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Non-Markovianity, Loschmidt echo, and criticality: A unified picture

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A simple relationship between recently proposed measures of non-Markovianity and the Loschmidt echo is established, holding for a two-level system (qubit) undergoing pure dephasing due to a coupling with a many-body environment. We show that the Loschmidt echo is intimately related to the information flowing out from and occasionally back into the system. This, in turn, determines the non-Markovianity of the reduced dynamics. In particular, we consider a central qubit coupled to a quantum Ising ring in the transverse field. In this context, the information flux between system and environment is strongly affected by the environmental criticality; the qubit dynamics is shown to be Markovian exactly and only at the critical point. Therefore, non-Markovianity is an indicator of criticality in the model considered here.

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The process of decoherence is not only of paramount importance to understand the emergence of the classical world from the microscopic domain of quantum mechanics, but it also constitutes the fundamental obstacle for the design of reliable quantum technologies [1–3]. By now, decoherence is understood to be the result of a relentless monitoring of the quantum system by an environment. In this description, a preferred set of states, known as the pointer basis, is singled out in a process known as environment-induced superselection, or einselection [4,5]. Essentially, interactions between the system and the environment lead to an entangled composite state, and phase relations between the pointer states are lost. In other words, the system is giving up information which otherwise would inhibit the emergence of classicality.

The dynamics of the reduced system state is typically described using the tools of open quantum systems theory [6]. A process in which the information flows out continuously from the system (and is, in fact, delocalized in the correlations) is known as a quantum Markov process. In such a case, the reduced state dynamics may be described using a semigroup of completely positive dynamical maps, equivalent to a quantum master equation in Lindblad form. The microscopic derivation of the master equation often requires severe approximations, such as weak interaction between system and environment, and crucially the assumption of a fast decaying temporal correlation function of the environment [7]. This approach has been remarkably successful to describe a wealth of processes in the field of quantum optics. However, when strong-coupling and structured or finite-size environments are concerned, one often encounters non-Markovian behavior, with a dynamics characterized by memory effects whereby during certain time intervals a reflux of information back to the system is observed.

The characterization of a given quantum dynamical process in terms of its non-Markovianity has recently been the subject of intense theoretical work resulting in the construction of a number of quantitative measures [8–11]. In particular, the measure by Breuer *et al*. [9], based on the trace distance, has been used to experimentally explore non-Markovianity in a linear optics setup [12], and to investigate non-Markovianity in a variety of different system-environment models, which include spin chains [13,14], biomolecules [15], and ultracold quantum gases [16].

Interacting many-body environments are highly structured and typically induce non-Markovian dynamics on systems interacting with them. Therefore, it can be difficult to obtain a full description of the reduced dynamics and of the information flux. Where pure dephasing of a qubit is concerned, one can express the reduced qubit state in terms of the Loschmidt echo [17], which gives a measure of the environmental response to the perturbation induced by the system [18,19]. The Loschmidt echo has been used extensively to explore decoherence dynamics for critical environments [20–26], where it has been shown that in the weak coupling regime, the decoherence rate is greatly enhanced.

Our aim is twofold: (i) we will study the general relationships between the Loschmidt echo, the measures of non-Markovianity, and the master equation for a qubit undergoing pure dephasing; and (ii) we will illustrate these concepts by revisiting, from the viewpoint of information flux, the hypersensitivity to environmental criticality of the decoherent dynamics of a qubit centrally coupled to a quantum spin chain in a transverse field [20]. We will express the information flow in terms of the Loschmidt echo and also use the quantum Fisher information to quantify the amount of (phase) information lost by the central spin. We will further show that the non-Markovianity measure of Breuer *et al*. can be used to pinpoint the critical value of the transverse field.

