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# FAILURE PROBABILITY ESTIMATION OF LONG PIPELINE BY SEGMENT-DEPENDENT-RELIABILITY MODEL

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**Key Words:** Pipeline, Large-scale series system, System strength distribution, System failure probability, reliability

**Abstract:** For the purpose of probabilistic risk assessment of long pipeline, pipeline was treated as a large-scale series system composed of a great number of segments. First of all, pipeline strength distribution was described with respect to different segment partitions. Then, the s-dependence among segment failures was expounded. Finally, a series system failure probability model was presented and the size-dependent variation of system failure probability was demonstrated. It was shown that there exists strong dependence among segment failures, and there exist an upper bound for the failure probability of a large-scale series system such as long pipeline.

## 1. Introduction

It is well known that, for the majority of pressurized pipelines, the load and resistance parameters have important uncertainties, and probabilistic approach should be applied to assess their behaviors. Concerning reliability estimation of passive components such as pipeline, there are two kinds of approaches - direct estimation using statistics of historical failure event data, and indirect estimation using probabilistic analysis of the failure phenomena of consideration. No matter which is applied, the statistical dependence among component failures has to be correctly treated.

Besides, estimating pipeline failure probability by traditional reliability engineering and statistical analysis principles is complicated also because no generally applicable component partition approach exists [1,2].

Asymptotic approach was also applied to evaluate large-scale system failure probability [3-6]. Mathematically, such an approach is based on the limit theorems of order statistics distribution. However, the s-dependences among component failures may violate the s-independent failure assumption underlying the limit theorems of order statistics distribution.

## 2. Segment partition and system strength distribution

One of the acceptable methods for modeling a run of a pipe in probabilistic risk assessment is to divide the pipe run into segments [7,8].

Pipeline is taken as a series system comprising of a great number of virtual elements in the present paper. The strengths of the individual components, denoted by  $X_i$  ( $i=1\sim n$ ), are independent and identically distributed random variables. Thus, the strength of the weakest link system, denoted by  $X$ , is equal to the smallest component strength in statistical sense, i.e.

$$X=\min(X_1, X_2, \dots X_n) \quad (1)$$

It indicates that the system strength equals to the smallest sample value of the  $n$  component strengths  $X_i$  ( $i=1, 2, \dots, n$ ). Obviously, it is more likely to achieve low system strength  $X$  if the component numbers  $n$  is large.

In order to estimate pipeline failure probability, firstly we divide the pipeline into  $n$  segments (virtual

components). Furthermore, it is assumed that the strengths of the  $n$  segments are i.i.d random variables. Then, in the condition that all the segment failures are independent of each other, the relationship between pipeline (a series system) failure probability  $P_n$  and its segment failure probability  $p$  can be expressed as:

$$P_n = 1 - (1 - p)^n \quad (2)$$

Consider a different partition to the pipeline by using larger segment, e.g., let  $k$  segments mentioned above make up one larger segment, the pipeline can be considered as composed of  $m$  ( $m = n/k$ ) larger segments. Obviously, the failure probability of the larger segment equals:

$$p_k = 1 - (1 - p)^k \quad (3)$$

Then the relationship between pipeline failure probability and segment failure probability is:

$$P_m = 1 - (1 - p_k)^m = 1 - (1 - p)^{k \times m} = 1 - (1 - p)^n \quad (4)$$

It turns out that the partition to the pipeline does not influence the estimation of its failure probability in the condition of s-independent failures.

As a system made up of statistically identical segments, the segments can be taken as the samples coming from the same population, and the strengths  $X_1, X_2, \dots, X_n$  of the segments in the system are i.i.d random variables. The order statistic  $X_{(k)}$  stands for the strength of the  $k^{th}$  weakest segment. Obviously, the strength of the series system is equivalent to the minimum order statistic  $X_{(1)}$ . According to probability theory [9], the cumulative probability function of the minimum order statistic  $X_{(1)}$  is:

$$G_1(x) = 1 - (1 - F(x))^n \quad (5)$$

Where,  $G_1(x)$  and  $F(x)$  are the cumulative distribution function (cdf) of the minimum order statistic  $X_{(1)}$  and that of the population  $X$ , respectively.

