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Nonlinear Analysis of a Class of Inversion-Based Compliant Cross-Spring Pivots*

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Abstract

This paper presents a nonlinear model of an inversion-based generalized cross-spring pivot (IG-CSP) using the beam constraint model (BCM), which can be employed for the geometric error analysis and the characteristic analysis of an inversion-based symmetric cross-spring pivot (IS-CSP). The load-dependent effects are classified in two ways, including the structure load-dependent effects and beam load-dependent effects, where the loading positions, geometric parameters of elastic flexures, and axial forces are the main contributing factors. The closed-form load-rotation relationships of an IS-CSP and a non-inversion-based symmetric cross-spring pivot (NIS-CSP) are derived with consideration of the three contributing factors for analyzing the load-dependent effects. The load-dependent effects of IS-CSP and NIS-CSP are compared when the loading position is fixed. The rotational stiffness of the IS-CSP or NIS-CSP can be designed to increase, decrease, or remain constant with axial forces, by regulating the balance between the loading positions and the geometric parameters. The closed-form solution of the center shift of an IS-CSP is derived. The effects of axial forces on the IS-CSP center shift are analyzed and compared with those of a NIS-CSP. Finally, based on the nonlinear analysis results of IS-CSP and NIS-CSP, two new compound symmetric cross-spring pivots are presented and analyzed via analytical and FEA models.

Keywords: Compliant cross-spring pivot; load-dependent effects; loading positions; center shift; nonlinear analysis

1. Introduction

A compliant generalized cross-spring pivot (G-CSP) consists of two flexure sheets, including the symmetric cross-spring pivot (S-CSP) [1–8] and the asymmetric cross-spring pivot (A-CSP). It can provide rotational motions with its rotational center at the intersection of the two flexure sheets. We classify the G-CSP as shown in Fig. 1.

The S-CSP has been widely studied as shown in [9–12], including the non-inversion-based (traditional) symmetric cross-spring pivot (NIS-CSP) and the inversion-based cross-spring pivot (IS-CSP). The beam constraint model (BCM) [13–18], pseudo-rigid-body model (PRBM) [6, 19–22], numerical approaches [23], and finite element analysis (FEA) are commonly used for modeling nonlinearities of such compliant mechanisms. The BCM is accurate enough when the deflection of an elastic flexure is in an intermediate range. A number of closed-form models of compliant mechanisms based on the BCM have been derived for quickly analyzing parameters and for providing design insights, such as the work by Hao et al. [24] and Zhao et al. [13]. In this paper, our analysis is based on the BCM and FEA verification. We derive the nonlinear analytical model of an inversion-based generalized cross-spring pivot (IG-CSP). This model can be used for the geometrical error analysis of the IS-CSP due to machining imperfections, and the nonlinear analytical model of an IS-CSP can be derived quickly. This is the first motivation of our paper.

Axial loads can lead to strong nonlinearities of the compliant mechanisms, motivating Zhao et al., along with other researchers, to have further analyzed the load-dependent effects of the NIS-CSP. Also the effects of the geometric parameters and bearing-direction loads on the rotational stiffness have been detailed in [25–29]. However, the loading position of bearing-direction loads is another important factor contributing to the load-dependent effects, and there are no reported papers investigating the loading-position effects on the rotational stiffness of the S-CSP, which is the second motivation of our paper.

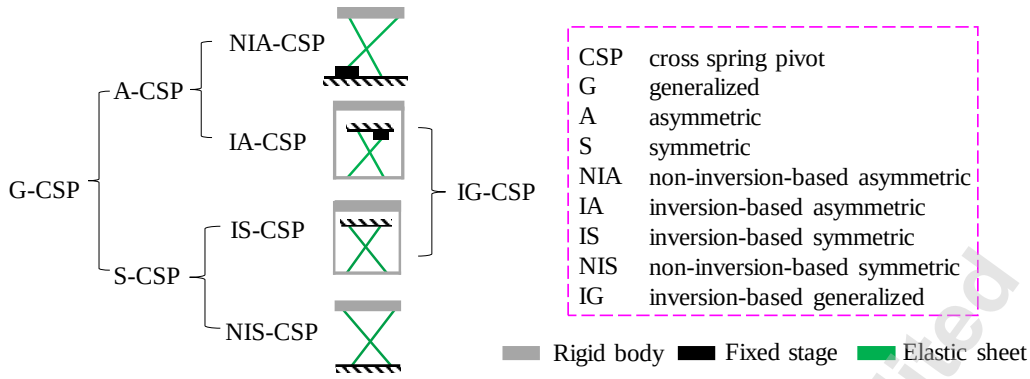


Fig. 1. The categories of the generalized cross-spring pivot.

In this paper, load-dependent effects [30] are divided into two categories, namely: 1) structure load-dependent; and 2) beam load-dependent effects. Here, the loading positions, geometric parameters of the sheets, and axial forces are three main contributing factors to such load-dependent effects, indicatively explained in Fig. 2.

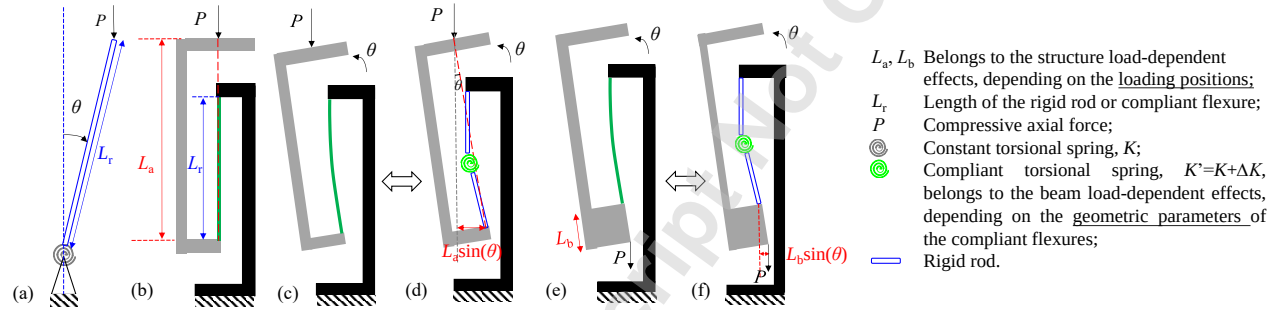


Fig. 2. Load-dependent effects: (a) a compressive load (P) acting on the rigid rod, (b) a compressive load (P) acting on an inversed compliant mechanism, (c) the inversed compliant mechanism rotating at a small angle with P on the top of the motion stage, (d) demonstrating the equivalent stiffness K_{eff} of Fig. 2(c), (e) P acting at the bottom of the motion stage, and (f) demonstrating the equivalent stiffness K_{eff} of Fig. 2(e).

The structure load-dependent effects can be explained as follows. In Fig. 2(a), a rigid rod connects with a constant torsional spring (stiffness denoted by K) on the ground, and a compressive axial force (denoted by P) acts on the free end of the rod. If the rod tilts, the effective rotational stiffness of the mechanism (denoted by K_{eff}) can be expressed as $K_{\text{eff}} = K - PL_r$ under a small angle assumption. K_{eff} decreases with the increase of P and L_r (the rod length). The beam load-dependent effects can be explained as follows. Let ΔK denote the rotational stiffness error correction term due to the beam load-dependent effects, whose absolute value depends on the geometric parameters of the flexure. When a tensile axial force acts on the free end of an elastic beam, the resulting rotational stiffness of the beam (i.e. $K' = K + \Delta K$) increases with the increase of the tensile axial force, i.e., $\Delta K > 0$; When a compressive axial force acts on the free end, K' decrease with the increase of the compressive axial force, i.e. $\Delta K < 0$ [31].

K_{eff} of the inversed compliant mechanism is a collective result of the structure and beam load-dependent effects. L_a (or L_b) and K' contribute to the structure and beam load-dependent effects, respectively. In Figs. 2(b) through (d), when P acts at the top of the motion stage of an inversed compliant mechanism, the effective rotational stiffness can

be expressed as $K_{\text{eff}} = K' - PL_a = (K + \Delta K) - PL_a$ under a small angle assumption. When $\Delta K > PL_a$, K_{eff} increases with the axial loads; when $\Delta K = PL_a$, K_{eff} keeps constant with the axial loads; when $\Delta K < PL_a$, K_{eff} decreases with the axial load. When a compressive force acts on the top of the inversed compliant mechanism, K_{eff} can increase, decrease, or be constant. Similarly, in Figs. 2(e) and (f), if P acts at the bottom of the motion stage, $K_{\text{eff}} = K' + PL_b = (K + \Delta K) + PL_b$ under a small angle assumption, which means K_{eff} can only increase with a compressive axial force. The equilibrium between structure and beam load-dependent effects of the inversed compliant mechanism is important for the stiffness control, which is the third motivation of our work.

K_{eff} decreasing to quasi-zero [32] (i.e., first-order buckling in the rotation direction) should be avoided in a compliant mechanism. When a compressive axial/bearing-direction force acts on the NIS-CSP, the rotational stiffness can increase, decrease, or be constant, which relates to its geometric parameters and the axial forces [25], but the possibility of increasing rotational stiffness is relatively low. However, when a compressive axial force acts on the IS-CSP, the possibility of increasing rotational stiffness rises, thus leading to a higher possibility of avoiding buckling, which is the fourth motivation of our paper. Therefore, by combining the NIS-CSP and IS-CSP in a parallel arrangement, we propose a novel compound S-CSP whose rotational stiffness being insensitive to the axial load (i.e. no first-order buckling), which is robust to second-order buckling in the bearing direction.

Zhao et al. [13] derived the closed-form center shift model of a NIS-CSP and designed several novel compound NIS-CSPs to reduce the center shift. They analyzed the effects of geometric parameters on the center shift and obtained the parameters combination that can produce the smallest possible shift. The effects of the axial forces on the center shift of the NIS-CSP with these special parameters are detailed in [13]. Bi et al. [27] also analyzed axial load effects of a cartwheel flexure pivot. It is worth analyzing the effects of the axial forces on the S-CSP center shift with general parameters and comparatively evaluating center shifts of NIS-CSP and IS-CSP. This is the inspiration of another novel compound S-CSP with the NIS-CSP and IS-CSP arranged in series, whose center shift is minimized significantly.

We briefly summarize these motivations as follows.

(1) The nonlinear analytical or closed-form models of an IG-CSP based on BCM have not been reported. The nonlinear models are greatly needed for analyzing the geometrical errors and quickly deriving the analytical models of an IS-CSP. For instance, such relations may be used to optimize devices such as the knee rehabilitation oriented joint presented in [33].

(2) There is a lack of investigations concerning the rotational stiffness of S-CSPs with respect to different loading positions.

(3) Problems concerning the equilibrium between structure and beam load-dependent effects, which controls the stiffness of an inversed compliant mechanism, shall be further analyzed.

(4) The analysis of loading-dependent effects and center shift of the IS-CSP and the NIS-CSP inspires us to propose two new compound S-CSPs with improved performances.

This paper is organized as below. Section 2 derives the nonlinear analytical model of the IG-CSP based on BCM, which can reduce to the closed-form model of the S-CSP. In Section 3, the load-rotation relationship of an IS-CSP is derived, and the effects of the loading positions, geometric parametric, and the axial forces on the rotational stiffness are analyzed. The equilibrium between structure and beam load-dependent effects of the IS-CSP are analyzed for

rotational stiffness regulation. The load-dependent effects of an IS-CSP and a NIS-CSP are compared. The closed-form center shift of an IS-CSP is derived, and the effects of the axial forces on the center shift of the IS-CSP are analyzed, which are also compared with those of a NIS-CSP. Section 4 presents two new compound S-CSPs, and each design consists of an IS-CSP and a NIS-CSP: design I is a parallel mechanism whereas design II is a serial mechanism. The conclusions are drawn in Section 5.

2. The Nonlinear Analysis of an IG-CSP with Two Sheets

In this section, the normalization-based analytical model of an IG-CSP is derived, followed by a closed-form load-rotation relationship of the IG-CSP under a small angle assumption. The IG-CSP can be modeled as two flexure sheets connected in a parallel arrangement. In line with [13], the analytical center-shift model of the IG-CSP is derived. The right-handed coordinate system and right-handed rule are used throughout this paper.

2.1 Normalization-Based Analytical Model

The normalized analytical model of an IG-CSP is derived as below. The two sheets in the IG-CSP are numbered as sheet 1 and sheet 2, respectively. The local coordinate systems of the two sheets are denoted as $o_1-x_1y_1z_1$ and $o_2-x_2y_2z_2$ with their origins locating at their free ends. L_i , T_i , and U_i denote the length, thickness, and width of sheet i ($i = 1$ or 2) along the x_i , y_i , and z_i -axes, respectively. $U_1=U_2$ and $T_1=T_2$ are required throughout this paper. External actual loads: F_{xi} , F_{yi} , and M_{zi} act at the origin, o_i . F_{xi} and F_{yi} denote pure forces along the x_i and y_i axes, respectively; M_{zi} denotes the pure moment about the z_i -axis. Sheet 1 with a local coordinate system $o_1-x_1y_1z_1$ is shown in Fig. 3.