Let us consider a two-level system (qubit) undergoing a purely dephasing dynamics due to its coupling to a spin environment. Assume the initial state of the composite system to be factorized, $\rho^{\mathrm{tot}}(0) = |\phi_s\rangle\langle\phi_s| \otimes \rho^{\mathrm{env}}(0)$, with a pure system state $|\phi_s\rangle = c_g|g\rangle + c_e|e\rangle$, $|c_g|^2 + |c_e|^2 = 1$. The dynamics of the environment splits into two branches with weights depending on the state of the qubit. The

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branches are characterized by effective Hamiltonians $H_\alpha = H_{\rm env} + \langle\alpha|H_{\rm int}|\alpha\rangle$, where $\alpha = e,g$ and $H_{\rm env}$ and $H_{\rm int}$ are the environment Hamiltonian and the interaction Hamiltonian, respectively. The effective Hamiltonian for each branch includes a perturbation induced by the system in state $|\alpha\rangle$, which gives rise to a backaction on the environment. Under the action of this environment, the qubit evolves as $\rho_s(t) = |c_g|^2|g\rangle\langle g| + |c_e|^2|e\rangle\langle e| + c_g^* c_e \nu(t)|e\rangle\langle g| + {\rm H.c.}$, where $\nu(t)$ is the so-called decoherence factor. If the initial environmental state is pure, $\rho^{\rm env}(0) = |\Phi\rangle\langle\Phi|$, the decoherence factor is simply the overlap between perturbed environmental states of the two branches $\nu(t) = \langle\Phi|e^{i\hat{H}_g t}e^{-i\hat{H}_e t}|\Phi\rangle$ (we set $\hbar = 1$). Finally, the square of the decoherence factor gives a quantity known as the Loschmidt echo:

$$L(t) = |\nu(t)|^2 = |\langle\Phi|e^{i\hat{H}_g t}e^{-i\hat{H}_e t}|\Phi\rangle|^2. \tag{1}$$

The Loschmidt echo describes the stiffness of the environment with respect to the specific system perturbation, and it relates to the decoherence of the qubit in the following simple way: consider the purity of the qubit $P(t) = {\rm Tr}_s(\rho_s^2)$; for an initial equatorial spin state, this reduces to $P(t) = [1 + L(t)]/2$. When $L(t) \to 0$, then $P(t) \to \frac{1}{2}$, indicating that the qubit is maximally entangled with the environment and signaling a complete loss of coherence.

In order to explicitly connect the Loschmidt echo to information flux, we use the approach to the quantification of non-Markovianity put forward by Breuer *et al.*, which is based on the information flow between a system and its environment [9]. This, in turn, is given by the rate of change of the trace distance between different system states. For a pair of system states $\rho_{1,2}$, the trace distance is $D(\rho_1,\rho_2) = {\rm Tr}|\rho_1 - \rho_2|/2$. It yields a natural metric on the state space, invariant under unitary transformations and not increasing under dynamical (completely positive and trace-preserving) maps, and tells us how distinguishable the two states are. In a Markovian process, the distinguishability between any two quantum states decreases monotonously, indicating a loss of information, whereas a non-Markovian process is characterized by its growth for some time interval in which information flows back to the system. In terms of the rate of change of the distinguishability, $\sigma[t,\rho_{1,2}(0)] = dD[\rho_1(t),\rho_2(t)]/dt$, where $\rho_{1,2}(t)$ are time-evolved states, a non-Markovian process must have $\sigma > 0$ for some time interval. One can define the degree of non-Markovianity of a quantum process by optimizing over all possible pairs of input states:

$$\mathcal{N} = \max_{\rho_1,\rho_2} \int_{\sigma>0} dt\ \sigma(t,\rho_{1,2}(0)). \tag{2}$$

For the purely dephasing process, such as the one considered in this paper, the optimization is achieved with equatorial, antipodal states [27]. Interestingly, these are eigenvectors of a spin component in a direction orthogonal to that of the pointer states. The trace distance of two such states under the dephasing noise described in the previous section is $D[\rho_s^1(t),\rho_s^2(t)] = \sqrt{L(t)}$. This gives a neat expression for the measure of non-Markovianity in terms of the Loschmidt echo:

$$\mathcal{N} = \sum_n \sqrt{L(b_n)} - \sqrt{L(a_n)}, \tag{3}$$

where $[a_n,b_n]$ are the time intervals over which $L'(t) > 0$, and $L(a_n)$ and $L(b_n)$ are the local minimum and maximum, respectively, of the Loschmidt echo. We have thus arrived at a simple relationship between the Loschmidt echo and the non-Markovianity measure introduced in Ref. [9]. Intuitively, a monotonously decaying $L(t)$ is a signature of Markovian dynamics, while the presence of a time oscillation is a direct sign (and its amplitude is a quantitative measure) of information backflow.

