

Title	The role of synchronization in digital communications using chaos - part III: performance bounds for correlation receivers
Authors	Kolumbán, Géza;Kennedy, Michael Peter
Publication date	2000-12
Original Citation	Kolumban, G., Kennedy, M.P., 2000. Communications using chaos - part III: performance bounds for correlation receivers. IEEE Transactions on Circuits and Systems I: Fundamental Theory and Applications, 47(12), pp.1673-1683. doi: 10.1109/81.899919
Type of publication	Article (peer-reviewed)
Link to publisher's version	10.1109/81.899919
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The Role of Synchronization in Digital Communications Using Chaos—Part III: Performance Bounds for Correlation Receivers

Géza Kolumbán, *Senior Member, IEEE*, and Michael Peter Kennedy, *Fellow, IEEE*

Abstract—In a digital communications system, data is transmitted from one location to another by mapping bit sequences to symbols, and symbols to sample functions of analog waveforms. The analog waveform passes through a bandlimited (possibly time-varying) analog channel, where the signal is distorted and noise is added. In a typical conventional system, the analog sample functions sent through the channel are weighted sums of one or more sinusoids, called basis functions; in a chaotic communications system, the sample functions are segments of chaotic waveforms.

At the receiver, the symbols may be recovered by means of coherent detection, where all possible sample functions are known, or by noncoherent detection, where one or more characteristics of the sample functions are determined based on the received signal.

In a coherent receiver, synchronization is the most commonly used technique for recovering the sample functions from the received waveform. These sample functions are then used as reference signals for correlators.

Synchronization-based coherent receivers have advantages over noncoherent receivers in terms of bandwidth efficiency (in narrow-band systems), data rate (in chaotic systems), and noise performance (both).

These advantages are lost if carrier synchronization cannot be maintained, for example, under poor propagation conditions. In these circumstances, communications without synchronization may be preferable.

This three-part paper shows in a tutorial manner how the theory of conventional telecommunications systems can be applied to chaotic modulation schemes. In addition, it discusses the latest results in the field of chaotic communications.

In Part I [1], the theory and operation of conventional communications systems are surveyed and possible fields of application of chaotic communications are identified.

In Part II [2], the theory of conventional telecommunications is extended to chaotic communications, chaotic modulation techniques and receiver configurations are surveyed, and chaotic synchronization schemes are described.

In Part III, examples are given of chaotic communications schemes with and without synchronization, and the performance of correlator-based systems is evaluated in the context of noisy, bandlimited channels.

Index Terms—Chaotic communications, chaotic correlation receivers, chaotic modulation, estimation problem, noise performance bounds.

Manuscript received December 20, 1999; revised September 5, 2000. This work was supported by the Information Technologies RTD Program of the European Commission under Esprit Project 31103 INSPECT.

G. Kolumbán is with the Department of Measurement and Information Systems, Budapest University of Technology and Economics, H-1521 Budapest, Hungary.

M. P. Kennedy is with the Department of Microelectronic Engineering, University College, Cork, Ireland.

Publisher Item Identifier S 1057-7122(00)10638-5.

I. INTRODUCTION

OVER THE past five years, much research effort has been devoted to the study of digital modulation schemes using chaotic basis functions [3]–[19].

It is now possible to make definitive statements about the noise performance of these schemes. The aim of this tutorial paper is to present theoretical performance bounds for correlator-based chaotic digital communications schemes, to summarize the performance of some representative schemes relative to these limits, and to highlight the expected best case performance in real applications.

In Section II, we extend the basis function approach to modulation and demodulation using chaotic basis functions and highlight the problem associated with estimating statistical properties of chaotic signals from sample functions of finite length.

The estimation problem, which results from an inherent characteristic of a chaotic communications system, namely that the basis functions vary from symbol to symbol even if the same symbol is transmitted repeatedly, potentially degrades the performance of every chaotic digital modulation scheme. We discuss this problem in Section III and show that it can be solved by using orthonormal basis functions.

The digital chaotic modulation schemes which we consider in this work are analyzed in the context of a receiver model, which is described in Section IV.

