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Multi-Objective Mixed Bus Fleet Charging Schedule Problem with Time-of-Use for Real-world Data-sets

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Abstract

As the effects of climate change are increasingly felt worldwide, the transition to Electric Buses (EB) presents an opportunity to decarbonize the transportation sector. Several issues exist that hinder the adoption of a green bus fleet. Such as the increased cost, reduced travel distances, and required recharge times, which may negatively impact service quality. This work proposes a Mixed Integer Programming model to solve a multi-objective mixed fleet charging schedule problem. The mixed fleet considers EBs and Internal Combustion Engine Buses (ICEBs) and minimizes daily costs, such as fuel price, the Social Cost of Carbon (SCC) produced by the bus fleet, and the Value of Time (VoT) of public transport users. Non-linear charging is considered as well as alternative approaches along with Time of Use (TOU) constraints for electricity price and SCC. Empirical evaluation shows that significant savings can be made, with reductions of over €20000 in fuel costs, and reductions of over 100 tCO_{2eq} per day. Consideration of VoT minimizes negative customer impact, limiting late arrivals to an average of 8.78 seconds per EB. The inclusion of non-linear charging makes minimal positive impact compared to limiting the total capacity of the battery, and while the inclusion of TOU constraints correlates to more savings, the amount saved is minute.

Keywords

Multi-objective, Mixed fleet, Time of use, Scheduling, Electric Bus

1. Introduction

The transportation sector is a major contributor to carbon emissions. Many countries work to reduce their emissions, such as the Irish government which has set a carbon budget for the transportation sector of 54 $MtCO_{2eq}$ for the 2021–2025 period. As such, the domain of electric vehicles (EVs) has been gaining more attention. In particular, the use of Electric Buses (EB) presents several benefits compared to Internal Combustion Engine Buses (ICEB), including lower levels of air pollution and reduced CO_2 emissions. Nevertheless, bus operators encounter several obstacles in electrifying transportation networks, such as creating schedules that maximise the benefit of EBs while minimising negative service impact due to recharging operations. As such, a mixed fleet approach is often adopted, especially considering the increased initial costs associated with EBs and the infrastructure required to operate them [1, 2].

This work presents a framework for a multi-objective charging scheduling problem for a mixed fleet of EBs and ICEBs. Our goal is to minimize the total fuel cost and the carbon emissions of the bus fleet. We also consider the impact on passenger service by using public transport users' Value of Time (VoT) to minimize negative deviations from the schedule.

Electricity prices and the carbon intensity of the national grid fluctuate throughout the day. As such, we consider the Time-of-Use (TOU) of the electricity consumed by EBs to minimize the total cost of the electricity used to power a fleet. The cost of the fleet's carbon emissions is modelled using the Social Cost of Carbon (SCC). Non-linear charging functions for batteries are considered as well as alternatives to ensure feasible schedules.

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2. Related Work

The Vehicle Scheduling Problem (VSP) assigns trips to a fleet of vehicles to create optimal routes to meet demand while satisfying operational constraints [3]. Electric VSP (EVSP) is a variation of the traditional VSP problem that includes additional resource constraints in the form of budgeting constraints to encapsulate the limited distance EBs can travel, this problem has been proven to be NP-hard [4]. Numerous variations exist of the problem, such as the Green Vehicle Routing Problem and the Fixed Route Vehicle Charging problem for EVs [5].

Numerous approaches have been proposed to tackle the transition to EBs problem with various assumptions. This includes assumptions that on-route charging occurs while passengers are embarking or disembarking while satisfying certain operational constraints [6, 7, 8]. In particular, the authors focus on scheduling the charging events with acceptable deviation times (from original timetables) and non-overlapping constraints to enforce a unary nature on charging stations. Additional works rely on dwell times to recharge the fleet [9, 10, 11]. Others use MIP to tackle the transition to EBs with a mixed fleet of electric and conventional buses [9, 12]. This related work has been limited to modelling approaches with detailed cost functions to tackle small-size instances using commercial solvers.

Previous works incorporate some charging technologies in the design of bus routes to reduce operational costs with several proposed approaches to minimize the number of required EBs needed to replace a conventional fleet [13]. Evaluation of charging technologies for EBs has shown that placement of charging stations at terminal stops proves to be the most cost-effective method [14].

The non-linearity of battery charging is an important aspect to consider when generating schedules for EBs. Batteries are charged using a constant current (CC)-constant voltage(CV) scheme. In the CC phase of battery charging is linear with time, however as the State of Charge (SOC) nears 100 the terminal voltage of the battery is reached. When this occurs the CV phase begins and the relationship between charge time and energy gained is no longer linear. Not modelling this feature of EBs may lead to infeasible schedules. Conversely, allowing EBs to charge to a high SOC results in a high variance in SOC during the operational day, which can negatively impact battery health [15, 16].

One of the main features of EBs is the lack of tailpipe emissions, as most EBs cannot emit any Greenhouse Gases (GHG). However, EBs still have life-cycle emissions, as the electricity used to power the EB may produce GHG [17]. As such, to truly optimise the emissions of a fleet of EBs the source of the electricity used to power the fleet must also be considered. Previous works have shown that when the source of electricity is taken into account, the usage of energy generated from cleaner sources such as renewables can be notably increased [7, 18].