Further insight into the information dynamics comes from the master equation for the qubit. From the form of $\rho_s(t)$, one obtains

$$\frac{d\rho_s(t)}{dt} = i\Lambda(t)[\sigma_z,\rho_s(t)] + \gamma(t)[\sigma_z\rho_s(t)\sigma_z - \rho_s(t)], \tag{4}$$

with $\gamma(t) = -L'(t)/[4L(t)]$ and $\Lambda(t) = -\phi'(t)/2$, where we use the notation $\nu(t) = |\nu(t)|e^{i\phi(t)}$. The effect of the environment is twofold: it gives rise to a time dependent Lamb shift $\Lambda(t)$ and to a time dependent decay rate $\gamma(t)$. Since the above master equation is exact, it allows us to draw a number of conclusions. It has a Lindblad-like form, but with a time dependent rate which may take temporarily negative values, occurring whenever the Loschmidt echo increases with time. When this happens, the dynamical map Φ describing the system dynamics $\rho_s(t) = \Phi(t,t_0)[\rho(t_0)]$ is nondivisible, that is, it cannot be written as a composition of two dynamical maps $\Phi(t,t_0) \neq \Phi(t,s)\Phi(s,t_0)$. This makes the dynamics of the qubit non-Markovian also according to a definition proposed in Ref. [10].

Indeed, the relation between environmental response to the system perturbation, quantified by the Loschmidt echo, and information flow can be established by using other non-Markovianity measures. In the proposal put forward in Ref. [10], one considers that the decohering qubit shares a maximally entangled state with an ancilla. By monitoring the time evolution of an entanglement measure, $E_m(t)$, one can define a non-Markovianity measure as $\mathcal{N}_E = \int_{\dot{E}_m>0} \dot{E}_m\, dt$. By choosing concurrence C as the entanglement measure, for a purely dephasing dynamics one finds that the two non-Markovianity measures exactly coincide:

$$C(t) \equiv \sqrt{L(t)} \Rightarrow \mathcal{N}_E = \int_{\dot{L}>0} \frac{d\sqrt{L(t)}}{dt}\, dt \equiv \mathcal{N}. \tag{5}$$

Yet another possibility to quantify the non-Markovian character of the time evolution is given by the Fisher information [28]. The idea is the following: assume that a phase gate $U_\theta = |g\rangle\langle g| + e^{i\theta}|e\rangle\langle e|$ is applied on the input state $|\psi_{\rm in}\rangle$ of the qubit at time t_0, before it starts interacting with the dephasing environment. After a time $t > t_0$, one can try to estimate the superimposed phase θ. The error performed in any kind of such unbiased estimations is lower bounded by the Cramér-Rao formula in terms of the quantum Fisher information, ${\rm var}(\theta,t) \geqslant 1/\mathcal{F}(t)$. The quantum Fisher information is the best possible accuracy achievable in estimating the parameter θ: it gives a quantitative description of how much the extractable (phase) information contained in the qubit has been deteriorated by the interaction with the environment.

In the absence of environment, the best choice is to select an eigenstate of σ_x as the input state. We assume the same initial qubit state even in the presence of decoherence, and we

obtain the quantum Fisher information flow in terms of the Loschmidt echo and the decay rate $\gamma(t)$ as

$$\mathcal{I}_F(t) \equiv \mathcal{F}'(t) = -4\gamma(t)\,L(t). \tag{6}$$

Since the Loschmidt echo $L(t)$ is always positive, this expression gives an intuitive relation between the quantum Fisher information flow and the decay rate: when $\gamma(t) > 0$, phase information flows out of the qubit to become nonlocal as it is recorded in the full system+environment state, while for $\gamma(t) < 0$ there is an information backflow toward the qubit. These results agree qualitatively with the information quantified in terms of distinguishability by the trace distance. To make the agreement quantitative, one could take $\tilde{D} = D(\rho_s^1(t), \rho_s^2(t))^2$ as the distance between evolved states and define a new information flow $\tilde{\sigma}(t) = \frac{d}{dt}\tilde{D}$. This newly defined flow, once optimized over the possible initial states of the system, coincides with the quantum Fisher information flow $\tilde{\sigma}(t) \equiv \mathcal{I}_F(t)$.