In Section V, we show by computer simulation that a chaotic modulation scheme with one basis function, referred to as antipodal chaos shift keying (CSK), can theoretically achieve the noise performance of binary phase-shift keying (BPSK). In practice, this performance cannot be reached because at least two problems must be overcome: the estimation problem and recovery of the basis function independently of the modulation. Recovery of chaotic basis functions independently of the modulation is difficult to achieve; failure to solve this problem to date has impeded the development of coherent demodulators for chaotic communications.

With no available solution to the problem of basis function recovery independently of the modulation, alternative modulation schemes have been proposed which exploit two basis functions. In Section VI, we show that CSK with two basis functions and a coherent receiver, referred to as chaotic switching, can theoretically achieve the noise performance of coherent frequency-shift keying (FSK).

This level of performance can be reached only if the estimation problem is solved and the basis functions can be recovered

at the demodulator. While the former problem can be readily solved, implementation difficulties associated with the recovery of chaotic basis functions cause published results for coherent CSK receivers to lag far behind their theoretical maximum noise performance.

Given the difficulty of recovering chaotic basis functions at the receiver, one may ask what is the best performance that can be expected without recovering the basis functions. It has been shown in [20] that the noise performance of CSK with two appropriately constructed basis functions and a differentially coherent correlation receiver is only 3 dB worse than that of differential phase-shift keying (DPSK) with autocorrelation demodulation [21].

The results of this study are summarized in Section VII and lead to a number of important conclusions (see Section VIII).

If it is possible to recover the basis functions at the receiver independently of the modulation, then antipodal CSK, where one basis function is used, offers the best noise performance of all known chaotic digital modulation schemes.

As shown analytically in [20], the choice of waveform of the basis function, even if it differs for every transmitted bit, makes no difference to the theoretical noise performance, provided that the energy per bit is constant.

In practice, the primary issue influencing the choice of a chaotic or conventional basis function in coherent communications is the robustness of the basis function recovery process. The current state of the art is that antipodal CSK has not been demonstrated. The reported performance of CSK with two basis functions lags significantly behind that of coherent FSK, which represents the theoretical upper bound on its performance.

If one considers only the noise performance of the modulation schemes in a bandlimited single-ray additive white Gaussian noise (AWGN) channel [22] under propagation conditions where the basis functions can be recovered at the receiver, then higher performance can be achieved by using a conventional narrow-band modulation scheme with periodic, rather than chaotic, basis functions.

If it is not possible to recover the basis functions at the receiver then, in a single-ray channel, differential chaos shift keying (DCSK) with differentially coherent detection can reach within 3 dB of the noise performance of DPSK with an autocorrelation demodulator; in a multipath channel, wide-band DCSK can offer significantly better performance.

II. CHAOTIC MODULATION AND DEMODULATION

Chaotic digital modulation is concerned with mapping symbols to analog chaotic waveforms. In CSK [23], information is carried in the weights of a combination of basis functions, which are derived from chaotic signals. Differential chaos shift keying is a variant of CSK, where the basis functions have a special structure and the information can be recovered from the correlation between the two parts of the basis functions.

In this paper, we concentrate on the transmission and reception of a single isolated symbol; problems arising from the reception of symbol streams are not treated here.

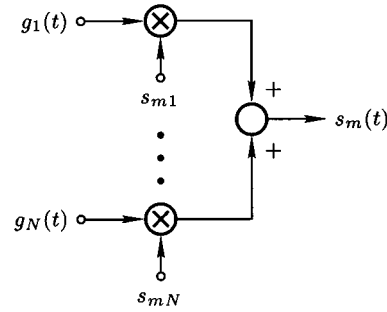


Fig. 1. Generation of the elements of signal set.

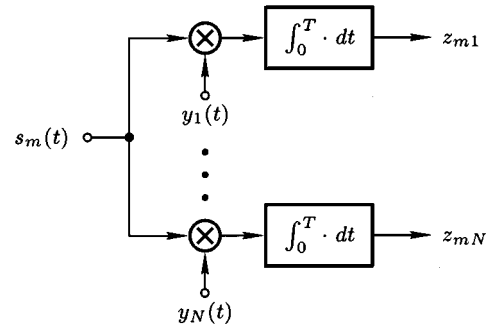


Fig. 2. Determination of the observation vector in a correlation receiver.