3. Problem definition

In this work, we proposed a multi-objective mixed fleet charging scheduling problem with time-of-use constraints to optimise the daily cost of fuel, carbon emissions, and impact on passengers' VoT. Preexisting bus routes are retained and assigned either an ICEB or EB. Charging schedules for EBs are generated, using terminal-based charging stations which can charge one bus at a time. Terminal-based charging is considered as it has shown to be the most cost-effective deployment of charging infrastructure for the overall lifecycle cost of an electric bus fleet [14]. EBs can charge any amount up to their maximum capacity, with a piecewise linear approximation used for non-linear battery charging. Strategies for restricting charging to the CC phase are also explored.

A static price for diesel used by ICEBs, while the cost of electricity used during re-charging is used for EBs[1]. For SCC we explore two scenarios, the first is based on a Paris agreement extended scenario for 2025 [19]. The second comes from the Irish government's appraisal guidelines for the "Shadow Price of Carbon" for 2050 [20]. Both works present a financial cost for the economic ramifications of burning one metric tonne of carbon dioxide equivalent (tCO_{2eq}). These values are converted into Euro (€) and adjusted for inflation. The resulting values are then multiplied by the Carbon Intensity of the Irish

[1]Electricity price available from https://www.semopx.com/

national grid to determine the SCC of 1 MWh of electricity at any given point of the day[2]. The values for electricity price and SCC are gathered for one year, and data for all weekdays in that period are averaged, resulting in values representative of an average weekday.

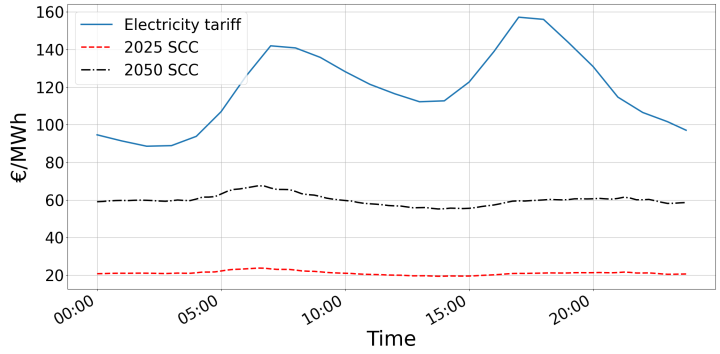


Figure 1: Cost of electricity and SCC throughout an average weekday in €/MWh

Figure 1 shows the electricity price and SCC of the national grid throughout the average weekday. Fluctuates occur throughout the day, with peak electricity prices 77.48% higher than their lowest value. Small fluctuations are also observed for SCC, with the 2050 scenario featuring more defined peaks and troughs.

The passengers' VoT is modelled to reduce the negative impact on customer service. A passenger's VoT is based on the Irish government non-work commuting value [20] adjusted for inflation. Deviations from the original schedule affect the VoT of the passenger in either a negative or positive way depending on if the passenger spends more or less time on public transport. Modifiers for VoT are also applied in the scenarios where a bus arrives early, departs early, arrives late, and departs late to reflect the user's sentiment of that particular scenario [21]. Naive passenger load information is modelled with peak operation times featuring a higher number of passengers compared to non-peak times. The total weight of the EB has a notable impact on energy consumption per KM travelled, as a result, higher passenger loads result in a higher kWh/km value. To the best of our knowledge, no other works consider SCC, fuel costs, and the impact on VoT for public transport users when scheduling mixed bus fleets.

4. MIP model

To create this problem we extend previous works [7]. We consider a set of buses $b \in B$, each of which has a predefined sequence of stops which it must visit $s_i \in S_b$, stops in this sequence are in a repeating order, resulting in the same stop being visited several times. Each stop i has a scheduled arrival time τ_{bi} and may have a charging station, denoted by X_i. Each charging station can only charge one b at a time, and the amount of energy gained by b at i is denoted by continuous variable e_{bi}. γ denotes the minimum allowed e_{bi}, so long as a charge event occurs, denoted by binary variable x_{bi}. Each b has a maximum and minimum battery capacity denoted by $C_{\max}$ and $C_{\min}$ respectively, the capacity of the bus (represented by variable c_{bi}) cannot go outside of these bounds. The type of bus is denoted by the binary variable eb_b, which is assigned the value of 1 if bus b is an EB, otherwise b is considered an ICEB.

The charge time of a bus is denoted by the variable ct_{bi}. We limit the deviation between the arrival time of the new schedule (continuous variable t_{bi}) and the original one (τ_{bi}) using $\Delta t_{\max}$. Variable ΔVoT_{bi} represents the total increase in VoT for passengers. λ_a^+, λ_a^-, λ_d^+, and λ_d^- are variables representing the amount of VoT associated with a late arrival, early arrival, late departure, and early departure for bus b at stop i. While ζ_a^+, ζ_a^-, ζ_d^+, and ζ_d^- are modifiers used in the calculation of λ_a^+, λ_a^-, λ_d^+, and λ_d^- respectively, representing public transport user's opinion of different types of schedule deviations.

Time taken to travel between two stops is provided (T_{ij}), as well as the energy required to make the journey between the i^{th} and j^{th} stop of S_b ($D_{b_{ij}}$). The energy required for the trip depends on the bus'

[2]Carbon intensity of national grid available from https://www.smartgriddashboard.com

passenger load. L_{bi} represents the passenger load of bus b when departing stop i and is depended on the scheduled arrival time at stop i (i.e., τ_{bi}) and the peak public transport operational hours.

