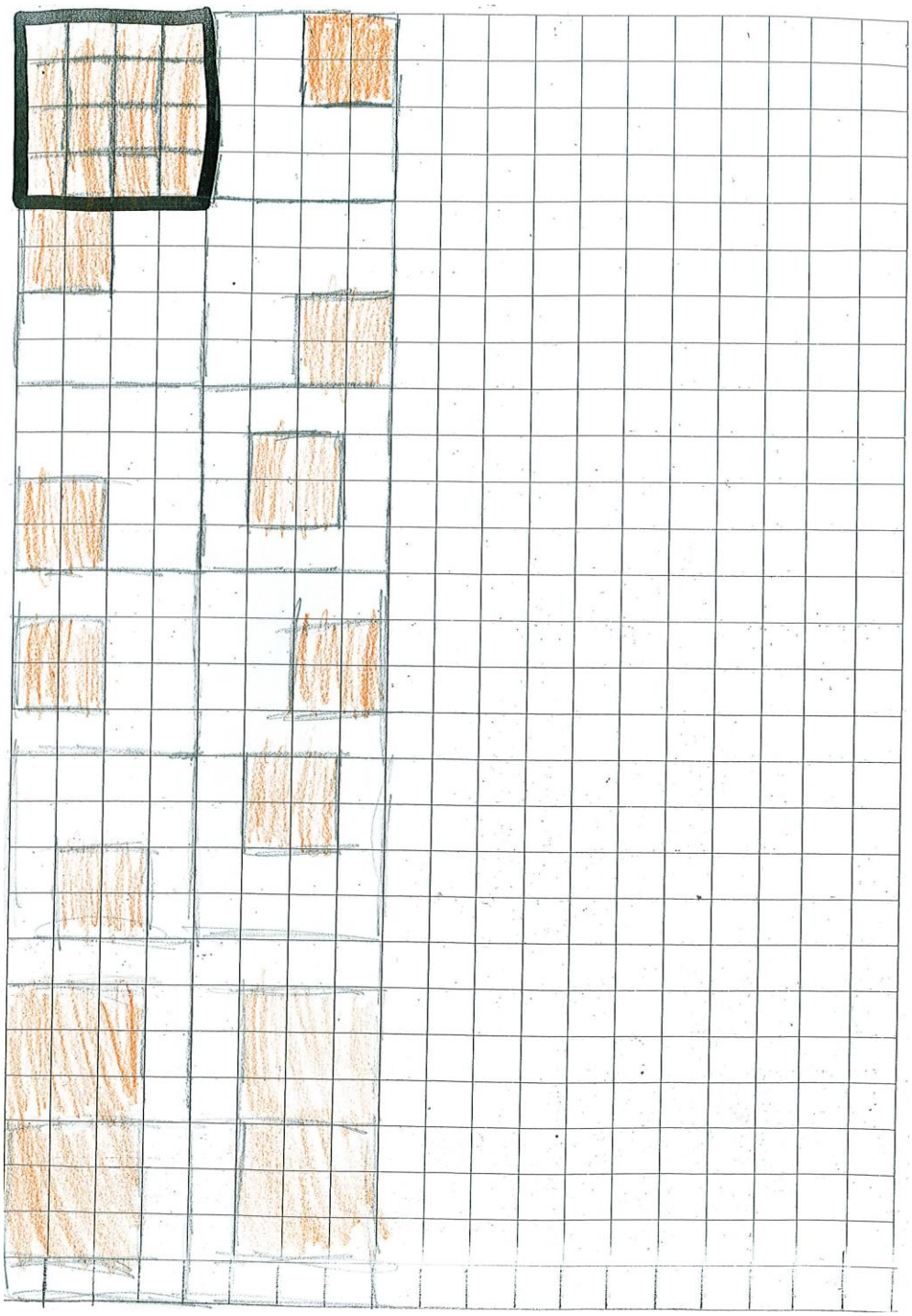
(Intercom message interrupts the lesson and then class resumes.)

Linda: The bottom right circle is grey.

Lisa: So if we had grey here, the red here, green here, then it makes sense. Okay, sorry about that. We're going to have to move on now to our next activity. So close your booklets and go back into your groups.

Appendix 11:

Nora's Groupwork from Lisa's First Lesson on 19.10.10

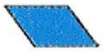
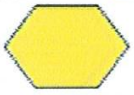
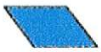
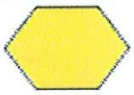
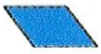


Appendix 12:

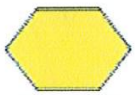
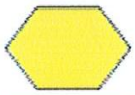
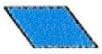
Lisa's Fraction Worksheet

Names; _____ **Date;** _____

Fun Fractions (2D shapes; area; fractions). Work with your partner to solve the following questions. Use the concrete materials available.

1. How many are in  ? _____
2. How many are in ? _____
3. How many are in ? _____
4. How many are in  ? _____-
5. How many  are in  ? _____
6. How many  are in ? _____

Based on these relations,

7. If  = 1, = _____ .
8. If = 1, = _____ .
9. If  = 1, = _____ .
10. If = 1,  = _____ .

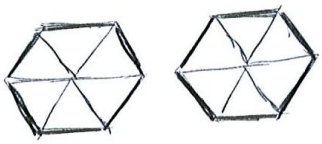
Extra Challenge! Get Your Thinking Hats on!!!

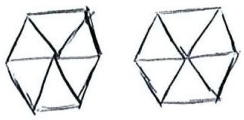
Write these shapes as fractions pentagon. Simplify your answers.

e.g. 4▲ $4/6 = 2/3$

ⓐ 3▲

ⓑ 2▲

ⓒ 7▲ 

ⓓ 9▲ 

ⓔ 3

ⓕ 5

ⓖ 10

Appendix 13:

Creating Open-Ended Tasks by Sue Cunningham

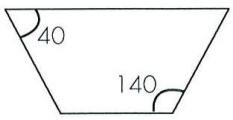
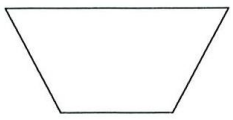
Creating Open-Ended Tasks in the Mathematics Classroom

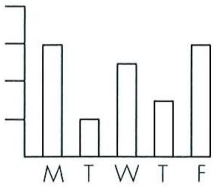
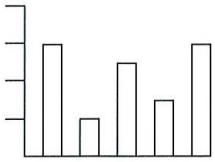
Any closed question can be reformulated to create an open-ended question using one of two methods (Sullivan & Lilburn 1997).

Method 1

Omit enough information so that, although the answer remains the same, the digits required to achieve the answer become variable. This can best be exampled by considering a standard algorithm.

Traditional	Open-Ended
249 + 173 422	2** + *7* 422

Traditional	Open-Ended
Find the missing angles on this trapezoid.  	What might the angles on this trapezoid be? 

Traditional	Open-Ended
If each mark on the Y-axis denotes 5 mm of rainfall, describe the rainfall from Monday to Friday.  	What might this be a graph of? Label the axes appropriately. Describe what the graph now tells us. 

Appendix 14:

Claire's Fourth Lesson Plan from 25.05.11

Introduction:

Problems on board – work as whole class to solve and discuss.

Lesson:

Give one problem at a time to groups, allowing time to figure it out.

After sufficient time, discuss results and record feedback.

Conclusion:

Get groups' opinion on tasks.

Appendix 15:

Claire's Challenge for the Week

Challenge for the Week

Find *seven* more ways of dividing a square in half.

Appendix 16:

Aoife's First Lesson Plan from 09.11.10

- **Date 9/11/'10**
- **Class level: 5th class**

Constructivist Maths lesson: Square puzzle

Learning objectives

(Informed by strand, strand unit, content objectives, the skills and concepts to be developed)

- The children will be able to work collaboratively to come to a solution
- The children will share their methods with the class as a whole.
- The children will be enabled to develop positive attitudes to Maths.
- The children will be enabled to document/record and verbalise their methods and solutions.
- The children will be able to internalise their learning through peer tutoring/mentoring.
- to enable the child to acquire an understanding of mathematical concepts and processes to his/her appropriate level of development and ability.
- to enable the child to acquire proficiency in fundamental mathematical skills and in recalling basic number facts.

Learning activities

(Informed by Approaches and Methodologies in Long-Term Plan)

Introduction:

Physical warm up.

Game to focus them: Concentration.

Questioning: Quick fire maths questions revising squares and square roots.

Revise what the word 'Inverse' means.

Revise lines: 180 degrees.
Horizontal
Vertical
Parallel
Perpendicular
Diagonal

Pair work:

Display the equation 5 + 5 + 5 = 550 on IWB.

Ask the class if this equation is true. Tell the class that the equation could be true by making an adjustment. Give the children a few minutes to come up with a solution in their pairs. Select groups to feedback to class with possible solutions.

Group work/Body of Lesson:

Remind groups of rules when working together.
Assign roles within groups (They choose roles from envelope on their desk).

Give out h/o – 100 square sheet.

Ask the class to look carefully at the sheet and together they must find out how many squares there are. Remind class that I want steps (and trial and improvements) recorded.

Group work will be monitored and assisted through questioning and feedback to their questions.

Activity will be timed (approx 20 mins).

Groups will feedback to the class with their answers.
Demonstrate solutions on IWB.

Conclusion:
Revise different solutions and show their relationship e.g 6 x 6 = 6 squared.
Revise square tables.

■ Linkage and Integration

SPHE: Teamwork, working collaboratively.

■ Differentiation

Mixed group abilities.
Extension work for early finishers.
Change of instruction or questioning for lower ability pupils KOR, ME.
More challenging extension work for JW and EM.

■ Assessment

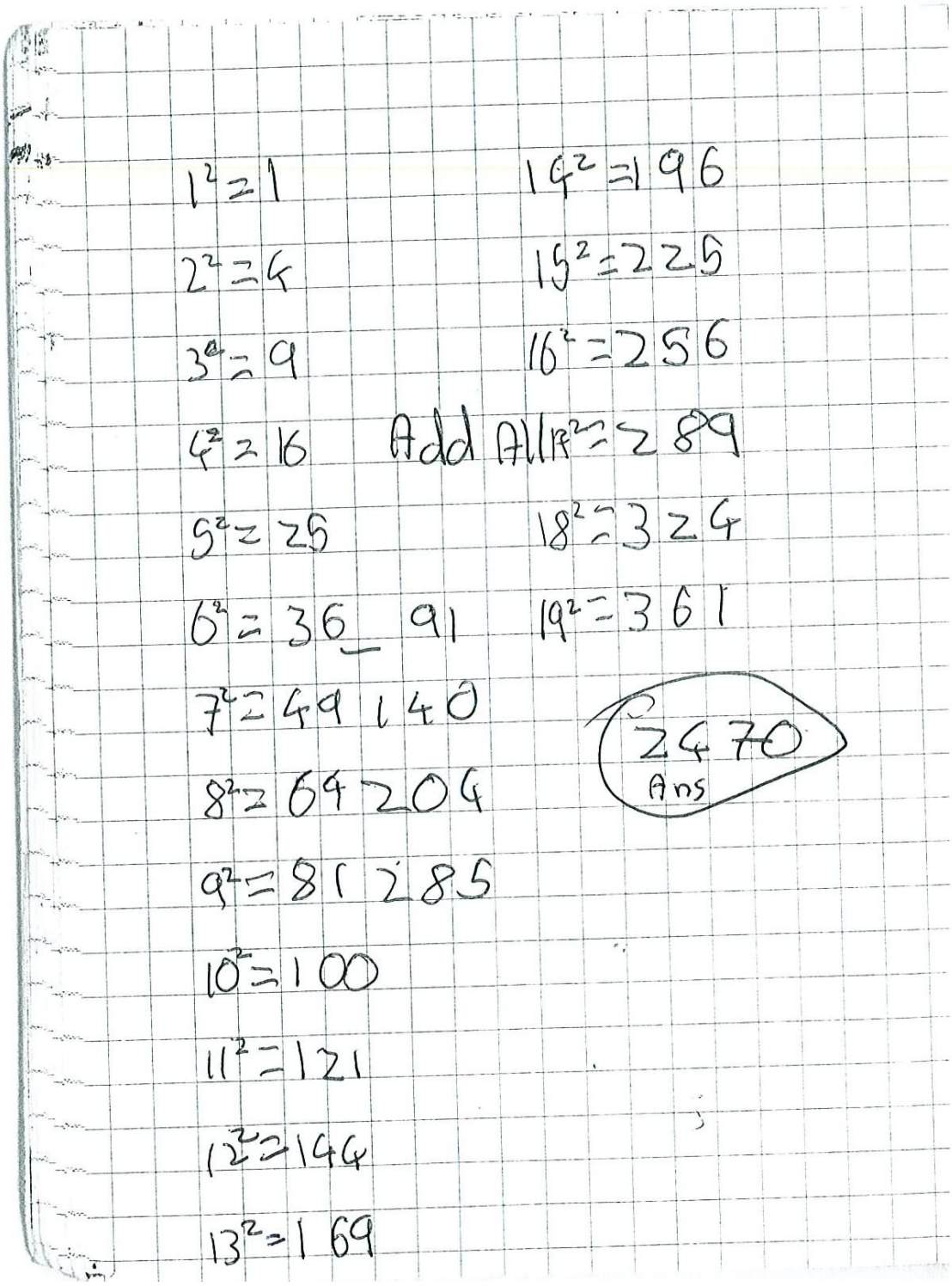
Through observing the group interactions.
Through questioning individuals
Test at the end of the week based on squares and square roots.
Reflecting myself personally on how the lesson went.

