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Smart Meter Based Two-Layer Distribution System State Estimation in Unbalanced MV/LV Networks

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***Abstract*—This paper presents a smart meter based two-layer state estimation technique, design to enable integrated monitoring of Medium Voltage (MV) and Low Voltage (LV) power distribution networks. The main contributions of this work include the development of a novel topology reduction technique for LV networks in order to carry out state estimation with a reduced number of smart meter based measurements, and a linear Low Voltage State Estimation (LVSE) technique which reduces convergence problems in LV networks. In addition, a detailed framework for integrated MV/LV network state estimation in realistic, unbalanced three-phase networks is proposed and demonstrated using the IEEE 13 bus and IEEE 906 LV networks. The results suggest that the proposed two layer state estimation technique is robust and provides improved Distribution System State Estimation (DSSE) accuracy compared to traditional approaches.**

***Index Terms*—Distribution grid monitoring, distribution system state estimation (DSSE), low voltage network, smart meters, topology reduction.**

I. Introduction

A Rapid growth in the deployment of smart meters worldwide is enabling many utilities to improve the monitoring and operation of power distribution networks [1]. The evolution of the electric distribution grid into a more complex system with the integration of numerous heterogeneous components has increased the importance of efficient monitoring of the network. Distribution Management System (DMS) rely heavily on the monitoring of the operating condition of the network. This monitoring can be provided by Distribution System State Estimation (DSSE) [2]. Distinctive levels of research have been performed on DSSE by various researchers in recent years however, most of these efforts are focussed on the Medium Voltage (MV) level for state estimation assuming pseudo-measurements; namely, forecasts data from the Low Voltage (LV) side of the network [3]–[6]. Predominantly, distribution utilities generally do not include the LV side of the network in their network models, which makes monitoring of the secondary side of the network more difficult. The reason for this approach is the lack of measurements and lack of accurate, up-to-date LV network models [7]. Due to this, LV systems pose entirely new problems to the DSSE approach. In spite of this, the inclusion of the LV side of the network for state estimation is essential as the secondary side of the network is closely related to the end-users' power quality and also the service interruption is often related with the issues on the LV network [8].

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While there is a growth of Advanced Metering Infrastructure (AMI), in the LV distribution network, there is also a gap of knowledge between the end of the network model and the customer meter connection, as a result, there is a constraint on the utility to extract the full value and functionality of AMI. This is further evidence that LV network modelling and state estimation is necessary for the secondary distribution network monitoring. Therefore, this paper focuses on monitoring of the entire distribution network, concentrating on the LV level in order to resolve the problem of grid observability.

Firstly, to the authors' knowledge, very few works to date have tackled the problem of including the LV network in the DSSE. A major limitation for researchers at present is the unavailability of standard European LV test feeders. The only IEEE test feeder currently available for LV network studies is the IEEE European Low Voltage Test Feeder [9]. However, due to lack of load information in the network, it is not possible to implement state estimation algorithm on it as it causes measurement redundancy. To solve this problem of network observability, topological reduction of the IEEE European Low Voltage Test network has been attempted in previous work as in [10], where the loads have been lumped together to reduce the size of the network and make the network observable but also making the network a balanced system. In [11], the European LV grid has been reduced by disaggregating the network into 128 small feeders. The reduction technique suggested in this paper cannot be used for applications like state estimation, as the algorithm must be run on all the feeders separately. Hence, this method is computationally inefficient and is time-consuming. With reducing the IEEE LV network, some other researchers have considered synthetic LV networks for LVSE with each load connected as a three phase load, while some others have applied the network as a three-phase balanced network [12]–[14]. These assumptions are practically invalid as most LV grids are three phase unbalanced networks. Therefore, this paper focuses on developing a novel LV topology reduction technique which maintains the integrity of the network and makes the already established network accessible to applications like state estimation.

The need for a novel topology reduction technique arises due to the limitations of previous techniques proposed in the literature to date. For example, the proposed reduction technique in [15] adds loops in the network, thus change the radial configuration of the LV network. The authors in [16] do not assure the equivalent voltage values after reduction, while the technique used in [17] implements assumptions like fixed current loads and the addition of synthetic load buses in the feeder. Other benchmark algorithms like the

Minimum Spanning Tree (MST) reduction technique [18] focus on retaining the three-phase laterals and not reducing the feeder. The algorithm proposed in [19] provides the lowest voltage errors for the equivalent reduced network; however, the technique is computationally burdensome. Hence, these limitations lead to the proposal of a novel topology reduction technique for obtaining the reduced equivalent LV network.

Secondly, due to the addition of smart meters at the customer side, certain novel state estimation techniques have been presented as solutions for the network observability with the help of smart meter measurements [20]–[23]. However, in [23], the author considers using smart meter measurement as synchronised measurement, which can be considered unrealistic because of limited available bandwidth. When explored further, many have assumed smart meter data on a distribution network consisting of transmission network properties [24]. Some other researchers consider smart meter measurement with 6-12 hours delay [1] which does not provide reliable results, as such a long delay leads to very high estimation errors. A certain number of recent research works have investigated techniques for smart meter placement to enhance grid observability [22]. Similar studies in these areas have taken aggregated smart meter readings for state estimation due to privacy issues where most of them have considered smart meter data with poor accuracy, implemented in combination with pseudo-measurements used for Medium Voltage State Estimation (MVSE).

Several recent publications which focus on considering the LV network in the DSSE [25], [26] presented their work with several assumptions, due to which their techniques seem difficult to implement in real European networks. A majority of these papers on LVSE [22], [27], [28] use WLS technique for state estimation. The authors in [28] have used this algorithm with gain matrix technique, conversely, due to the high R/X ratio of the network this WLS technique will have convergence problems if applied in LV networks [29] with considerable time delays, resulting in inaccurate estimates of current LV network states. In [30], a distribution system three phase state estimation is proposed using hybrid particle swarm optimisation. This presented technique is also computationally inefficient and time consuming as it requires 1000 iterations. The requirement for a computationally-efficient LV algorithm is imperative, as it is not economically feasible to require high-performance computing resources for every LV network. Other constraints, such as considering 100% smart meter availability in the network and the assumption that all loads are connected as three-phase loads, makes this algorithm suitable only in certain special cases, and not in the general case. In this paper, these assumptions are not required in order to apply the proposed state estimation technique. This is implemented in realistic European networks, where smart meter data is only available at a limited number of network locations, as per the current condition in many European networks. Therefore, this enhances the functionality of smart meters for monitoring of the entire distribution network including MV and LV feeders.