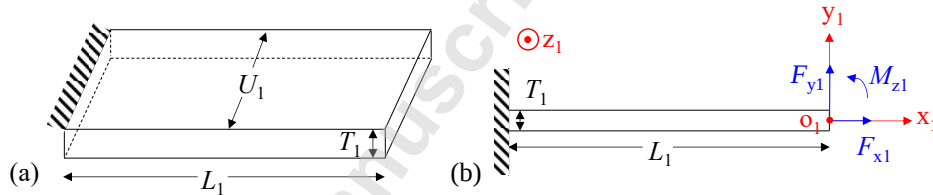


Fig. 3. The definitions of sheet 1: (a) the sheet geometry, and (b) the local coordinate system and loads acting at the free end.

Throughout this paper, we use capital symbols to denote dimensional parameters and use lower-case symbols to denote normalized (dimensionless) ones if not specified otherwise. All translational displacements and length parameters are divided by L_1 (we assume that L_1 is longer than L_2 in the IG-CSP). Forces and moments are divided by EI_{z1}/L_1^2 and EI_{z1}/L_1 , respectively [34], where I_{z1} denotes the cross-section moment of inertia about the z_1 -axis and can be expressed as $U_1T_1^3/12$, and E is Young's Modulus of the material. Therefore, for sheet i ($i=1$ or 2), we use f_{xi} and f_{yi} to denote normalized forces, m_{zi} to denote normalized moments, d_{xi} and d_{yi} to denote normalized tip displacements; θ_{zi} to denote a rotational angle. The analytical models of the two sheets are shown in Eqs. (1) and (2) based on the BCM in [31].

$$\frac{d_{xi}}{r_i} = \frac{f_{xi}}{12} \left(\frac{T}{L_1} \right)^2 + \begin{bmatrix} d_{yi}/r_i & \theta_{zi} \end{bmatrix} \begin{bmatrix} -3/5 & 1/20 \\ 1/20 & -1/15 \end{bmatrix} \begin{bmatrix} d_{yi}/r_i \\ \theta_{zi} \end{bmatrix} + r_i^2 f_{xi} \begin{bmatrix} d_{yi}/r_i & \theta_{zi} \end{bmatrix} \begin{bmatrix} 1/700 & -1/1400 \\ -1/1400 & 11/6300 \end{bmatrix} \begin{bmatrix} d_{yi}/r_i \\ \theta_{zi} \end{bmatrix} \quad (1)$$

$$\begin{bmatrix} r_i^2 f_{yi} \\ r_i m_{zi} \end{bmatrix} = \begin{bmatrix} 12 & -6 \\ -6 & 4 \end{bmatrix} \begin{bmatrix} d_{yi}/r_i \\ \theta_{zi} \end{bmatrix} + r_i^2 f_{xi} \begin{bmatrix} 6/5 & -1/10 \\ -1/10 & 2/15 \end{bmatrix} \begin{bmatrix} d_{yi}/r_i \\ \theta_{zi} \end{bmatrix} \quad (2)$$

where, $i=1$ or 2 , and $r_1=1$; $r_2=L_2/L_1$.

The IG-CSP is described in Fig. 4(a). O_s - $X_s Y_s Z_s$ denotes the mobile coordinate system of the IG-CSP, which locates at the motion stage with the Y_s -axis passing through the rotation center in the non-deformed configuration. The origin O_s also denotes the loading position. The directions of the three axes of the O_s - $X_s Y_s Z_s$ coordinate system remain constant over the motion of the IG-CSP (see Figs. 4(a) and (b)). The axes of each local mobile coordinate system (O_i - $x_i y_i z_i$) are defined as similar as the O_s - $X_s Y_s Z_s$ coordinate system.

The independent normalized parameters to define the IG-CSP include r_2 , α_1 , α_2 , λ_1 , and h . The first four ones are independent geometric parameters, and the last one h relates to the loading position with a positive or negative sign. The sign of h can be decided as shown in Fig. 4(b). When the loading position is above the free ends of the two sheets, h is positive; otherwise, h is negative. The absolute value of h is equal to H/L_1 , where H denotes the dimensional vertical distance between the free end of the sheet and the loading position. λ_2 can be derived from the equation $\lambda_1 \cos(\alpha_1) = \lambda_2 r_2 \cos(\alpha_2)$. We use f_{xs} , f_{ys} , and m_{zs} to denote the normalized loads of the IG-CSP acting at the origin, O_s , and use d_{xs} , d_{ys} , and θ_{zs} to denote the normalized displacements and rotational angle of the IG-CSP's motion stage (at point O_s) shown in Fig. 4(c).

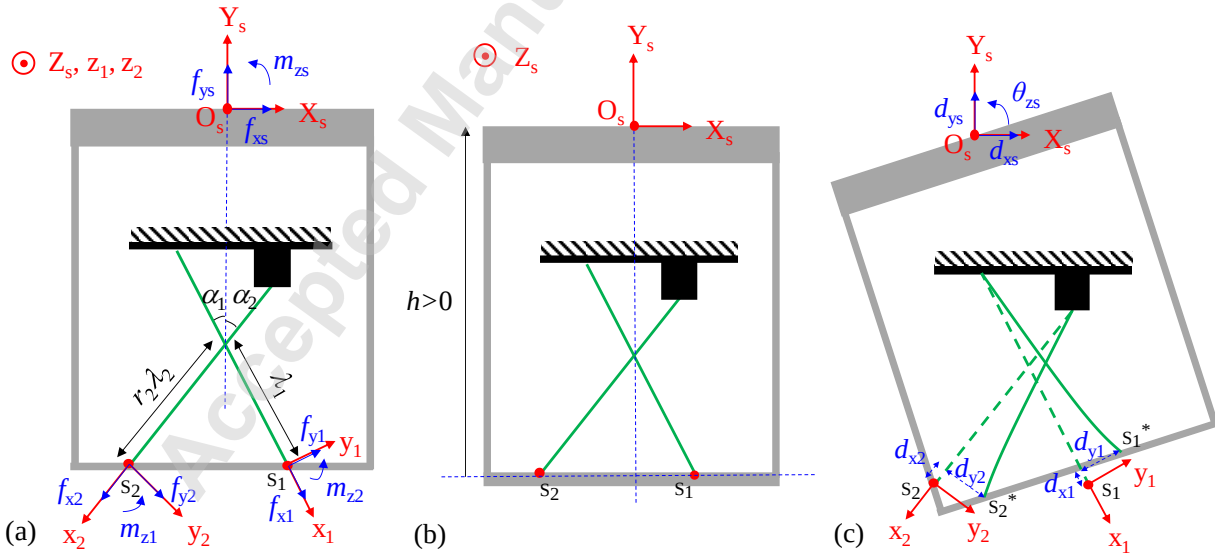


Fig. 4. The description of the IG-CSP: (a) normalized geometric parameters and loads, (b) the sign of h being positive when the loading position is above the free ends of the two sheets, and (c) the normalized displacements.

The compatibility conditions can be described as Eqs. (3) and (4) (derivation details can be seen in [24]).

$$\begin{bmatrix} d_{x1}, d_{y1}, \theta_{z1} \end{bmatrix}^T = \mathbf{R}_{z1} (\mathbf{R}_{z3} \mathbf{S}_1 - \mathbf{S}_1) + \begin{bmatrix} d_{xs}, d_{ys}, \theta_{zs} \end{bmatrix}^T \quad (3)$$

$$\begin{bmatrix} d_{x2}, d_{y2}, \theta_{z2} \end{bmatrix}^T = \mathbf{R}_{z2} (\mathbf{R}_{z4} \mathbf{S}_2 - \mathbf{S}_2) + \begin{bmatrix} d_{xs}, d_{ys}, \theta_{zs} \end{bmatrix}^T \quad (4)$$

where, $\mathbf{S}_1$ and $\mathbf{S}_2$ denote the coordinates of points S_1, S_2 (Fig. 4) with regard to the global coordinate system, and are represented as follows. $\mathbf{S}_1 = [\lambda_1 \sin(\alpha_1), -h, 0]^T$, $\mathbf{S}_2 = [-\lambda_2 r_2 \sin(\alpha_2), -h, 0]^T$. $\mathbf{R}_{zj}$ ($j=1, 2, 3$, or 4) is a rotational matrix

about the Z_s -axis, which is designated as $\mathbf{R}_{z1} = \begin{bmatrix} \cos \beta_j & -\sin \beta_j & 0 \\ \sin \beta_j & \cos \beta_j & 0 \\ 0 & 0 & 1 \end{bmatrix}$. $\beta_1 = \pi/2 - \alpha_1$, $\beta_2 = \pi/2 + \alpha_2$, $\beta_3 = \theta_{z1}$, and $\beta_4 = \theta_{z2}$.

The load-equilibrium conditions of the motion stage in the deformed configuration can be expressed Eq. (5) (See details in [24]).

$$\begin{bmatrix} f_{xs}, f_{ys}, m_{zs} \end{bmatrix}^T = \mathbf{D}_{p1}^T \mathbf{R}_{z1}^T \begin{bmatrix} f_{x1}, f_{y1}, m_{z1} \end{bmatrix}^T + \mathbf{D}_{p2}^T \mathbf{R}_{z2}^T \begin{bmatrix} f_{x2}, f_{y2}, m_{z2} \end{bmatrix}^T \quad (5)$$

where, $\mathbf{D}_{pi}$ ($i=1$ or 2) denotes a translational matrix, $\mathbf{D}_{pi} = \begin{bmatrix} 1 & 0 & -\mathbf{S}_i^*(2,1) \\ 0 & 1 & \mathbf{S}_i^*(1,1) \\ 0 & 0 & 1 \end{bmatrix}$. $\mathbf{S}_i^*$ ($i=1$ or 2) denotes the coordinates

of point S_i relative to the global coordinate system after only the rotation of the motion stage, and $\mathbf{S}_i^* = \mathbf{R}_{zs} \mathbf{S}_i$. $\mathbf{R}_{zs}$ is a

rotational matrix about the Z_s -axis, $\mathbf{R}_{zs} = \begin{bmatrix} \cos(\theta_{zs}) & -\sin(\theta_{zs}) & 0 \\ \sin(\theta_{zs}) & \cos(\theta_{zs}) & 0 \\ 0 & 0 & 1 \end{bmatrix}$.

Given the independent parameters and three loading inputs (f_{xs} , f_{ys} , and m_{zs}), the outputs of the motion stage (d_{xs} , d_{ys} and θ_{zs}) are solved by using Eqs. (1) through (5). These solutions refer to analytical models in this paper.

To derive the closed-form rotations (Eq. (7)) of the IG-CSP, we use Eq. (6) to simplify the results of d_{y1} and d_{y2} [13, 27], and use small-angle approximations to derive the closed-form rotational angle of the IG-CSP (Eq. (7)). The small-angle approximations are based on $\sin(\theta_{zs}) \approx \theta_{zs}$ and $\cos(\theta_{zs}) \approx 1$ while other higher orders associated with θ_{zs} are neglected.

$$d_{y1} = \lambda_1 \theta_{zs} \text{ and } d_{y2} = r_2 \lambda_2 \theta_{zs} \quad (6)$$

$$\theta_{zs} = \frac{\cos^2 \alpha_2 r_2^3 (A_1 f_{xs} + A_2 m_{zs})}{A_3 f_{xs} + A_4 f_{ys} + A_5 m_{zs} + A_6} \quad (7)$$

where A_1 through A_6 are the expressions of the independent geometric parameters and the loading position, as elaborated in Eq. (8).