A. Modulation

Using the notation introduced in [1], the elements of the CSK signal set are defined by

$$s_m(t) = \sum_{j=1}^N s_{mj} g_j(t), \quad j = 1, 2, \dots, N$$

where the weights s_{mj} are the elements of the signal vector and the basis functions $g_j(t)$ are chaotic waveforms. The signals $s_m(t)$ may be produced conceptually as shown in Fig. 1.

Note that the shape of the basis functions is not fixed in chaotic communications. This is why the signal $s_m(t)$ which is transmitted through the channel has a different shape during every symbol interval of duration T , even if the same symbol is transmitted repeatedly. As a result, the transmitted signal is never periodic.

To achieve the best noise performance, basis functions must be orthonormal [22]. In the general case, chaotic basis functions are orthonormal only in the mean, i.e.,

$$E \left[\int_0^T g_j(t) g_k(t) dt \right] = \begin{cases} 1, & \text{if } j = k \\ 0, & \text{otherwise} \end{cases} \quad (1)$$

where T is the bit duration and $E[\cdot]$ denotes the expectation operator.

Equation (1) identifies another important characteristic of chaotic modulation schemes: the basis functions are not fixed waveforms, but can be modeled only as sample functions of stochastic processes.

Consequently, the cross-correlation and autocorrelation of basis functions evaluated for the bit duration become random

numbers which can be characterized by their mean value and variance.¹ The consequences of this property, called the *estimation problem*, will be discussed in Section III.

B. Demodulation

The message may be recovered at the receiver by correlating the received signal with reference signals $y_1(t), y_2(t), \dots, y_N(t)$, and forming the corresponding observation signals $z_{m1}, z_{m2}, \dots, z_{mN}$, as shown in Fig. 2.

The reference signal $y_j(t)$ can be generated in a number of different ways: it can be the received signal itself, or a delayed version of the received signal, or a basis function recovered from the received signal.

In a coherent correlation receiver, the reference signals $y_j(t)$ are locally regenerated copies of the basis functions $g_j(t)$. When signal $s_m(t)$ is transmitted and $y_j(t) = g_j(t)$, the j th element z_{mj} of the observation vector emerging from the j th correlator is given by

$$\begin{aligned} z_{mj} &= \int_0^T s_m(t) y_j(t) dt \\ &= \int_0^T \left[\sum_{k=1}^N s_{mk} g_k(t) \right] g_j(t) dt \\ &= s_{mj} \int_0^T g_j^2(t) dt + \sum_{\substack{k=1 \\ k \neq j}}^N s_{mk} \int_0^T g_j(t) g_k(t) dt \end{aligned}$$

where $E[\int_0^T g_j^2(t) dt] = 1$ and $E[\int_0^T g_j(t) g_k(t) dt] = 0$ for $k \neq j$.

If the bit duration T is sufficiently long, then $\int_0^T g_j^2(t) dt \approx 1$ and $\int_0^T g_k(t) g_j(t) dt \approx 0$. In this case,

$$z_{mj} \approx s_{mj}. \quad (2)$$

Thus, in the case of a distortion- and noise-free channel, and for a sufficiently long bit duration, the observation and signal vectors are approximately equal to each other.

In this way, the elements s_{mj} of the signal vector can be recovered (approximately) by correlating the received signal with the reference signals $y_j(t)$.

In real applications, the elements z_{mj} of the observation vector are random numbers because of the estimation problem and additive channel noise; in addition, their values are influenced by a number of factors, including channel filtering and distortion. This is why the observation vector can be considered only as an *estimation* of the signal vector.

While filtering, distortion, and noise effects are common to all communication systems, the estimation problem results from using chaotic basis functions. In the next section, we explain the two sources of the estimation problem and indicate how to solve it.

¹By contrast, for example, sine and cosine basis functions can be made orthonormal by appropriate scaling and by choosing the ratio of the bit duration and the period of each basis function to be an integer.