To ensure the unary nature of charging stations we use the non-overlapping constraints from [7] where binary variable z_{bdij} is used to denote when bus b and d share a charging station at stop i and j respectively. Binary variables z_{bdi}^{BD} and z_{dbi}^{DB} are used to denote if b arrives after d finishes charging and if d arrives after b has finished charging respectively.

A piecewise linear approximation is used to represent the non-linearity of charging batteries. To this end, three charge rates $\mathcal{R}_0$–$\mathcal{R}_2$ are used depending on the value of c_{bi}. C_0–C_2 denote the maximum battery capacity b can reach before changing to the next charge rate. For example, if $c_{bi} \leq C_0$ charge rate $\mathcal{R}_0$ is used, likewise if $C_0 \leq c_{bi} \leq C_1$, $\mathcal{R}_1$ is used. We define C_0, C_1, and C_2 to be 85, 95, and 100 SOC respectively and define charge rates $\mathcal{R}_1$ and $\mathcal{R}_2$, to be 46% and 14% of $\mathcal{R}_0$ respectively. These are values based on contemporary literature involving non-linear charging [22].

Electricity usage is recorded with TOU constraints to model the fluctuating electricity price and SCC during the day. The day is split into k TOU windows ($|\Gamma|$) each with an associated start time (P_k), end time (Q_k), electricity price (F_k), and SCC (V_k). Variable scc_b denotes the total SCC for bus b, where DV_b is the fixed SCC cost of b if it is an ICEB, otherwise variable scc_{bi} represents the SCC of the electricity consumed by bus b at stop i. Similarly, variable f_b represents the total fuel cost of bus b, DF_b is the fuel cost if b is an ICEB, otherwise variable fe_{bi} is the cost of the electricity used by bus b at stop i. Binary variable o_{bikr} denotes if the charge time of bus b at stop i overlaps with the TOU window k during piecewise function section r. Variables e_{bikr} and ct_{bikr} are then used to express the total electricity used and charge time used during the associated k and r.

This problem minimizes the total daily operational cost of the fleet by reducing fuel and emission costs and the negative impact on VoT. We define the objective function of the full problem as:

$$\min : \sum_{b=0}^{|B|} \sum_{i=0}^{|S_b|} \Delta VoT_{bi} + \sum_{b=0}^{|B|} scc_b + \sum_{b=0}^{|B|} f_b \tag{1}$$

S.T.

$$\forall_{b \in B} \forall_{s_i \in S_b} :$$

$$C_{\min} \leq c_{bi} \tag{2}$$

$$c_{bi} + e_{bi} \leq C_{\max} \tag{3}$$

$$ct_{bi} \leq x_{bi} \tag{4}$$

$$\gamma \cdot x_{bi} \leq e_{bi} \leq x_{bi} \tag{5}$$

$$x_{bi} \leq X_i \cdot eb_b \tag{6}$$

$$\forall_{b \in B} \forall_{s_j \in S_b \setminus s_0, i=j-1} :$$

$$c_{bj} \leq c_{bi} + e_{bi} - D_{b_{ij}} \cdot eb_b \tag{7}$$

$$t_{bj} \geq t_{bi} + ct_{bi} + T_{ij} \tag{8}$$

$$\lambda_{abj}^{+} \geq (t_{bj} - \tau_{bj}) \cdot (VoT \cdot \zeta_a^{+}) \tag{9}$$

$$\lambda_{dbj}^{+} \geq (\tau_{bi} - t_{bi} - ct_{bi}) \cdot (VoT \cdot \zeta_d^{+}) \tag{10}$$

$$\lambda_{abj}^{-} \leq (\tau_{bj} - t_{bj}) \cdot (VoT \cdot \zeta_a^{-}) \tag{11}$$

$$\lambda_{dbj}^{-} \leq (t_{bi} + ct_{bi} - \tau_{bi}) \cdot (VoT \cdot \zeta_d^{-}) \tag{12}$$

$$\Delta VoT_{bi} \geq (VoT_{bi} + \lambda_{abi}^{+} + \lambda_{dbi}^{+} - \lambda_{abi}^{-} - \lambda_{dbi}^{-}) \cdot L_{bi} \tag{13}$$

$$\Delta Vot_{bi} \leq VoT_{bi} + ((\Delta t_{\max} \cdot VoT) \cdot \zeta_d^{-}) \cdot L_{bi} \tag{14}$$

$$\forall_{b,d \in B | b \neq d} \forall_{s_i \in S_b, \forall_{s_j} \in S_d | s_i = s_j} :$$

$$x_{bi} + x_{dj} \leq z_{bdij} + 1 \tag{15}$$

$$t_{bi} \geq t_{dj} + ct_{dj} - M \cdot z_{dbi}^{DB} \tag{16}$$

$$t_{dj} \geq t_{bi} + ct_{bi} - M \cdot z_{bdi}^{BD} \tag{17}$$

$$z_{dbi}^{DB} + z_{bdi}^{BD} - (1 - z_{bdij}) \leq 1 \tag{18}$$

$$\forall_{b \in B} \forall_{s_i \in S_b} \forall_{k \in \Gamma} \forall_{r \in \mathcal{R}} :$$

$$e_{bikr} \leq C_r - c_{bi} - \sum_{h=0}^{k}\sum_{l=0}^{r-1} e_{bihl} - \sum_{h=0}^{k-1}\sum_{l=0}^{r} e_{bihl} + C_r \cdot (1 - o_{bikr}) \tag{19}$$