■ Resources

IWB
Hundred square sheet.
Envelopes with group roles.

Appendix 17:

Jeremy's Work on Counting the Squares



$1^2 = 1$

$2^2 = 4$

$3^2 = 9$

$4^2 = 16$

$5^2 = 25$

$6^2 = 36$ 91

$7^2 = 49$ 140

$8^2 = 64$ 204

$9^2 = 81$ 285

$10^2 = 100$

$11^2 = 121$

$12^2 = 144$

$13^2 = 169$

$14^2 = 196$

$15^2 = 225$

$16^2 = 256$

Add All $17^2 = 289$

$18^2 = 324$

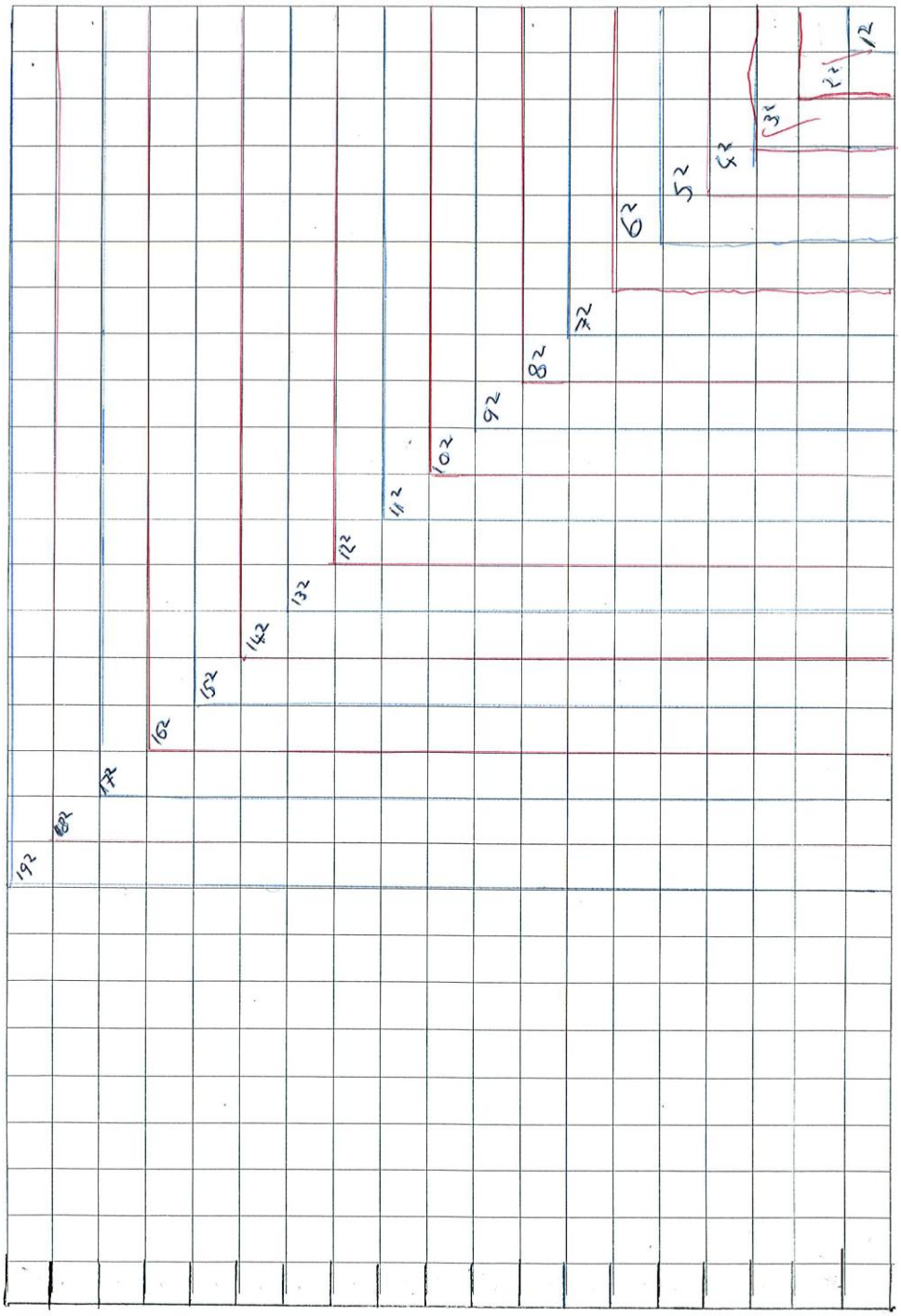
$19^2 = 361$

2470

Ans

Appendix 18:

Emma's Work on Counting the Squares



Appendix 19:

Aoife's Second Post-Lesson Interview from 09.03.11 (Abridged)

Interviewer: Okay, Aoife, how do you feel it went?

Aoife: They were all doing the group work and I suppose with the working together in the pairings and collaborative work; that always seems to go well for them. I think what... if I was to change anything the next time it would be the time management on my own part just because the feedback needs more, nearly as much time as the group work does because usually when we do Maths from 'Action Maths' there is a kind of given solution and you know what's the answer; it's either going to be the answer or not. I mean you might just show them on the board this is how you do it move directly onto the next one. But because this one needs more discussion and kids discussion at that age they all want to tell you how they did it and what they would do and what they would do better. Ahm, I kind of felt when I was rushing them, do you know, skipping them and they feel awful hard done by that age; they are big into fairness; oh you know that group got a go and we didn't and that kind of thing. So that's one thing that if I was to do that again I would allow for the feedback. I would give it nearly equal parts as the discussion; group work. And I suppose that's because I'm so used to doing the kind of run of the mill 'What's the answer?' It's right or it's wrong move on and you know or just show them on the board myself; this is how it's done. But when you have to actually ask them for their own opinions, and even sometimes they are very slow at verbalising and explaining and trying to connect; to trying to drag it out of them, that takes an awful lot of time and I suppose with the assembly today as well you know that I had to be kind of rigid, like oh no, we have to stop now and get out. Whereas I could see I could have gone on for about another twenty minutes.

Interviewer: Yeah.

Aoife: You know, if time had allowed for it. So that was one area that kind of strikes me. I'm so used to doing things one way and I know that this is how I pace myself for maths. It's like 'Was the answer good?' 'Did you get that?' and then I just help any children with difficulties and we just move to the next one and it's kind of you know, very regimented because there is only one way of getting the answer really. It's the right way or the wrong way. Whereas this, there was a load of possibilities. That was the…the one thing that stood out. The good things then; they are a great little class to work , em, together, and you know argue out their ideas, and eh, but I do find still that I have some of my quieter children who will try and take the back seat when they can. I was glad today of the reporters panned out because two of them; the boy at the top of the room Hugh and Fiona; they're very very quiet, they're very afraid, I think to explain their answers but they didn't mind today. She went up to the board and drew that out now, generally, if she was her own work and she was doing her own thing she'd be like 'Oh I don't want to go up' You know, so because she had the backing of the group she kind of had the guts I suppose to go up and to do it. I like that there is group work involved in it and that they worked together and that they're not noisy; they don't seem to be messing. They seem to be trying and they do listen to each other and then they, you know, shout each other down … (inaudible)… and it was good for the weaker ones who may not have understood how to do some to have another child explain to them as opposed to me explaining it. You know in a way means something to me but it mightn't to them. So the peer tutoring is a great concept. Ahm, I found it hard today as I was going around to the groups to not tell them what to do. I found it very hard. (Laughs)

Interviewer: Okay

Aoife: And I found myself starting to do it and then I'd have to just walk away from the table just because you know, ahm, they came up...some group came up with the suggestion about the bridge and I felt like going 'No, but if you've got twenty people'… but I couldn't, you know. So I was just trying to tell them... think about it again but I didn't want to be pushing them. I found that very difficult. To try and not to tell them what to do.

Interviewer: Very good.

Aoife: And em,

Interviewer: That is a dilemma, isn't it?

Aoife: It is. That is the hardest part of it coz like I just wanted to go 'No, you can't do that' but I suppose you could. Like it was their way of thinking it out. I really wanted to go 'Ah stop and start that again' and I couldn't and that was the hardest thing. What I found was very good by then this time and there was a few things and not just maths. They don't ask me for help as much now or they would have said' What do we do?' or just tell me what to do. Whereas they've kind of…

Interviewer: Do you think their confidence might be growing a bit?

Aoife: Yeah. I think they're taking to the whole constructivism faster than I am.

Interviewer: Right. Interesting.

Aoife: That they…that they were going 'Right, she's not going to help… obviously I'm not going to sit down and do nothing but she's not going to show us what to do. So we'll just have to try and figure it out for ourselves. Because before they were going 'What do we do with this number?' and I'm going I just want to tell them but I didn't and I still find that quite hard. But they seem to have just gone with it. No, they seemed to have adapted to it a lot faster than me. It's a challenge for me because I suppose we like to tell the people what to do. I think they love the ownership of it. They love that they came up with the ideas.

Interviewer: Right.

Aoife: And I think what they love as well is that there's not necessarily a wrong answer in these type of things. That, you know, their way is as good as the other group over there, you know, the group along side them. And they're very proud, you might have seen them there when they were holding up their sheets saying 'This is what we did!'

Interviewer: Yeah.

Aoife: And oh look… you know there's great kind of pride in it. But I think the onus is on myself now, kind of, to let it go because I feel that are they learning, you know, are they doing anything? So I feel that I would always (inaudible) it when I'm doing something.

Interviewer: Well what I've noticed there is that you've very good routines built up in the class where they do work productively and they do listen to each other as well. You know, there is good discipline and they seem to listen to one another and that, do you know? You wouldn't have that in every classroom.

Aoife: But I had to kill that class when I had them first because they came in; they had maternities, they had teachers emigrating. They had maybe about three teachers last year and the year before that as well so they were a little bit wild so I had to give them structure and because, God, like that now when I went outside the door with you and they had nothing to do they'll just…off they'll go and they'll just cause a ruction and whatnot.

Interviewer: Yeah, I see you've a lot of routines I'd call them; with the structure you know, across classroom (inaudible)

Aoife: What I do sometimes I change them around now and then just to keep them interested. What (they) call it, 'What gimmick are we doing this week?'

Interviewer: (Laugh)

Aoife: You see, excuse me? What gimmick are we doing this week?

Interviewer: Okay. One thought for you, Aoife, you feel, let's say you know, we could cut down on the amount of activities and maybe take one of the problems and do it in great detail.

Aoife: Yeah, because the feedback as I've said, seems to take an awful lot of time. And because I had three activities today I just like, because I wanted to hit the three of them I just kind of like skimmed them and I didn't give them the opportunity to discuss or to argue , you know to kind of say Oh why did ye do that?

You know I didn't give them the opportunity because I was so conscious of getting them up here. And em, they feed off my panic as well so they just going to say okay just let's go. So definitely I think, start with the one maybe, and then as I… as we get better at it maybe up to two and then maybe if they're getting better at explaining and you know, giving feedback maybe move it up to three.

Interviewer: You could always have one in reserve in case they got finished early, you know.

Aoife: Yeah, yeah. Exactly, yeah.

Interviewer: Like the average one now; there's a huge amount of teaching could go on there about averages.

Aoife: Yeah. They found averages difficult, not the adding and dividing by three but those kind of sums where you know they might be given the average and maybe some of the numbers and what's the other number.

Interviewer: Yeah.

Aoife: You know and they found that kind of a concept difficult actually. It would be nice to use something like that when I'm doing averages again with them.

……

Interviewer: So you're appealing to their different zones of development, you know.

Aoife: Okay, so, fair enough. Yeah, I do agree to put it back down to one activity and give them say fifteen minutes working on it. And then give them another five minutes to talk to the reporter about you know how they're going to explain it or what because sometimes the reporter is the child who sits there like this and at the end they're handed a sheet and they have to explain it paying no attention to what was going on because I could see that there with Sarah Kate, she's just up there and I'd say if I asked her to explain what her table were doing she wouldn't have been able to because she wasn't engaging herself. So maybe to give them two or three minutes doing the actual activity; five minutes would go to the reporter, how are we going to explain this. Maybe they might take notes and then to take the feedback and then let them argue or discuss things like that. So that would be another, I'd say, fifteen to twenty minutes on that.