The probability density function of the minimum order statistic  $X_{(1)}$ , which is just the probability density function (pdf) of the series system, is:

$$g_1(x) = n[1 - F(x)]^{n-1} f(x) \quad (6)$$

Where,  $g_1(x)$  and  $f(x)$  are the pdf of the minimum order statistic and that of the population, respectively.

Considering a different partition of  $m$  ( $m = n/k$ ) larger segments, the cdf and pdf of the larger segment strength are respectively:

$$H_1(x) = 1 - (1 - F(x))^m \quad (7)$$

$$h_1(x) = m[1 - F(x)]^{m-1} f(x) \quad (8)$$

Therefore,

$$g_1(x) = n[1 - F(x)]^{n-1} f(x) \quad (9)$$

This proved that the segment partition scheme does not influence the system strength distribution. Nevertheless, one should remember in mind that such a conclusion is only hold true with the underlying precondition that the pipe material is continuous (no defect) and its strength is uniform along the length.

### 3. Pipeline failure probability estimation and failure dependence analysis

It is well known that the conventional assumption "component failures are s-independent of each other in a system" is not usually valid since common cause failure (CCF in short) exists in the majority of systems [10-14]. Subsequently, failure dependence has to be taken into account when estimate system failure probability.

By means of order statistics, we can develop a pipeline (series system) failure probability model without

any assumption on failure dependence. As mentioned above, pipeline can be taken as an  $n$ -segment series system, and its failure probability is equal to the probability that system strength (i.e. the minimum order statistic of segment strengths) is less than the load. That is, the failure probability of a series system made up of  $n$  segments equals to

$$P_s^{seri} = \int_0^\infty h(y) \left[ \int_0^y g_1(x) dx \right] dy \quad (10)$$

where,  $h(y)$  is the pdf of the load subjected to the segments.

#### 4. The upper limit of large-scale series system failure probability

In the following, discussed is the effect of component numbers on system failure probability. For a system such as pipeline, if the size of the segment is small, then the numbers of segment in the system will be very large. According to conventional s-independent series system failure probability model, system failure probability will approach to one quickly with the increase of component numbers. It means that for a series system comprised of a huge number of components, e.g. 10000 or more, its failure probability will approach to one, even though the individual component failure probability is very low. Obviously, the estimated failure probability (closed to one) is not reasonable even for a very long pipeline. Fortunately, different conclusions can be drawn with s-dependent system failure probability model.

For example, let load pdf  $h(y) \sim N(300, 50)$  (i.e. the load  $y$  follows the normal distribution with the expectation of 300 and the std of 50), segment strength pdf  $f(x) \sim N(600, 50)$ , which yield segment failure probability of  $1.105 \times 10^{-5}$ , the relationship between the failure probability of series system and segment numbers is shown in Fig.1. It shows that system failure probability increases with the increase of segment numbers, the system failure probability estimated by the conventional

s-independent system model approaches to one quickly, but that estimated by the s-dependent system model does not show such a strong tendency.

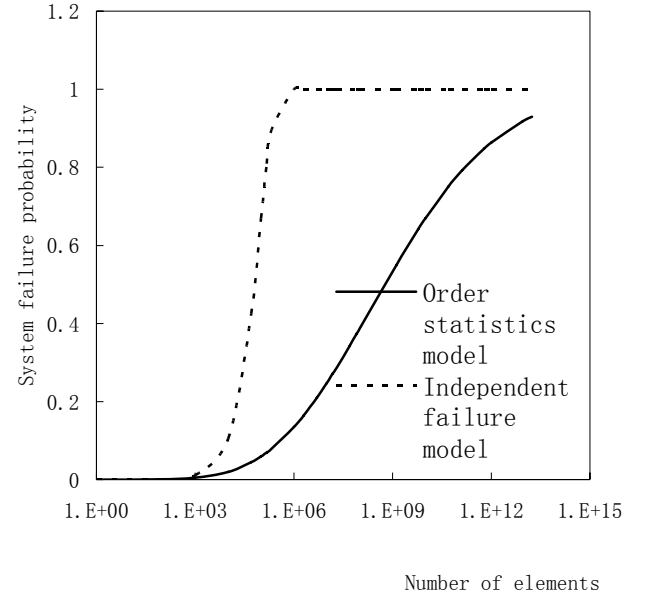


Fig.1 The relationships between series system failure probability and segment numbers in condition of normal distributed load and segment strength