$$A_1 = -\cos \alpha_2 \sin \alpha_1 h - \cos \alpha_1 \sin \alpha_2 h + \lambda_1 \cos \alpha_1 \sin \alpha_1 \cos \alpha_2 + \lambda_1 \cos^2 \alpha_1 \sin \alpha_2 \quad (8a)$$

$$A_2 = \cos \alpha_2 \sin \alpha_1 + \cos \alpha_1 \sin \alpha_2 \quad (8b)$$

$$A_3 = \frac{6}{5}(-\cos \alpha_2 \cos^2 \alpha_1 \sin \alpha_2 \sin \alpha_1 r_2^2 \lambda_1^2 + \cos^3 \alpha_1 \cos^2 \alpha_2 r_2^2 \lambda_1^2 - \cos^3 \alpha_1 r_2^2 \lambda_1^2 - \cos^2 \alpha_1 \cos^2 \alpha_2 h r_2^2 \lambda_1$$

$$+ \cos \alpha_1 \cos^2 \alpha_2 \sin \alpha_2 \sin \alpha_1 r_2^3 \lambda_1^2 - \cos^2 \alpha_2 \sin \alpha_2 \sin \alpha_1 r_2^3 h \lambda_1 - \cos^3 \alpha_2 r_2^3 \lambda_1 + \cos^3 \alpha_2 \cos \alpha_1 h r_2^3 \lambda_1$$

$$+ \lambda_1 \cos^2 \alpha_1 \cos \alpha_2 r_2^3 + \cos \alpha_2 \cos \alpha_1 \sin \alpha_2 \sin \alpha_1 h r_2^2 \lambda_1 - \cos^3 \alpha_2 \cos^2 \alpha_1 r_2^3 \lambda_1^2) \quad (8c)$$

$$+ \frac{2}{15}(\cos^3 \alpha_2 r_2^3 + 1.2 \cos^3 \alpha_2 r_2^3 \lambda_1^2 - r_2^4 \cos^2 \alpha_2 \cos \alpha_1)$$

$$A_4 = \cos^3 \alpha_2 \sin \alpha_1 r_2^3 h - \cos \alpha_1 \cos^3 \alpha_2 \sin \alpha_1 r_2^3 \lambda_1 - \cos^2 \alpha_1 \cos^2 \alpha_2 \sin \alpha_2 r_2^3 \lambda_1$$

$$+ \frac{6}{5}(\cos^2 \alpha_2 \sin \alpha_2 r_2^3 \lambda_1 + \lambda_1 \cos \alpha_1 \sin \alpha_1 \cos \alpha_2 r_2^3 - \cos^2 \alpha_1 \sin \alpha_1 r_2^2 \lambda_1^2 - \cos^2 \alpha_2 \sin \alpha_2 r_2^3 \lambda_1^2) \quad (8d)$$

$$- \frac{2}{15}(r_2^4 \cos^2 \alpha_2 \sin \alpha_1 + \cos^2 \alpha_2 \sin \alpha_2 r_2^3 + \cos \alpha_1 \cos^2 \alpha_2 \sin \alpha_2 r_2^3 h)$$

$$A_5 = \frac{6}{5}(\cos^2 \alpha_2 \sin \alpha_2 \sin \alpha_1 r_2^3 \lambda_1 - \cos^3 \alpha_2 \cos \alpha_1 r_2^3 \lambda_1 + \cos^2 \alpha_2 \cos^2 \alpha_1 r_2^2 \lambda_1 - \cos \alpha_2 \cos \alpha_1 \sin \alpha_2 \sin \alpha_1 r_2^2 \lambda_1) \quad (8e)$$

$$A_6 = 4(r_2^2 \cos^2 \alpha_2 \sin \alpha_2 \cos \alpha_1 + r_2^2 \cos^3 \alpha_2 \sin \alpha_1 + \cos^3 \alpha_2 \sin \alpha_1 r_2^3 + \cos \alpha_1 \cos^2 \alpha_2 \sin \alpha_2 r_2^3)$$

$$+ 12(\sin \alpha_1 \cos^2 \alpha_1 \cos \alpha_2 \lambda_1^2 - \cos \alpha_1 \cos^2 \alpha_2 \sin \alpha_2 r_2^3 \lambda_1 + \cos^3 \alpha_2 \sin \alpha_1 r_2^3 \lambda_1^2 + \cos \alpha_1 \cos^2 \alpha_2 \sin \alpha_2 r_2^3 \lambda_1^2) \quad (8f)$$

$$- \lambda_1 \cos^2 \alpha_1 \sin \alpha_2 \cos \alpha_2 r_2 + \cos^3 \alpha_1 \sin \alpha_2 \lambda_1^2 - \lambda_1 \cos \alpha_1 \sin \alpha_1 \cos^2 \alpha_2 r_2 - \cos^3 \alpha_2 \sin \alpha_1 r_2^3 \lambda_1)$$

An FEA model of an IG-CSP is built in COMSOL 5.0. We assume the sheets are elastic and the motion stage is rigid. The maximum meshing size of the sheets is 0.814 (mm). The material of each elastic sheet is Aluminum: Young's modulus $E=69 \times 10^9$ (Pa); Poisson's ratio $\nu=0.33$ and density is 2700 (kg/m³). Let us consider an example to evaluate the accuracy of the analytical and closed-form models of the IG-CSP. We fix L_1 , L_2 , U , T , λ_1 , α_1 , and α_2 at 50 (mm), 30 (mm), 5 (mm), 0.5 (mm), 0.5, $\pi/4$, and $\pi/6$. A series of rotations range from 0 to 0.3 (rad) with a 0.02 (rad) step are prescribed on the rotational center of the IG-CSP. The results for the analytical, FEA, and the closed-form models are illustrated in Fig. 5. In Fig. 5(c), when -0.3 (rad) $\leq \theta_{zs} \leq 0.3$ (rad), the maximum error between analytical and FEA models is less than 5%; when -0.1 (rad) $\leq \theta_{zs} \leq 0.1$ (rad), the maximum errors between the analytical and FEA models, closed-form and FEA models are less than 1.5% and 5%, respectively.

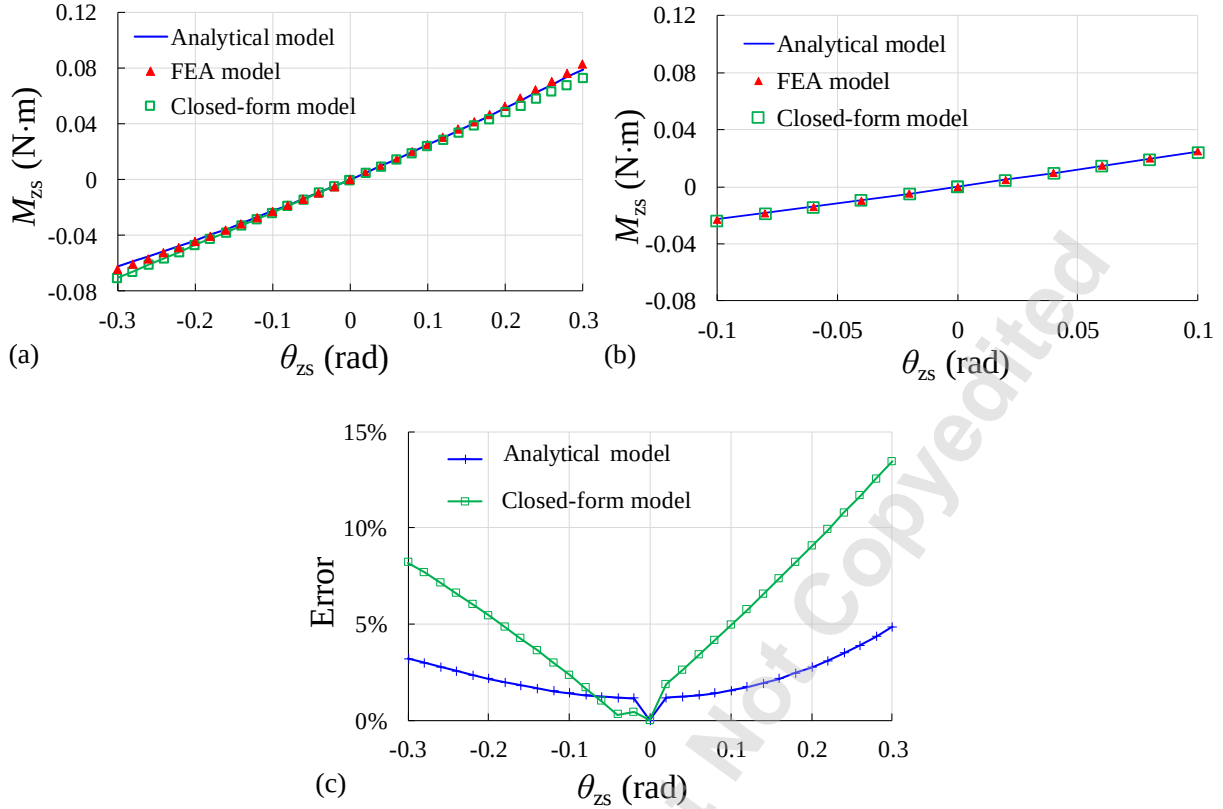


Fig. 5. Comparisons of rotations for the analytical, closed-form and FEA models: (a) $-0.3 \text{ (rad)} \leq \theta_{zs} \leq 0.3 \text{ (rad)}$, (b) $-0.1 \text{ (rad)} \leq \theta_{zs} \leq 0.1 \text{ (rad)}$, and (c) the errors between analytical and FEA models, and the errors between the closed-form and FEA models.

2.2 The Center Shift Model

d_{xc} and d_{yc} denote the normalized center shift along the X_s -axis and Y_s -axis, respectively. In Fig. 4(c), the displacement along the X_s -axis of point S_1 results from two contributions: the center shift d_{xc} and the resulting displacement along the X_s -axis due to a rigid pivot at the intersection point. The displacement along the Y_s -axis of point S_1 can be derived similarly. So the normalized center shift based on point S_1 can be derived as Eq. (9).

$$d_{xc} = d_{x1} \sin \alpha_1 + d_{y1} \cos \alpha_1 - \lambda_1 \sin(\alpha_1 + \theta_{zs}) + \lambda_1 \sin \alpha_1 \quad (9a)$$

$$d_{yc} = -d_{x1} \cos \alpha_1 + d_{y1} \sin \alpha_1 - \lambda_1 \cos \alpha_1 + \lambda_1 \cos(\alpha_1 + \theta_{zs}) \quad (9b)$$

Similarly, the center shift based on point S_2 can also be derived as Eq. (10).

$$d_{xc} = -d_{x2} \sin \alpha_2 + d_{y2} \cos \alpha_2 - \lambda_2 r_2 \sin \alpha_2 + \lambda_2 r_2 \sin(\alpha_2 - \theta_{zs}) \quad (10a)$$

$$d_{yc} = -d_{x2} \cos \alpha_2 - d_{y2} \sin \alpha_2 + \lambda_2 r_2 \cos(\alpha_2 - \theta_{zs}) - \lambda_2 r_2 \cos \alpha_2 \quad (10b)$$

From Section 2.1, we have the outputs of the motion stage with given loading conditions and substituting these results into Eqs. (3) and (4) to obtain d_{xi} and d_{yi} ($i=1$ or 2). Then d_{xc} and d_{yc} are solved from Eq. (9) or (10). We use the same example in Section 2.1 to verify the accuracy of the center shift, and use D_{xc} and D_{yc} to denote the dimensional

center shift, where $D_{xc}=d_{xc}L_1$ and $D_{yc}=d_{yc}L_1$. The comparison of center shift between the analytical and FEA models is shown in Fig. 6. The maximum errors of D_{xc} and D_{yc} are 1.7% and 5.2%, respectively.

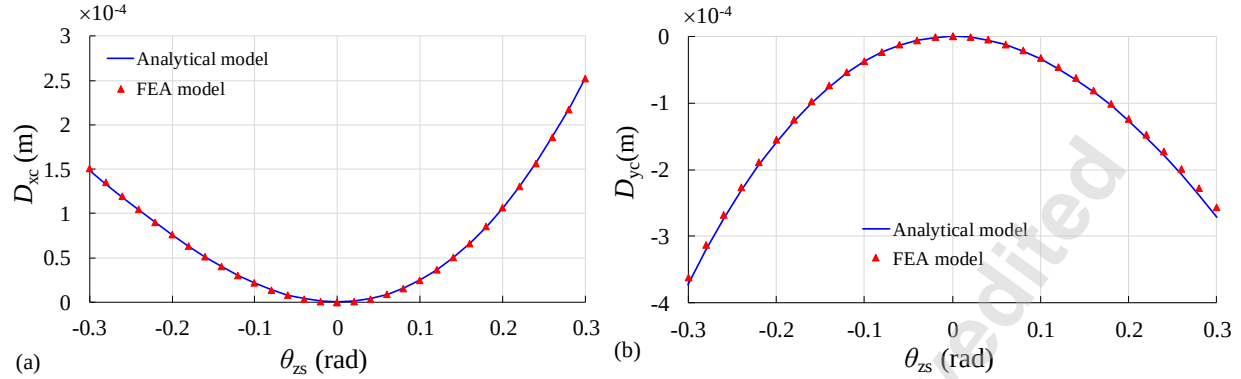


Fig. 6. The comparison of the center shift between the analytical and FEA models: (a) D_{xc} , and (b) D_{yc} .

3 The Nonlinear Analysis of the IS-CSP

In this section, the closed-form load-rotation relationships of an IS-CSP and a NIS-CSP are derived, respectively. Then we analyze the effects of the geometric parameters, loading positions, and axial forces on the rotational stiffness of the IS-CSP. The load-dependent effects of an IS-CSP and a NIS-CSP are compared.