The main contributions of this paper are: (1) Development of a novel topology reduction technique for LV networks by allowing state estimation to be carried out accurately with a reduced number of available measurements; (2) the proposal of a linear technique for Low Voltage State Estimation (LVSE) to solve convergence problems in LV networks; (3) a detailed framework for including realistic, unbalanced LV networks in DSSE, which increases the accuracy of the state estimation (compared to using pseudo-measurement data), and reduces the requirement for installation of high-cost network monitoring at the MV level.

This paper is structured as follows: Section II provides the problem formulation used in the paper which explains the novel network topology reduction technique, the linear LVSE technique, and the overall DSSE algorithm used. Section III deals with the test environment exploring the case studies using various simulation environments and a comparison has been shown to showcase the performance of the novel DSSE technique over other techniques. Section IV includes the results with discussion and the paper is concluded in Section V.

II. Problem Formulation

A. Low Voltage Network Preprocessing

In this paper, an LV network has been selected to implement the proposed linear state estimation algorithm as described in Section I. The IEEE 906 LV test network is the only available IEEE European test feeder for LV network studies. There are several limitations of using the one and only available standard IEEE European grid. One such limitation is that the execution of LVSE is unfeasible on this network because it consists of only 55 load buses out of 906 total number of nodes [15]. Additionally, it is impractical to install smart meters on no-load buses to gain the measurement data. As a result, the problem of redundancy and observability makes this network impractical for LVSE. Therefore, a topology reduction technique has been suggested in this paper to modify the standard IEEE European LV network.

B. Reduced Topology Modelling

In this section, a generic LV distribution grid branch is used to explain the concept of topology reduction, as shown in Fig. 1. The main aim of this technique is to obtain a reduced version of the network with equivalent parameter properties. As seen from Fig. 1, the number of no-load buses which are on the same branch are aggregated together. All the lines between the no-load buses with the same line configuration (i.e. having the same resistance and reactance) are added together. As per Fig. 1, $x1, x2, x3....xn$ are the no load buses and $y1, y2, y3....yn$ indicate the line length. Whereas, $l1, l2, l3....ln$ are the line length of a different branch configuration. In the second step, all the line lengths with same line configuration are added together, giving X no-load buses and the number of branches Y to be aggregated as seen from the pseudocode, Table. I. The branches having the same line configuration are added together to form a single no-load bus $s1$ which is a cluster of aggregated no-load buses. The aggregating centre, which is considered as the position for the aggregated no load bus is selected by taking the midpoint of the reduced network node's line length. This same procedure can be used on all

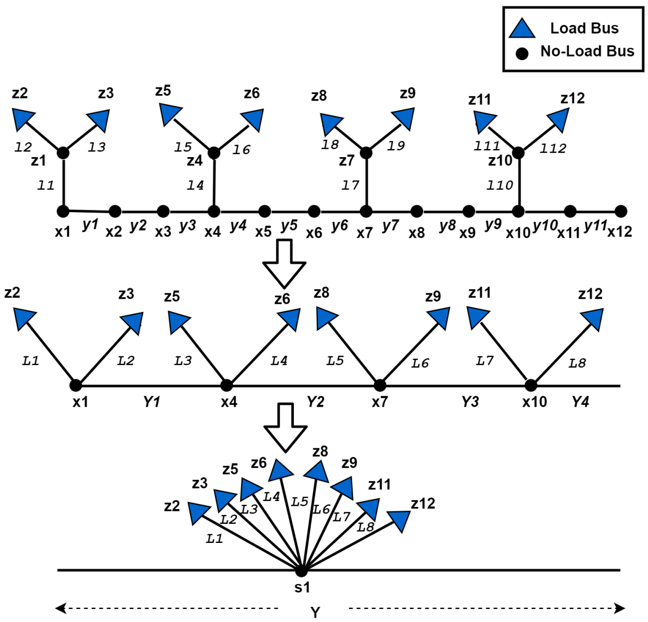

Fig. 1: Topology reduction technique

the branches of the network to obtain an equivalent reduced LV network model. The load flow technique is then run on the whole network to compare the novel aggregated reduced network with the standard available network.

TABLE I: Equivalent topology reduction Model Pseudocode

Input: $X = \{x_1, x_2, \cdots, x_k\}$ // Set of no-load nodes to be aggregated
K // Number of nodes
$Y = \{y_1, y_2, \cdots, y_l\}$ // Set of branches to be aggregated
l // number of branches
$C = c_1, c_2, c_3, \cdots, c_i$ // Set of branch configuration with i sets of configuration type

Output: $S = \{\mathbf{s_1}, \mathbf{s_2}, \cdots, \mathbf{s_n}\}$ // Set of aggregating nodes
$v = [1, 2, 3]$ // Set of phase labels for loads

function CREATEAGGREGATINGNODES (X, Y, C)
repeat
  foreach $\mathbf{s_i} \epsilon S$ **do**
  forall branches Y **do**
  foreach $c_i \epsilon C$ **do** // Set of configuration type
    if $Y == c_1 \epsilon C$ **then** // if Y has same line config
      $\sum_{y \epsilon Y} \mathbf{y}$
  ; //repeat for all c_i types
  Take node location at each branch: $s_i = \frac{1}{x_k} \sum_{y \epsilon Y} \mathbf{y}$
until *All the nodes for same branch configuration are not aggregated;*
for $j = 2{:}S \forall v$ **do**
  Calculate the power flow with aggregated nodes
end

In this novel LV topology reduction technique, no additional load is included, and no loads are modified, as seen in some previous work related to aggregation techniques (e.g. [10], [11], [31]). The aim is to propose an LV topology reduction technique that can produce a reduced electrically-equivalent network, for a realistic, unbalanced three-phase LV system such as the IEEE 906 LV network. All the line data has been taken into consideration by only eliminating the no-load buses. In previous work like [10], LV loads are aggregated together, making the network balanced. In [11], the IEEE 906 LV network is disaggregated into small feeders, and in [15], a reduction technique is used which changes the LV topology from radial to a meshed network by adding new synthetic lines in the network. All these aggregation techniques do not provide any benefit if the LV network is to be used for applications such as state estimation. Therefore, one advantage of the proposed LV topology reduction technique over previous work is that it maintains the integrity of the network (e.g. it does not make the network balanced, keeps the network's radial configuration, and does not disaggregate the network into smaller feeders).