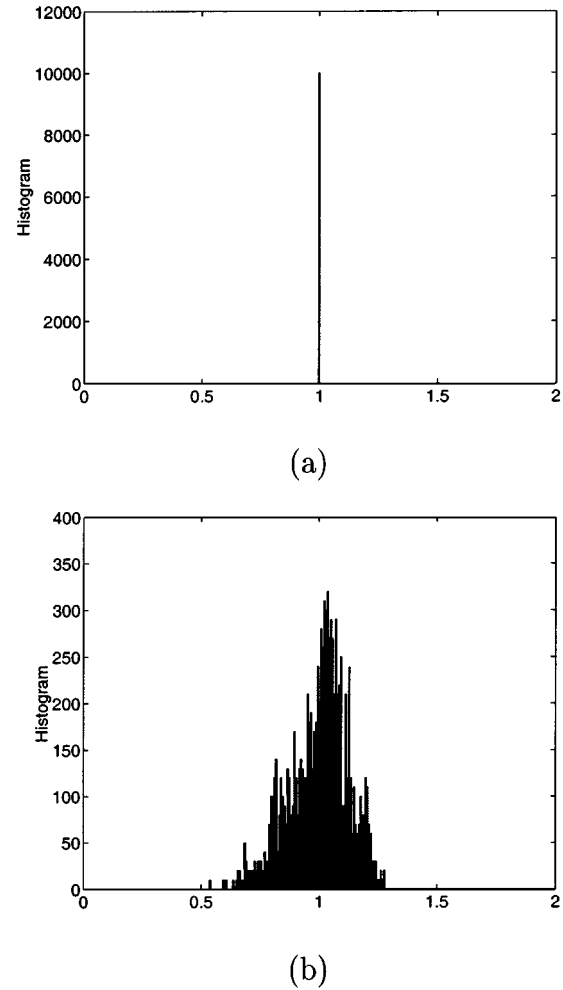


Fig. 3. Samples of $\int_0^T g_j^2(t) dt$ for (a) periodic and (b) chaotic basis functions $g_j(t)$.

III. THE ESTIMATION PROBLEM

A. Autocorrelation Estimation Problem

In a typical conventional modulation scheme, the basis functions are periodic and the bit duration T is an integer multiple of the period of the basis functions; hence, $\int_0^T g_j^2(t) dt$ is constant. By contrast, chaotic signals are inherently nonperiodic and $g_j(t)$ is different in every interval of length T . Consequently, $\int_0^T g_j^2(t) dt$ is different for every symbol, even if the same symbol is transmitted repeatedly.

Fig. 3(a) and (b) shows histograms of samples of $\int_0^T g_j^2(t) dt$ for periodic and chaotic waveforms $g_j(t)$, respectively. In the periodic case, all samples lie at $\int_0^T g_j^2(t) dt = 1$. By contrast, the samples in the chaotic case are centered at $E[\int_0^T g_j^2(t) dt] = 1$, as before, but have nonzero variance.

This nonzero variance causes the components z_{mj} of the observation vector to differ from the corresponding components s_{mj} of the signal vector and consequently causes errors in interpreting the received signal. The consequence is a considerable degradation in noise performance, as will be seen in Figs. 10 and 11.

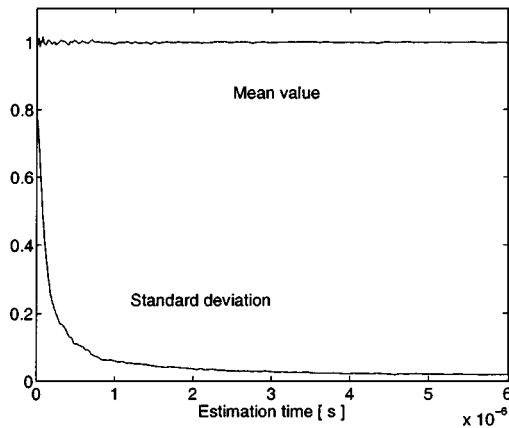


Fig. 4. Mean and standard deviation of the estimation of $\int_0^T g_1^2(t) dt$ versus the estimation time.

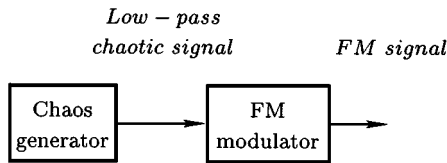


Fig. 5. Block diagram illustrating the generation of chaotic sample functions with constant energy per bit by means of a frequency modulator.