3.1 Load-rotation relationship

The closed-form load-rotation relationship of the IS-CSP is expressed as Eq. (11) by substituting $r_2=1$, $\alpha_1=\alpha_2=\alpha$, $\lambda_1=\lambda_2=\lambda$, into Eq. (7). λ , α and h are the independent parameters of the IS-CSP.

$$\theta_{zs} = \frac{15 \cos \alpha (f_{xs} \lambda - f_{ys} h + m_{zs})}{(-18\lambda^2 + 18\lambda - 15\lambda \cos^2 \alpha + 15h \cos \alpha - 2) f_{ys} + 120 \cos \alpha (3\lambda^2 - 3\lambda + 1)} \quad (11)$$

In addition, the description of the NIS-CSP is shown in Fig. 7. The analytical model of a NIS-CSP is derived as below. Similar to the derivation in Section 2.1, we use $\mathbf{S}_{ni}$ to replace $\mathbf{S}_i$, $\mathbf{R}_{zni}$ to replace $\mathbf{R}_{zi}$ ($i=1$ or 2). $\mathbf{S}_{ni}$ denotes the free ends of the sheets in a NIS-CSP. $\mathbf{S}_{ni}$ denotes the coordinates of point $\mathbf{S}_{ni}$ with respect to the global coordinate system after only the rotation of the motion stage. $\mathbf{S}_{n1}=[-\lambda \sin(\alpha), -h, 0]^T$, and $\mathbf{S}_{n2}=[\lambda \sin(\alpha), -h, 0]^T$. $\mathbf{R}_{zni}$ denotes the

rotational matrix about the Z_s -axis, $\mathbf{R}_{zni} = \begin{bmatrix} \cos \delta_i & -\sin \delta_i & 0 \\ \sin \delta_i & \cos \delta_i & 0 \\ 0 & 0 & 1 \end{bmatrix}$. $\delta_1=-\pi/2-\alpha$ and $\delta_2=-\pi/2+\alpha$. The closed-form load-

rotation relationship of the NIS-CSP is expressed as Eq. (12).

$$\theta_{zs} = \frac{15 \cos \alpha (-\cos \alpha f_{xs} \lambda - f_{ys} h + m_{zs})}{(18\lambda^2 - 18\lambda + 15\lambda \cos^2 \alpha + 15h \cos \alpha + 2) f_{ys} + 120 \cos \alpha (3\lambda^2 - 3\lambda + 1)} \quad (12)$$

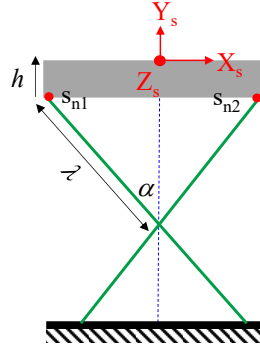


Fig. 7. The description of a NIS-CSP.

3.2 Load-Dependent Effects

k_{zm} denotes the normalized rotational stiffness due to the normalized moment m_{zs} of an IS-CSP, which is expressed as Eq. (13) by rearranging Eq. (11). The rotational stiffness due to the bending force f_{xs} can be discussed in a similar way.

$$k_{zm} = A_m f_{ys} + 8(3\lambda^2 - 3\lambda + 1) \quad (13)$$

where $A_m = A_{mgeo} + h = -2(9\lambda^2 - 9\lambda + 1)/(15\cos\alpha) - \lambda\cos\alpha + h$. A_{mgeo} denotes the value of A_m due to the geometric parameters. h denotes the value of A_m due to the loading positions.

A_m is an expression of the independent geometric parameters (beam load-dependent effects) and loading positions (structure load-dependent effects). If $A_m = 0$, $A_m f_{ys} = 0$, f_{ys} has less effect on k_{zm} when a compressive axial force ($f_{ys} < 0$) acts on the IS-CSP. Meanwhile if $A_m > 0$, $A_m f_{ys} < 0$, k_{zm} decreases with f_{ys} , and vice versa. To analyze the effect of f_{ys} on k_{zm} , it is necessary to analyze the signs of A_m when λ , α , and h take different values.

A_m only relates to the geometric parameters, i.e. $A_m = A_{mgeo}$, when h is equal to 0. A_{mgeo} of an IS-CSP and a NIS-CSP are shown in Fig. 8, when λ and α range from 0 to 1 and 0 to $\pi/2$, respectively. When the geometric parameters of the IS-CSP are in the B region, $A_{mgeo} > 0$, the IS-CSP can be regarded as a non-inversed compliant sheet as k_{zm} decreases with a compressive f_{ys} . When the geometric parameters of the IS-CSP are in the C region, $A_{mgeo} < 0$, the IS-CSP can be regarded as an inversed compliant sheet. We can draw similar conclusions for a NIS-CSP operating in the D and E regions. In this way, if the geometric parameters are specifically given, A_{mgeo} can be determined from Fig. 8.

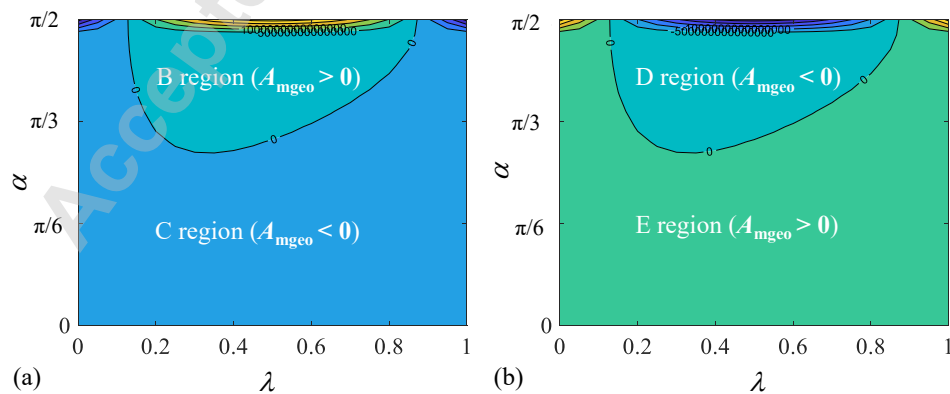


Fig. 8. The effects of λ and α on A_{mgeo} of (a) an IS-CSP, and (b) a NIS-CSP.

On the other hand, the sign of A_m depends on the dominant position of A_{mgeo} and h as $A_m=A_{mgeo}+h$. When λ and α are specifically given in the C region, A_{mgeo} is determined and less than 0, and k_{zm} increases with a compressive f_{ys} for the beam load-dependent effects ($A_{mgeo}f_{ys}>0$). Under this condition, we analyze the structure load-dependent effects in C region. y_j ($j=1, 2, \dots$, or 7) denotes loading positions along the Y_s -axis as shown in Tab. 1 and Fig. 9(a). When the loading positions move from y_1 to y_6 , $h > 0$, and k_{zm} decreases with a compressive f_{ys} for the structure load-dependent effects ($hf_{ys}<0$). If $h > -A_{mgeo}$, i.e., $A_m=A_{mgeo}+h>0$, the structure load-dependent effects dominate k_{zm} . If $h < -A_{mgeo}$, i.e., $A_m=A_{mgeo}+h<0$, the beam load-dependent effects dominate k_{zm} . If $h = -A_{mgeo}$, i.e., $A_m=0$, the equilibrium between structure and beam load-dependent effects is balanced, so when λ and α are specified, the load-dependent effects can be reduced significantly by regulating the loading positions. When the loading positions move from y_6 to y_7 , $h < 0$, and A_m is always less than 0. k_{zm} increases with a compressive f_{ys} for both the beam and structure load-dependent effects. The equilibrium of beam and structure load-dependent effects in other regions can be analyzed similarly.

Moreover, if the loading position fixes at y_6 , the region where $A_m<0$ for an IS-CSP is increased. When the compressive axial forces ($f_{ys}<0$) act on the IS-CSP, the region where $A_mf_{ys}>0$ is increased, which means that the possibility of increasing k_{zm} rises. When the IS-CSP and NIS-CSP share the same λ , α , and h , the absolute values of A_m are equal and the signs of A_m are opposite.

The effects of y_j ($j=1, 2, \dots$, or 7) on A_m of an IS-CSP are shown in Fig. 9(b) and Fig. 10. $\lambda=1/2\pm\sqrt{5}/6$ are two roots of $9\lambda^2-9\lambda+1=0$ (Eq. (13)). In Fig. 9(b), when the loading positions range from y_1 to y_7 , the curves representing $A_m=0$ are enclosed gradually following the arrows' direction. The possibility of the geometric parameters is on the curve representing $A_m=0$ can be increased. When the axial forces act on the rotational center (y_4) and λ is $1/2-\sqrt{5}/6$ or $1/2+\sqrt{5}/6$, A_m is equal to 0 without depending on the value of α . In Fig. 10, when the loading positions range from y_1 to y_7 , the region where $A_m<0$ is increased. If h at y_1 is large enough, A_m is always greater than 0, which means that k_{zm} decreases with a compressive f_{ys} . Similarly, if the absolute value of h at y_7 is large enough, A_m is always smaller than 0, which means k_{zm} increases with f_{ys} . When h is specified, the load-dependent effects can be reduced significantly by taking any geometric parameters on the curve representing $A_m=0$.

Table 1. The examples of loading locations y_j ($j=1,2,\dots$ or, 7). ($\Delta_1=20(\text{mm})/L$, $\Delta_2=10(\text{mm})/L$).

y_j	1	2	3	4	5	6	7
h	$y_2+\Delta_1$	$\cos\alpha$	$0.5(y_2+y_4)$	$\lambda\cos\alpha$	$0.5(y_4+y_6)$	0	$-(y_6+\Delta_2)$

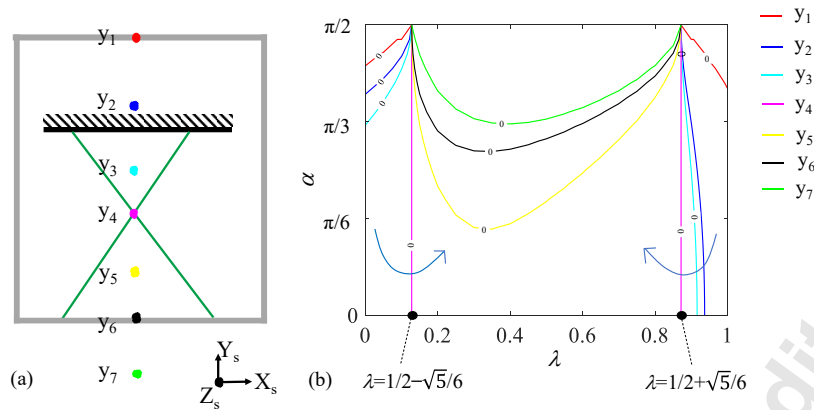


Fig. 9. The effects of the loading positions on A_m of an IS-CSP: (a) the loading positions, and (b) the curves representing $A_m=0$.

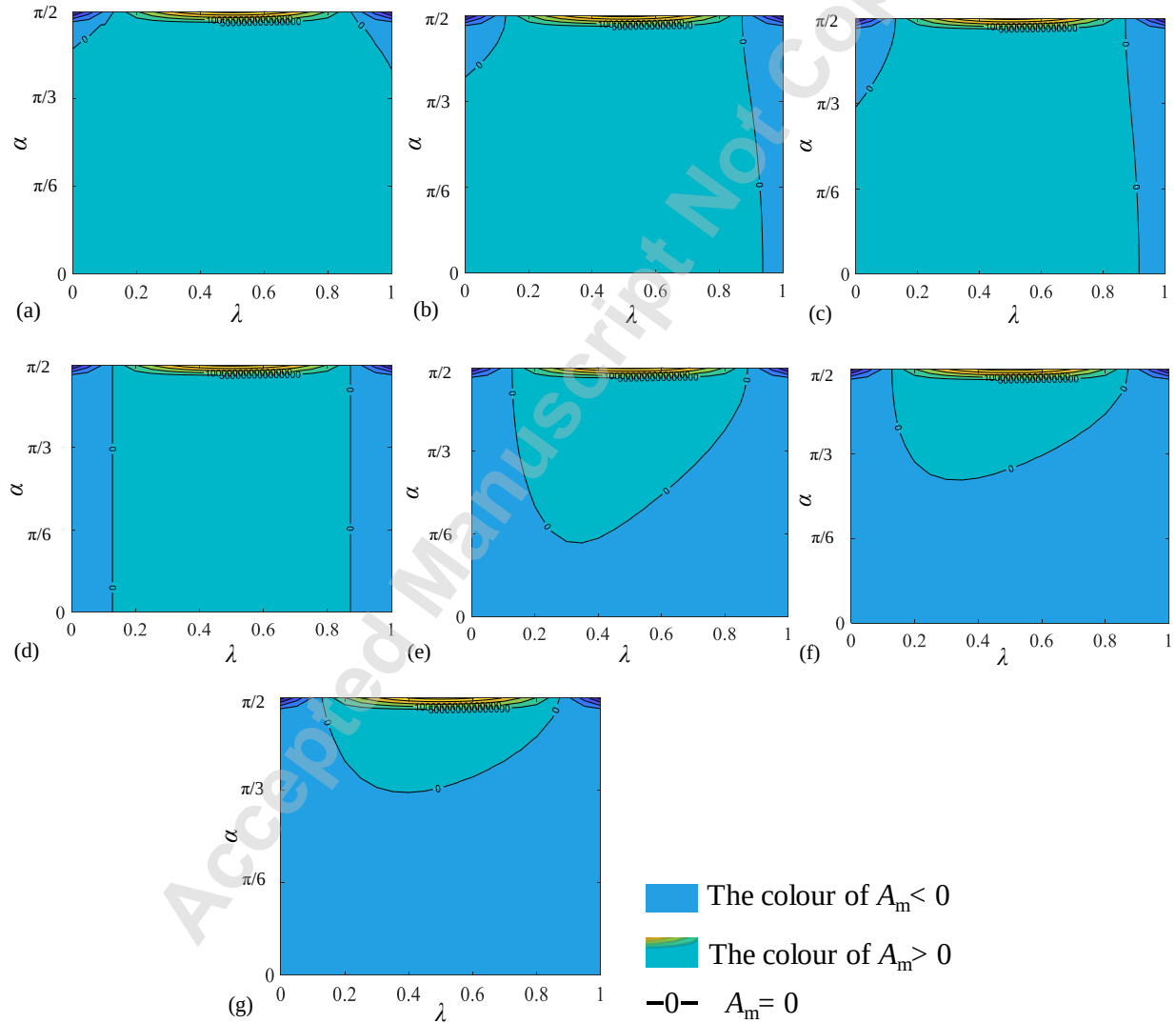


Fig. 10. The effects of the loading positions on A_m of an IS-CSP: (a) y_1 , (b) y_2 , (c) y_3 , (d) y_4 , (e) y_5 , (f) y_6 , and (g) y_7 .