The proposed LV topology reduction also reduces customer privacy concerns, as the utility reads the data only at the concentrator level, where the smart meter measurement has already been aggregated, unlike in [32] where the load data is measured from individual smart meters. The proposed technique results in an equivalent, but a reduced model of the original network. This is demonstrated by detailed time-series load flows on both the original network and the reduced modified network (Section III.A). This LV topology reduction technique provides an efficient means of achieving enough redundancy to ensure network observability without interfering with customer privacy. This novel reduction technique can be applied in all unbalanced three-phase networks without disturbing its standard properties.

The topology reduction technique has been used in this paper for LV networks in order to address the observability problem before implementing the state estimation algorithm. This technique enables the use of the LV distribution networks, such as the IEEE 906 network for state estimation.

C. Proposed Linear State Estimation Algorithm

The widely used Weighted Least Squares (WLS) state estimation technique is not suitable for LV networks due to the unavailability of measurement datasets, which results in negative redundancy as explained in section I. The typical measurement equipment available in the LV network are the smart meters connected at the consumer end. However, utilising the power consumption data for monitoring purposes directly from the consumer end is prohibited due to privacy issues. The other most important aspect is to consider state estimation for the unbalanced three-phase, as the LV network are usually asymmetric.

The purpose of proposing this novel state estimation technique is to monitor the LV network with the available smart meter measurements. The major problem in the LV network, as stated in the previous section, is the unbalanced nature and lack of measurement availability. Thus, a linear state estimation technique is adopted in this paper considering the unbalanced network since, the symmetric state estimation technique is not suitable as it does not detect the single-phase violation. Furthermore, the classical WLS state estimation technique uses an iterative method as the state estimation equations are non-linear. This technique tends to have a non-convergence problem in distribution networks [33]. Another reason for using this proposed linear state estimation technique is that it is computationally efficient since it is a non-iterative solution.

For linear LVSE, the voltage magnitude, active and reactive power injections from the installed smart meters are used as inputs for the state estimation algorithm. The main merit of this proposed algorithm is its computational speed, and, it is not prone to convergence problem as explained above. Since the loads in the LV network are asymmetric, the state estimation technique must be applied on the three-phase network. In this paper, LVSE measurement data is taken at one-minute intervals over the course of 24 hours.

Assuming a network with N nodes, where the node at the substation level is considered to be a slack bus. The system state vector is defined as:

$$x = [U_{1,r}, U_{2,r}, \cdots\cdots U_{N,r}; U_{2,im}, \cdots\cdots U_{N,im}] \quad (1)$$

where $U_{N,r}$ is the real part and $U_{N,im}$ is the imaginary part of the complex voltage node. For considering the algorithm to be linear, the imaginary part of the above voltage complex equation has been neglected. This assumption is assumed to be valid as the voltage angles in the LV network are considered to be very small, only up to 2 degrees. The measurement vector depends on the available input measurements. The input measurement taken in this paper are voltage magnitude, active and reactive power injections from the concentrator. The active and reactive power in the input measurement are non-linear function of x. Therefore, Taylor series has been applied for linearized approximation of the measurements.

The complex power flow for three phase with $v \in [1, 2, 3]$ is given in the equation below:

$$S_v^{ij} = U_v^i \cdot (I_v^{ij})^* = U_v^i \cdot \sum_v^3 [Y_{v,w}^{ij} \cdot (U_w^i - U_v^j)]^* \quad (2)$$

Where S_v^{ij} is the power flow equation between node i and j. U_v^i is the node voltage and I_v^{ij} is assumed to be line current. Since the network is assumed to be linear, the $Y_{v,w}^{ij}$ is the admittance matrix in phase sequence Y_{123}^{ij} which is transformed from the symmetrical components Y_{012}^{ij} as shown in [34]. The intent for choosing the symmetrical component method first is because of the resulting LV network matrix, which is mostly sparse. Therefore, it can be efficiently inverted by a numerical method. A further benefit of using this technique is that the neutral conductor data can be easily deduced from the zero-sequence current.

For the linear measurement model, in this algorithm, the voltage angle φ for phase v, w and node i, j are considered as shown below.

$$\begin{aligned} \Delta\varphi_{v,w}^{i,j} &= \varphi_v^i - \varphi_w^j \\ s.t \quad v, w &\in [1, 2, 3] \end{aligned} \quad (3)$$

The equation above is considered as an evaluation point of linearization for the LVSE algorithm by considering the following conditions.

$$\Delta\varphi_{v,w}^{ij} = \begin{cases} 0 & if\ (v - w) = \quad 0 \\ -\frac{2\pi}{3} & if\ (v - w) = -2 \vee 1 \\ \frac{2\pi}{3} & if\ (v - w) = -1 \vee 2 \end{cases} \quad (4)$$

With the help of equation (4), the Taylor series can be applied to the conventional power flow equations, as used in [35]. Therefore, the linearized approximation for the power flow equation is given below.

$$(P_v^{ij})_U = \sum_{w=1}^{3} U_r ef \cdot [(G_{v,w}^{ij} \cdot \cos(\Delta\varphi_{v,w}^{ij})_{ref} + B_{v,w}^{ij} \cdot \sin(\Delta\varphi_{v,w}^{ij})_{ref}] \quad (5)$$

$$(P_v^{ij})_\varphi = \sum_{w=1}^{3} U_{ref}^2 \cdot [(G_{v,w}^{ij} \cdot \sin(\Delta\varphi_{v,w}^{ij})_{ref} - B_{v,w}^{ij} \cdot \cos(\Delta\varphi_{v,w}^{ij})_{ref}] \quad (6)$$

$$(Q_v^{ij})_U = \sum_{w=1}^{3} U_{ref} \cdot [(G_{v,w}^{ij} \cdot \sin(\Delta\varphi_{v,w}^{ij})_{ref} - B_{v,w}^{ij} \cdot \cos(\Delta\varphi_{v,w}^{ij})_{ref}] \quad (7)$$

$$(Q_v^{ij})_\varphi = \sum_{w=1}^{3} U_{ref}^2 \cdot [-(G_{v,w}^{ij} \cdot \cos(\Delta\varphi_{v,w}^{ij})_{ref} - B_{v,w}^{ij} \cdot \sin(\Delta\varphi_{v,w}^{ij})_{ref}] \quad (8)$$