Let the equivalent statistical bandwidth² [25] of the chaotic signal $g_j(t)$ be defined by

$$BW_{eq} = \frac{1}{S_C(0)} \int_{-\infty}^{\infty} S_C(f) df$$

where $S_C(f)$ is the power spectral density associated with the stationary chaotic stochastic process [24]. Then, the standard deviation of samples of $\int_0^T g_j^2(t) dt$ scales approximately as $1/(BW_{eq}T)$, as shown in Fig. 4. Note that the variance of estimation can be reduced by increasing the statistical bandwidth of the transmitted chaotic signal or by increasing the bit duration T [25]. Alternatively, one may solve the autocorrelation estimation problem directly by modifying the generation of basis functions such that the transmitted energy for each symbol is kept constant.

1) *Sample Solution:* Recall that the instantaneous power of an FM signal does not depend on the modulation, provided that the latter is slowly varying compared to the carrier. Therefore, one way to produce a chaotic sample function with constant energy per bit E_b is to apply a chaotic signal to a frequency modulator; this can be achieved as shown in Fig. 5.

2) *Conclusion:* A necessary condition for chaotic digital modulation schemes to reach their maximum noise performance is that the chaotic sample functions should have constant energy per bit [20]. Therefore, in the remainder of this paper, we will assume that the chaotic sample functions have constant

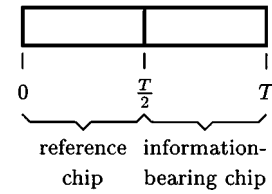


Fig. 6. A DCSK basis function $g_j(t)$ consists of two segments called the reference and information-bearing chips.

E_b . In particular, we normalize the basis functions such that, for all j

$$\int_0^T g_j^2(t) dt = 1.$$

B. The Cross-Correlation Estimation Problem

The estimation problem also arises when evaluating the cross-correlation between different chaotic basis functions of finite length. Although $E[\int_0^T g_j(t)g_k(t) dt] = 0$, for sufficiently large T , $\int_0^T g_j(t)g_k(t) dt \neq 0$ in general, unless $g_j(t)$ and $g_k(t)$ are orthogonal in $[0, T]$.

1) *Sample Solution:* Consider the basis functions defined by

$$\begin{aligned} g_1(t) &= \begin{cases} +\frac{1}{\sqrt{E_b}} c(t), & 0 \leq t < T/2 \\ +\frac{1}{\sqrt{E_b}} c(t - T/2), & T/2 \leq t < T \end{cases} \\ g_2(t) &= \begin{cases} +\frac{1}{\sqrt{E_b}} c(t), & 0 \leq t < T/2 \\ -\frac{1}{\sqrt{E_b}} c(t - T/2), & T/2 \leq t < T \end{cases} \end{aligned} \quad (3)$$

where $c(t)$ is derived from a chaotic waveform and has the property that $\int_0^{T/2} c^2(t) dt = E_b/2$. Each basis function consists of two segments, called the reference and information-bearing chips, respectively. This is shown schematically in Fig. 6.

Because the digital information to be recovered is also carried in the correlation between the reference and information-bearing chips, we call these differential CSK (DCSK) basis functions.

Although $c(t)$ is not periodic, the DCSK basis functions $g_1(t)$ and $g_2(t)$ are orthogonal, i.e., $\int_0^T g_1(t)g_2(t) dt = 0$.

Note that, in addition, $\int_0^T g_1^2(t) dt = \int_0^T g_2^2(t) dt = 1$ (see Section III-A-2). Therefore, the DCSK basis functions are *orthonormal*.

2) *Conclusion:* By using orthogonal basis functions, the cross-correlation estimation problem can be solved.

IV. RECEIVER MODEL

Noise performance is the most important characteristic of a modulation scheme and receiver configuration. In Part II [2], we have shown that all of the chaotic modulation techniques discussed in this work can be considered under the unifying umbrella of the basis function approach. Here, we consider their

²In a chaotic stochastic process, the ensemble of sample functions is generated by the same chaotic attractor starting from all possible initial conditions [24].