$$C_{r-1} \cdot o_{bikr} \leq c_{bi} + \sum_{h=0}^{k}\sum_{l=0}^{r-1} e_{bihl} + \sum_{h=0}^{k-1}\sum_{l=0}^{r} e_{bihl} \tag{20}$$

$$e_{bikr} \leq \mathcal{R}_r \cdot ct_{bikr} \tag{21}$$

$$t_{bi} + \sum_{h=0}^{k}\sum_{l=0}^{r} ct_{bihl} - M \cdot (1 - o_{bikr}) \leq Q_k \tag{22}$$

$$P_k \cdot o_{bikr} \leq t_{bi} + \sum_{h=0}^{k}\sum_{l=0}^{r-1} ct_{bihl} + \sum_{h=0}^{k-1}\sum_{l=0}^{r} ct_{bihl} \tag{23}$$

$$ct_{bikr} \leq (Q_k - P_k) \cdot o_{bikr} \tag{24}$$

$$o_{bikr} + o_{bik-1r+1} \leq 1 \tag{25}$$

$\forall_{b \in B} \forall_{s_i \in S_b}$

$$ct_{bi} \geq \sum_{k=0}^{|\Gamma|}\sum_{r=0}^{|\mathcal{R}|} ct_{bikr} \tag{26}$$

$$e_{bi} \geq \sum_{k=0}^{|\Gamma|}\sum_{r=0}^{|\mathcal{R}|} e_{bikr} \tag{27}$$

$$scce_{bi} \geq \sum_{k=0}^{|\Gamma|}\sum_{r=0}^{|\mathcal{R}|} e_{bikr} \cdot V_k \tag{28}$$

$$fe_{bi} \geq \sum_{k=0}^{|\Gamma|}\sum_{r=0}^{|\mathcal{R}|} e_{bikr} \cdot F_k \tag{29}$$

$$\sum_{i=0}^{|S_b|} scce_{bi} \leq ev_b \tag{30}$$

$$\sum_{i=0}^{|S_b|} fe_{bi} \leq ef_b \tag{31}$$

$\forall_{b \in B}$

$$ev_b + DV_b \cdot (1 - eb_b) \leq scc_b \tag{32}$$

$$ef_b + DF_b \cdot (1 - eb_b) \leq f_b \tag{33}$$

Constraints 2 and 3 ensure that c_{bi} is within range of the maximum and minimum capacity. Constraint 4 sets an upper bound for ct_{bi}, which is limited to one hour if a charge event occurs. Constraint 5 enforces a lower limit on the e_{bi} to γ if a charge event occurs. We remark that this protects the longevity of the batteries by avoiding overheating and regular tiny charges. Constraint 6 enforces that b can charge (x_{bi}) only if i is a charging station and if b is an EB.

Constraint 7 enforces the energy expenditure of EBs, while constraint 8 sets stop arrival times to maintain time linearity. Constraints 9–12 defines the deviations in VoT for passengers representing late arrival, early departure, early arrival, and late departure respectively. Constraint 13 applies the deviations to the total VoT deviation for bus b at stop i considering the number of passengers on the bus. Here we define late arrival and early departure as increasing VoT and early arrival and late departure as decreasing VoT. This is because the former represents additional time spent on public transport, while the latter results in the opposite. Constraint 14 defines the upper limit to the total VoT deviation for bus b at stop i which depends on the maximum allowed deviation time per stop Δt_{bi}.

Constraints 15–18 enforce the unary nature of charging stations. Where constraint 15 denotes if two buses charge at the same stop, and constraints 16–18 use the big-M model to model a logical OR statement with disjunctive constraints.

Constraints 19–25 defines the TOU constraints for electricity price, SCC, and the piecewise linear

charging function. Constraint 19 defines the maximum energy b can gain at stop i during r, considering any charging from elapsed TOU windows and previous piecewise linear function sections. Here C_r is used in place of big-M as it is a smaller value that proves suitable for the constraint. Constraint 20 ensures the viability of piecewise linear function section r for b at i. Constraint 21 associates the energy gained in k and r with a linear sum of $\mathcal{R}_r$ and ct_{bikr}. Constraint 22 enforces the maximum allowed charge time in r before the end of k. Likewise, constraint 23 denotes that charge time can only occur in k if charging happens after the start of window k. Constraint 24 ensures that charging only occurs if there is an overlap with the time b spends at i, TOU window k, and charge section r. Finally, constraint 25 denotes that there can be no overlap with TOU window k and charge section r if a higher charge rate section was previously used to charge bus b at stop i.

Constraints 26 and 27 associate the charge time and energy gained via the TOU constraints with the total charge time and energy gained by b at i. Constraints 28 and 29 set the values for SCC and fuel cost based on the values from the TOU constraints, while constraints 30 and 31 calculate the total amount of SCC and fuel cost for b if it is an EB. Constraints 32 and 33 then decide if the EB values for SCC and fuel cost are used, or the SCC and fuel cost associated with ICEB depending on the type of bus assigned to b's route.

$$\forall_{b\in B}\forall_{s_i\in S_b}:$$

$$e_{bi} \leq ct_{bi} \cdot \mathcal{R}_0 \tag{34}$$

$$scce_{bi} \geq e_{bi} \cdot V' \tag{35}$$

$$fe_{bi} \geq e_{bi} \cdot F' \tag{36}$$

We explore decomposition and alternative approaches for non-linear charging. For the decomposition approach, we do not initially consider the TOU constraints. Constraints 19–29 are instead replaced with constraints 34–36 which enforce a linear charge rate and use a daily average value for SCC and electricity cost represented by V' and F' respectively. We limit charging to the first section of the piecewise linear charging function, that is, for the first part of the decomposition approach, $C_{\max}$ is set to C_0. The solution is translated into a solution for the full problem with TOU constraints and non-linear charging by setting key variables such as eb_b, t_{bi}, c_{bi}, ct_{bi}, and e_{bi}. The resulting solution is then used as a warming solution for the full problem.