Where P_v^{ij} and Q_v^{ij} are active and reactive power flow from node i to node j for all the three phases. U_r is the voltage at the secondary transformer, which is taken as the reference voltage, and φ is the reference angle. $G_{v,w}^{ij}$ and $B_{v,w}^{ij}$ is the conductance and susceptance of the admittance matrix at row v and column w.

The equation (5)−(8) is implemented for power injection, as shown in the equation below where the number of grid nodes is denoted by N.

$$P_v^i = -\sum_J^N (P_v^{ij})_{U,\varphi} \quad , \quad Q_v^i = -\sum_J^N (Q_v^{ij})_{U,\varphi} \quad (9)$$

Now, the measurement vector model with all the linear measurement functions are expressed as shown below.

$$z = H \cdot x + e \quad (10)$$

Where H is a measurement function matrix formed based on (5) − (9) and e is the measurement error vector, which is often assumed to be white Gaussian noise, i.e. $e \sim N(0, R)$.

For the linear state estimation, the estimated state $\hat{x}$ can be implemented, as shown below.

$$\hat{x} = (H^T R^{-1} H)^{-1} H^T R^{-1} z \quad (11)$$

$$R = diag\ (\sigma_1^2, \cdots\cdots\cdots, \sigma_m^2) \quad (12)$$

Here R is the diagonal matrix consist of the input measurement variances [36].

Now assuming the objective function $J(x)$ from the classical WLS method with the measurement value z_k and the measurement variance σ_k^2.

$$J(x) = \sum_{k=1}^{M} \frac{(z_k - \hat{z}_k)^2}{\sigma_k^2} \quad (13)$$

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As the state estimation is linear, the equation (13) can be written as:

$$J(x) = \sum_{k=1}^{M} \frac{\left(z_k - h_k^T \cdot x\right)^2}{\sigma_k^2} \tag{14}$$

Here the row vector h_k^T relates the estimated measurement value z_k to the state vector.

In LV networks, there are many no-load buses. Therefore, virtual measurements are assumed on those no-load buses, which does not have any measurement meter installed to it.

The Standard Deviation (SD) for virtual measurements is set to zero, and for voltage magnitude from the smart meter, it is set as 0.1 v. With these values for the classical WLS technique, the condition of the gain matrix will tend to infinity, and hence SE instability will occur. To overcome the numerical instability, the Augmented Matrix Approach (AMA) for SE is used [36]. In AMA approach, the input measurement is divided into smart meter's measurements and virtual measurements and takes them as equality constraints. Representing the regular measurement as H_R and virtual measurement $C.x$, the above objective function can be minimised as shown below:

$$min\ J(x) = r^T \cdot R^{-1} \cdot r \quad \backslash s.t\ C \cdot x \ = \ 0 \backslash \& r - z + H_R \cdot x = 0 \tag{15}$$

Here r represents the residual vector, which is the difference between the estimated and true measurement and H_R represent the Jacobian matrix.

Since the above equation has two equality constraints, it can be formulated as a Lagrangian function [36] with two sets of multipliers denoted as μ and λ for equality constraints.

$$\mathcal{L} = J(x) - \lambda^T \cdot (C \cdot x) - \mu^T \cdot (r - z + H_R \cdot x) \tag{16}$$

For including the Lagrangian function in state estimation algorithm, the equation (16) is written as a linear matrix optimization problem in the equation below.

$$\begin{bmatrix} \alpha^{-1} \cdot R & H_R & 0 \\ H_R^T & 0 & C^T \\ 0 & C & 0 \end{bmatrix} \cdot \begin{bmatrix} \mu \\ x \\ \lambda \end{bmatrix} = \begin{bmatrix} z \\ 0 \\ 0 \end{bmatrix} \tag{17}$$

Here, the coefficient matrix is called as the augmented matrix. In this matrix, the additional weighting factor α can be applied for adjusting the input variance diagonal matrix. Also, the H_R represent the measurement Jacobian matrix in the symmetrical component. This variable is similar to the nodal admittance matrix. Therefore, after solving the optimization problem, the outputs have to be retransformed to three-wire values. The computational effort for transforming in symmetrical components is compensated by the good solution and analytical properties, especially for asymmetric network states with high unsymmetrical loads.

D. Two Layer Distribution System State Estimation

As seen in Fig. 2, the DSSE technique consists of two different state estimation layers. At first, a topology reduction technique has been implemented on a standard IEEE 906 European LV network. The topology reduction of the standard 906 network is performed because of the relatively large number of nodes per customer in the network, which creates a network observability problem. Hence, the LV topology reduction technique suggested in Section II.B, aims to improve the LV network observability, thereby increasing state estimation reliability. Using smart meter data on load buses and considering virtual measurements on no-load buses, a time series load flow analysis is performed with 1 minute time interval for 24 hours on the modified IEEE 906 network using EPRI OpenDSS software [37]. Gaussian noise is added in the load flow results to use this as the input measurements for linear state estimation.