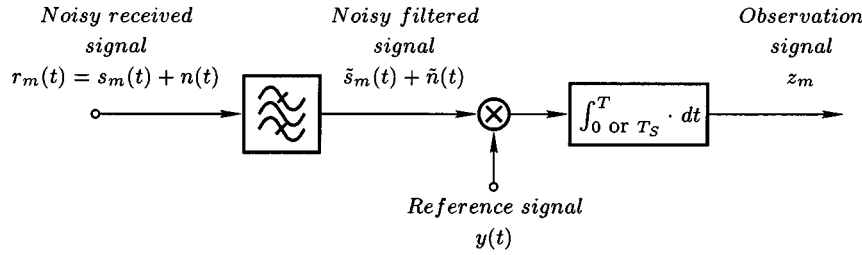


Fig. 7. General block diagram of a digital chaotic communications receiver.

noise performance, assuming the receiver block diagram shown in Fig. 7, where $r_m(t) = s_m(t) + n(t)$ and $\tilde{s}_m(t) + \tilde{n}(t)$ denote the noisy received signal before and after filtering, respectively. Note that the channel (selection) filter, which is an ideal band-pass filter with a total RF bandwidth of $2B$, is included explicitly in this figure.

This model can be used to characterize the performance of noncoherent, differentially coherent, and coherent correlation receivers. The difference between these schemes is primarily due to the way in which the reference signal $y(t)$ is generated at the receiver.

In the following sections, we use this model to develop performance limits for CSK with one and two basis functions.

V. CSK WITH ONE BASIS FUNCTION

A. Modulation

In the simplest case of binary CSK, a single chaotic basis function $g_1(t)$ is used, i.e.,

$$s_m(t) = s_{m1}g_1(t).$$

At least two types of CSK based on a single basis function can be imagined: chaotic on-off keying (COOK) and antipodal CSK.

In COOK, symbol "1" is represented by $s_1(t) = \sqrt{2E_b}g_1(t)$ and symbol "0" is given by $s_2(t) = 0$. Equivalently,

$$s_{11} = \sqrt{2E_b}; \quad s_{21} = 0$$

where E_b denotes the average energy per bit and we have assumed that the probabilities of symbols "1" and "0" are equal.

The upper limit on the noise performance of a modulation scheme is determined by the separation of the message points in the signal space; the greater the separation, the better the noise performance. Fig. 8 shows the signal-space diagram for COOK.

The distance between the message points in COOK is $\sqrt{2E_b}$.

In antipodal CSK, symbol "1" is represented by $s_1(t) = \sqrt{E_b}g_1(t)$ and symbol "0" is given by $s_2(t) = -\sqrt{E_b}g_1(t)$. Fig. 9 shows the signal-space diagram for antipodal CSK.

The distance between the message points is $2\sqrt{E_b}$. Consequently, the noise performance of antipodal CSK is potentially superior to that of COOK.

While the modulator determines the distance between the message points, the noise performance of the system depends on the efficiency with which the demodulator exploits this separation.

In principle, the best noise performance in an AWGN channel can be achieved by using a coherent receiver. In practice, the

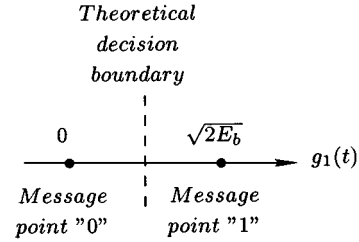


Fig. 8. Signal-space diagram for binary COOK.

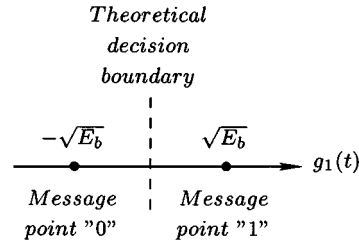


Fig. 9. Signal-space diagram for binary antipodal CSK.

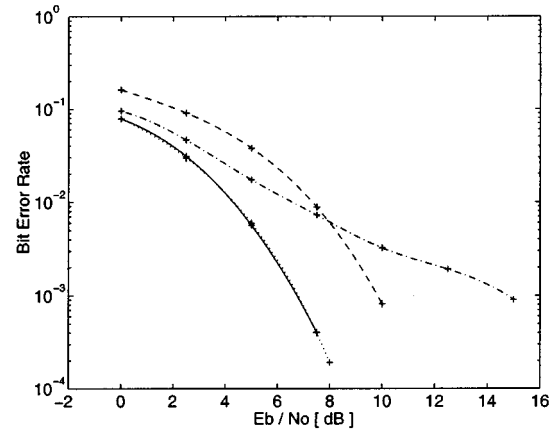


Fig. 10. Simulated optimum noise performance of COOK and antipodal CSK with a coherent correlation receiver: antipodal CSK with constant bit energy E_b (solid curve); COOK with equiprobable symbols and constant bit energies 0 and $2E_b$ (dashed curve); antipodal CSK with nonconstant bit energy (dash-dot curve). Coherent BPSK is shown (dotted) for comparison.

propagation conditions may be so poor that it is difficult, if not impossible, to regenerate the basis functions at the receiver. Under these conditions, a noncoherent or differentially coherent receiver may offer better performance.

