Within-host competition can prevent the invasion of rare parasites

SUPPLEMENTARY INFORMATION

Classic SI Model

Equations 5.1 from the main manuscript are shown below:

$$\begin{aligned}\frac{dS}{dt} &= b(S+I_w) - S(\lambda_w+\mu) + cI_w \\ \frac{dI_w}{dt} &= S\lambda_w - (\mu+v+c)I_w\end{aligned} \qquad [5.1]$$

Where S refers to numbers of susceptible hosts, and I_w refers to number of hosts infected with a wild-type pathogen. b= host birth rate, μ= host death rate, v= extra mortality of host caused by parasite infection, c= rate of parasite clearance from the host, and β= parasite transmission rate. λ_w is the force of infection of the wild-type parasite λ_w (= βI_w,). Setting the rate of change to 0 i.e. dS/dt=0 and dI_w/dt=0, and solving for of S and I_w respectively gives the co-existence equilibrium conditions shown in equations 5.2 from the main manuscript.

$$\begin{aligned}S^* &= \frac{\mu+v+c}{\beta} \\ I_w^* &= \frac{(b-\mu)(\mu+v+c)}{\beta(\mu+v-b)}\end{aligned} \qquad [5.2]$$

Substituting these values back into equations 5.1, we can construct a Jacobian matrix, which looks at how each differential equation changes as each variable changes, and can be used to test the stability of this co-existence equilibrium (Otto & Day, 2011). If the eigenvalues of this matrix have negative real parts, the system is asymptotically stable (i.e. the equilibrium will be restored after slight perturbations). The Routh-

Herwitz criteria (Britton, 2003) can be used to test this condition, which specifies that if the trace of a Jacobian matrix is negative and its determinant is positive, its eigenvalues will have negative real parts. This condition is satisfied when:

$$\mu < b < \mu + v \qquad \text{Condition 5.3}$$

(Condition 5.3 in main manuscript). If these conditions hold, the system is locally asymptotically stable.

Introducing Rare Mutant

Following the description in the main manuscript, the full system is described by the following set of differential equations (5.5 in main manuscript):

$$\begin{aligned}
\frac{dS}{dt} &= bS - S(\lambda_w + \lambda_m + \mu) + c(I_w + I_m) \\
\frac{dI_w}{dt} &= S\lambda_w - I_w(\mu + v + c + \lambda_m) + cI_{wm} \qquad [5.5] \\
\frac{dI_m}{dt} &= S\lambda_m - I_m(\mu + v + c + \lambda_w) + cI_{wm} \\
\frac{dI_{wm}}{dt} &= I_w\lambda_m + I_m\lambda_w - I_{wm}(2c + \mu + v)
\end{aligned}$$

Assuming that the mutant is rare, values of I_m and I_{wm} can be set to 0 in the above equations. Constructing a Jacobian matrix of the augmented system produces a block triangular matrix. The eigenvalues of a block triangular matrix are given by the eigenvalues of the blocks on the diagonal (Britton, 2003). We have already demonstrated that the eigenvalues of the upper left block have strictly negative real parts i.e. the eigenvalues from the Classic SI equilibrium (above) are negative when condition A is satisfied. For the resident S and I_w equilibrium to be stable against invasion from a rare I_m, then the eigenvalues of the bottom right block must also be negative. The stability of this bottom right block is determined by two eigenvalues. One always remains negative, whilst the other eigenvalue can vary between positive or negative, depending on the parameter values. This eigenvalue therefore determines stability. We can take this eigenvalue and ask under what conditions is it positive i.e. under what conditions *can* the mutant invade?

$$\frac{\beta_{wm}}{\beta} > \frac{(v+\mu)^2}{(c+\mu+v)(b+2v+\mu)} \qquad \text{Condition 5.6}$$

Condition 5.6 (from the main manuscript) must be satisfied for the eigenvalue to be positive i.e. when condition 5.6 is true, the augmented system becomes unstable and the rare mutant can invade.