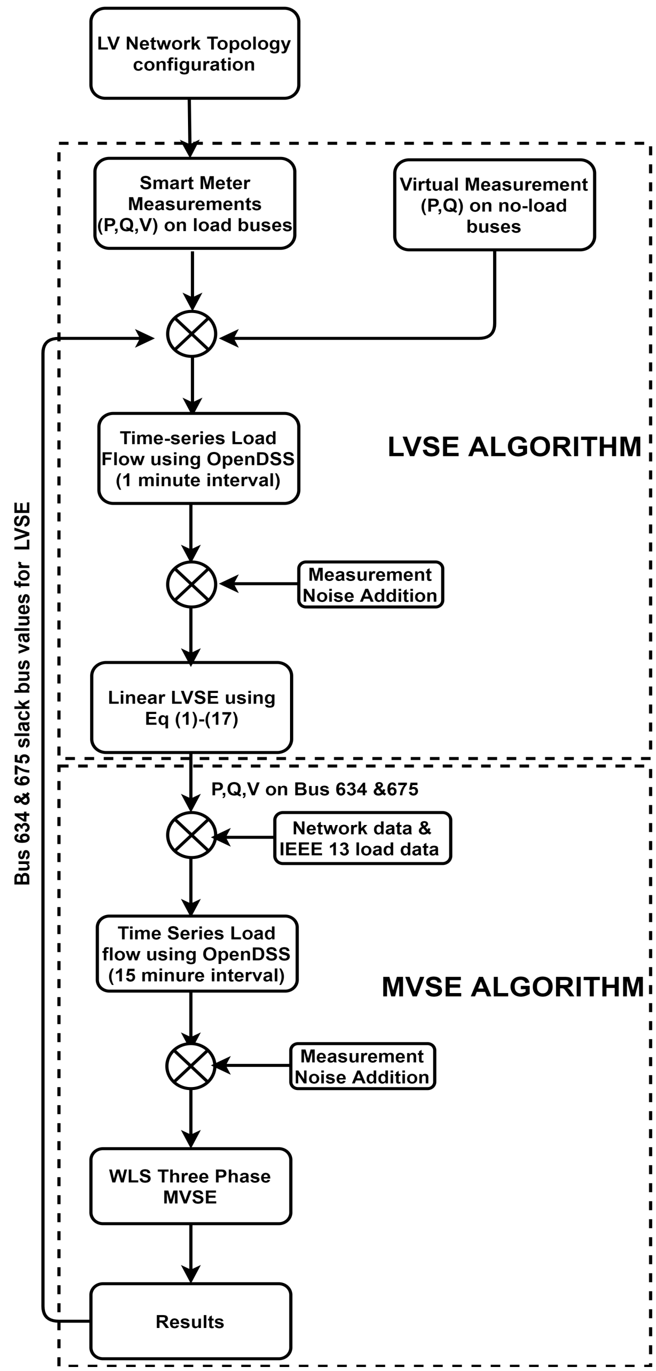

Fig. 2: Distribution system state estimation model

The novel linear LVSE technique proposed in Section II.C

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solves the problem of non-convergence, which occurs while using traditional techniques like WLS for LVSE. Also, it is computationally efficient and has a much faster convergence due to the proposed algorithm's non-iterative nature. The two LV feeders are assumed to be connected at two different nodes of the MV network, as seen in IEEE 13, Fig. 4. It receives the results of voltage magnitude and total active and reactive power injection (with their uncertainties) from the LV estimators on two of its buses in the MV/LV substations. In addition, it also receives the measurements from possible other smart meters deployed in the rest of the nodes on the MV grid. The time-series power flow is again run for the MV network which contains modified IEEE 906 LV network on two of its nodes and the MV smart meter load data on the rest of its buses, for 24 hours with 15 minutes interval. All the true measurements from power flow output are subjected to random noise before implementing traditional WLS state estimation on the IEEE 13 MV network. Since MVSE is receiving real time LVSE output on two of its buses, therefore, the novel algorithm is expected to be more accurate than the SE with forecast or pseudo-measurement input data. The estimated state vector at bus 634 and 735 in MVSE are fed back to the LV feeder to serve as the new slack bus measurement of the reduced IEEE 906 bus network for LVSE after each iteration.

III. Test environment

The simulation of the two-layer DSSE has been evaluated using the reduced IEEE 906 low voltage network for LVSE and modified IEEE 13 bus network for MV level state estimation.

Three different case studies are performed which are inter-related with each other to showcase the entire DSSE simulation. The overall performance of the DSSE is assessed by simulating the given case studies.

Case1 — Model validation. In the first case, the reduced IEEE model is validated with the standard IEEE 906 network. A daily mode power flow simulation is run in OpenDSS with 1 minute time step to compare the reduced model with the standard network.

Case2 — Base case. In this case, the true value from the OpenDSS simulation environment, which are considered as the true values, are additionally subjected to random Gaussian noise. These measurements with noise are fed as an input in the linear LVSE which is performed on 24-hour basis with 1-minute time interval.

Case 3 — DSSE case. In this case, the traditional WLS state estimation is performed for 24 hours with 15 minutes time interval for MVSE. It also uses every 15th minute LVSE estimated values on particular buses as an MVSE inputs for 1 full day.

The above case studies are simulated on one LV network and one MV network models shown in Table II.

A. Modified IEEE 906 Low voltage reduced network

This modified IEEE 906 network is supplied by 400 kVA MV/LV transformer having *Dyn* connection. The nominal voltages of the primary and secondary windings are 11 kV and 0.4 kV. This model is an equivalent network of standard

TABLE II: Network Models for DSSE

Grid Model	Total Nodes	Total Loads	Network Type
Reduced IEEE 906 Network	12	8	Unbalanced Three-phase LV network
Modified IEEE 13 Network	13	7 MV Loads/2 LV Feeder Loads	Unbalanced Three-phase MV network

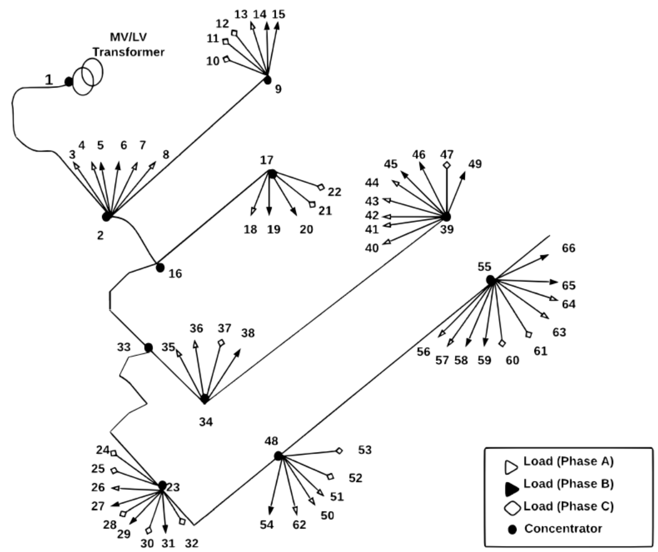

Fig. 3: Reduced IEEE 906 Network

IEEE 906 network with all the loads aggregated at various specific nodes, as shown in the Fig. 3. The algorithm used for reducing the network has already been explained in section II.B.