Interviewer: So you can see how your full lesson would go then, all right, you know.

Aoife: Because I don't think a lot of them didn't get time to do the third one.

Interviewer: Yeah.

Aoife: You know, so that was difficult. I would definitely do one for the future. And then if they get really good you could go back up to two or something like that.

Interviewer: Yeah. There's a phrase that came to me; 'skimming'. That if we take on two much we're skimming.

Aoife: (inaudible)... impact

Interviewer: Yeah, going for depth maybe, you know.

Aoife: Yeah, no, I agree with that.

Interviewer: Anything else so, Aoife?

Aoife: Well, I think that's it.

Interviewer: Great. Thanks very much Aoife.

Appendix 20:

Aoife's Fourth Lesson Plan from 26.05.11

Short-Term Plan

- **Date 26/5/'11**
- **Class level 5th**

Constructivism: Challenge Find 7 more ways of dividing a square in half.

Learning objectives

(Informed by strand, strand unit, content objectives, the skills and concepts to be developed)

- The children will be required to problem solve without following a fixed formula.
- They will learn from one another.
- They will be encouraged think 'outside the box'.
- The class will express and explain their solutions to their peers using the correct mathematical language.
- The pupils will revise and reinforce their knowledge of 2D shapes

Learning activities

(Informed by Approaches and Methodologies in Long-Term Plan)

Introduction:

The class will revise basic 2 D shapes. They will compare and contrast a square and a rectangle. They will be encouraged to use the correct terminology.
Elicit other quadrilateral shapes. Discuss the angles these shapes have.

Recall the different triangles there are, compare and contrast them.

Identify shapes that tessellate.

Recall symmetry and rotational symmetry – ask for examples.

Challenge.

Ask the class to draw squares using their rulers and pencils. Ensure that their peers have indeed drawn a square (sides are all of equal length with right angles)

The pupils must attempt to find 7 or more ways of dividing a square in half. Show one example on the board.

Allow them to work in pairs or groups if they wish.

Take feedback on the groups/individuals work. Have them describe and demonstrate their solution on the IWB.

Extension activity: Could as many be found with a rectangle?

Linkage and Integration

Revision of 2D shapes and symmetry.

Differentiation

Peer work.

One to one assistance for ME and K O' R.

Assessment

Observation and feedback.

Resources

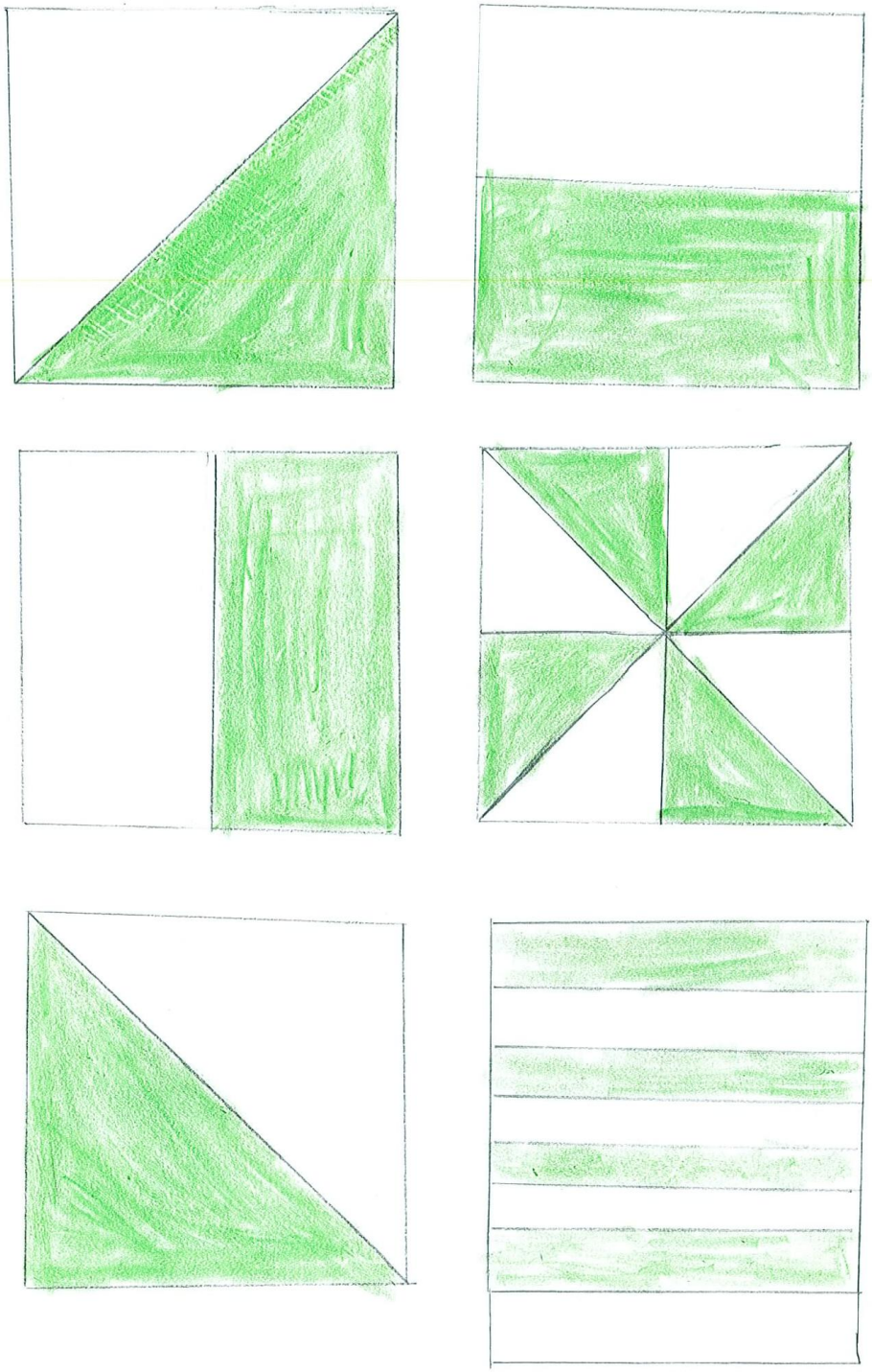
Paper, IWB, Rulers, Pencils

Reflection

After the lesson with Mr. McCarthy.

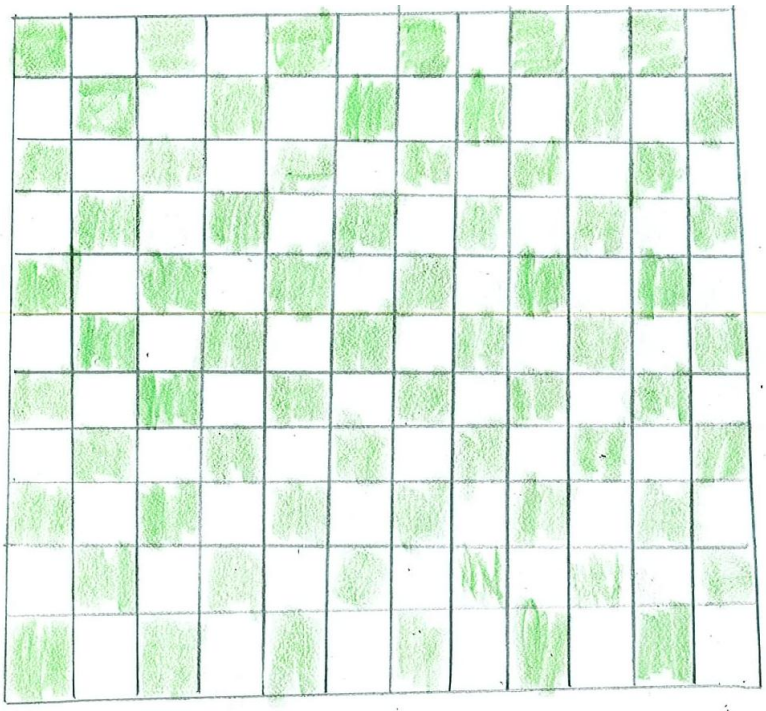
Appendix 21:

Naomi's First Work on Halving the Squares



Appendix 22:

Naomi's Second Attempt at Halving the Squares



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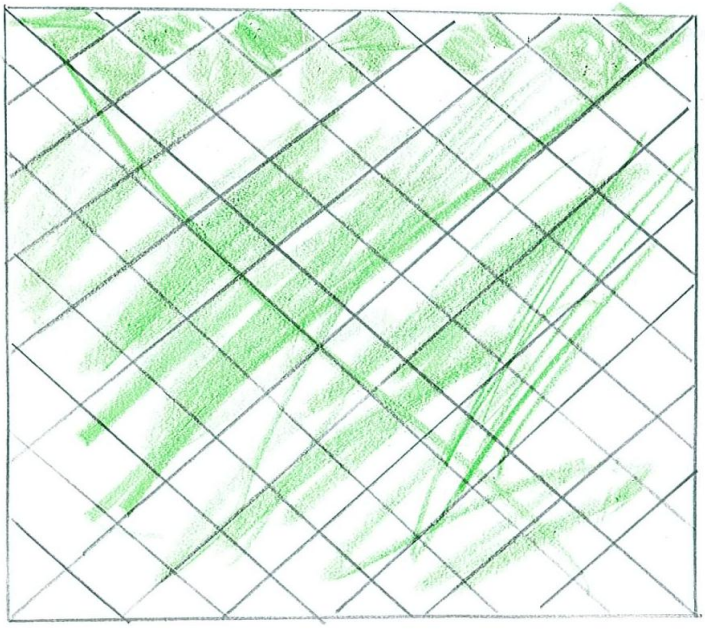

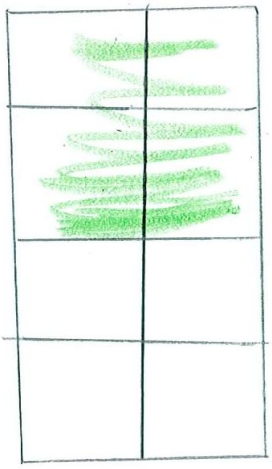

Appendix 23:

Pre-Lesson Interview Questionnaire

Interviewee:_____ Date: _____ Time: _____

1. If you have read p. 3-4 of the Mathematics Teacher Guidelines and p.5 of the Mathematics Curriculum can you comment on any aspects which struck you in particular?

_____

_____

_____

_____

_____

_____

2. Can you give examples of any implications for the teacher in adopting a constructivist approach?

_____

_____

_____

_____

_____

_____

3. In your opinion how does constructivism affect what pupils do in classrooms?

4. "Work on open-ended problems, where the emphasis is placed on using skills and discussion rather than seeking a unique solution, is recommended" (Teacher Guidelines, p.4). What do you think of this statement?

5. Can you give examples of what you think are the main impediments to teachers adopting a constructivist approach in Irish primary mathematics classrooms?

6. Words like “scaffolding” appear in the Teacher Guidelines. Can you give an example of what this means to you?

_____

_____

_____

_____

7. From where has your own knowledge of constructivism come?

_____

_____

_____

8. To what extent would you describe your current practice as constructivist? Please give examples.

_____

_____

_____

_____

9. Would you like to add any more comments on how you believe constructivism influences classroom practice?

_____

_____

Appendix 24:

Exit Interview Questionnaire

Interviewee:_____ Date:_____ Time:_____

1. Are there any aspects of the project which struck you in particular?

_____

_____

_____

_____

_____

_____

2. What do you now think are the implications for you in adopting a constructivist approach?

_____

_____

_____

_____

_____

_____

3. What do you now think are the implications for the pupils if the teacher adopts a constructivist approach?

4. What do you now think of working on open-ended problems?

5. Have the impediments to adopting a constructivist approach changed for you in any way?

6. What does the word “scaffolding” now imply for you?

_____

_____

_____

_____

7. Did you work on any open-ended problems off camera? If so, describe the experience.

_____

_____

_____

_____

_____

8. Has the project influenced your practice in any way?

_____

_____

_____

9. Any other general comments on the project?

_____

_____

Thank you for your participation!

Appendix 25:

The Group Interview Transcript from 16.06.11 (Abridged)

Interviewer: Could I start with you Claire?

Any thoughts on how the project has gone for you, let's say? And what, if anything you might have learned about a constructivist approach from it?

Claire: Yeah, mmm, yeah. I suppose once I started I quite enjoyed it, I enjoyed watching the kids figure things out but I did find, you know, the preparation; it is still hard to find content, and hard to tie in I suppose with what you're doing that week and I know it shouldn't be completely tied to the book but at the same time as I said, we are under constraints and things like that; the heavy curriculum in fifth.

Interviewer: Uh huh?

Claire: Mmm, but as I say once it started and they got into it they loved working in groups and I do think it benefited especially the weaker pupils who aren't involved day to day in the, you know, I've quite a few who don't do our maths book at all. So mmm I suppose it was nice maybe for them to be included as well. And it's good to have it for the brighter ones to keep everybody working.