To verify k_{zm} varying with the sign of A_m when $f_{ys} < 0$, we take four cases of IS-CSPs to compare K_{zm} between the analytical and the FEA models, where K_{zm} denotes the rotational stiffness due to the moment M_{zs} . Table 2 lists the independent parameters of A_m (λ , α , h), the signs of A_m , and the predictions of the load-dependent effects. The geometric parameters (λ , α) of Cases 1 and 2 are the same while the loading positions (h) of Cases 1 and 2 are y_4 (the rotational center) and y_6 , respectively. In Cases 3 and 4, the load-dependent effects of Case 2 can be reduced by regulating the loading position or the geometric parameters, respectively. When L is constant at 30(mm), θ_{zs} ranges from -0.1 (rad) to 0.1 (rad), K_{zm} for the four cases are shown in Fig. 11. The load-dependent effects of the predictions, analytical model, and FEA model are consistent. The maximum error of K_{zm} between the analytical and FEA models is 1.2%.

Table 2. Parameters of IS-CSPs ($\downarrow$, $\uparrow$, or c denote that k_{zm} decreases, increases, or remains constant with f_{ys} , respectively).

Cases	λ	α	h	A_m	k_{zm}
1	0.5	$\pi/6$	$\lambda \cos \alpha$	>0	$\downarrow$
2	0.5	$\pi/6$	0	<0	$\uparrow$
3	0.5	$\pi/6$	$-A_{mgeo}$	0	c
4	0.5	$54.735\pi/180$	0	0	c

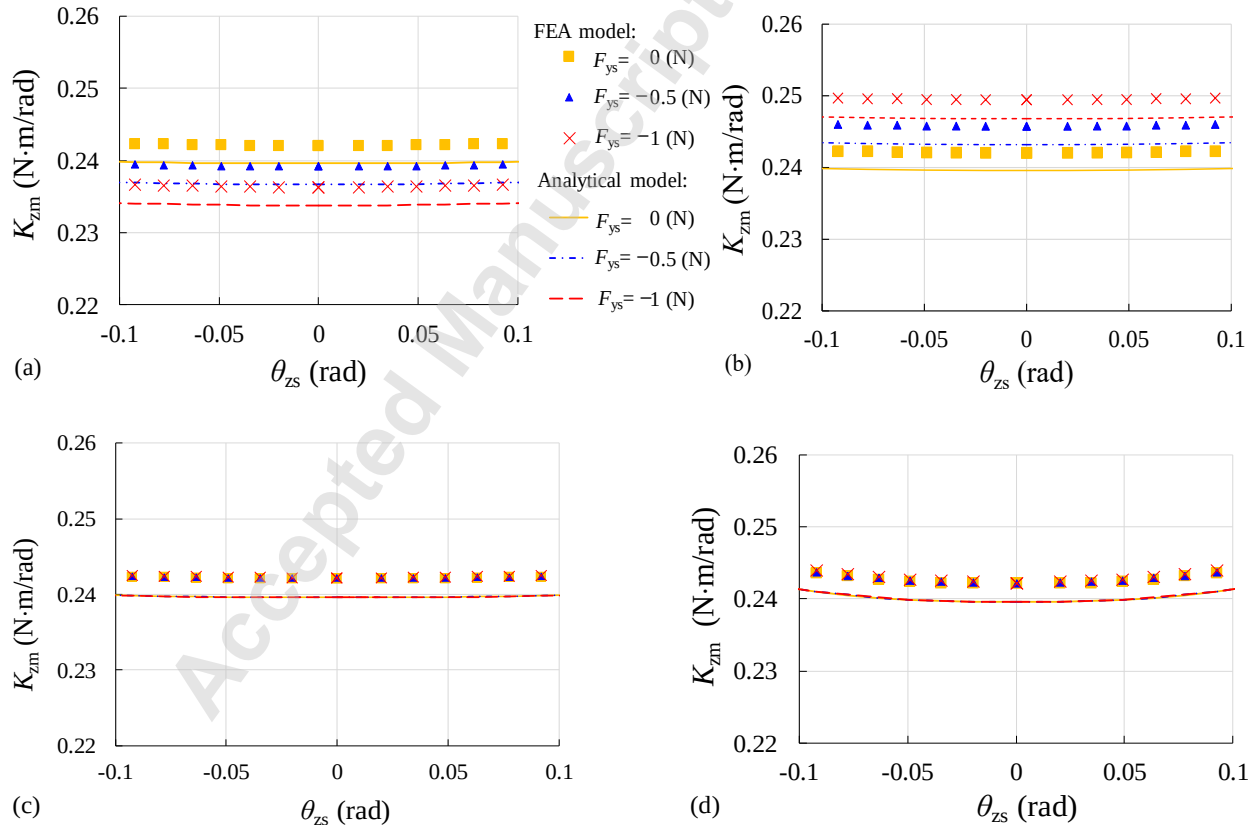


Fig. 11. K_{zm} of: (a) Case 1, (b) Case 2, (c) Case 3, and (d) Case 4.

3.3 The closed-form model of the center shift

We derive the closed-form center shift of an IS-CSP using the method that Zhao et al. [13] have introduced. d_{xc} is derived as Eq. (14) for any values of λ . When λ is smaller than 0.5, the closed-form d_{yc} can be directly derived from Eq. (6) as shown in Eq. (15), because the accuracy of Eq. (6) is highly acceptable. When λ is greater than 0.5, an accurate d_{yc} with a more complex form is shown in Eq. (16) (derivation details can be seen in [13]), as the accuracy of Eq. (6) is compromised slightly. Besides, we do not discuss the center shift for the bending force f_{xs} , and assume f_{xs} is equal to 0 in this section.

$$d_{xc} = \frac{-(12\lambda - 1)(9\lambda^2 - 9\lambda + 1)\theta_{zs}^3}{150 \cos \alpha} + \frac{[12(-2\lambda + 1) \cos \alpha \theta_{zs}] \left[\frac{1}{6300} (9\lambda^2 - 9\lambda + 11) \theta_{zs}^2 + \frac{t^2}{12} \right]}{2 \sin^2 \alpha} + \frac{\left[\frac{1}{10} (12\lambda - 1) f_{ys} \theta_{zs} \right] \left[\frac{1}{6300} (9\lambda^2 - 9\lambda + 11) \theta_{zs}^2 + \frac{t^2}{12} \right]}{2 \sin^2 \alpha} \quad (14)$$

$$d_{yc} = \frac{1}{-15 \cos \alpha} (-9\lambda^2 + 9\lambda - 1) \theta_{zs}^2 + \frac{f_{ys} \left[\frac{1}{6300} (9\lambda^2 - 9\lambda + 11) \theta_{zs}^2 + \frac{t^2}{12} \right]}{2 \cos^2 \alpha} - \frac{\theta_{zs}^4}{3000 \cos \alpha} (2592\lambda^4 - 3024\lambda^3 + 1338\lambda^2 - 241\lambda + 2) \quad (15)$$

$$d_{yc} = \frac{1}{-15 \cos \alpha} (-9\lambda^2 + 9\lambda - 1) \theta_{zs}^2 + \frac{f_{ys} \left[\frac{1}{6300} (9\lambda^2 - 9\lambda + 11) \theta_{zs}^2 + \frac{t^2}{12} \right] \left[\frac{11 - 12\lambda}{20} \left(\frac{1}{\cos^2 \alpha} - \frac{1}{\sin^2 \alpha} \right) \theta_{zs}^2 + \frac{1}{2} \left(\frac{1}{\cos^2 \alpha} + \frac{\theta_{zs}^2}{\sin^2 \alpha} \right) \right]}{2 \cos^2 \alpha} - \frac{\theta_{zs}^4}{\cos \alpha} \left[\frac{(2592\lambda^4 - 3024\lambda^3 + 1338\lambda^2 - 241\lambda + 2)}{3000} - \frac{\cot^2 \alpha (1 - 2\lambda)}{1050} (9\lambda^2 - 9\lambda + 11) \right] \quad (16)$$

where, $t = T/L$.

We use an example to evaluate the center shift between the closed-form and FEA models of an IS-CSP. L , U , T , α and λ are constant at 30 (mm), 5 (mm), 0.5 (mm), $\pi/6$, and 0.5 respectively. A series of prescribed rotations, ranging from -0.1 (rad) to 0.1 (rad) and $F_{ys} = 0$ (N), act on the rotational center of the IS-CSP. In Fig. 12, the maximum errors of D_{xc} and D_{yc} between closed-form and FEA models are 1.77% and 1.85%, respectively.

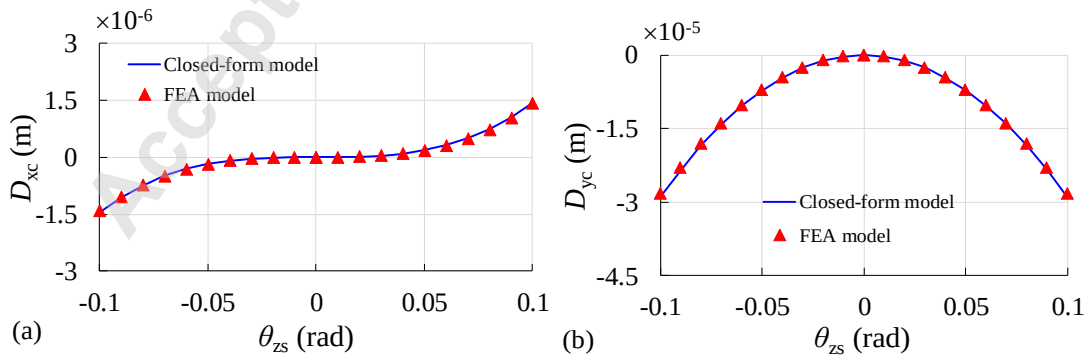


Fig. 12. The center shift of the closed-form and FEA models: (a) D_{xc} , and (b) D_{yc}

Comparing the coefficients of $9\lambda^2-9\lambda+1$ and f_{ys} in Eqs. (14) through (16), $9\lambda^2-9\lambda+1$ is a dominant item for both d_{xc} and d_{yc} . We use the same geometric parameters of Fig. 12 except λ . When $9\lambda^2-9\lambda+1=0$, D_{xc} and D_{yc} decrease significantly, and the axial force has less effect on D_{xc} but a more significant effect on D_{yc} as illustrated in Figs. 13(a) and (b). Zhao et al. reached the same conclusions in their research [13].

When $9\lambda^2-9\lambda+1 \neq 0$, the axial force does not influence both D_{xc} and D_{yc} significantly, and it is worth minimizing D_{yc} because D_{yc} is approximately ten times larger than D_{xc} . For example, when λ is 0.4 or 0.7, the results of the center shift are shown in Figs. 13(c) through (f). We compare the center shifts between an IS-CSP and a NIS-CSP. The geometric parameters of Fig. 12 are used for both an IS-CSP and a NIS-CSP. A series of prescribed rotations, ranging from -0.1 (rad) to 0.1 (rad) and an axial force of -1 (N), act on their rotational centers. Their center shifts are compared as shown in Fig. 14. Note that Figs. 14(a) and (b) share the same legend. When an IS-CSP and a NIS-CSP are under a same rotational angle, the absolute values of their center shifts are close, and directions are opposite.