The reduced network consists of 12 nodes with a total number of 55 loads aggregated at specific nodes based on the topology reduction technique. The line configuration and the time-series load data are kept the same as the IEEE 906 network. The time series is with 1440 time steps, each covering active and reactive power for different loads having 1 minute time interval for 24 hours [9]. In the IEEE 906 network, the reduction is done by aggregating all the lines having same line configuration and placing the aggregated loads at a mean distance on the branch, as explained in the previous sections. Afterwards, with the help of OpenDSS, the three-phase unbalanced load flow is run for each aggregated node, and the load flow results for the full modified network are used as the true measurements for linear state estimation. The modified IEEE 906 network consists of 8 reduced load buses and 3 no-load buses as can be seen in Fig. 3. The Standard Deviation (SD) for the voltage measurement is 0.01v along with 1W and 1var for active and reactive power injections. The no-load buses are assumed to have virtual measurements with 0 SD. After adding the Gaussian measurement noise in these measurement values, the state estimation is run in MATLAB. Table III shows the three-phase power flow output comparison between IEEE 906 network and the reduced IEEE 906 network

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in OpenDSS, which proves that reduced modified network can be used as an equivalent for the standard network for state estimation purposes.

TABLE III: Load Flow comparison

Circuit Summary	IEEE 906 Bus Network	Reduced 906 Network
Buses	907	66
Loads	55	55
Nodes	2721	198
Computational Time	7.62	1.28
Total Iterations	3	2
Max Pu. Voltage	1.0499	1.0499
Min Pu. Voltage	1.045	1.0474
Total Active Power (MW)	1.05441 x 10^{-2}	1.05613 x 10^{-2}
Total Reactive Power (Mvar)	3.4673 x 10^{-3}	3.4760 x 10^{-3}
Total Active Losses	0.2703	0.1939
Total Reactive Losses	0.00109987	0.0018786

B. Modified IEEE 13 Bus Network

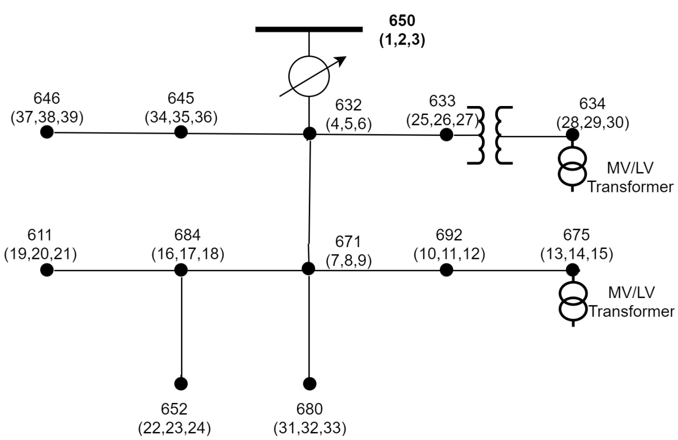

Fig. 4: Modified IEEE 13 Network with LV Feeders

The IEEE 13 bus network is modified to test the DSSE by integrating the LV feeder on particular nodes. Bus 650 is selected as the slack bus providing the reference voltage. The voltage bases are set for three voltage levels for delta loads, star loads and the load 634 on the secondary side of the XFM transformer. The transformer turns ratio at the MV/LV substation at bus 634 and 675 is neglected in the paper after using the per unit value of the LVSE output. The LVSE estimates the voltage state and the overall power injection resulting at the MV/LV substations. The power injection and the voltage state of the LV network is directly taken as the input at the MV side of the transformer neglecting the coupling effect caused amongst the phases. To use the time varying load data for running the continuous load flow, the IEEE 13 standard dataset is multiplied with suitable factors for P and Q separately. Since the load data available in the IEEE 13 bus document is for the peak load, it is multiplied by a factor less than 1 to distribute it for 24-hour dataset.

C. Key Performance Indices

To assess the performance of the two-layer estimation technique, three different performance indices are proposed in the paper.

The error between the estimated state and the true measurement for a time span of 24 hours in LV network is calculated as an Absolute Percentage Error (APE).

$$APE = \left| \frac{E_n(t) - \hat{E}_n(t)}{E_n(t)} \right| \times 100 \quad \forall n = 1, \cdots\cdots N_{bus} \quad (18)$$

Where E_n is the true measurement and the $\hat{E}_n$ is the estimated value for all the buses (N_{bus}) in the network.

For considering the accuracy of all the nodes of the LV network, the maximum error occurring in each bus for a total time span of 24 hour with 1 minute time step is calculated an the maximum Mean Absolute Percentage Error (MAPE)

$$Max\ MAPE = \frac{1}{T} \sum_{t=1}^{T} \left| \frac{E_n(t) - \hat{E}_n(t)}{E_n(t)} \right| \times 100 \qquad \forall n = 1, \cdots N_{bus} \quad (19)$$

The impact of integrating LV network outputs in the MVSE is been investigated by estimating the accuracy percentage of DSSE. The estimator performance in each case for 24-hour time span is been formulated using Mean Absolute Percentage Error (MAPE) averaged over a 1 full day.

Therefore, for each node n of the IEEE 13 bus network and at each 15-minute time step t, the estimated error between the daily load flow measurement and the estimated state is calculated as MAPE.

$$MAPE = \frac{1}{N} \sum_{N=1}^{N} \frac{1}{T} \sum_{t=1}^{T} \left| \frac{E_n(t) - \hat{E}_n(t)}{E(t)} \right| \times 100 \quad (20)$$

Where E_n is the true measurement and $\hat{E}_n$ is the estimated value for each node N and each time step t. Here T is the total number of time steps for 24-hour day with 15 minutes time interval as explained above and N is the total number of buses in the network.