Interviewer: All right. Lisa, anything on that?

Lisa: Mmm, well I agree with what Claire said.

Interviewer: Course you do. (Laughs)

Lisa: Yeah, I suppose like that it was good for their confidence and their attitude towards maths was another thing I noticed. They weren't as worked up about solving problems. Even, you know, the textbook problems, it kind of gave them a bit more confidence that way. They had a better attitude towards maths and they definitely like working in pairs or groups. It kind of helps them along. The weak ones especially enjoyed it. And the more able ones as well, they…

Interviewer: Yeah? (interjects)

Lisa: They like working in groups. It's less mmm, scarier or intimidating, I suppose for them.

Interviewer: Yeah.

Lisa: They know they'll get there in the end.

Interviewer: But you know your kids there today the word 'funner' you know, seemed to come up all right. They seem to like the approach, I suppose.

Aoife: Like the girls now I don't want to reissue, reiterate, we need to talk about English (Laughs), what they said. But what I found at the start Joe was you didn't tell us what constructivism was. Now I know that was probably part of your plan. When you were describing it to our staff the last day it was very clear. We had finally arrived at what it was ourselves but mmm, that was the hardest thing for me. I didn't know what was expected of me so mmm, I didn't have the confidence to perform, you know, if you want to call it that, to teach the lessons on the day, because I didn't know if I was going completely off on a tangent, completely off from what you wanted from me. So I found that very difficult. Mmm, I found that, you know when you were saying the weaker students benefited (looking at Lisa) I thought that the stronger ones benefited because my strong pupils tend to be the ones who think outside the box anyway, where my weaker ones need assistance; they want me to give them, you know, just tell me what to do. So, mmm, they…..it took a while but once they got into it they became a bit more comfortable but the stronger ones definitely got more out of it, I thought than the weaker ones. I differentiated greatly for them. And that's a good thing because it's very easy differentiate for the weaker pupils; the support, changing tasks and everything like that. But sometimes it's harder to find material for the stronger ones but there's scaffolding kind of inbuilt into these exercises that while my weaker ones were still working away at their level, you know, trying to find, trying to count for me 16 boxes or 17 boxes you

have the others who were managing to find the pattern of the squares, so like, it did help my differentiation there; it was ready-made differentiation which was good. Definitely the group work they enjoyed it immensely. So they loved anything where they were doing pair work or group work, and as you said, whenever they saw Joe coming, it was the maths man, puzzles is what they called it. Puzzles, they didn't see it as a maths exercise, they just saw it as a puzzle exercise. So, mmm, you know, yeah, it was good. It was just at the start I was a bit frustrated because I didn't know was I completely off the mark or not. I didn't know what it was but when you explained it to our staff the last days like we had cottoned on at that stage but I wonder if we'd known at the start would we have, you know, done better, do you know? I don't know.

Interviewer: Yeah, yeah.

Aoife: Maybe that's part of your project or skill.

Interviewer: Yeah, I suppose I didn't want to tell ye what to do because constructivism is kind of, you know, about people making their own of things as well, you know. So 'tis very hard to say this is what it is because ah, there's no set idea of what constructivism is; so it's kind of learning by doing, it really is what I was hoping, really.

Aoife: But I'd say the first few attempts were disastrous. Coz I didn't know. It was only towards the end that I started linking in the curriculum with what I was doing. Like literally, I was going 'Here's a square. Try and find as many squares as you..'(didn't finish sentence). That's all I was doing whereas the last time, the second last time maybe, I was linking in the circles before they were cutting the 'pi's. But like that didn't dawn on me at the beginning.

Interviewer: I think another thing for you Aoife, would be, you know you mentioned the handing over of authority. I think that was a huge issue for you.

Aoife: I'm a huge disciplinarian. That is why I got that class, I think. Mmm, like I'm regimental. I was called by a parent a benevolent dictator. That I run like it has to be, you know, a certain way and then to give over this control. Here's an exercise, go, run with it. I want to tell them stop doing that. Do it this way. Do it my way and I find that very difficult, to hand over that control.

Interviewer: Yeah, yeah.

Aoife: They loved it. But some of them, the weaker ones, the weaker ones still wanted that relationship with me. They still wanted me to tell them what to do and when I was kind of pulling back from them like you could see now Kay whom you interviewed as well; she became quite anxious about that. Maybe Nimrod did as well. Because they were used to and not just me but I presume all up along were doing this as well.

Claire: I suppose it's up to the stronger children to try and explain things but some of mine weren't. They had the answer and that was it. They didn't explain it to the rest of the group to help them along, at all.

Aoife: And I found my stronger kids, like John whom I had you interview as well. He knew how to do it but found it very difficult to explain to the others how to do it because he didn't get why they didn't get it.

Interviewer: Yeah, yeah.

Aoife He was like 'It's just this way.' And they're just like 'but where?' He would miss out with explanation steps and he would be frustrated then with them, you know. But I don't understand why they don't understand me.

Interviewer: (Interjects) Anita, what would have been an issue for yourself now let's say with the construction?

Anita: Well, as Aoife said at the start I did find it hard coz I just thought it was just up in the air, you know what I mean, and then I didn't know was I doing the right

thing or was I not, so that was something I totally agree with you. One thing I felt in my class, the really strong kids weren't kind of willing to share their ideas with their group, or let's say, now not all of them, but say some of them because they were used to the normal typical maths lesson they work independently.

Aoife: (Interjects) This kind of thing like, yeah. (Makes a hand and elbow movement as if covering a copybook.)

Anita: So the whole discussion thing took a few lessons to actually start in my class, if you know what I mean.

Claire: (Interjects) They are working as a group, not just one girl against their….

Anita: (Interjects) Yeah, but then my weaker kids did. They were very relaxed with those lessons because they always knew there was someone in their group who'll help them or give them guidance or a bit of support, you know.

Interviewer: That came up in the children's interviews. They seemed to like the idea that if they didn't know the right answer there was somebody there to help them in the group not necessarily the teacher, you know. So they were kind of being scaffolded, to use the buzz word, by somebody else in the group. You know, 'twas interesting.

Anita: Yeah, mmm, And another issue for me, I think, was kind of aah, time management.

Interviewer: Yeah.

Anita I found that the, when I was, kind of, let's say had the lesson down on paper I was thinking yeah, that's fine, I'll get a good forty five minutes out of that and I could have maybe divided the actual lesson into two lessons because children were coming up with answers that I didn't even think existed.

Interviewer: Yeah?

Anita: And then they wanted to show them off to the class because that was another thing, the children were very proud, you know, if they came up with….

Aoife: (Interjects) They had ownership over it!

Anita: Yeah. If they came up with aah, a solution and no one else came up with it in the class, then there's a real sense of, you know, achievement and pride but I thought that the lessons really like, you could have had them for an hour or an hour and a half which is not a normal, typical maths lesson then as well.

Interviewer: Yeah.

Anita: And then as you said, that the curriculum is set and there is, you know there's a lot.

Claire: A normal day you can't let it go for an hour and a half like.

Anita: No, you can't. And then you have to think about the other subjects as well, you know, so...

Claire: It's time management actually, yeah.

Interviewer: That came up, yeah. I wonder as Irish teachers do we like to have a set amount done in each lesson whereas in this approach you can drift over several lessons with an investigation. What would ye comment on that? Is it maybe like, the way we were trained or what? Do we like, you know...

Claire: We want it to end at a certain point each day.

Interviewer: I've done multiplication fractions now today; now that's the end of that.

Anita: But I suppose there are so many subjects to get through. We're trying to touch on everything.

Claire: And you do, kind of, stick to a chapter a week. You know you'll get finished the whole book.

Anita: And do you know what the end of the month and if you're filling in your cuntas míosúil you realise that you don't have things, alarm bells start to go off. Or even standardised testing, you realise that you haven't got circles covered and they're coming up on the exam. I think it's just instilled; it's just a teachers mind.

Aoife: But you do have to teach the formulas. You do have to teach the methods and the steps, because without them the children won't be able to think laterally or they won't be able to try out different things so you do like need to make sure you've got your chapters or your units done in a week or a fortnight. Mmm, this would be definitely be something to boost what you're doing.

Interviewer: Yeah, yeah.

Aoife: So you're just going to be voluntary?

Claire: Yeah, I suppose like May or June maybe, for the likes of those problems would mean that from September on is probably the way we always do them as well. I suppose they're only just out of fourth class as well and they've all different names so…

Aoife: (Interjects) You tend to beat through the curriculum from September 'til after Christmas with the senior classes or you won't cover it.

Anita: Yeah.

Lisa: Then there's the whole issue of the whole paper of word problems in the Sigma-T so you know you have to do a lot of the written word problems. You have to do them.

Aoife: They're a problem in our sigmas as well.

Lisa: They go on forever as well, trying to get that across.

Interviewer: Yeah.

Anita: It's another huge problem, yeah. (Inaudible after that)

Aoife: You have to teach the key words and you know…

Lisa: That takes a lot of time as well so you're kind of, it's overloading really. You can only cover so much stuff in the time.

Interviewer: Yeah, I think the linkage came up as well with you, Lisa; you came up with that, kind of yourself, which is one of my own tentative findings. I think this approach might work better as a half-way house if we can try, and you were doing it Aoife, with the circles as well, if we can try and link it in to something you're doing. And to try to pick the problems then based on something that is in the curriculum, you know what I mean. I think there might be less guilt, because as teachers we're often guilt-ridden; we have the Sigma-T to do, we've the books to do, whatever. I don't know. Any comments on that, try and link it in more with what we are doing already? Now, it's not easy, let's say, to find problems all the time, but in the amount of times you do it then, would it be an idea to try and link it in with what you're doing already?

Aoife: It would be more beneficial. (**Claire:** Yeah).It would consolidate the learning that you did teach, directly teach them. And then they could do this as the scaffolding, you know, kind of especially in differentiation. But like the last time with the circle, as you say the guilt, I felt much better about doing because I felt at least some of them would be doing a bit of revision. If they got nothing else out of this they'd revise the different aspects and the components of the circle and we've gone over like and we've gone over diameter and all that and then this will be to kind of, especially for my stronger ones, to push them a little bit further. So I, as you said, there's a guilt thing; I didn't feel as guilty then. I kind of felt, oh well, that's my circle revision done for this year.

Interviewer: Yeah, yeah.

Aoife: So I felt that.

Interviewer: How did that go for you as well, Lisa? You mentioned about linkage.

Lisa: (Interrupts) I was saying that like that the first two lessons, the squares and everything, it was lovely but, you know, they would survive without doing it.(laughs)

Interviewer: Yeah, yeah.

Lisa: And then the last, the last lesson on the fractions, if you remember, finding the shapes and the fractions; I felt that was worthwhile because I was revising shape, fractions and they were doing higher order thinking skills and group work and all the rest. And the last lesson with the, mmm, can I remember it, what was that?

Anita: Table.

Lisa: Table! They were doing algebra in that so I thought it was worthwhile as well as solving the puzzle; and the dice game of chance. We were covering chance (Interviewer: Yeah) so I felt better about doing things that are on the curriculum anyway, and it's kind of, I know we shouldn't be guilty about it when it comes to skills but when it comes to the end of the month and you don't have anything to write down.

Claire: At the end of the day I agree about pupils using skills, so yeah!

Interviewer: And hopefully you're developing the problem solving skills.

Lisa: You're doing a few things at the one time, so definitely that's the way I would work it.

Interviewer: Okay, yeah, yeah.

Lisa: And it doesn't have to be something you're doing that week. When I did the fractions and shape I had done that months before that but it was a good way to come back to it rather than just doing it for the sake of it as well, you know, if it's something you've already taught.

Interviewer: Okay.

Lisa: Like going back over it.

Aoife: Yeah! (Nods)

Lisa: It's worthwhile in that way too.

Interviewer: So if I was to say to ye, now, we've done this project, okay. You've been observed now for four lessons and interviewed. You've had your reflections and I must go through your reflections now in more detail myself, okay, and analyse those. The linkage, I think, is one thing that might help. So if you were to say in an Irish context, with the constraints we're under, you know, okay, linkage might be one thing. Is there anything else that would help, let's say, to help teachers adopt a constructivist approach to their work; an open-ended, if you like, investigative approach, coz I suppose now you've realised that's kind of what's meant by constructivism; this kind of more open-ended, different methods, problem solving, that kind of thing, you know? Any thoughts on that Anita?

Anita: Well I suppose if the material was there in front of you. Let's say, like, you're doing your chapter on chance. If there was like, a few possible activities that could be done.

Interviewer: Yeah.

Anita: Which would reinforce what they have already learned and then allow them, you know, take part in open-ended problems as well because…

Interviewer: (Interjects) So sourcing the content?