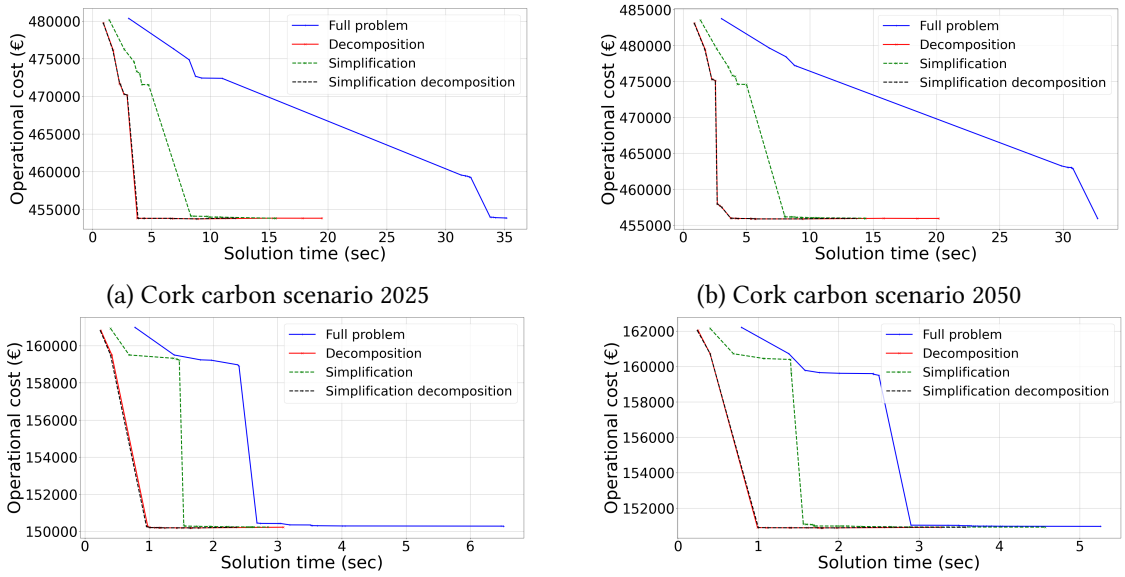
In the alternative approach for the piecewise linear charging function, we use a simplification where TOU constraints are considered but limit charging to the first section of the piecewise linear function. To this end, we remove constraints 19 and 20, change $|\mathcal{R}|$ to 1 (i.e., only consider $\mathcal{R}_0$), and set $C_{\max}$ to C_0. Theoretically, this approach will have a worse global minima compared to the others outlined above. We also consider a decomposition approach to this simplification, the first part follows the same definition as the decomposition for the full problem. However, instead of translating the resulting solution into the full problem with TOU and piecewise linear charging, the solution is used as a warming solution for the simplification outlined above.

5. Results

We empirically evaluate the performance of our model using bus data-sets from three Irish cities (i.e., Limerick, Cork, and Dublin), with the largest data-set featuring 1169 buses and 4279 unique bus stops[3].

We assume that the charging station infrastructure is already in place, and charging stations are placed at terminal stops of each bus route, which is consistent with works investigating the cheapest deployment method for charging stations [14]. Terminal stop information is readily available for the Cork and Limerick data-sets, for Dublin it is assumed that a terminal stop occurs whenever a bus must

[3]The GPS location of the bus stations and timetables for Cork and Limerick are available at https://www.buseireann.ie. GPS data for Dublin data-set available at https://data.gov.ie/dataset/dublin-bus-gps-sample-data-from-dublin-city-council-insight-project



(a) Cork carbon scenario 2025

(b) Cork carbon scenario 2050

(c) Limerick carbon scenario 2025

(d) Limerick carbon scenario 2050

Figure 2: Solution quality (€) vs solution time (seconds) for Cork and Limerick data-sets for the four examined approaches with carbon scenarios 2025 and 2050

wait 30 minutes before its scheduled departure time. This results in 13, 28, and 410 charging stations for Limerick, Cork, and Dublin respectively.

The starting capacity of EBs (c_{bi} where $i = 0$) is assigned a random value between 40–60% $C_{\max}$. $C_{\max}$ is assigned the value of 0.454MWh and a $\mathcal{R}_0$ of 0.15MWh is used which is consistent with values used for EB's employed by Dublin Bus. $C_{\min}$ is 0.0454 (i.e., 10% of $C_{\max}$), an average speed of 20 km/h is used, and a γ value of 0.01 MWh. During peak operation hours we assume the passenger load for each bus is 88, which is the maximum passenger capacity of EBs employed by Dublin bus, during off-peak hours we assume the passenger load is 23, this results in an average passenger load of 33.83 over a 24 hour period, which is consistent with average passenger load for Dublin bus in 2018 [23]. Outside peak hours, the energy expenditure of EBs is set to be 0.0016 MWh per km, otherwise this increases to 0.0032, this is due to the increased total weight from the additional passenger load, which has been identified as a significant factor influencing the operational range of EBs [24]. We assume peak operating hours are between 7–10 and 17–19. We assign the value of 3 to big-M, as it is sufficiently large enough to deal with time-based constraints, as all times are represented in an hour decimal format (i.e. 1:45 P.M. is represented as 13.75). While the normal representation of energy used by EBs is kWh, MWh is used in this work to avoid scaling issues between energy and time-related variables.