Model Variations

The result we derive is true for system described by equations 5.5. Variations of the model give qualitatively similar results. For example, if we assume that all classes of individuals give birth to susceptible hosts, rather than just susceptibles, the full system is now described by the following set of differential equations:

$$\begin{aligned}
\frac{dS}{dt} &= b(S+I_w+I_m+I_{wm}) - S(\lambda_w+\lambda_m+\mu) + c(I_w+I_m) \\
\frac{dI_w}{dt} &= S\lambda_w - I_w(\mu+v+c+\lambda_m) + cI_{wm} \qquad \text{[SI 1]} \\
\frac{dI_m}{dt} &= S\lambda_m - I_m(\mu+v+c+\lambda_w) + cI_{wm} \\
\frac{dI_{wm}}{dt} &= I_w\lambda_m + I_m\lambda_w - I_{wm}(2c+\mu+v)
\end{aligned}$$

Following the same procedure as described in the previous section, the condition that must be satisfied for the rare mutant to invade becomes:

$$\frac{\beta_{wm}}{\beta} > \frac{(b-v-\mu)(v+\mu)}{(b-2v-\mu)(c+v+\mu)} \qquad \text{Condition SI 2}$$

Modifying equations 5.5 such that the doubly-infected host has a recovery rate of *c* instead of 2*c*, the full system is now described by the following set of differential equations:

$$\begin{aligned}
\frac{dS}{dt} &= bS - S(\lambda_w + \lambda_m + \mu) + c(I_w + I_m) \\
\frac{dI_w}{dt} &= S\lambda_w - I_w(\mu + v + c + \lambda_m) + cI_{wm} \qquad \text{[SI 3]} \\
\frac{dI_m}{dt} &= S\lambda_m - I_m(\mu + v + c + \lambda_w) + cI_{wm} \\
\frac{dI_{wm}}{dt} &= I_w\lambda_m + I_m\lambda_w - I_{wm}(c + \mu + v)
\end{aligned}$$

Two conditions must now be satisfied for the mutant to invade:

$$\frac{\beta_{wm}}{\beta} > \frac{v+\mu}{b+2v+\mu} \qquad \text{Condition SI 4.1}$$

and

$$c > \frac{(v+\mu)(v(\beta-2\beta_{wm}) + \beta\mu - \beta_{wm}(b+\mu))}{b\beta_{wm} + v(\beta+2\beta_{wm}) + (\beta+\beta_{wm})\mu} \qquad \text{Condition SI 4.2}$$

If we introduce density dependence on susceptibles, the full system is now described by the following set of differential equations:

$$\begin{aligned}
\frac{dS}{dt} &= S(b(1-\frac{S+I_w+I_m+I_{wm}}{K} - S(\lambda_w+\lambda_m+\mu)+c(I_w+I_m) \\
\frac{dI_w}{dt} &= S\lambda_w - I_w(\mu+v+c+\lambda_m)+cI_{wm} \\
\frac{dI_m}{dt} &= S\lambda_m - I_m(\mu+v+c+\lambda_w)+cI_{wm} \\
\frac{dI_{wm}}{dt} &= I_w\lambda_m + I_m\lambda_w - I_{wm}(2c+\mu+v)
\end{aligned} \qquad \text{[SI 5]}$$

Where *K* is the carrying capacity. The two conditions that must now be satisfied for the mutant to invade are:

$$\beta_{wm} > \frac{\beta(v+\mu)(K\beta(v+\mu)+b(c+v+\mu))}{(c+v+\mu)(K\beta(2v+\mu)+b(c+v+K\beta+\mu)} \qquad \text{Condition SI 6.1}$$

and

$$K > \frac{b(c+\mu+v)}{\beta(b-\mu)} \qquad \text{Condition SI 6.2}$$

Further variations of the model give qualitatively similar results.

References

Britton, N. F. (2003). *Essential mathematical biology*. Springer Verlag.
Otto, S. P., & Day, T. (2011). *A Biologist's Guide to Mathematical Modeling in Ecology and Evolution*. Princeton University Press