Aoife: Yeah! (All the others nod in agreement)

Anita: If I was, I personally, if it was there in front of me, and I had a few foolscap sheets of ideas I would do it no problem but if I had to go scanning on the internet and going to the library and go looking at maths books or something like that it would kind of turn me off the idea.

Interviewer: Yeah, yeah.

Claire: Yeah, definitely, yeah whereas if there was this book where, let's say, chapter that there was a few, just pick one, you know, or try one out, it may or not work.

Interviewer: So yeah, if it was integrated with the textbook or… (pause), yeah, okay; that's a good idea actually, yeah, yeah. Any thoughts on that Aoife?

Aoife: No, actually, I think you kind of hit it there (turning to Anita). That was myself and Claire's, like, you had to keep giving us books because you know we exhausted the internet, and you're trying to find problems that are age appropriate for them and you know then I had to figure out was I able to do it myself.

Lisa: Yeah (laughs).

Aoife: And I was trying to find the solutions and I was going Claire can you find another way of doing this and it was like homework then at that stage. It was... We don't have history books or we don't have science books so we're already trying to source materials and resources for that so it was just like an added thing that, you know the blue book with the light bulb that you gave us?

Interviewer: Oh yeah?

Anita: Yeah!

Aoife: If we had one of those up on our shelves that you can go, right, at the end of this unit I'm going to use this activity like if it was just… (pauses)

Claire: Yeah, that's it.

Aoife: Readymade, trialled and everything and ready to go. Mmm that would be ideal.

Lisa: Mmm (nods)

Interviewer: Ahuh, ahuh.

Lisa: If they were broken up into the strands as well.

Aoife: Yes, exactly, yeah.

Lisa: Anything that would make things easier but you're right.

Interviewer: Say that again, Lisa. That's an important point.

Lisa: If there were each strand doing these activities then you can just know you've covered everything, and just as you say (looking at Aoife), going through the internet and the solutions have to be there as well.

……

Lisa: I suppose if there was in-service where people came into the classroom modelling and stuff.

Interviewer: It'd work, yeah, yeah.

Lisa: Coz, that would like, at the start, it was very hard to know what was or what was not accepted.

Interviewer: I deliberately stayed away from the modelling for ye because I didn't want to (**Claire:** next week now Joe) influence it. Yeah, I didn't want to influence it too much.

Lisa: Come in and show us how it's done.

Interviewer: Yeah, oh yeah, it is difficult. Ye did a fantastic job. Ye know the classes as well which helps and ye know which kids to… (Doesn't finish sentence) I suppose that's another thing, maybe, to kind of wrap up as well; the groupings. Would you say mixed ability, Anita, or did you, you know?

Anita: Yeah, at the start, I had said, yeah, mixed ability but I think it depends on the actual activity as well. I think it's nice mixed ability because you have the weaker ones who can rely on the strong ones from it but then sometimes it's good to have the more well able kids grouped together because they can actually, maybe, expand more on an activity. They could be quite limited with their people. There's less able people in the group and they don't make a huge contribution.

Interviewer: Mmm, Mmm.

Anita: You know, I think it's good to vary it as well because I know, was it the first lesson; I divided them into groups of four.

Interviewer: Ahuh.

Anita: And then I decided to divide them into pairs, I think for the second or third one it was actually, I think, they did more work when they were in pairs (Interviewer: Okay) because it was only let's say me and you but if (**Claire:** They can't switch off, yeah) it was a group of three and four, two of them could be, you know, (nods head), I don't know, kind of, having a good gawk around the classroom…

Claire: Yeah, yeah.

Anita: While the other two do all the work.

Lisa: Definitely, yeah.

Interviewer: So the pair work might help them to keep on task?

Anita: Yeah, but it depends on the class as well, do you know, if you have a lively class you might be better off putting them in pairs and then you could be pairing them off according to, you know (pauses), their attention span as well.

Claire: Yes, yeah.

Anita: Generally you're not going to be putting two kids who find it very hard to stay on task.

Interviewer: Yeah, yeah.

Anita: So I think it's kind of, the teacher uses their own, maybe, initiative in deciding.

Interviewer: Yeah, okay. What do you think of that, Aoife?

Aoife: I had them mixed ability, I had them same, same ability pairings, mmm, and I gave them out their little, you know, you're the recorder, you're the reporter. And?

Interviewer: What roles had you again? Recorder? Reporter?

Aoife: Recorder, reporter and then you had things like encourager. That person wasn't going to be doing very much (smiles) and a timekeeper. And ahm, the chairperson then would try to tell everyone to cop on and to pay attention so they had all the different jobs but once I put them into the mixed abilities, even if let's say my strong child now James was made encourager, like you know 'well done' he literally stood up at one stage and he started like me dictating to them and then they just went 'yeah' whatever he says' and they wrote it down. So then the next time we came in we did a pair. It was the time where we were doing, don't know was it the circle time, I can't remember, I put them in pairs and ahm, they were a bit quieter, they weren't so noisy. You're still talking and they were allowed to talk to other people, but they were in pairs and the strong ones just drove on. You know, they didn't have to waste time explaining things. They were like 'oh my gosh James, that's a good idea' and they were going back and forth whereas my weaker ones just floundered in their pairings. They were like fish literally flapping and I had to spend a lot of time at their tables and ahm they spent a lot of time colouring in their lines and, you know, beautifying it and you know, as opposed to actually solving it.

Interviewer: Were the pairings then similar ability?

Aoife: They were similar ability, the pairings.

Interviewer: Ah, yeah, yeah.

Aoife: So mmm, that's why I think it works very well for the stronger child (pauses), for me, but not so well for the weaker child.

Interviewer: Okay, okay.

Aoife: Because they rely either on myself or on the strong child too much.

Interviewer: Okay (nods).

Aoife: And it gives them a safety net but it doesn't necessarily mean that they are actually taking part actively in it.

Interviewer: Okay, okay (nods).

Aoife: They might be as passive as they are when I'm dictating to the whole class, you know, so… (Shrugs shoulders)

Interviewer: Lisa, any comment on that. Pairings or the groupings we'll say?

Lisa: Yeah, I think I used pairings the whole time except first; no, I used pairings the whole time but… (Pauses).

Interviewer: Mixed ability or?

Lisa: It was mostly mixed ability but I wouldn't have put anyone who was, you know, good at maths with someone who was really, really, really, really weak. I'd pair really good – average; do you know, average – weak?

Interviewer: Okay, okay.

Lisa: My classroom was really lively so I wouldn't have got away with it.

Interviewer: (Interjects) So one higher ability and one lower ability? (Gestures up and down with hand)

Lisa: Not two extremes, either, do you know what I mean? I think if you have a very strong child and someone in the first percentile or whatever, it's not going to work.

Interviewer: No, no (agreeing).

Lisa: So it's kind of, and personality came into it a lot.

Aoife: Yeah.

Lisa: And attention span and dominant personalities versus laid back, you know what I mean.

Interviewer: Ahuh, ahuh.

Lisa: 'Twas kind of, you'd want to know your class very well really to know who'd work well together.

Aoife: And my bunch were very competitive. Like they, like that, you know, this whole idea of covering it up.

Anita: Covering it up (chorusing)

Interviewer: Yeah, yeah.

Anita: Coz they're so used to, because I know, when we'd be doing maths lessons when I'd want them to be doing something I'd say 'look into your own copy now'.

Claire: Do your own; that's it, yeah.

Aoife: Whereas if it's spellings you don't go 'C'mere that's how you spell that' (smiles). It's like a test, so yeah.

Interviewer: Yeah.

Claire: Yeah, the opposite of what they've been … (pauses)

Aoife: The opposite of what they've been entitled to do.

Interviewer: Claire, what do you think of the grouping yourself?

Claire: I suppose the same. I suppose a variety really. I think I did mostly groups. Ahm, like it was all mixed ability coz mine are so used to that it was hard to go same, I think (laughs) same and same; we wouldn't have gone anywhere with that but ahm, but yeah, I suppose a lot comes into it. But I suppose to vary the pairs and we tried different things as well and it does work well.

Aoife: The problem is though at that age they are very aware of same ability groups.

Claire: They are, yeah.

Claire: But that's the same as any area I suppose.

Aoife: Yeah, yeah, not just maths.

Interviewer: I think one thing in it for me, as well is, don't be afraid if you have to stop and direct teach something as part of the problem. Let's say they don't understand how to convert a mixed number or a top heavy fraction to a mixed number.

Aoife: You see we didn't know we could do that. We didn't know was it under the constructivism umbrella.

Interviewer: Yeah, yeah. It's just my thing really that helps children to learn is kind of under the umbrella of ah, the theory is vague. That's the problem really, that's why I'm trying to, trying to…

Aoife: Pinpoint it.

Interviewer: Flesh it out a bit, yeah, yeah. Anything else now, you've been very good, it's half past three. Is there anything anyone would want to add before we wrap up?

Claire: I don't think so.

Anita: A lot has been discussed there.

Interviewer: No, that's great. Okay, thanks very much girls. That's fantastic. I can take those off you so if you're finished.

(Teachers begin to hand up their interview notes)

Interviewer: Yeah. Thanks Lisa, thanks Aoife, thanks Anita. Al right girls, mass has ended, go in peace. (All laugh). And thanks very much for everything.

Claire: Thanks, Joe. Bye, bye.

Lisa: No problem.

Interviewer: If you discover any other bits of paper or whatever or, you know, you can give them to me.

(Teachers put pupil chairs on which they were sitting back on top of tables).

Appendix 26:

Long Term Impact Questionnaire

Name: _____ Class: _____ Date: _____

1. To what extent, if any, did the project influence the way you introduce mathematical challenge to pupils?

_____

_____

_____

_____

_____

_____

2. To what extent, if any, did you become more sensitive to pupils' mathematical needs?

_____

_____

_____

_____

_____

_____

3. To what extent, if any, did you change the way you manage your classroom?

_____

_____

_____

_____

4. To what extent, if any, did the project impact on your practice?

5. What advice would you give to another teacher attempting to adopt a constructivist approach to their work?

6. Please write any other comment you may have on the project.

THANK YOU!

Appendix 27:

Long Term Impact Interview Transcript

(Each participant teacher's contribution has been colour-coded for salient extracts.)

Interviewer: Okay Girls. We'll start away. So the first question there is to what extent if any did the project influence the way you introduce mathematical challenge to pupils? Was there anything on that Anita?

Anita: Em… I suppose I am more aware of putting them into pairs or groups when you're doing problem solving in the class. Em…and getting them to kind of work together .. to work through it… together.

Interviewer: Right, right.

Anita: You know, em… and… for them to explain different methods.

Interviewer: Okay.

Anita: I think I'm more open to that. That there isn't just one religious method .. that like another child might come up with another alternative which is also correct. But I suppose I'm allowing … allowing that more inside in my classroom.

Interviewer: Okay.

Anita: You know as opposed to tell them 'oh, just stick to this method'.

Interviewer: Alright. Aoife, Anything on that?

Aoife: Like Anita I would agree with that you know it would be my higher achievers now would find the other way or they'd have another way and I'd let them come up and demonstrate that to the class and again like the paired work you know… they're no, like, no longer kind of doing the work this; that they're not copying from one another but that they're kind of sharing the ideas but at the same time I'd give the work and say this is how you do it before…before I would have had done the… the … I was going to say the course.

Interviewer: The project?

Aoife: The project or the programme or whatever the theme. I would have put up the work and say 'This is how you do it' I would put up the work and say 'how would you think you might do this sum?'

Interviewer: Eh mmm.

Aoife: But em..besides that I suppose I still kind have stayed in my comfort zone a little bit. At the same time because…

Interviewer: What would be your comfort zone?

Aoife: My comfort zone would be the 'Modh direach' I suppose the old fashioned way… and I think I answered that further down that my high achievers are able for this, kind of, you know, 'how do you think you'll do it?' and work together where there would be children who would be weaker and they need to be told the formula, the methodology and they feel safe with it. And because you're teaching and you know you're testing and you've SIGMA's and you've work …a load of course work to get through sometimes you kind of nearly have to abandon the constructivism because it can be very time consuming at times and you know you have to go 'this is how you do it' but then as you said that you're more open-minded to the paired work and you're more open-minded to…you know that there are going to be more than one way 'to skin a cat' and that you know to allow them to express that.

Interviewer: Okay, Okay. You're nodding there Claire.

Claire: Yeah. I'm just agreeing I suppose. When I have time to do it I do try and branch off but it's where (laughs) I'm very honest.

Interviewer: I know. When you branch off.. when you branch off..

Claire: Well like that if a small child was coming up with something then I would be much more aware of saying 'that's ok too' and..

Interviewer: Ok.

Claire: How did they get to that conclusion. It does happen regularly. I suppose they figured out some other way.

Interviewer: Ok. Ok.

Claire: But as for the groups and the pairs and the problem solving like we had done I haven't really… I suppose started.

Interviewer: Yeah. 'Twould be more whole class teaching and that?