B. Demodulation

1) *Coherent Correlation Receiver:* In a coherent correlation receiver, the reference signal $y(t)$ at the receiver is the basis

function which has been recovered from the noisy filtered received signal. The observation signal is given by

$$\begin{aligned} z_m &= \int_{T_S}^T [\tilde{s}_m(t) + \tilde{n}(t)]y(t) dt \\ &= \int_{T_S}^T [s_{m1}\tilde{g}_1(t) + \tilde{n}(t)]y(t) dt \\ &= s_{m1} \int_{T_S}^T \tilde{g}_1(t)y(t) dt + \int_{T_S}^T \tilde{n}(t)y(t) dt \end{aligned} \quad (4)$$

where we assume that the synchronization transient lasts at most T_S seconds per symbol period. In the best case, where synchronization of $y(t)$ with $g_1(t)$ is maintained throughout the transmission $T_S = 0$.

Note that z_m is a *random variable*, whose mean value depends on the energy per bit of the chaotic signal and the “goodness” with which the basis function has been recovered [see the first term in (4)].

In a noise-free channel with exact recovery of the basis function, a sufficiently wide-band channel filter, and permanent synchronization, $y(t) = g_1(t)$, $\tilde{g}_1(t) = g_1(t)$, and $T_S = 0$. The observation variable in this case is

$$z_m = s_{m1} \int_0^T g_1^2(t) dt = s_{m1}.$$

Therefore, this receiver can be used to demodulate both COOK and antipodal CSK, provided that the basis function $g_1(t)$ can be recovered from the received signal $s_m(t)$.

Figure 10 shows, by simulation, the theoretical upper bounds on the noise performance of coherent COOK and coherent antipodal CSK with one basis function. Note that the noise performance of antipodal CSK (solid) exceeds that of COOK (dashed) by 3 dB; this is a consequence of the greater separation of the message points (by a factor of $\sqrt{2}$) at the modulator.

Figure 10 also highlights the autocorrelation estimation problem in the case of antipodal CSK when the bit energy has nonzero variance. In this case, the effective noise level at high E_b/N_0 is dominated by the variance of the energy per bit $E_b \int_0^T g_1^2(t) dt$. If $\int_0^T g_1^2(t) dt$ is kept constant, the problem disappears.

2) *Conclusion:* The noise performance of an antipodal CSK modulator and coherent correlation receiver can theoretically match that of BPSK. This performance can be achieved only if the following necessary conditions are satisfied:

- energy per bit is kept constant;
- basis function $g_1(t)$ is recovered exactly at the receiver, independently of the modulation.

The first condition can be satisfied in the case of chaotic basis functions by using FM, for example, as described in Section III.

Although several strategies for recovering the basis function $g_1(t)$ have been proposed in the literature under the title “chaotic synchronization” [3], we are not aware of any chaotic synchronization technique which can regenerate the basis function exactly, *independently of the modulation*. If the basis function cannot be recovered exactly, the noise performance of antipodal modulation is degraded significantly.

3) *Noncoherent Correlation Receiver:* Although antipodal CSK cannot be demodulated without recovering the basis function, COOK can still be demodulated by means of a noncoherent receiver.