D. Comparison to Conventional Estimation

The majority of the papers dealing with LVSE do not implement their proposed algorithm on the standard IEEE 906 European network because of the limitations of the standard network, as explained in the previous sections. These papers have assumptions such as requiring a balanced network, taking synchronised smart meter measurements, or considering 100% smart meter availability on all LV load nodes, which are not practical for LVSE. Therefore, a direct comparison from other papers implementing LVSE is not feasible as the algorithms used in these published papers are implemented on different, often synthetic network model with different network properties as explained in Section I. However, in [4], the MVSE takes the aggregated smart meter data from the IEEE 906 network, and uses this as the input measurement for MVSE. Therefore, a comparison has been demonstrated in the next section, which compares the performance, efficiency, and computational time difference between the proposed technique and [4]. To give a better insight, the performance of the proposed method

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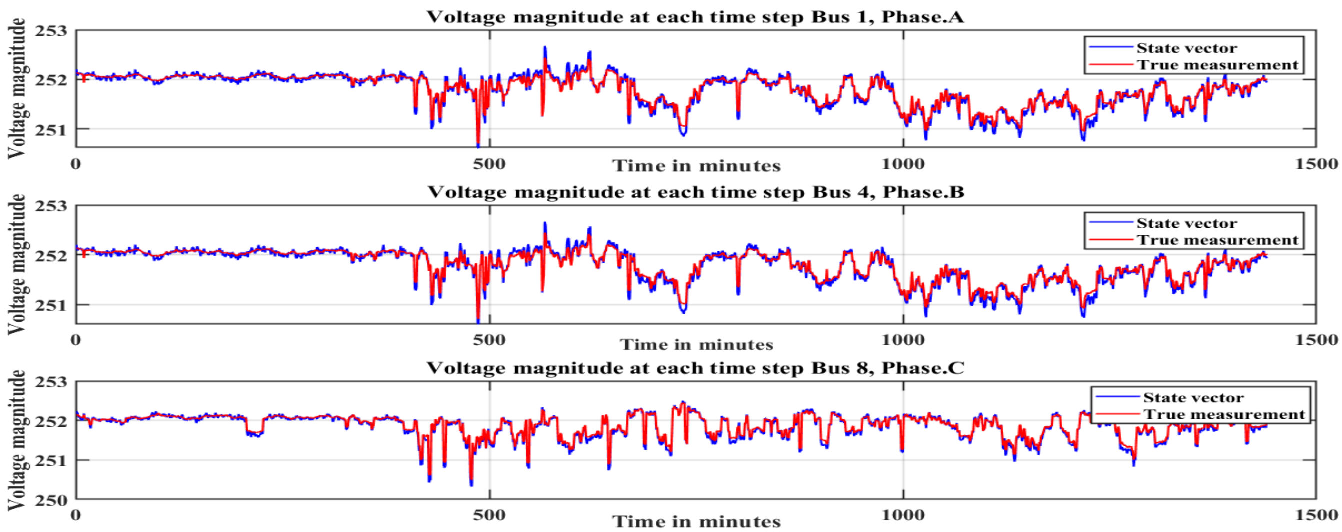

Fig. 5: Voltage Magnitude for three phase at different nodes

is compared with the benchmark smart meter aggregation technique by calculating the DSSE error using equation (21)

$$\varepsilon = \sqrt{\sum_{i=1}^{n} |(v_i^{est} - v_i^t)|^2} \tag{21}$$

Where ε is the estimated error defined over all the bus voltages with n number of buses , v_i^{est} is estimated voltage of the ith bus and v_i^t is the true bus voltage. Since the measurement noise is assumed to be random, the DSSE error simulation has been run over 1000 times. Each voltage variable in this equation is represented in per unit.

Also, the evaluation metrics comparison are being made by comparing the maximum absolute relative voltage magnitude error $|RVME|_{max}$ and the maximum absolute relative voltage angle error $|RVAE|_{max}$ of the proposed technique and the smart meter aggregated pseudo measurement DSSE for each bus for 24 hours with 15 minutes time interval, i.e. 96 time instants [38].

$$\begin{aligned} |RVME|_{max} &= max\ |v_i^j - \hat{v}_i^j|/v_i^j \\ |RVAE|_{max} &= max\ |\theta_i^j - \hat{\theta}_i^j| \end{aligned} \tag{22}$$

Where v_i^j and θ_i^j are the true voltage magnitude and voltage angle at ith bus and jth time instant, $\hat{v}_i^j$ and $\hat{\theta}_i^j$ are the estimated voltage magnitude and voltage angle at ith bus and jth time instant for MV bus network. The performance of DSSE is considered satisfactory when the maximum RVME and RVAE error of more than 95% of cases (each bus with 96 time instants) are under 0.7% and 0.7 crad (1 crad=0.01 rad) [38].

IV. Results and discussion

A. Reduced IEEE 906 Network

The time series load flow is run on the reduced topology IEEE 906 network in an open source simulator, OpenDSS. The integrated aggregated load and linear state estimation algorithm is applied to estimate the operating state of the LV network. The estimated voltage magnitude at each node is compared to the true measurement derived from the time-series load flow. In Fig 5, the true measurement and state estimated

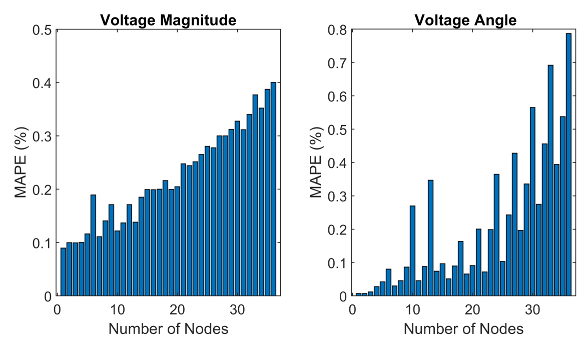

Fig. 6: Maximum MAPE for voltage magnitude and voltage angle at LV network

values are compared at three different buses for 24 hours with 1-minute time interval. The Absolute Percentage Error (APE) in equation (18) for the three phases of Fig 5 varies from 0.01% to 0.03% in 24 hour time interval depending upon the location of the bus in the radial network.