Claire: Yeah. And again time; confirmation this year and things as well.

Interviewer: Ok. Confirmation. Was there anything peculiar about this year; just kind of as a constraint?

Claire: Well I know this year follows on from last year there was more of a rush on things.

Interviewer: Yeah.

Claire: With, we found it hard to do stuff last year so for a second year in a row so…

Interviewer: How about yourself Lisa?

Lisa: Ahm… I agree all of what she was saying maybe em…when I say I have done a little more it's more word problems really than puzzles and things that we would have done last year. Definitely a lot more working in pairs and …more of a different way of doing word problems.

Interviewer: Ok.

Lisa: More than em… I can't think of the word for ..

Interviewer: Open-ended attitude?

Lisa: Open-ended attitude problems; nothing that amazing now but at the same time.. you know you were saying the weaker children …. I don't think… I don't know…

Interviewer: They find it difficult?

Lisa: Maybe, maybe it just confuses them; maybe the connection, the, or you know it can be a bit confusing: this is how you do it or how I do that.

Interviewer: They want the structure ?

Lisa: But you need to like… teach the whole class…

Interviewer: Ok. Ok. Great. And in the second question then it just says then did you become any more sensitive to a pupils' mathematical needs? What would you say to that Anita?

Anita: I find that the weaker children really like working in groups and in pairs because it takes the kind of limelight off them and the responsibility and that they kind of have let's say a 'buddy' if you want to call it. Someone to help them along.

Interviewer: A maths buddy?

Anita: Yeah, d'you know and em.. I think they kind of get a sense of achievement as well when they get it right and they can explain it to somebody less able. D'you know the high achievers when they are able to figure something out and teaching it to the, to their child, but as I say, similar to Lisa it's only … I'm only doing it with word problems do you know, I'm not coming up with these open ended puzzles. Realistically coz there's such a demand on the curriculum and…

(Other teachers agree in background.)

Interviewer: Would you feel the word problems have to be covered first?

Anita: Yeah. Because at the end of the day at the end of the year the children have to have thirty something word problems and they need to…

Interviewer: They find them hard. (interjects)

Anita: They find them very hard. They need to get into…

Interviewer: You mean the Sigma T's and that kind of thing?

Anita: Yeah. Standardised tests you know they kind of have to get into the routine of underlining, highlighting, figuring out whether … what kind of computation it is.

Aoife: I definitely have become more aware. I suppose I don't underestimate them as much you know that they probably… a lot of the time they do know how to solve the problem and I don't need to feed it to them. That you know… that they work together and that they are capable of coming up with the solution or a method all by themselves but as you said you know we're dictated by things like the Sigmas and we literally have to train them. You know using RUDE as you said, read, underline, draw, estimate, em, because they don't have the luxury or the time when they're in a tested environment to be able to explore options and trial and error.. time isn't on their side for things like that.

Interviewer: Yeah, yeah, okay. Claire?

Claire: Ahm.. I think the same I suppose; just that it's different brains work differently I suppose and that's huge in my class so the higher achievers as well.

Interviewer: Okay. Alright and Lisa?

Lisa: I find it hard to both the …It's just what the girls were saying really like you know they all operate in lots of different ways and just try to support each other in the best way you can, getting them to support each other, working in pairs sometimes you'll have some with it, chance to figure it out.

Interviewer: Okay.

Lisa: Though we have… it's the time. It would be so much easier to take small groups because d'you know there's always people who have it solved fast; frustrated then waiting together, hard to get cooperation.

Interviewer: It's hard to get it right… to get the mix right.

Lisa: Yeah.

Interviewer: And in classroom management… ah Anita moving on to the next question in the way you organise your classroom … any changes there?

Anita: Ahm I suppose the key there is having space inside in your classroom because if you're putting them into groups the tables need to be laid out in a certain way, you know, so that's something you have to be kind of vigilant of maybe at the start of the year when you're changing… you're rotating around, em.. and then I suppose is how are you going to pair them up then as well? Are you just doing a random selection or are you going to put a well able child with a less able child?

Interviewer: Yeah. What did you find? Did you have to vary that or what did you find worked for you?

Anita: It varied but to be honest the day goes so quickly it's something that I kind of do randomly d'you know 'oh pair off there with the child that sits next to you'.

Other teachers: Mmm, mmm, yeah.

Anita: You know, I wouldn't really have had time to… to pick a high with a lower achiever.

Interviewer: Lower or whatever.

Anita: You know and it was fine. Now in saying that I haven't done it an awful lot either. It's kind of been incidental during the maths lesson. Do you know if we came up with a problem and they found it hard then I… I'd ask the children ' do you know right turn to your neighbour next to you and have a little chat about it and come up with a solution if you can'.

Interviewer: That's good though. That's … you know, that's constructivist all right, yeah. Aoife?

Aoife: Pretty much as Anita was saying again. Em because I have such a small space in the room, it's the same room as I was in. They're in groups anyway but they're in mixed ability groups not done mathematically or by the Sigma scores or anything that. They're in mixed ability groups so that you'd have your recorder, your writer and things like that but em I found when I did try it… to have them in the same or

similar ability groups… that it didn't work because the high achievers kind of flew on ahead and we were throwing out ideas and where the weaker ones were just looking for me to assist them the whole time. And em.. they just didn't have the confidence, or the know-how, or the lateral thinking whatever you want to call it to be able to do the problems …so like you said it's just kind of a general right turn to your partner and it's whoever your partner seems to be at that day/ week and I found with the group work … high achievers do well together; the low achievers… the weaker mathematical children don't. They need to be mixed but then it's the stronger pupils that lead the rest of them and they just go along with whatever they say.

Interviewer: Okay. How would you have found Claire?

Claire: Yeah I'd say the same I suppose. Say from my top three to my bottom three there's no comparison anyway so if they're… if they're paired with each other the lower end haven't a hope really of even reading the problem …say and the others would.

Interviewer: A different problem (suggesting)… if you had a different…

Claire: Yeah. Again comes back to differentiation and then say time and the materials and that to keep going, I suppose really.

Interviewer: Yeah. Yeah. Okay. Lisa?

Lisa: Right. Start of maths lesson I put, eh, different problems in different groups organised; but that rarely happens, once in a blue moon. Every so often target lesson with problems, that's all.

Interviewer: And would that be because of other subjects; obviously the maths as well?

Lisa: School is busy to do the other subjects is for all of them and trying to find, you know, within every class, you have in my class, anyway, a girl totally different

curriculum between her and the rest of them, trying to keep her occupied and what to do and tell her that she's very, very weak. So..

……

Interviewer: Okay. So I suppose the summary questions then are coming up. Ahm… Anita, to what extent if any , did the project impact on your practice we'll say as of now if you know what I mean?

Anita: Yeah. Well I suppose as I said before I am more open and I'm more aware of it.

Interviewer: Right.

Anita: You know. I wouldn't be implementing it an awful lot into my daily routine either. Ahm, I definitely do use pairwork and teamwork more …

Interviewer: Okay.

Anita: … in the classroom. Ahm… but to be honest it's not… it's not something that stands out in me that I think has huge importance inside in my classroom.

Interviewer: Yeah. Yeah. That's grand. Aoife?

Aoife: Again to agree with Anita, to add to it I suppose, I'd allow more for trial and error whereas before if the answer's wrong I'd go like 'Come on, I've shown you how to do this before' whereas now I see well they'd learn 'that's not it' so they're eliminating it by process of elimination to get to it. It's just a slower process but I suppose I'm just more… more lenient with the trial and error and more aware of it and you know you try to implement the groups and the pairs but as you said you know, you know you still have your own style you tend to gravitate towards that what works for you within the dynamic of the class. Constructivism isn't always the thing I would try… to be quite honest . Yeah. That's it really.

Claire: Again I have two kids in my classroom, third class level, you know, and then you've ones you're trying to push on to try and get them ready for next year and ahm… I suppose I definitely am more aware when I am at the board if they want to I and they're adamant they got the same answers somehow. You know I do listen to them now I suppose whereas I before might probably cut them off a bit…

Interviewer: Alright. Mmmm. Mmmm.

Claire: Do you know I would have realistically, like Anita was saying, I would have said no. Only this way you know definitely any of even the ones in the book, often even the straightforward ones they might see it a different way. And to be honest it seems to be the same kids all the time find it in a different way.

Anita: Yeah.

Interviewer: Maybe the brighter children?

Claire: Not always. No. No.

Interviewer: The more mathematical children?

Claire: Yeah. But it is the same few who ask as to how we got it one way and they'll put up their hands or listen to their way, you know. about it.

Interviewer: Alright. Lisa?

Lisa: Ahm, Like you're just saying, like I must say that I'm … constructivist because (laughs)…

I suppose I would have never used teamwork before in maths. I would have seen it as em an individual kind of go through everything as a whole class and they just go off and do it themselves and you'd check it.,

Interviewer: Grand.

Lisa:… and I'd help them on or whatever or they might, they'd be working on their own. So definitely use a lot more time for discussion in maths and… exploring different options and… discussing which is a better approach and you know…

definitely doing a lot of as well but I suppose I could have a lot better if I had time you know.

Interviewer: The Japanese go into that a lot, discussion of the different ways to solve a problem.

……..

Interviewer: Okay. And do you feel then that you might be confusing the weaker ones when you're doing this?

Lisa: Yeah. Exactly. That the other ones are going 'oh yeah' and then they're lost .It's the mathematically-minded children definitely… that it… Yeah. They can see what you're on about; know where you're going. The rest of them are going, like waiting at work, like you know.

Interviewer: Alright. Lisa we might start with you this time. And again it's just a summary you know. What advice, if you had to give a sentence or two or whatever, to someone who was setting out on a similar project or were attempting to adopt a constructivist approach, what advice would you give them?

Lisa: Em… well… I think you need to integrate it into your existing class work, you know whatever you're doing one month to try and integrate it into that. The lessons that we did last year while they were great fun and they really enjoyed it. There's no way really of … time to be planning lessons like that. Hours later you know.

Interviewer: Yeah.

Lisa: And they really enjoyed them and everything but there's not the time … or you know to make them or the time to do them. So your best bet is to try and do it within whatever you're trying to teach at the moment. But having open-ended problems to do with the work you are doing, to do that just a lot of the time. Just to be a bit more…

Interviewer: Yeah. You got into the linkage in one area of maths within another alright.

Lisa: Yeah. And tie it all together.

Interviewer: Okay. Is there room in the market so for a publication; kind of outline a lot of open-ended problems?

Lisa: Yeah. Definitely.

Interviewer: I might make my next million alright!

Lisa: It would help if it was linked to the curriculum as well.

Interviewer: If it was linked to the curriculum?

Lisa: There's no point … I know you've your maths for fun and that.Yeah it's just while you're… but like I think they just saw them as puzzles and games without any relation to the actual curriculum, you know what I mean?

Interviewer: Okay. Okay. Right. Em.. Claire?

Claire: Ahm… I think day to day, like I suppose if we get the time to maybe dedicate to group work or pair work, or maybe it might be later in the year or maybe we might decide one day we'll do something and…

Interviewer: Why do you say later in the year?

Claire: I suppose they've more material covered you're not under so much pressure; the book you know that … em…

Interviewer: That's a real constraint isn't it? The book?

Claire: It is. Yeah, 'tis awful. Em… and like it was great in that they didn't even realise that they were doing maths really I suppose for those lessons which is good and it is good for the children to do…I suppose who enjoy that kind of learning.

Interviewer: When I was outside the office in your school the other day and a guy says to me, he says to me eh… 'you're the maths for fun guy' which I thought was interesting you know (laughs). …the 'maths for fun guy'…okay.

Aoife: In a way because it's such a deviance from the norm that as you said like they don't really see it… they don't really connect it to the maths at all.

Interviewer: Yeah. They seemed to…attitudinally from the pupils viewpoint they seem to enjoy it.

Teachers: Oh yeah!

Interviewer: Which was interesting you know. That even if it didn't we'll say improve their maths ability it seemed to make them more open to more maths.

Interviewer: Okay. Sorry Aoife.

Aoife: Ahm I suppose the advice I'd have for someone would be em… to allow for the release of control. I found that very difficult. I swear I was a tyrant inside in my room; that you know it was more about facilitating than actual teaching…

Interviewer: Yeah.

Aoife:…and I found that hard sometimes. Especially the weaker groups where I just wanted to go push them and do this way, go that way and you couldn't. I suppose from… I suppose I'm a bit old school that way. You know I'd teach the maths and learn your formulas and Pie R squared. Just go ahead and do that and then you're completely handing it over to the children and you see them making the mistakes and you see them discussing things and you see them going down the wrong track…the whole trial and error… and you just want to grab them and pull them back, but you have to kind of release that control and I found that hard as I said to you several times myself.

Interviewer: You used to organise your groups very well; different roles again what were they?

Aoife: Ahm recorder, reporter, ahm timekeeper and eh like the chairperson, the chairperson made sure everyone had a fair go.