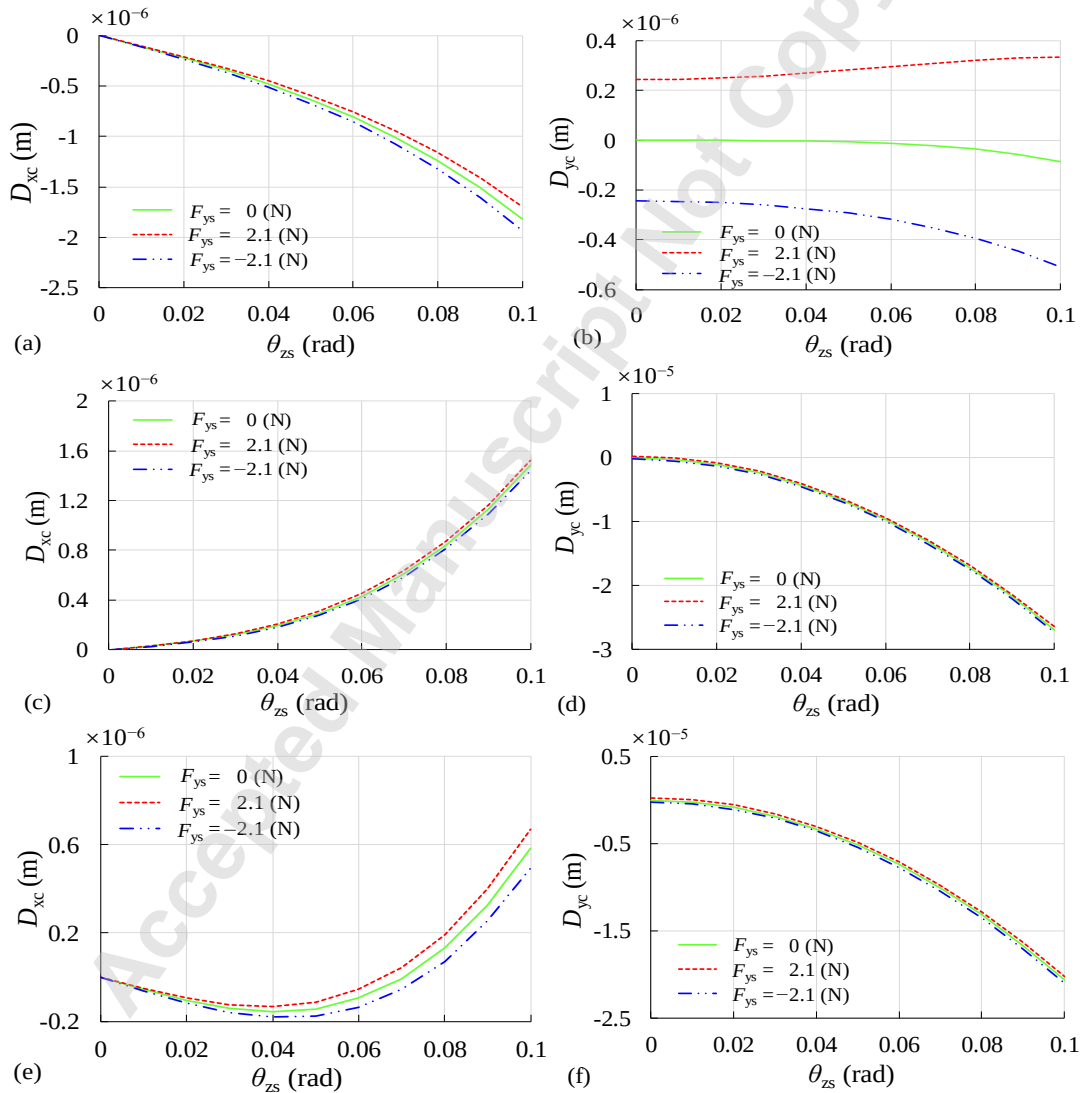


Fig. 13. The effects of F_{ys} on D_{xc} and D_{yc} of the closed-form models: (a) D_{xc} when λ is $1/2 + \sqrt{5}/6$, (b) D_{yc} when λ is $1/2 + \sqrt{5}/6$, (c) D_{xc} when λ is 0.4, (d) D_{yc} when λ is 0.4, (e) D_{xc} when λ is 0.7, and (f) D_{yc} when λ is 0.7.

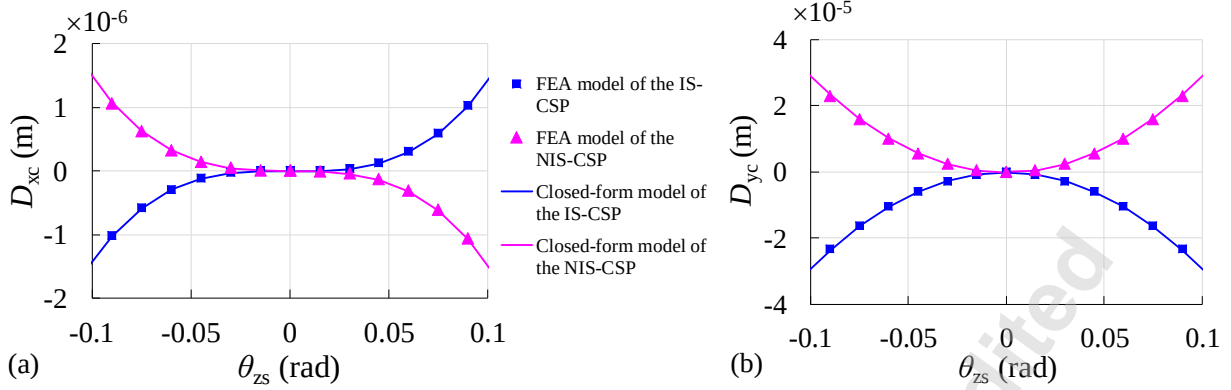


Fig. 14. The center-shift comparison between an IS-CSP and a NIS-CSP: (a) D_{xc} , and (b) D_{yc} .

4 Design of The Compound S-CSP

In this section, we present two novel compound S-CSPs, including a parallel design and a serial design, and each design consists of an IS-CSP and a NIS-CSP, whose geometric parameters are the same correspondingly. They are regarded as two basic units, and each design can be modeled as the two basic units connected in a parallel (or serial) arrangement. The analytical models of an IS-CSP and a NIS-CSP are used in Sections 2.1 and 3.1, respectively. O_{s1} - $X_{s1}Y_{s1}Z_{s1}$ and O_{s2} - $X_{s2}Y_{s2}Z_{s2}$ denote the local coordinate systems of the IS-CSP and NIS-CSP, respectively. O_{DI} - $X_{DI}Y_{DI}Z_{DI}$ and O_{DII} - $X_{DII}Y_{DII}Z_{DII}$ denote the global coordinate systems of design I and II, respectively. Their origins locate at the rotation center, so the displacements with regard to the global coordinate system are the center shift of each design.

We evaluate the characteristics by the analytical and FEA models, and fix L , U , T , α and λ for each design, which are 30 (mm), 5 (mm), 0.5 (mm), $\pi/6$, and 0.5, respectively.

4.1 A Parallel Design: Design I

Inspired by [35, 36] and Sections 3.2, the description of design I is shown in Fig. 15(a). The rotational stiffness of design I can be insensitive to axial forces when an applied moment is constant. The additional benefit of this design lies in the minimized center shift. The analytical model of design I is derived as follows. The compatibility conditions and load-equilibrium equations are illustrated in Eqs. (17) and (18). d_{xDI} , d_{yDI} , and θ_{zDI} are solved with given f_{xDI} , f_{yDI} , and m_{zDI} .

$$d_{xDI} = d_{xs1} = d_{xs2}; d_{yDI} = d_{ys1} = d_{ys2}; \theta_{zDI} = \theta_{zs1} = \theta_{zs2} \quad (17)$$

$$f_{xDI} = f_{xs1} + f_{xs2}; f_{yDI} = f_{ys1} + f_{ys2}; m_{zDI} = m_{zs1} + m_{zs2} \quad (18)$$

where, f_{xDI} , f_{yDI} , m_{zDI} , f_{xs1} , f_{ys1} , m_{zs1} , f_{xs2} , f_{ys2} , and m_{zs2} denote the normalized loads of the design I, IS-CSP, and NIS-CSP, respectively, as shown in Figs. 15(b) and (c). d_{xDI} , d_{yDI} , θ_{zDI} , d_{xs1} , d_{ys1} , θ_{zs1} , d_{xs2} , d_{ys2} and θ_{zs2} denote the normalized displacements and rotation angles correspondingly.

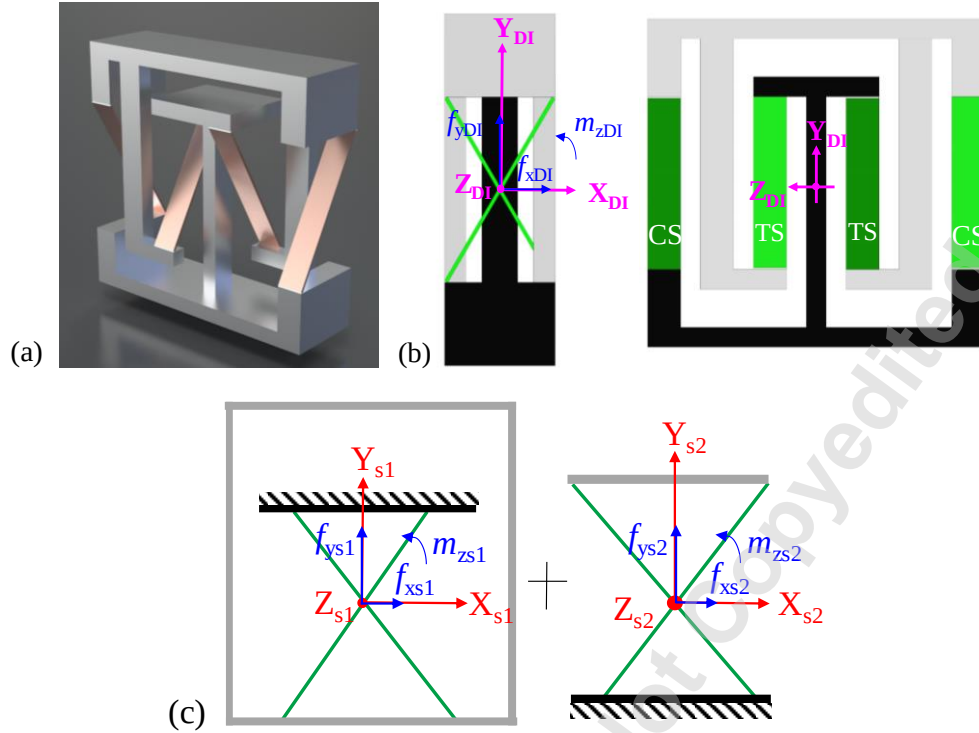


Fig. 15. Descriptions of the design I: (a) a 3D model, (b) the global coordinate system ('CS' denotes a compressive sheet, 'TS' denotes a tensile sheet), and (c) the local coordinate systems.

Similar to the discussion in Section 3.2, the load-dependent effects of design I depend on A_m of design I (denoted by A_{mDI} in this section). A_{mDI} approximates the result of adding each CSP's A_m when the two CSPs are arranged in parallel to form design I, i.e., $A_{mDI} \approx A_{mgeo} + h$ of IS-CSP' + $A_{mgeo} + h$ of NIS-CSP'. Because the center shifts of two individual CSPs move in the opposite direction, and the actual normalized rotational stiffness of design I (denoted by k_{zmDI}) is larger than the simple stiffness addition of two individual CSPs (i.e., load-stiffening effect). ' $A_{mgeo} + h$ of IS-CSP' refers to Eq. (13), and ' $A_{mgeo} + h$ of NIS-CSP' can be derived from Eq. (12). A_{mgeo} of IS-CSP is always counteracted by that of the NIS-CSP when they have the same geometric parameters. Therefore, the load-dependent effects of design I due to geometric parameters almost disappear. However, h of the two CSPs are not always counteracted with each other. The load-dependent effects of design I due to loading positions are analyzed as follows.

The rotations of design I obtained from the analytical and FEA models under different loading positions and axial forces are shown in Fig. 16. L_{cy} denotes a directional distance between the rotational center and the loading position, where $L_{cy} > 0$ and $L_{cy} < 0$ mean that the loading positions are above and below the rotational center, respectively. L_{cy} ranges from 30 (mm) to -30 (mm) with a -10 (mm) step. F_{yDI} ranges from -4 (N) to 4 (N) with a 0.5 (N) step acting on the loading positions, and M_{zDI} is kept unchanged at 0.04 (N·m). The maximum error between analytical and FEA models is 2.5%.

When $|L_{cy}| \leq 20$ (mm), the maximum error of θ_{zDI} between $|F_{yDI}| = 4$ (N) and $F_{yDI} = 0$ is 5.0%. However, when $|L_{cy}| = 30$ (mm), the maximum error of θ_{zDI} between $|F_{yDI}| = 4$ (N) and $F_{yDI} = 0$ is 7.7%. With the same M_{zDI} , axial forces influence θ_{zDI} slightly within a specified $|L_{cy}|$ but they affect θ_{zDI} significantly if $|L_{cy}|$ is large enough.