The maximum allowed deviation time per stop ($\Delta t_{\max}$) is 0.066666667 (4 minutes). VoT is set to 10.87 € per hour [20] (Commuting non-work price adjusted for inflation), and the values for ζ_a^+, ζ_a^-, ζ_d^+, and ζ_d^- set to 1.97, 0.92, 0.33, and 0.43 respectively [21]. The cost of diesel fuel is 0.3393274 €/km, and emissions for ICEBs are set to 1360 gCO_{2eq}/km. As a result, the SCC for ICEBs is 0.120598 and 0.3430396 €/km for the 2025 and 2050 scenarios respectively. For decomposition approaches F' is set to 119.6795 €/MWh, while V' is 21.0858 €/MWh for the 2025 scenario and 29.9783 €/MWh 2050 scenario.

The MIP model is implemented in C++ and uses CPLEX version 12.10. All experiments are executed five times with a different random seed. Experiments were conducted on a server with 62GB of RAM, and a 2.5GHz Intel Xenon W-2175 CPU. A timeout of 1200 seconds is used for the Cork and Limerick data-sets, while a timeout of 3600 is used for Dublin. When using a decomposition approach, each part is limited to half the total timeout value[4].

We report that all experiments for Limerick and Cork find optimal solutions. Figure 2 shows the solution quality in € vs solve time in seconds for all approaches for both data-sets using the 2025 and

[4]Code and post-processed data-sets available upon request

Table 1
The mean search time and the mean, median, standard deviation of heuristic score, and the score difference between the solution found and a fully ICEB fleet for the Limerick and Cork data-set for both carbon scenarios, and all approaches

City	Config.	Time (s)	Mean Score (€)	Median Score(€)	STD score(€)	ΔICEB(€)
Limerick	2025 F	6.38	150980.6811	151066.9628	**571.8977**	8617.7915
	2025 D	4.78	150943.1586	151019.0605	586.3493	8655.3140
	2025 S	4.73	150943.2969	**151016.9693**	584.0584	8655.1757
	2025 SD	**4.09**	**150941.9809**	151016.9608	585.8825	**8656.4917**
	2050 F	6.03	151729.0736	151813.7086	**616.5176**	9097.2574
	2050 D	4.87	151689.3326	151757.4044	630.2135	9136.9984
	2050 S	4.86	151690.3671	151760.5031	627.5853	9135.9639
	2050 SD	**4.05**	**151688.7073**	**151757.4044**	630.3015	**9137.6237**
Cork	2025 F	37.65	452864.3374	452711.5825	1082.7145	24148.2844
	2050 D	42.408	452861.7713	452704.9805	1073.3037	24150.8505
	2025 S	**15.55**	**452848.7509**	452700.2076	**1071.2811**	**24163.8709**
	2025 SD	23.424	452857.0186	**452698.0427**	1072.8210	24155.6032
	2050 F	40.26	454970.1445	454679.7893	1278.3058	25427.9982
	2050 D	40.95	454880.3992	**454676.1836**	1148.4161	25517.7435
	2050 S	**15.40**	**454874.2333**	454681.2138	1151.0025	**25523.9094**
	2050 SD	22.096	454887.9297	454694.1202	**1146.4702**	25510.2130

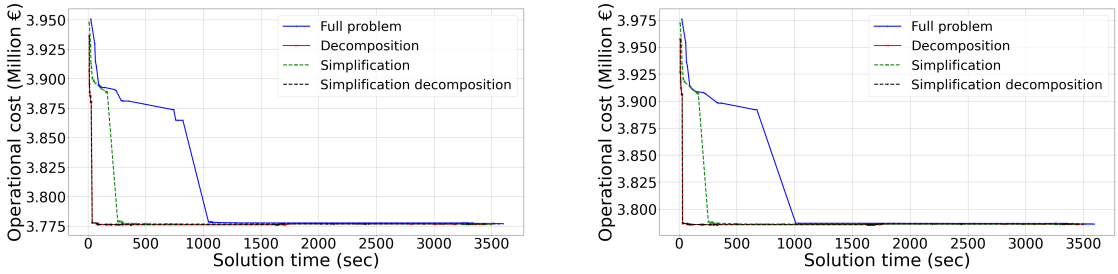
2050 carbon scenarios for one of the five executions. Figures 2a and 2c show that both decomposition approaches find optimal solutions before their respective non-decomposition approaches. It is observed that the performance of both decomposition approaches progresses along the same trajectory during the initial stages of the search process. This is because phase one of both approaches is the same, with variance occurring during phase two of the decomposition.

These findings are also observed in Figures 2b and 2d, with both decomposition approaches able to find an optimal solution in less than 20 seconds for the Cork data-set and 4 seconds for the Limerick data-set. Of note the performance of the full problem approach in the Cork data-set is worse than other approaches examined, indicating it scales poorly with data-set size. It is also observed that the score of the solutions found by both decomposition approaches worsened at a particular point during the search process. This signifies the transition from phase one of the decomposition approaches to phase two, where the average electricity price and SCC are exchanged for their realised values and TOU constraints are introduced. The scores for both decomposition approaches before this point are approximate.