Interviewer: Okay.

Aoife: And if there was an extra person I'd just…an encourager or something like that rather than…

(Teachers laugh.)

Aoife: 'You're doing great, you're doing great, keep it up'. Yeah but I suppose I got that from the mentoring just because a lot of teachers coming out of Mary I now are big into group work and are big into pair work and the inspectors are looking for it the whole time.

Interviewer: Mmm.

Aoife: So it does lend to constructivism…

Interviewer: Yeah. Anita?

Anita: Something that crossed my mind there now you know you're saying that the kids didn't associate… they didn't think they were actually doing maths. I think it's because it's as well there was no kind of sense of failure in it either. D'you know, in the way like in a normal maths classroom the sum is either right or wrong. And with the constructivism it was like 'oh well there's no real set answer here now or there's no real right or wrong answer'. So I think then the kids were more relaxed because…

Aoife: And I had a competitive bunch last year, the crowd who are in sequence now and work it they were like 'oh miss what's the answer?' and I'm like 'but there's no'. 'But what is the answer?' they wanted to know were they right.

Anita: So then I think the weaker kids felt more relaxed doing that because they weren't being highlighted as being the group that got it wrong.

Aoife: That's true, yeah.

Anita: But em for advice for somebody else I'd definitely say yeah it's very time consuming so make sure that if you are doing it you are aware that it could take a long time in the class. It's not really a five minute job. You know it could take up a whole lesson. Do you know with the discussion. Yeah and em…

Interviewer: And one of the things that came across really in the research to me was how busy classrooms are; the constraints we're under really you know. I have some interesting texts ye sent me saying 'we've this on today or that on today or whatever'. That's very rich because it shows like how busy we are and I think that's worth for me to write down how busy we are as teachers. But ahm… even for the public outside. Okay. And just to finish up look…any other comments that you might have maybe spotted in there or anything that came up today now from our discussion. Ahm… that you want to conclude on?

Aoife: I just said that we do constructivism to a degree but I don't… without the forward planning if you spent more time and more thought into it but we do it to a degree but…

Interviewer: Yeah.

Aoife: …but it's not something that's thought out. It's not planned like I suppose.

Interviewer: Yeah. Yeah. And I think the constructivism allows for direct teaching as well. I think when I started out I was kind of thinking it didn't but I think it does coz you have to teach pupils directly to move on and then maybe try a bit of problem solving and whatever you know. But there are times when you have to tell them.

You need a balance.

Interviewer: Yeah you need a balance. You have to tell. And like constructivism is all about giving them a chance to construct their own knowledge. So if you have to tell them certain things to do that, then that's fine too you know.

Aoife: You need some prior knowledge.

Interviewer: Yeah. They need some prior knowledge. Okay. Any other thing there Anita on your last comment there? Is there…?

Anita: I suppose it's been repeated lots but the maths curriculum especially in the higher classes, it's so vast…

Interviewer: Okay.

Anita:…So like I suppose from a teacher's perspective our priority is to get the course content covered.

Interviewer: Claire, anything on that? Any last comments there?

Claire: No, I suppose they need to, they need knowledge. They also need to learn how to work.

Interviewer: Yeah.

Claire: As well, you know, so if they're not used to it. And again if the room isn't laid out ideally, all their stuff and their boxes and everything They do need to learn that skill as well, which is important too.

Interviewer: True. True. And final comment from you Lisa?

Lisa: Yeah, well, you can implement it to a degree but sometimes I think you have to get really old school, as you were saying, trying to teach. I'd say three weeks this year teaching the maths that you are teaching and, you know, I suppose, really trying in a way how to make that constructivist you know. That's only one way to try…Like surprising so many of them could not do it at all!

Interviewer: Yeah. Yeah.

Lisa: So I mean it depends what you're at. Some areas will lend themselves to it more than others. The maths group are just… this is it. They need to know, too like: the fractions, the decimals, the percentages. I remember when you came in first we thought how are we going to be able to, as it was so unrelated to what we were doing to do this at all. Shouldn't be but it was. There's so much done in fifth related to what we were doing. How do I add or how do I multiply?

Interviewer: That's great girls. Listen, thanks for taking the time to come up.

Appendix 28:

Children's Views on the Project

Lisa's Children's Interview 16/06/11 (Lauren, Kelly1, Kelly2)

Interviewer: Okay girls… I'll start with you there Lauren. What did you think of the maths project?

Lauren: Em… I thought that it was easier than the usual maths, eh, because Miss Higgins explained it more precise.

Interviewer: Right. Okay. And what did you think, Kelly?

Kelly1: I thought it was like different because like teamwork and… like all the different problems and like (inaudible)…

Interviewer: Right, right. And yourself, Kelly?

Kelly2: I thought it was like, it was really like fun and like she made it like… really… like easier.

Interviewer: Okay. How…why would you say that Kelly 'she made it easier'? How do you think she made it easier?

Kelly2: She like put us into pairs and em… groups and like… she writ like stuff on the sheet.

Interviewer: Okay. Okay. What would you think of that Kelly? Did she make it easier or harder for you or…?

Kelly1: Em I thought it was easier.

Interviewer: Okay. Why would you say that?

Kelly1: Because like if… in the usual maths it's like all… em like … different like where, the problems and stuff.

Interviewer: Okay.

Kelly1: But this was different.

Interviewer: And what was different about it?

Kelly1: Like we've never done these things before…

Interviewer: Okay.

Kelly1: … and like teamwork.

Interviewer: Okay. So the teamwork was different, was it?

Kelly1: Yeah.

Interviewer: Okay. What did you find about it, Lauren?

Lauren: I thought some of the questions were harder but it made it kind of easier working in pairs and stuff.

Interviewer: Okay. Okay. That's right …yeah, what..what… questions… did ye feel they were hard or… you know easy… or you know? What…? You found some of them hard, Lauren. Yeah? That's a very honest answer.

Lauren: The squares.

Interviewer: The squares?

Lauren: Yeah.

Kelly2: Yeah.

Interviewer: Okay. Counting how many squares? Was that…that was the first activity, was it?

Kelly1: Yes. Some of them were like hard enough but then some of them were okay.

Interviewer: Okay. And you enjoyed working in groups, did you?

Girls: Yeah. Yeah.

Interviewer: Okay. What about you Kelly? How did you find the maths problems, we'll say?

Kelly 1: The questions were hard and like… and like it was easy like when we get into groups and did … do you know the one with the... the sums… you had to make the sums…

Interviewer: Right.

Kelly2: …that was kind of easy.

Interviewer: Okay. Okay. Where there was a question mark and you had to put in the missing numbers, is it?

Kelly2: Yeah.

Interviewer: Okay. Okay. And do ye think the teacher behaved differently now during this than she normally would? I know Lauren you've an opinion on that anyway.

Lauren: Yeah, I think she did because usually we…well sometimes anyway, she'd just say 'okay, do this' because she expects us to know some things and then she was explaining way like… (Inaudible)…

Interviewer: Okay. And in 'normal' maths we'll call it, would she explain it as much as that?

Lauren: She wouldn't explain like… she wouldn't go over everything. She'd just go over some bits of it and then you know get down to it.

Interviewer: So are you saying she did more explaining with this, is it?

Lauren: Yeah.

Interviewer: Okay. Okay. What did you think, Kelly?

Kelly1: I thought she was a bit different… em, the same kind of…

Interviewer: Yeah?

Kelly1: … Because like… she like gives us the, like sheets and we just work… to get like … we work… we like put our heads down and work.

Interviewer: Okay. Okay. So with this then you were working in pairs rather than just working on your own, were you?

Kelly1: Yeah.

Interviewer: Okay. But normally would you work on your own, let's say in maths?

Kelly1: Yeah.

Kelly2: Sometimes

Kelly1: … Mostly, yeah.

Interviewer: Mostly you'd work on your own? Is it?

Kelly1: Yeah.

Interviewer: What about you, Kelly?

Kelly2: I thought it was different like the way she act and like…

Interviewer: What was different so, do you think?

Kelly2: Like it was because the camera was in front of her.

Interviewer: Right. She was trying to be an actor, maybe or something?

(All girls laugh.)

Kelly2: And em, like she made us… like when in normal maths she… she just says like… she doesn't explain it like…she does explain it but not that…as the way she did in the… the... maths…the thing.

Interviewer: Okay. Okay. All right. So you thought there was more explanation went on.

Kelly2: Yeah.

Interviewer: Okay. Is there anything else ye'd like to say about it? …. Anything at all? …. Just your own experiences of it now?

Lauren: I thought it was good the way that she did the warm up. Coz it can give you an idea of what you did before and that would pop into your head if it comes up in some of the questions.

Kelly2: Yeah.

Kelly1: I liked the dice games.

Other girls: Yeah.

Interviewer: You like the warm up is it?

……

Kelly2: Yeah. Coz it got you into your… like usually we just do … get straight into maths … but like…

Interviewer: Yeah. What did you like about the warm up Kelly so?

Kelly2: Em… like that… like sometimes all maths… like it kind of makes it funner if… like when you em… when you have the warm up.

Interviewer: Okay. Okay. And would it get you interested in the maths that's coming up then, is it?

Kelly2: Yeah.

Interviewer: Would it get your brain switched on?

Kelly2: Yeah (laughs)

Kelly1: Like coz when we see you, we're like 'Yes!' but usually like with normal maths we're like 'oh no maths!'

Interviewer: Mmm. Mmm. And why would you be saying 'yes!' when you see me?

Kelly1: Because like they're really fun like…

Interviewer: Okay.

Kelly2: …and I especially love the games and…

Interviewer: Okay.

Kelly1: It doesn't feel you're doing it for that long; like today Miss Higgins said em… 'We did loads of maths there yesterday so I don't think we should do it today' but it felt like it was only ten minutes.

Other girls (agreeing): Yeah.

Interviewer: Whereas it was about… 'Twas about fifty minutes, was it?

Girls: Yeah.

Kelly2: Time flies when you're having fun.

Interviewer: Girls that's great. Thanks very much for that. That's fantastic.

Interview with Anita's children 16/06/11 (Isabella, Sharon, Bria)

Interviewer: Okay, Isabella. I might start with you. What did you think of working with this kind of maths?

Isabella: I thought it was fun coz like we got to work together. And it was fun to find more than one answer.

Interviewer: Okay. Sharon, what…?

Sharon: I thought it was like helpful because like if you didn't understand something you'd have something… you'd have another person to help you. It's very helpful to have… being in a group and not by yourself…

Interviewer: Okay.

Sharon: …and it's good that you could figure out more than one answer.

Interviewer: All right. Very good. Okay. Bria?

Bria: I liked it because it was different to all the other maths we did. Like… in pairs like… it was better because we're usually working alone by yourself and then like you get to talk about the way other people can figure out answers… like how different they think about maths than we do.

Interviewer: And do you feel you learned different ways from the other girls in your group?

Bria: Yeah.

Interviewer: Were there times when you would have said 'ah yeah, yeah, I wouldn't have done it that way'?

Bria: Yeah.

Interviewer: Yeah? Okay. Okay. Right. What did you think Isabella of the different ways of doing things?

Isabella: Em… I thought it was good because then like… one time I didn't know like what to… and then my partner knew what to do and I was like 'I wouldn't have done it like that'.

Interviewer: Yeah.

Isabella: And then that was like a better way to do it.

Interviewer: All right. So you were able to learn and help one another, is it?

Isabella: Yeah.

Interviewer: Okay. Sharon, any comment on that?

Sharon: When I had like an answer I would discuss with my partner and… so most of the times we have different answers and we'd like compare 'em and see do we still get the same answers and we did so … (inaudible)…same way.

Interviewer: Okay. Thanks Sharon. Bria?

Bria: Yeah, I really kind of … the same as what they thought. Like it was helpful to know what they thought about that sum… how they would do it. Would it be 'taking' or 'division'? Would it have decimal point or not and stuff like… if I didn't have a decimal maybe my partner would have it. Maybe then we can write up one of the answers and then see what… what difference is it between that answer with the decimal point than without the decimal point.

Interviewer: Okay. Okay. Were there any of the activities in particular ye remember or that ye liked doing?

Isabella: I liked the one where she'd give us like two numbers and then at the bottom another two numbers and then she'd give us the answer and then we'd have to find out what the other numbers were.

Interviewer: The missing numbers? Okay.

Isabella: Yeah.

Sharon: Coz like you could go in the groups like you can…. one person could do the top and the other person could try at the bottom. You could find out all different ways… (inaudible) like...

Interviewer: Right. Okay.

Bria: You don't have to just get that answer; once we try doing answers that was just maybe a bit bigger or a bit smaller than that answer. Like once we got four nine seven …another time we got … four nine five… and they were all different. We wouldn't get just the right answer.

Interviewer: Okay. You liked the idea of getting different answers.

Bria: Yeah, we were allowed experiment on it.