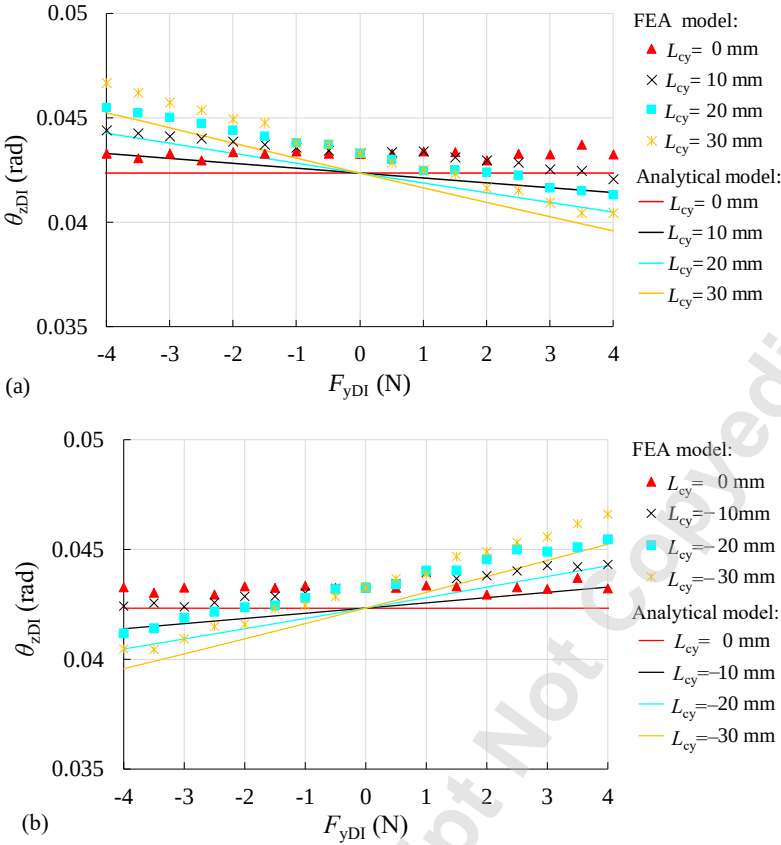


Fig. 16. The effects of loading positions and axial forces on θ_{zDI} with $M_{zDI} = 0.04$ (N·m): (a) above the rotational center ($L_{cy} > 0$), and (b) below the rotational center ($L_{cy} < 0$).

We use A_{mDI} to explain how θ_{zDI} varies with F_{yDI} in Fig. 16. When $|L_{cy}| = 0$ or 20 (mm), A_{mDI} , and the predictions of k_{zmDI} and θ_{zDI} , corresponding to different F_{yDI} , are illustrated in Tab. 3. When the loading position is the rotational center (i.e., $L_{cy} = 0$), A_m of IS-CSP is counteracted by that of NIS-CSP, leading to $A_{mDI} \approx 0$. Therefore, k_{zmDI} and θ_{zDI} remain constant with different F_{yDI} . When $L_{cy} = 20$ (mm), $A_{mDI} > 0$, so k_{zmDI} decreases and θ_{zDI} increases under a compressive F_{yDI} . When $L_{cy} = -20$ (mm), k_{zmDI} and θ_{zDI} can be similarly analyzed.

Table 3. Load-dependent results of design I when $|L_{cy}| = 20$ (mm) or 0 (mm). ($\downarrow$, $\uparrow$, or c denote that k_{zmDI} decreases, increases, or remains constant with f_{yDI} , respectively. ‘+’ denotes ‘ $f_{yDI} > 0$ ’, and ‘-’ denotes ‘ $f_{yDI} < 0$ ’.).

L_{cy}	CSP	A_{mgeo}	h of each CSP	A_m of each CSP	A_{mDI}	f_{yDI}	k_{zmDI}	θ_{zDI}
0	IS	-0.24	$\lambda \cos(\alpha) = 0.43$	0.19	0	-	c	c
	NIS	0.24	$-\lambda \cos(\alpha) = -0.43$	-0.19		+	c	c
20	IS	-0.24	$\lambda \cos(\alpha) + L_{cy}/L = 1.10$	0.86	1.33 > 0	-	$\downarrow$	$\uparrow$
	NIS	0.24	$-\lambda \cos(\alpha) + L_{cy}/L = 0.23$	0.47		+	$\uparrow$	$\downarrow$
-20	IS	-0.24	$\lambda \cos(\alpha) + L_{cy}/L = -0.23$	-0.47	-1.33 < 0	-	$\uparrow$	$\downarrow$
	NIS	0.24	$-\lambda \cos(\alpha) + L_{cy}/L = -1.10$	-0.86		+	$\downarrow$	$\uparrow$

4.2 A Serial Design: Design II

Inspired by Section 3.3, design II has a small center shift as shown in Fig. 17(a), and the compatibility conditions and the load-equilibrium equations of the analytical model are illustrated in Eqs. (19) and (20), respectively. d_{xDII} , d_{yDII} , and θ_{zDII} are solved with given f_{xDII} , f_{yDII} , and m_{zDII} .

$$\begin{aligned} d_{xDII} &= d_{xs2} + d_{xs1} \cos \theta_{zs2} - d_{ys1} \sin \theta_{zs2} \\ d_{yDII} &= d_{ys2} + d_{xs1} \sin \theta_{zs2} + d_{ys1} \cos \theta_{zs2} \end{aligned} \quad (19)$$

$$\theta_{zDII} = \theta_{zs1} + \theta_{zs2}$$

$$\begin{aligned} f_{xDII} &= f_{xs1} \cos \theta_{zs2} - f_{ys1} \sin \theta_{zs2} \\ f_{yDII} &= f_{xs1} \sin \theta_{zs2} + f_{ys1} \cos \theta_{zs2} \\ f_{xs2} &= f_{xs1} \cos \theta_{zs2} - f_{ys1} \sin \theta_{zs2} \\ f_{ys2} &= f_{xs1} \sin \theta_{zs2} + f_{ys1} \cos \theta_{zs2} \end{aligned} \quad (20)$$

$$m_{zDII} = m_{zs1} = m_{zs2}$$

where, f_{xDII} , f_{yDII} , and m_{zDII} denote the normalized loads of the design II as shown in Figs. 17(b) and (c). d_{xDII} , d_{yDII} , and θ_{zDII} denote the normalized displacements and rotation angles correspondingly.

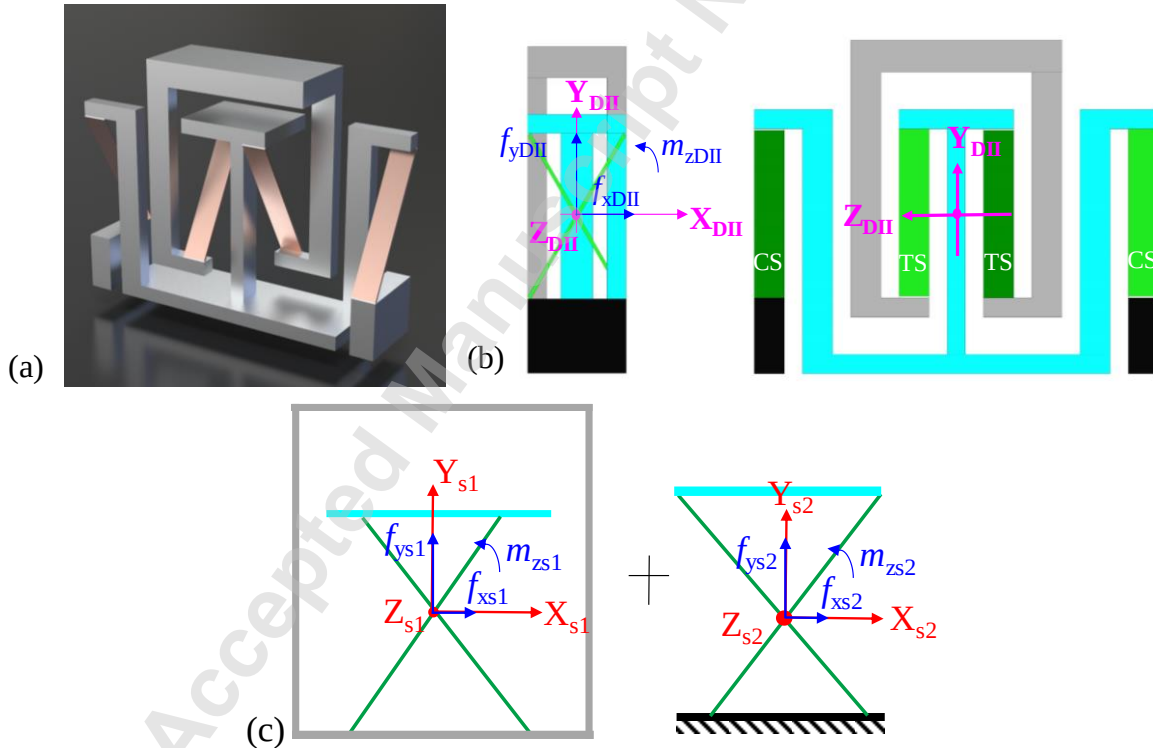


Fig. 17. Descriptions of design II: (a) a 3D model, (b) the global coordinate system, and (c) the local coordinate systems.

When $F_{xDII}=0$, $F_{yDII}=-0.5$ (N), M_{zDII} ranges from 0.001 to 0.015 (N·m), the results between analytical and FEA models are shown in Figs. 18(a) through (c). We use D_{xDII} , D_{yDII} to denote the displacements of design II and $D_{xDII}=d_{xDII}L$, $D_{yDII}=d_{yDII}L$. The maximum errors of D_{xDII} , D_{yDII} , and θ_{zDII} are 5.7%, 4.7%, and 1.67%, respectively, as shown in Fig. 18(d). The center shift along the X_{DII} -axis and Y_{DII} -axis of the design II are reduced by 10 and 100 times, respectively, compared with those of an S-CSP (Fig. 14).

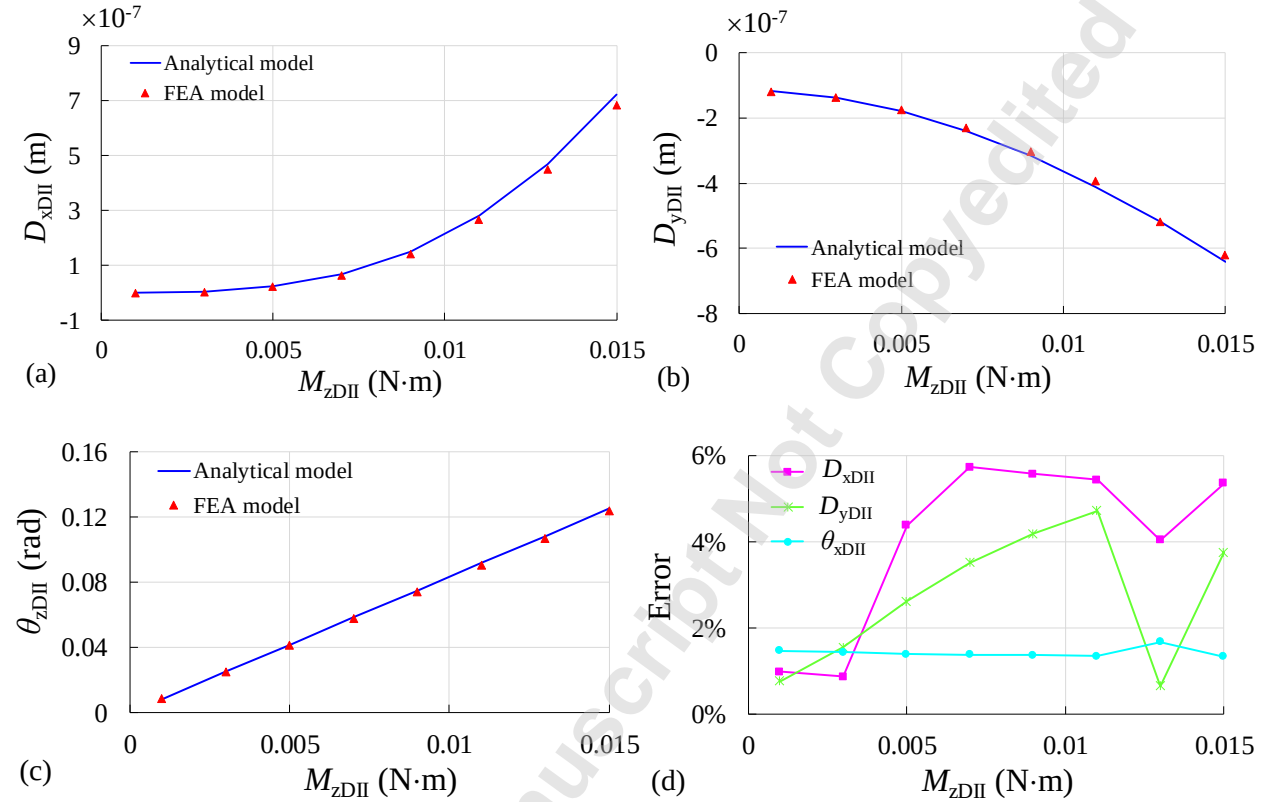


Fig. 18. The results of analytical and FEA models of the design II with $F_{yDII}=-0.5$ (N): (a) D_{xDII} , (b) D_{yDII} , (c) θ_{zDII} , and (d) errors of the results between the analytical and FEA models.