We now specify the discussion above to the case in which the environment is described by a one-dimensional Ising model in a transverse field [20],

$$H_{\rm env}(\lambda) = -J \sum_j \sigma_j^z \sigma_{j+1}^z + \lambda \sigma_j^x, \tag{7}$$

where J is the exchange interaction strength and λ describes the strength of the transverse field. We assume that the qubit is centrally coupled to the spin chain via the interaction term

$$H_{\rm int}(\delta) = \delta |e\rangle\langle e| \sum_j \sigma_j^x, \tag{8}$$

leading to a state-dependent transverse field strength $\lambda^* = \lambda + \delta$, where δ is the strength at which the qubit couples to the environment. By means of a Jordan-Wigner transformation, the Hamiltonians $\hat{H}_\alpha$, with $\alpha = g, e$, may be diagonalized by suitable sets of fermion creation and annihilation operators $\hat{c}_k^\alpha$ such that

$$H_\alpha = \sum_{k=1}^{N} \epsilon_k^\alpha \left(c_k^{\dagger\alpha} c_k^\alpha - \frac{1}{2} \right). \tag{9}$$

Operators c_k^α are connected to each other by means of a Bogoliubov transformation $c_k^e = \cos(\beta_k) c_k^g - i \sin(\beta_k) c_{-k}^g$. After some algebra, it can be shown that the Loschmidt echo for the Ising chain is

$$L(\lambda^*, t) = \prod_{k>0} \left[1 - \sin^2(2\beta_k) \sin^2\left(\epsilon_e^k t\right) \right], \tag{10}$$

where β_k are the Bogoliubov angles and ϵ_e^k are the single quasiparticle excitation energies of the system with central spin in the state $|e\rangle$; see Ref. [20].

For this spin environment, the exchange interaction and the external field tend to make the state rigid in orthogonal directions. When the two balance, the environment is at a critical point, where it is most susceptible to perturbations. Decoherence of the central qubit is strongest at this point [25], and the decay of the Loschmidt echo is strongly enhanced [20]. We demonstrate this effect in Fig. 1(a). The Loschmidt echo oscillates strongly outside the critical point, particularly for small values of the transverse field when the exchange interaction dominates the dynamics of the spin chain.

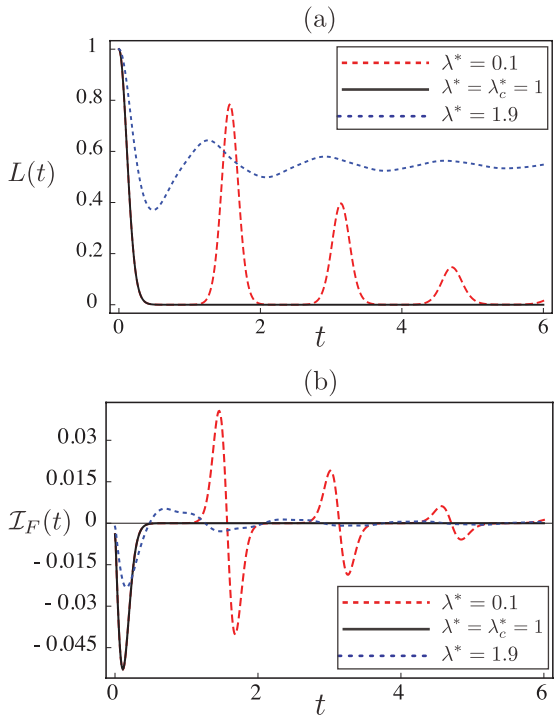


FIG. 1. (Color online) (a) The Loschmidt echo and (b) the Fisher information flow induced by the coupling of a central spin to the transverse Ising model consisting of $N = 4000$ spins for three different values of the transverse field. We choose $J = 1$ and $\delta = 0.01$. The sensitivity to the critical point $\lambda_c^* = 1$ is dramatic. Note that in (a), the solid black curve and the dashed red curve overlap for initial times.