Table 1 shows the mean solution time in seconds, and the mean, median, the score standard deviation, and difference between the mean score and the score of an entirely ICEB fleet across all five random seeds for all Limerick and Cork experiments. Best-performing results are highlighted in bold, and the carbon scenarios and approaches are abbreviated into config. column (i.e., carbon scenario 2025 full problem is 2025 F, carbon scenario 2050 simplification decomposition is 2050 SD, etc.). Slight differences in optimal solutions are reported. This is due to CPLEX's relative MIP gap tolerance, where solutions with a gap $\leq$1e-04 are considered optimal. Optimal solutions report relative gap values between approximately 1e-05 and 9e-05, this causes slight variations in the final score of the solution. However, the authors note that this variation is tiny in proportion to the scores reported. The standard deviation for solution scores reports limited variance, further reinforcing this. Experiments with the Cork data-set consistently find solutions where 69.63% of the bus fleet consists of EBs, while results for Limerick show that at most 60% of the fleet can be made electric.

From Table 1 we see both simplification approaches often outperform other approaches in search time and resulting solution score. The improved score performance is noteworthy, as the global optima should be worse due to the lack of non-linear charging. This is due to the instance size and the selected constant's values. As optimal solutions using these data-sets do not require the capacity of an EB to exceed C_0. As a result, the global optima for the full problem and the simplification approach are the same. This is due to several factors, such as the battery capacity used, the number of charging stations available, and the maximum allowed deviation time per stop, which limit the total amount of time an EB can charge.

Comparing the generated mixed fleet schedules to a fleet of fully ICEB significant savings can be



(a) Dublin carbon scenario 2025

(b) Dublin carbon scenario 2050

Figure 3: Solution quality (€) vs solution time (seconds) for Dublin data-set for the four examined methods with carbon scenarios 2025 and 2050

Table 2
The mean score, gap, and the score difference between the solution found and a fully ICEB fleet for the Dublin data-set for both carbon scenarios, and all approaches

Config.	Score (€)	Gap(%)	Δ ICEB(€)
2025 F	3776397.0628	0.000288	173631.7756
2025 D	**3776191.9620**	0.000226	**173836.8765**
2025 S	3776244.1452	0.000278	173784.6932
2025 SD	3776194.2948	**0.000172**	173834.5437
2050 F	3786121.4762	0.000417	189862.5576
2050 D	**3785539.2134**	0.000221	**190444.8204**
2050 S	3785546.3044	0.000265	190437.7294
2050 SD	3785541.0956	**0.000170**	190442.9382

observed for the Cork data-set, with up to €25523.90 saved per day due to decreases in fuel price and SCC, as well as VoT reductions. Excluding VoT, total savings of up to €4909.10 and €6946.42 per day are observed for carbon scenarios 2025 and 2050 respectively. This highlights the importance of VoT when generating schedules, as it has a high level of impact. The differences between carbon scenarios 2025 and 2050 are also highlighted by table 1 with the solution scores several thousand lower in the former compared to the latter. This is due to the increased cost of carbon (21.08 €/MWh for 2025 and 59.97 €/MWh for 2050), however, the use of carbon scenario 2050 accomplishes an overall reduction in emissions. For the 2025 carbon scenario, emission reductions of up to 8.31 tCO_{2eq} are observed for the Cork data-set per day, when considering the 2050 scenario emissions reduce further by 0.0025 tCO_{2eq}.

Figure 3 shows the performance of the four approaches for the Dublin data-set for both carbon scenarios from a randomly selected seed. None of the Dublin experiments find an optimal solution before the timeout. This includes both phases one and two of the decomposition approaches. However, both decomposition approaches outperform simplification and full problem approaches by a large margin, with most decomposition experiments finding solutions with a relative GAP of less than 2% within two minutes for both carbon scenarios. This coupled with the poorer performance of the full problem approach highlights the need for decomposition approaches when dealing with larger data-sets to find good-quality solutions.

Table 2 shows the mean score, relative MIP gap, and score difference from a fully ICEB fleet for all Dublin experiments. Here, the decomposition approach consistently reports the best mean score and savings, while the simplification decomposition obtains the best relative MIP gap. As previously outlined, the global optima for full problem and decomposition are the same if the optimal solution does not require non-linear charging. For the Dublin experiments, the best-performing solutions for the full problem use non-linear charging. As a result, while simplification decomposition reports a better relative gap, the mean score reported is worse due to a different global optima. However, it should be noted that the difference between the mean scores for both decomposition approaches is only €2.33 and €1.88 per day for carbon scenarios 2025 and 2050 respectively. This would imply that there is very little difference between non-linear charging and limiting capacity to 85% of maximum rated capacity when

Table 3
The mean change in VoT, Fuel, SCC and Score when introducing TOU constraints in phase two of the decomposition approaches for the Dublin data-set for both carbon scenarios

Config.	ΔVoT(€)	ΔFuel(€)	ΔSCC(€)	ΔScore(€)
2025 D	15.8522	187.2486	8.6491	211.7500
2025 SD	15.5240	183.3760	7.0935	205.9934
2050 D	13.8591	183.6443	24.9140	222.4175
2050 SD	17.7716	181.8571	21.2320	220.86

