University College Cork, Ireland
Coláiste na hOllscoile Corcaigh

Supporting Information: Jones-Todd, C. M., E. Pirotta, J. W. Durban, D. E. Claridge, R. W. Baird, E. A. Falcone, G. S. Schorr, S. Watwood, and L. Thomas. Discrete-space continuous-time models of marine mammal exposure to Navy sonar. *Ecological Applications.*

Appendix S4. Likelihood

This section derives the negative log-likelihood discussed in Section Discrete-space continuous-time Markov model. The transition probability matrix is given by $\mathbf{P}(t)$, where each element $p_{rs}(t)$ is the probability that, given an individual is currently in state r, they will be in state s at time t in the future. This transition probability matrix can be calculated by taking the matrix exponential of the scaled transition intensity matrix as follows:

$$\mathbf{P}(t) = \text{Exp}(t\mathbf{Q}).$$

The likelihood, $L(\mathbf{Q})$, is calculated as the product, over all individuals and all transitions, of the probabilities that individual k is in state $S(t_{j+1})$ at time t_{j+1} given they were in state $S(t_j)$ at time t_j, evaluated at time $t_{j+1} - t_j$ (for $j = 1, ..., n_k$):

$$L(\boldsymbol{Q}) = \prod_{kj} L_{kj} = \prod_{kj} p_{S(t_j)S(t_{j+1})}(t_{j+1} - t_j).$$

Parameter estimates are obtained via minimization of the negative log-likelihood, $-\log(L(\mathbf{Q}))$.