Figure 19 illustrates the performance differences between designs I and II. When only the rotational angle is fixed at 0.04 (rad) acting on the motion stage, the center shifts of the FEA models of design I and II are as follows: $D_{xDI}=3.58 \times 10^{-19}$ (m), $D_{yDI}=-1.24 \times 10^{-19}$ (m), $D_{xDII}=2.28 \times 10^{-8}$ (m), and $D_{yDII}=4.56 \times 10^{-10}$ (m). The magnitudes of the center shift of design I are much smaller than those of design II. However, when the moment is fixed, design II' rotational range is much larger than that of the design I. Design II is an ideal candidate for the application requiring the rotational ranges with a minimized center shift. On the other hand, the rotational stiffness of design I with any λ can be insensitive to axial loads, when M_{zDI} is fixed. If the axial forces act on the rotational center ($L_{cy}=0$), the loading positions and axial forces do not influence θ_{zDI} . If the axial forces do not act on the rotational center, axial forces can influence θ_{zDI} slightly within specified loading positions.

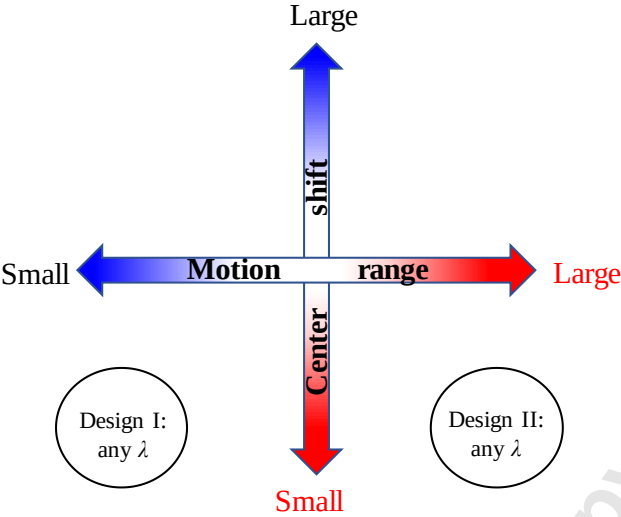


Fig. 19. Center-shift and rotational-range comparisons between designs I and II.

5 Conclusions

The closed-form model of an IG-CSP based on BCM is derived, along with the closed-form load-rotation relationship of the IS-CSP. We take the axial force (f_{ys}), geometric parameters, and the loading positions into consideration of the load-rotation relationship of the S-CSP for analyzing the load-dependent effects.

The load-dependent effects include the beam load-dependent effects and the structure load-dependent effects. The rotational stiffness can increase, decrease, or remain constant with the axial forces (f_{ys}) depending on the equilibrium of the beam load-dependent effects and the structure load-dependent effects. The coefficient of f_{ys} of the S-CSP load-rotation relationship is A_m , which is an expression of geometric parameters (λ , α) and the loading positions (h). The load-dependent effects can be designed by regulating the positive or negative sign of $A_m f_{ys}$. In other words, the equilibrium between the structure and beam load-dependent effects can be controlled by regulating λ , α and h . If an IS-CSP and a NIS-CSP are subjected to the same geometric parameters, axial loads, and loading positions, the absolute values of A_m are equal and the signs of A_m are opposite.

The closed-form center shift of the IS-CSP is derived and verified by analytical and FEA models. $9\lambda^2-9\lambda+1$ is a dominant term of the closed-form center shift solution. When $9\lambda^2-9\lambda+1$ is not equal to 0, the axial forces have less effect on the center shift. When an IS-CSP and a NIS-CSP have the same geometric parameter (required $9\lambda^2-9\lambda+1 \neq 0$) and are subjected to the same loading conditions, their absolute values of the center shift are close and their directions are opposite correspondingly.

Based on the above nonlinear analysis, two compound S-CSPs are proposed. When the applied moment is constant, the loading positions and axial forces can slightly influence the rotational stiffness of design I. Compared with an S-CSP, design II enlarges the rotations and minimizes the center shift along the X_{DII} -axis and Y_{DII} -axis by 10 and 100 times, respectively. In the future, the load-dependent effects and the effects of the axial forces on the center shift of the IG-CSP will be discussed.

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Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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593

594 **Nomenclature**

A_1 through A_6	The expressions of the independent geometric parameters and the loading position.
A_m	An expression for analyzing the loading depending effects of an IS-CSP. It consists independent geometric parameters (beam load-dependent effects) and loading positions (structure load-dependent effects).
A_{mDI}	A_m of design I.
A_{mgeo}	The value of A_m due to the geometric parameters.
D_{pi}	The translational matrix for the analytical analysis of IG-CSP ($i=1$ or 2).
d_{xc}, d_{yc}	Normalized center shift of the IG-CSP (or S-CSP)
D_{xc}, D_{yc}	Center shift of the IG-CSP (or S-CSP)
d_{xi}, d_{yi}	Normalized displacements of the sheet i 's free end along x_i and y_i axes, respectively ($i=1$ or 2).
d_{xs}, d_{ys}	Normalized displacements of the IG-CSP's motion stage (at point O_s) along the X_s and Y_s axes, respectively.
d_{xsi}, d_{ysi}	Normalized displacements of the IS-CSP's rotational center ($i=1$) or NIS-CSP's rotational center ($i=2$).
d_{xDI}, d_{yDI} d_{xDII}, d_{yDII}	Normalized displacements of design I and II at point O_D , respectively.
D_{xDI}, D_{yDI} D_{xDII}, D_{yDII}	Displacements of design I and II at point O_D , respectively.
E	Young's modulus.
f_{xi}, f_{yi}	Normalized forces of sheet i acting at the sheet's free end along x_i and y_i axes, respectively ($i=1$ or 2).
F_{xi}, F_{yi}	Actual forces of sheet i acting at the sheet's free end along x_i and y_i axes, respectively ($i=1$ or 2).
f_{xs}, f_{ys}	Normalized forces of the IG-CSP acting at the motion-stage center (at point O_s) along the X_s and Y_s axes, respectively.
f_{xsi}, f_{ysi}	Normalized forces of the IS-CSP ($i=1$) or NIS-CSP ($i=2$) acting at the rotational center.
f_{xDI}, f_{yDI} f_{xDII}, f_{yDII}	Normalized forces of design I and II acting at their rotational centers, respectively.
F_{xDI}, F_{yDI} F_{xDII}, F_{yDII}	Actual forces of design I and II acting at their rotational centers, respectively.
h	A parameter depends on the loading position.
K, K'	The constant torsional spring and compliant torsional spring, respectively.
K_{eff}	The effective stiffness of an inversed compliant mechanism.
k_{zm}	The normalized rotational stiffness due to the normalized moment m_{zs} of an IS-CSP.
K_{zm}	The rotational stiffness due to the moment M_{zs} of an IS-CSP.
k_{zmDI}	The actual normalized rotational stiffness due to the normalized moment m_{zsDI} of the design I.

K_{zmDI}	The actual rotational stiffness due to the moment M_{zsDI} of the design I.
ΔK	The rotational stiffness error correction term due to the beam load-dependent effect.
L	The sheet length of IS-CSP or NIS-CSP.
L_a, L_b	The lengths relate to loading positions of an inversed compliant mechanism.
L_{cy}	A directional distance between the rotational center and the loading position.
L_r	The length of the rigid rod or compliant flexure.
L_i	The length of sheet i along the x_i -axis ($i=1$ or 2).
m_{zi}	A normalized moment of sheet i acting at the sheet's free end ($i=1$ or 2).
M_{zi}	An actual moment of sheet i acting at the sheet's free end ($i=1$ or 2).
m_{zs}	The normalized moment of the IG-CSP acting at the motion-stage center (point O_s).
M_{zs}	A actual moment of the IG-CSP acting at the motion-stage center (point O_s).
m_{zsi}	The normalized moment of the IS-CSP ($i=1$) or NIS-CSP ($i=2$) acting at the rotational center.
m_{zDI}, m_{zDII}	Normalized moments of design I and II acting at their rotational centers, respectively.
M_{zDI}, M_{zDII}	Actual moments of design I and II acting at their rotational centers, respectively.
$O_i-X_iY_iZ_i$	The local coordinate system of the sheet i with the origin locating at the free end of the sheet i ($i=1$ or 2).
$O_s-X_sY_sZ_s$	The mobile coordinate system of the IG-CSP with the origin locating at the motion-stage center.
$O_{si}-X_{si}Y_{si}Z_{si}$	The local mobile coordinate system of the IS-CSP ($i=1$) or NIS-CSP ($i=2$) with the origin locating at their rotational centers, in design I or design II.
$O_{DI}-X_{DI}Y_{DI}Z_{DI}$	The global mobile coordinate system of design I.
$O_{DII}-X_{DII}Y_{DII}Z_{DII}$	The global mobile coordinate system of design II.
P	An axial force.
$\mathbf{R}_{zj}$	A rotational matrix about the Z_s -axis in compatibility conditions of an IG-CSP ($j=1, 2, 3$ or 4).
$\mathbf{R}_{zni}$	A rotational matrix about the Z_s -axis in compatibility conditions of a NIS-CSP ($j=1, 2, 3$ or 4).
$\mathbf{R}_{zs}$	A rotational matrix about the Z_s -axis in load-equilibrium conditions of an IG-CSP.
r_1, r_2	$r_1=1, r_2=L_2/L_1$.
S_i	The free end of the sheet i of an IG-CSP ($i=1$ or 2).
S_{ni}	The free end of the sheet i of a NIS-CSP ($i=1$ or 2).
S_i	The coordinate of point S_i with regard to the global coordinate system of an IG-CSP ($i=1$ or 2).
S_{ni}	The coordinates of point S_{ni} relative to with regard to the global coordinate system of a NIS-CSP ($i=1$ or 2).
S_i^*	The coordinates of point S_i relative to the global coordinate system after only the rotation of the motion stage ($i=1$ or 2).
T	The sheet thickness of an IS-CSP or a NIS-CSP.
T_i	The sheet i 's thickness of an IG-CSP along the y_i -axes ($i=1$ or 2).

U	The sheet width of an IS-CSP or a NIS-CSP.
U_i	The sheet i 's width of an IG-CSP along the z_i -axes ($i=1$ or 2).
y_j	Examples of loading locations on the IS-CSP ($j=1,2,\dots$ or, 7).
α_i	The tilt angle of sheet i of an IG-CSP ($i=1$ or 2).
α	The tilt angles of sheets of an IS-CSP or NIS-CSP.
θ_{zi}	The normalized rotation of sheet i ($i=1$ or 2).
θ_{zs}	The normalized rotation of the IG-CSP's motion stage.
θ_{zsi}	The normalized rotation of the IS-CSP ($i=1$) or NIS-CSP ($i=2$).
$\theta_{zDI}, \theta_{zDII},$	Normalized rotations of design I and II, respectively.
λ_i	The sheet-length parameter of sheet i of an IG-CSP ($i=1$ or 2).
λ	The sheet-length parameter of an IS-CSP or NIS-CSP.

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Fig. 13	The effects of F_{ys} on D_{xc} and D_{yc} of the closed-form models: (a) D_{xc} when λ is $1/2 + \sqrt{5}/6$, (b) D_{yc} when λ is $1/2 + \sqrt{5}/6$, (c) D_{xc} when λ is 0.4, (d) D_{yc} when λ is 0.4, (e) D_{xc} when λ is 0.7, and (f) D_{yc} when λ is 0.7.
Fig. 14	The center-shift comparison between an IS-CSP and a NIS-CSP: (a) D_{xc} , and (b) D_{yc} .
Fig. 15	Descriptions of the design I: (a) a 3D model, (b) the global coordinate system ('CS' denotes a compressive sheet, 'TS' denotes a tensile sheet), and (c) the local coordinate systems.
Fig. 16	The effects of loading positions and axial forces on θ_{zDI} with $M_{zDI} = 0.04 \text{ (N}\cdot\text{m)}$: (a) above the rotational center ($L_{cy} > 0$), and (b) below the rotational center ($L_{cy} < 0$).
Fig. 17	Descriptions of design II: (a) a 3D model, (b) the global coordinate system, and (c) the local coordinate systems.
Fig. 18	The results of analytical and FEA models of the design II with $F_{yDII} = -0.5 \text{ (N)}$: (a) D_{xDII} , (b) D_{yDII} , (c) θ_{zDII} , and (d) errors of the results between the analytical and FEA models.

Fig. 19 Center-shift and rotational-range comparisons between designs I and II.

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