# Modeling acousto-optical tomography with optical phase conjugation using Transmission Matrix formalism

**Michael Raju**[1,2], **Jacqueline Gunther**[1] **and Stefan Andersson-Engels**[1,2]

[1] *Tyndall National Institute, Lee Maltings, Dyke Parade, Cork, T12 R5CP, Ireland*
[2] *Department of Physics, University College Cork, Cork, T12 YN60, Ireland*
*michael.raju@tyndall.ie*

**Abstract:** A 2D electromagnetic wave transport modeling with optical phase information for simulating acousto-optical tomography is formulated using T-Matrix (translation matrix) and transmission matrix formalisms by cascading slabs of periodic collections of infinite cylinders.

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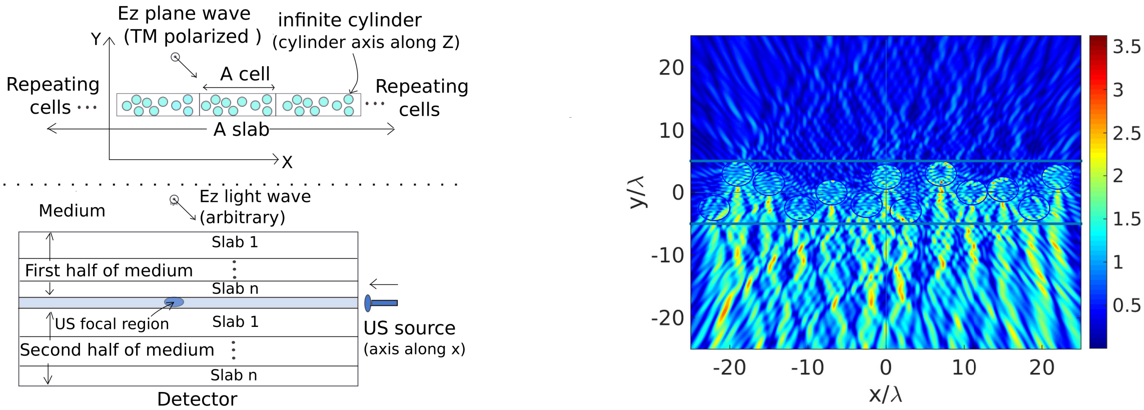
## 1. Introduction

Acousto-optical tomography (AOT) is based on acousto-optical modulation technique where a focused beam of ultrasound (US) is used to modulate a coherent optical wave propagating inside the tissue, resulting in the generation of Doppler shifted light [1]. Optical phase conjugation is able to refocus the Doppler shifted light back into the US focal zone [2] which requires the estimation of its optical phase. Conventional light transport models used in the context of AOT [3] are not well suited for understanding the optical phase conjugation problem as these models implement light diffusion theory which does not account optical phase. Although there exist Monte Carlo models associating optical phase to the photon to obtain the interference phenomena from multiple scattering, there are several criticisms to the approach [4]. Hence to implement the modeling for acousto-optical modulation phenomena in turbid media together with optical phase conjugation, we propose the use of T-matrix (translation matrix), scattering and transmission matrix formalisms for electromagnetic wave scattering of collections infinite cylinders as explained in the following section.

## 2. Method

Turbid media is approximated as a collection of infinite dielectric cylinders thereby considering this as a 2D electromagnetic scattering problem for the TM (Transverse Magnetic) polarized incident electric field. Inspired from Katz et al [5], the turbid media is divided into two regions linked together by an US interaction plane (shown in Fig 1a). Each half region represents a collection of slabs containing infinite dielectric cylinders with periodically repeating cells. Solution to the electromagnetic scattering problem for a collection of periodically repeating cells forming a slab is well known as a T-matrix solution implemented using cylindrical harmonic functions [6]. Computational frameworks developed by Jin et al [7, 8] are found to be particularly useful for us in estimating the Scattering matrix ($S$ matrix) of these slabs. In order to estimate the response of the first half of the medium to an incident wave, $S$ matrices of slabs enclosed need to be cascaded to evaluate the effective $S$ matrix for the first half. As the effective $S$ matrix has a block structure, the $S_{21}$ portion of it (otherwise called as transmission matrix) could undergo a singular value decomposition (SVD). Performing SVD on transmission matrix yield the eigenvalue distribution and the singular vectors. Eigenvalue distribution of the transmission matrix of the first half estimates transmission coefficients of various optical modes possibly reaching the thin US interaction plane. The US scattering matrix ($S_{US}$ matrix) defined in between the two half regions is a frequency dependent matrix which effectively selects the $E$ field from US focal zone for modulation. The efficiency of the US modulation at the focal zone depends on the various factors such as piezo-optic coefficient, optical properties, and US amplitude [3]. $S_{US}$ matrix tailors the incident wave for the second half which eventually propagates out of the medium. The second half contains non-absorbing dielectric cylinders and hence its effective $S$ matrix has time reversal and reciprocal symmetries. Therefore optical phase conjugation can be performed by defining the phase conjugate of the wave exiting out of the medium and using the same effective scattering matrix of the second half for focussing the light back to the US focal zone.




(a) (b)

Fig. 1: (a) Structure of the media (b) Modulus of the scattered (outside cylinder) and transmitted (inside cylinder) wave ($|E_z(x,y)|$) for a cell of non-absorbing dielectric cylinders with normal incidence of a TM plane wave ($\lambda = 632.8$ $nm$). All cylinders have the same relative dielectric constant $\varepsilon_r = 3$ and relative permeability $\mu_r = 1$, surrounded by medium with $\varepsilon_0 = 1$ and $\mu_0 = 1$. Incident wave is not shown.

## 3. Results

The problem is formulated as given in the method section for the structure explained in Fig 1a. Fig 1b gives the simulation results for a cell defined in Fig 1a. We plan to estimate the $S$ matrix for a slab made out of this cell and $S_{US}$ matrix for cascading these layers to estimate the effective $S$ matrix of the entire media. Once the effective $S$ matrix for the medium is known, optical phase conjugation can be implemented using it. Relationship between effective $S$ matrix, $S_{US}$ matrix, transmission matrix and its eigenvalue distribution quantifies the propagation of transmission modes in the sample through the US focus. Relationship of this framework with that of Random Matrix Theory (RMT) will be explored in future.

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