Fig. 6 illustrates the bar plots for maximum MAPE distribution of the estimated voltage magnitude and estimated voltage angle for all the three phases of the bus network. The plot for the voltage magnitude shows that the maximum MAPE value is approximately 0.4% at the last node downstream of the LV radial network. Similarly, for the voltage angle, the maximum MAPE value is around 0.8%.

B. Medium Voltage Network Testbed

In the case of the modified IEEE 13 bus network, Fig. 7 illustrates a 3D MAPE plot which shows the distribution of the estimated three phase voltage magnitude error of all the 13 bus over a 24-hour period. In Fig. 7 and Fig 8, the x axis represents the number of buses with all the three phases, y axis represents the 24 hour time period with 15 minutes time-step and z axis represents the MAPE in percentage. As it can be seen in the plot, the maximum mean MAPE error of 0.038% is at bus 675 (node-13,14,15). Also, from Fig 8, the maximum

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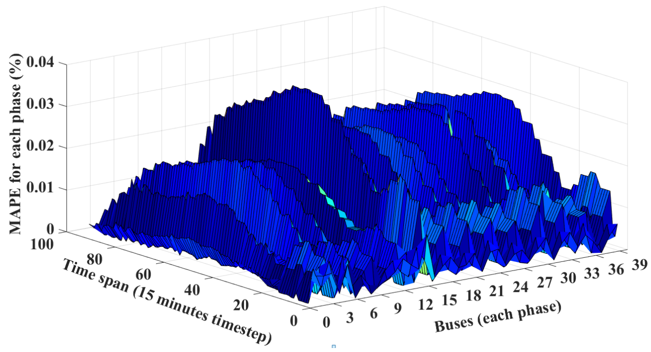

Fig. 7: Mean MAPE of IEEE 13 MV network for voltage magnitude at different time step

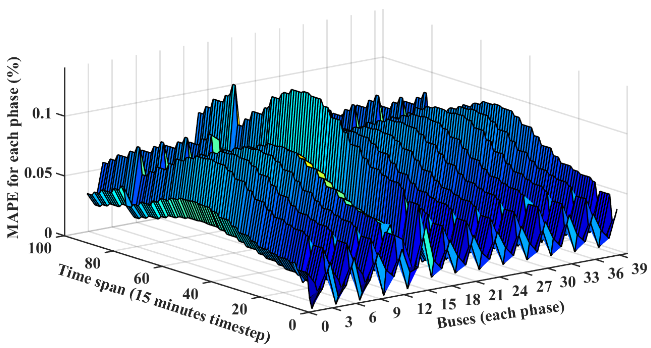

Fig. 8: Mean MAPE of IEEE 13 MV network for voltage angle at different time step

error of 0.1% can be seen at bus 675 for voltage angle. The reason for this maximum error at bus 675 is due to its distance from the slack bus. For the case of time series, as it can be seen from the y axis in the Fig. 7 and Fig 8, it can be seen that the MAPE varies from 0.001% to a maximum of 0.035% in voltage magnitude and in voltage angle from 0.001% to 0.1%, between the time-span 00:00 to 23:45 hours. Fig. 7 and Fig 8 show that the time series error increases in middle of the day time and starts to decrease in the later part of the day as the load on the network reduces.

C. Accuracy Comparison

For analysing the accuracy of this novel technique, a comparison has been made with the centralised WLS technique containing aggregated LV smart meter data as referred in Section III.D. Considering the uncertainty of the computation speed and using the random measurement noise for DSSE, 1000 Monte Carlo simulations have been performed on 2.5 GHz processor with 12 GB of RAM. The maximum absolute relative voltage error, DSSE error, maximum MAPE and execution time of the whole MV/LV framework is tested and compared to the centralised WLS SE technique containing aggregated smart meter measurements [4]. The complete comparison has been shown in Table IV.

This demonstrates that the proposed DSEE outperforms already established techniques in computational efficiency and time performance while keeping the maximum MAPE almost equal. For the maximum relative error comparison, the smart

TABLE IV: Performance comparison

Performance Index	Proposed DSSE LV/MV	Centralised WLS DSSE
$\lvert RVME\rvert_{max}$	78%	97%
$\lvert RVAE\rvert_{max}$	98%	99%
Mean DSSE error(ε)	9.94	12.78
Maximum MAPE	0.038%/0.035%	0.0483%
Total Iterations	1/3	5
Computational time	92 millisec/ 2.3 sec	11.9 sec

meter aggregated technique's performance is not satisfactory as only 78% of maximum RVME lies below the minimum acceptable threshold limit of 0.7%. In comparison, 97% of RVME lies under 0.7% in the proposed DSSE technique. For RVAE, the difference is only 1% between the two compared techniques which falls below the 0.7crads limits. Also, when mean DSSE is considered, as one can see, the proposed approach improves the accuracy of DSSE by an average of 25%. These results suggest that the proposed DSSE framework is a valid solution to providing accurate estimation performance, exploiting LVSE measurements while guaranteeing sufficiently low execution times with less computational burden.

V. Conclusion

A novel two-layer state estimation technique for realistic, unbalanced MV/LV distribution systems is presented in this paper. A linear state estimation technique is developed using smart meter measurements and demonstrated using the European LV network. The novel topology reduction technique reforms the problem of using LV network, which commonly consists of a limited number of measurement meters for state estimation purposes. A linear state estimation algorithm is proposed, which solves the convergence problems in radial LV networks caused by high R/X ratios, and is computationally-efficient. The utilisation of LVSE estimated states at MV/LV transformer as MVSE inputs increases the accuracy when compared with previous pseudo-measurement based approaches. The LVSE output at feeder buses of MV network also reduces the use of high-cost MV meters for monitoring purposes. The maximum MAPE error of estimated voltage magnitude and the estimated voltage angle at the end of the feeder, along with the time series error variation depending upon the load consumption pattern validates the accuracy of the two-layer state estimation algorithm. A comparison has been made with already established techniques to showcase the efficiency and performance of the proposed work.

Based on the results obtained, future work will focus on implementing the effects of the MV/LV transformer on LVSE estimated states used in the MVSE algorithm, and the accuracy of MVSE will be studied after replacing all the MV meters on each bus of the network with LVSE feeder measurements.

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