In light of our earlier results, these oscillations correspond to highly non-Markovian qubit dynamics, characterized by many intervals of time when the direction of the information flux is reversed. This is best seen in the flow of quantum Fisher information, plotted in Fig. 1(b). As pointed out before, such behavior can be expected when a system is in contact with a structured, interacting environment such as the spin chain in a transverse field that we consider here. The information exchange is greatest when the environment is strongly interacting, that is, the effect of the transverse field is small in comparison to the spin-spin interaction.

Instead, at the critical value of the transverse field, the Loschmidt echo decays monotonically, pointing to purely Markovian dynamics of the qubit. The hypersensitivity of the Loschmidt echo to the critical point translates in a striking way to the non-Markovianity measure, which is shown in Fig. 2. Indeed, the qubit dynamics becomes Markovian exactly at the critical point, while outside the critical point we always observe non-Markovian effects.

Notice that $\mathcal{N}$ is zero for $\lambda^* = 1$ even for finite-sized spin environments (in fact, $\mathcal{N} = 0$ for any value of N). Thus, strictly speaking, $\mathcal{N}$ is not a critical quantity. However, it gives an indication of the fact that the environmental fluctuations are slowing down at criticality; the environment is becoming stiff and is not able to react on the qubit within a time window smaller than the recurrence time. This results in a monotonous decay of the Loschmidt echo and, as a consequence of the relation we have found, Markovian dynamics for the qubit.

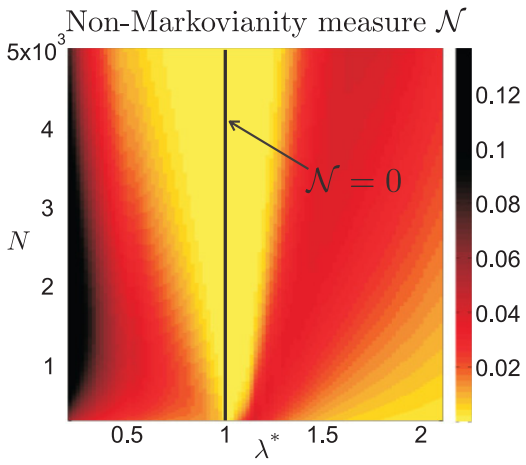


FIG. 2. (Color online) Non-Markovianity measure $\mathcal{N}$ as a function of the particle number N and the renormalized field λ^*. The time integral in $\mathcal{N}$ has been truncated to avoid the finite-size effects. A black line has been added to highlight the area where the measure is exactly zero, corresponding to purely Markovian dynamics.

This slowing down only occurs at the quantum phase transition point, and therefore, in this sense, the non-Markovianity measure $\mathcal{N}$ is an indicator of criticality.

In conclusion, our results provide a unified picture of decoherence in spin environments by connecting the Loschmidt echo to the time-dependent dephasing rate of the exact master equation, and to three different measures of non-Markovianity. This connection sheds new light on dephasing and einselection in spin environments, relating these phenomena to non-Markovianity and memory effects. The dynamics of the Loschmidt echo is shown to be directly linked to information flux between a qubit and the environment. Most notably, the connection to the Fisher information reveals how phase information about the qubit is lost, and when it can be temporarily regained.

We further explored this connection in the context of a qubit centrally coupled to an Ising spin chain and discovered that the non-Markovianity measure has a strong imprint of the quantum phase transition of the Ising model, even for a finite-sized environment. Indeed, $\mathcal{N}$ has the remarkable property of being able to pinpoint the critical value of the transverse field even away from the thermodynamic limit where the quantum phase transition truly takes place. This may be very useful in quantum simulations of the Ising model in finite systems such as trapped ions, where the number of spins is small [29].

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