If load pdf  $h(y) \sim W(200, 3, 100)$  (i.e. load follows the Weibull distribution of which the minimum load equals to 100), and segment strength pdf  $f(x) \sim W(300, 3.5, 420)$  (i.e. segment strength follows the Weibull distribution of which the minimum strength equals to 420), which yield a segment failure probability of  $1.575 \times 10^{-5}$ , the relationship between series system failure probability and segment numbers is shown in Fig.2. In the situation of which the minimum of the segment strength random variable is greater the minimum of load, system failure probability will not exceed a limit of less than one with the numbers of segment approaching infinite (Fig.3). The limit equals to the probability of load random variable exceeding the minimum order statistic of segment strength. The reason is simply that, in the situation of infinite segments, the strength minimum order statistic becomes into a deterministic constant which is equal to the minimum strength parameter of the Weibull-distributed random

variable.

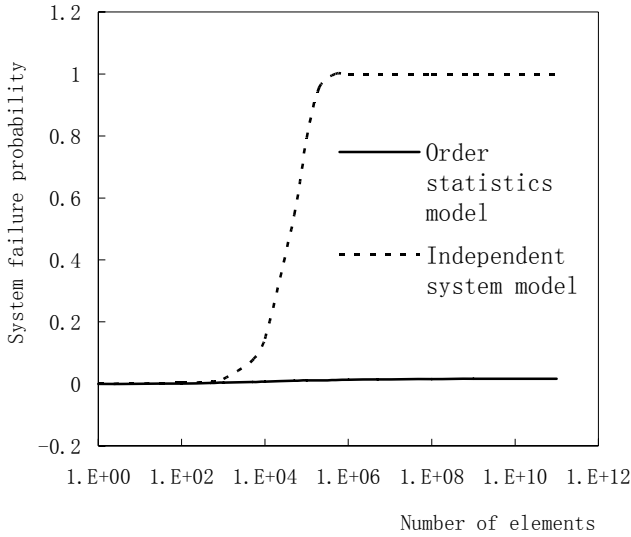


Fig.2 The relationships between series system failure probability and segment numbers in condition of Weibull distributed load and segment strength

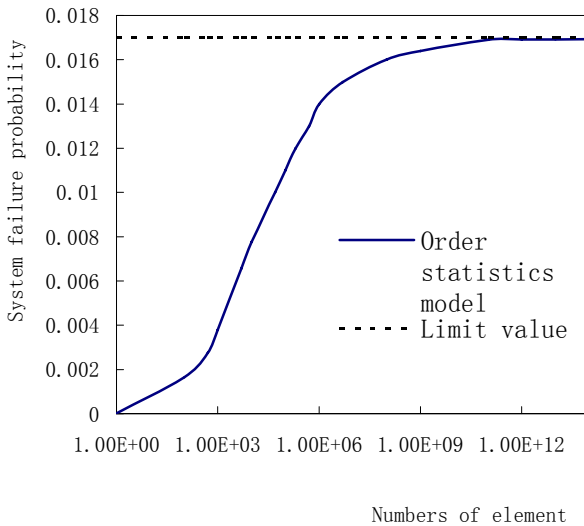


Fig.3 The upper limit of large-scale series system failure probability in the condition of Weibull distributed load and segment strength (the upper limit = 0.017)

## 5. Conclusion

Aimed at failure probability estimation of pipeline type continuous system, the present paper discussed the issues on segment partition including segment size selection, relationship between segment strength and its size, relationship between system strength and segment strength/numbers, failure probability model for s-dependent series system, and limit of large-scale series system failure probability.

The investigation showed that segment size/numbers selection does not normally affect system failure probability estimation. In situation that segment strength distribution has finite lower limit, series system failure probability has an upper limit of less than one. That is, with the increase of segment numbers, series system failure probability will not approach to one, but to a limit value much less than one, which is equal to the probability that the lower limit of the segment strength is less than load random variable.

It is also shown that there are considerable differences between the independent system failure probability model and dependent system failure probability model. It means that failure dependence plays an important role in system of which the components are subjected to the same random load. Obviously, here the load has a general meaning, it can be mechanical stress, temperature, corrosion intensity, and so on. Correspondently, the component strength will be the property against mechanical stress, temperature, corrosion, and so on.

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