Interviewer: Yeah. Yeah. Which is an unusual word for maths, isn't it, that… when you're allowed experiment? Yeah. Yeah. Okay. And the last question then is did ye feel that the teacher behaved any way differently during this to the normal way she does during maths, we'll say? What did you think, Isabella?

Isabella: Yeah. She kind of did because like she was asking us to work together and usually she asks us to work independently.

Interviewer: Okay. All right. Anything on that, Sharon?

Sharon: Yeah, I think it was different as well because we'd usually like work on our own and this time we actually went in groups and discussed it. But like when we were by ourselves we didn't discuss it. We'd do it on our own. And it's just better like… go with someone for a change and she was kind of different about that.

Interviewer: Right. Okay.

Bria: We're usually doing it all in our copies… mostly from the book, 'The Maths Magic'. And this time we got to do it on the whiteboards and we got to… all different type of sums….different to like division or fractions. They were like

maybe… problem solving and stuff. So it was better. And she act kind of different too… (inaudible).

Interviewer: All right. And do you feel the teacher, did she have to do more or less explaining with this kind of maths?

Isabella: I think she had to do less coz she'd just em… give us numbers and then we had to figure it out.

Interviewer: Okay. What do you think, Sharon?

Sharon: I think she had to give us less because we were in groups and it would be easier for us to… try work it out differently and then compare our answers and then she'd like tell us a small bit of information and we'd go and try and figure out the rest of it ourselves.

Interviewer: Okay. Okay. Bria?

Bria: Usually when you're doing (inaudible) you can't think straight away. And it's easier sometimes like when you're with someone. You can kind of get information from them as well. And then you can have your information and their information.

Interviewer: Did it take a bit of pressure off you so working in the groups?

Bria: Yeah. It was easier.

Interviewer: You're not put on the spot to give a particular answer?

Bria: Yeah.

Interviewer: Okay. Right. That's great girls. Anything else about it that… ye'd like to say… anything at all… good or bad?

(Girls shake their heads.)

Interviewer: No? You're okay? Anything else? Girls ye've been great. That's fantastic. Thanks a million.

Aoife's Children's Interview (Katie, James and Gavin)

Interviewer: Okay. Katie, could I start with you? What did you think of the maths project and working…?

Katie: It was em, easy like… in most of bits and like… it was fun.

Interviewer: Okay. What did you find fun about it, we'll say?

Katie: Em….Because it was like different activities that we wouldn't normally do.

Interviewer: Okay, okay. How about you, James? How did you find it?

James: It was great fun because it was like a challenge rather than like having to do sums constantly. And it was like trying to get at it from a different angle rather than just doing it the way we're supposed to, just kind of a different approach.

Interviewer: Okay. So did you feel there were different ways of doing the sums, is it? Or try out different approaches?

James: Yeah. Like rather than….You were free to try whatever way you wanted rather like you know in the subjects you would have to do it one way, what the teacher says.

Interviewer: Okay. Okay. No, I'll come back to you on that, James. Gavin, what did you think of it?

Gavin: I thought it was really… it was very fun coz em, you would never get anything wrong like, you're always right because you (inaudible).. few ideas.

Interviewer: Okay.

Gavin: And do you know one idea wasn't right and you'd get the rest of them wrong. They were all right.

Interviewer: They were all right. Okay. So you could make more of a contribution. Okay. And was there anything ye found difficult now about it? Anything…. Any of the activities ye found hard?

Katie: Mmm… I didn't find anything hard.

Interviewer: You didn't find anything hard, Katie. How about you, James?

James: No, It was really easy but like that… It wasn't… it wasn't hard but it was kind of like you couldn't do it straight away. It wasn't that kind of easy. It was the kind of easy where you had to take your time but you'd still get it done. Usually.

Interviewer: Okay. And on that, James, did you find it would help to write down as you were going along what you were doing?

James: Yeah, coz it takes time that way and em, coz you could just forget what you had written five minutes ago if you continuing the … (inaudible).

Interviewer: Yeah, yeah, yeah. And that was something I learned as well that it might be best to write down as you're going along what you're doing so you can keep track of it in your mind. Okay. What did you think, Gavin?

Gavin: Em… nothing was hard for me but it was a tiny bit awkward… (inaudible).

Interviewer: With the camera? You were just conscious that the camera was there, is it?

Gavin: Because I was just so close to it.

Interviewer: Okay. Okay. Well that's what it's like being a movie star now, you see Gavin. You'll have to get used to that if you want to be the next Colin Farrell, you know what I mean. Okay. And did ye feel that the teacher behaved in a different way during… during these lessons? You're smiling, James.

James: Yeah. (laughs). I think she did because like… normally like… as soon as em… the camera's turned on she's started like… like acting better like… trying to be the best teacher she could…

Interviewer: (Laughs). In what way James?

James: She was like do you know the way she'd say, em… like the 'one, two, three' thing and you'd have to reply and stuff.

Interviewer: Yes.

James: She'd never do that normally. She just does it….

Gavin: Showing off.

James: Yeah. Showing off.

Interviewer: Okay. Very good. What did you think, Katie of the teacher behaving differently?

Katie: Yeah. She definitely behaved a bit differently because like she wouldn't normally go like 'class, class, class' and like she was doing different activities that we like normally wouldn't do.

Interviewer: Okay. Okay. So you feel she was doing her different tricks, is it? Okay. Okay. What about you, Gavin? Did you notice any difference in the teacher's behaviour?

……

Gavin: Well it was… it was different with all the activities you'd usually have to do… adding, subtracting, multiplying and dividing. But with that we had to find like … we had to find like a diameter of a triangle…I mean of em... of a… circle.

Interviewer: Of a circle, yeah.

Gavin: And we had to find like you had to divide a quadrant into six sections to get as many…You had six lines to make in any sections as you could.

Interviewer: Yeah. Yeah. You liked that activity did you?

Gavin: Yeah.

Interviewer: Building it up? Okay. Okay. So is there anything else now let's say that ye'd like to comment on yourselves about the project?

James: It was kind of fun because like, do you know like the way like normal maths, you'd have to like constantly do the same. It kind of …(inaudible).. but em… it would challenge you in a different way coz like …like if you're doing sums you'd look… you'd look at one sort of way and then figure it out. And then you'd have to

do the next sum the same way. But because we were doing the lines, we had to… we had six lines and then the next one was seven down so you'd be looking it from a different angle again… (inaudible)... Coz you had an extra line.

Interviewer: Okay. Okay. Katie?

Katie: You had to kind of use your mind a lot. You had to think about it before you'd kind of do it.

Interviewer: Okay. So any other comments now? Ah… we spoke about what ye thought of it. We spoke about the teacher's….ah… behaviour… okay, during it. Ah… ahm… so…. yeah, I suppose one other question for ye, did ye do any of these lessons then when I wasn't there? Or did ye just go back to what ye used do normally?

Katie: We kind of went back to normal like em… multiplying, dividing.

Interviewer: Okay.

James: So it's the same with maths, like.

Interviewer: Okay. What about you, Gavin?

……

Gavin: Well, ahm...for the first one or two days she kept doing the different like sums and different approaches. We went back to the normal adding and subtracting. And now and again we just do the 'different thinking', like eh… the ways you would think to do different sums.

Interviewer: Okay. So you would do some of it now and again, is it?

Gavin: Yeah….. (inaudible)…

Interviewer: All right. We'll leave it at that. You've been fantastic. Thanks a million. All right.

Interview with Claire's children (David, Isaac and George) 13/06/11

Interviewer: Okay. If I start there so, we'll say David. What do you think of this approach to maths that we were… we were using?

David: Em, It was kind of… it was kind of easy and hard at the same time.

Interviewer: Right.

David: It was medium and it's great…it's really fun.

Interviewer: Okay. Okay. And then what did you think, Isaac?

……

Isaac: So if you didn't know it then somebody else would figure it out. If they didn't know then you would know it coz you were always in groups.

Interviewer: You were always in groups. And did you like working in groups, Isaac?

Isaac: Yeah.

Interviewer: Okay. Would you prefer that to working on your own?

Isaac: Yeah.

Interviewer: Okay. Okay. What did you think, George?

George: I thought it was like, do you know, fine in the middle but some were… some questions were hard and some questions were easy.

Interviewer: Okay.

George: And it was really fun like working in groups because with normal maths you're doing it by yourself.

Interviewer: Mmm.

George: But like in a group if you get something wrong some of the others might have gotten it right…

Interviewer: Okay

George:… and then you can correct yourself.

Interviewer: All right. So did it take the pressure off you a little bit to perform on your own?

George: Yeah.

Interviewer: Okay. Okay. Very good. And what let's say you know… was it more fun doing maths this way or what did you think?

David: Yeah.

Interviewer: David?

David: Yeah.

Interviewer: Why would you say that?

David: I like working in groups and you're having a bit of fun then, you know at the same time, as well like learning and getting… getting your mistakes.

Interviewer: And at the same time you can talk to your pals and that. Okay. What about you, Isaac?

Isaac: Yeah. I like … (inaudible) and because we don't usually do… (inaudible) and problems. We just do sums (inaudible)...and nothing else.

Interviewer: Right. With just one answer.

Isaac: Yeah.

Interviewer: Did you like the idea that there could be more than one answer or more than one way to do it?

Isaac: Yeah.

Interviewer: Yeah. Why did you like that, Isaac?

Isaac: Coz if you got it wrong then you'd be disappointed but then there could be other answers then if … (inaudible)

Interviewer: Okay. Okay. What about you, George?

George: I thought … (inaudible) it was definitely a lot of fun coz em you could… like you could talk to people.

Interviewer: Mmm.

George: And em, it wasn't just like writing in your copy book. You could be like discussing it.

……..

Interviewer: Okay. Did you find it handy George; let's say to record as you go along… what you were doing?

George: Yeah.

Interviewer: Do you think that was helpful?

George: Yeah.

Interviewer: Okay. What about you there, Isaac on the recording as you go along?

Isaac: Yeah.

……..

Interviewer: Yeah, yeah. That's one thing I've been learning. That it's very hard to keep it all in your head.

Isaac: Yeah. Yeah.

Interviewer: So it's best to maybe make a little note as you're going along. Okay. I'm going to ask you lads, did the teacher behave differently during this to the way she normally behaves in the… in the classroom for this kind of maths? What do you think?

David: Yeah, kind of. That … (inaudible) she was explaining more stuff. That like some bits are hard and stuff and like she would… she does explain stuff in the other maths and she doesn't like… she leaves us do most of it ourselves but in this maths she did… she helped us a lot in the sums.

Interviewer: Right. So did it force her to go round…?

David: Yeah, kind of.

Interviewer: …and be helping ye more, is it?

David: Yeah.

Interviewer: Okay. Interesting. What would you think, Isaac?

Isaac: Yeah. Because she usually …she… em for our normal maths she wouldn't explain the questions but then for this she would, so then we'd know what to do.

Interviewer: Okay. And do you feel she had to? You know, was that the important thing? That like these things were so different the teacher had to explain it maybe a bit more?

Isaac: Yeah.

……

Interviewer: Okay. That's interesting. How about you, George?

George: Yeah, a bit. She wasn't like completely like different but, yeah, she was explaining more of them than ahm she would… like our more normal maths.

Interviewer: Okay. Okay. And why do you think that was?

George: I thought some parts of it were a bit more difficult.

……

Interviewer: Yeah. Yeah. 'Twasn't just straight add, subtract, multiply and divide. You might be bringing in different things into it like the lines now, you know. The lines cutting each other.

George: The 'cuts' and …

Interviewer: Yeah. Yeah. Do you remember any of those activities that you enjoyed actually, any of the ones you have done?

George: The cuts was fun coz there was drawing and the game with the… tables, I think it was.

Interviewer: Okay.

George: Like you would call out like if you… if you had the… if you had an answer like somebody said like 'two plus five plus eight' and you say 'I've fifteen' and you say 'three multiplied by eight' … (inaudible)… ' I've twenty four'.

…….

Interviewer: It's 'loop games' I think we call them. 'Loop Games'. Okay. You enjoyed those. Isaac, anything… of the activities you remembered that you enjoyed?

Isaac: Yeah. I enjoyed the em… magic squares and the… em the table tennis.

Interviewer: Okay. Okay. And what about you, David?

David: I em... it was like… you know the first day you came in like the triangles and the symmetry and stuff.

…….

Interviewer: That's the one way you liked the best. Okay. Any other comments lads … about the project?

David: Mmm, no.

Interviewer: Okay. You all… you feel you enjoyed it, David anyway?

David: Yeah.

Interviewer: How about you, Isaac?

Isaac: Yeah.

Interviewer: George?

George: Yeah.

Interviewer: Okay. Okay. Well, hopefully in secondary school now this kind of maths is becoming more… more popular as well. They're calling it 'project maths'. So it's going to come into secondary schools as well. Lads, thanks for your help. You've been fantastic. That's great altogether.