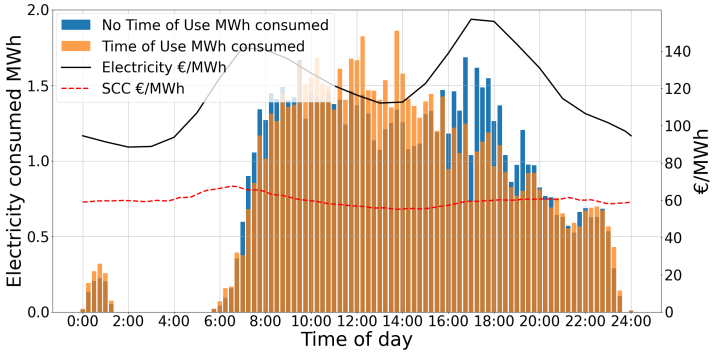


Figure 4: The electricity consumption for decomposition phase 1 and 2 throughout the day vs electricity price and SCC for the Dublin data-set with carbon scenario 2050

using realistic data-sets and on-route charging. We also report that the mixed fleet consists of 88.6% EBs, which is significantly more than the EB penetration reported for the Cork and Limerick data-sets, this is attributed to the increased number of charging stations.

Up to €173836.87 and €190444.82 can be saved daily for carbon scenarios 2025 and 2050 respectively across VoT, fuel prices, and SCC. Excluding VoT, €24661.88 can be saved for carbon scenario 2025 and €33979.49 for carbon scenario 2050 when transitioning from a fully ICEB fleet. This includes a carbon emission reduction of 101.52 tCO_{2eq} per day for carbon scenario 2025, with a further saving of 59.5 $kgCO_{2eq}$ when considering the increased weighting of SCC in carbon scenario 2050.

Table 3 shows the difference in VoT, Fuel, SCC, and score between the best solution for phases one and two of both decomposition approaches. The difference between the best solution without TOU and with TOU is greater for the full problem decomposition. Both full problem decomposition and simplification decomposition are subject to the same constraints in phase one. As such, this difference reflects the better global optima that phase two of the full problem decomposition has, as the score difference for the two decomposition approaches is consistent with the score differences seen in table 2. We also observe that when introducing TOU constraints, savings of up to an additional €211.75 and €222.41 occur for carbon scenarios 2025 and 2050 respectively. However, it should be noted that these additional values are small compared to the €24661.88 and €33979.49 saved per day for each scenario. Further reduction of carbon emissions is observed when introducing TOU constraints, with additional reductions of 95 $kgCO_{2eq}$ for carbon scenario 2025 and 98.8 $kgCO_{2eq}$ for 2050.

Figure 4 shows the energy consumption for both phases of the full problem decomposition approach, with carbon scenario 2050 for the Dublin data-set with electricity price and SCC. Introducing TOU constraints moves energy used to charge the mixed fleet from during the electricity price and SCC peaks to the associated pits. This shows that TOU constraints have an observable change when energy is consumed, even if the score difference is relatively low.

Figure 5 shows the mean late arrival, late departure, early arrival, and early departure per EB vs passenger load throughout the data for the Dublin data-set with carbon scenario 2050 and the decomposition approach. Here we see that the delays caused to each EB are relatively small for the savings made, with the average late arrival per EB peaking at 0.1464 minutes (8.78 seconds). The maximum deviation for late departure is 1.1453 minutes (68.71 seconds), this shows the negative service

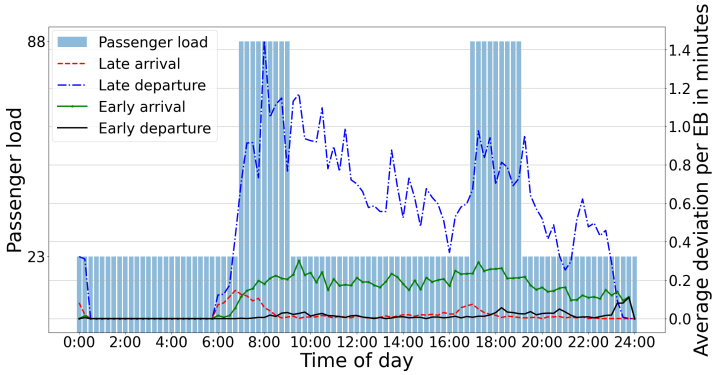


Figure 5: The mean late arrival, late departure, early arrival, and early departure per EB throughout the day vs passenger load for Dublin data-set with carbon scenario 2050 and decomposition approach

quality impact is insignificant compared to the potential savings from transitioning to a mixed fleet.

6. Conclusion

In this work, we presented an MIP model to create a multi-objective mixed fleet charging schedule problem to minimise total fuel cost, carbon emissions, and negative service impact. Considerations include non-linear battery charging, fluctuating electricity prices, and national grid carbon intensity. Evaluation of three Irish cities shows significant savings can be made by adopting mixed fleet schedules, with up to €20017.48 saved per day on fuel costs, and a reduction of 101.52 tCO_{2eq} per day. Negative impacts on service quality are minimized, with the maximum late arrival delay peaking at 8.78 seconds per EB. While non-linear charging affords higher-quality solutions, limiting total capacity to the CC phase of battery charging shows minimal differences and results in less SOC variance, which promotes battery health. Likewise, while additional savings are observed when considering TOU constraints, the relative improvement is modest compared to overall savings. This work treats travel times and energy consumption as deterministic, which may lead to challenges when fully implementing these approaches. In future work, we plan to study the impact of this assumption and explore potential mitigation strategies.

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