In a noncoherent correlation receiver, the reference signal $y(t)$ is equal to the noisy filtered signal $\tilde{s}_m(t) + \tilde{n}(t)$, and the observation signal can be expressed as

$$\begin{aligned} z_m &= \int_0^T [\tilde{s}_m(t) + \tilde{n}(t)]^2 dt \\ &= \int_0^T \tilde{s}_m^2(t) dt + 2 \int_0^T \tilde{s}_m(t)\tilde{n}(t) dt + \int_0^T \tilde{n}^2(t) dt \\ &= s_{m1}^2 \int_0^T \tilde{g}_1^2(t) dt + 2s_{m1} \int_0^T \tilde{g}_1(t)\tilde{n}(t) dt \\ &\quad + \int_0^T \tilde{n}^2(t) dt. \end{aligned} \quad (5)$$

In the noise-free case, if the signal $s_m(t)$ emerges unchanged from the channel [$\tilde{g}_1(t) = g_1(t)$], the observation signal is equal to the energy of the transmitted symbol, i.e.,

$$z_m = s_{m1}^2 \int_0^T g_1^2(t) dt.$$

Since $s_{11}^2 = s_{21}^2 = E_b$ in antipodal CSK, the symbols cannot be distinguished at the receiver. By contrast, the observation signals z_1 and z_2 of the two COOK symbols differ by $2E_b$, where E_b is the average bit energy. Figure 11 shows the noise performance for noncoherent COOK.

Figure 11 shows that the autocorrelation estimation problem manifests itself if $\int_0^T g_1^2(t) dt$ is not constant but disappears, as expected, when $\int_0^T g_1^2(t) dt$ is constant.

The noise performance of noncoherent COOK is worse than that of coherent COOK, due to the second and third terms in (5). Although $E[\int_0^T \tilde{g}_1(t)\tilde{n}(t) dt] = 0$, $E[\int_0^T \tilde{n}^2(t) dt] > 0$. Hence, z_m is a biased estimator of s_{m1}^2 and the decision threshold must be adjusted depending on the signal to noise ratio (SNR) at the demodulator input.

4) *Conclusion:* For a given energy per bit, a single basis function, and a noncoherent correlation receiver, the best noise performance can be achieved by COOK. However, COOK suffers two significant drawbacks:

- transmitted energy per bit varies between zero for symbol “0” and $2E_b$ for symbol “1”;
- optimum decision threshold at the receiver depends on the SNR.

The design of a digital communications receiver can be simplified considerably if the decision threshold at the demodulator is independent of the SNR. By using two basis functions, this condition can be satisfied.

VI. CSK WITH TWO BASIS FUNCTIONS

A. Modulation

In CSK with two basis functions, the elements of the signal set are given by

$$s_m(t) = s_{m1}g_1(t) + s_{m2}g_2(t)$$

where the basis functions $g_1(t)$ and $g_2(t)$ are derived from chaotic sources.

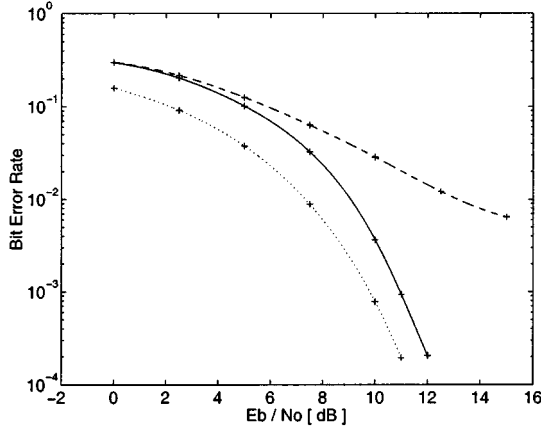


Fig. 11. Simulated noise performance of noncoherent COOK with constant (solid) and varying (dashed) energy per symbol. Coherent COOK is shown (dotted) for comparison.

In a special case of binary CSK, also called “chaotic switching” [19], the two elements of the signal set are simply weighted basis functions; the transmitted sample functions are $s_1(t) = \sqrt{E_b}g_1(t)$ and $s_2(t) = \sqrt{E_b}g_2(t)$, representing symbols “1” and “0”, respectively. The corresponding signal vectors are $(s_{11} \ s_{12}) = (\sqrt{E_b} \ 0)$ and $(s_{21} \ s_{22}) = (0 \ \sqrt{E_b})$, where E_b denotes the average energy per bit.

The signal-space diagram for chaotic switching is shown in Fig. 12. Note that the Euclidean distance between the two message points is $\sqrt{2E_b}$, which is the same as for COOK but is less than that of antipodal CSK with one basis function. This implies that the noise performance of chaotic switching is at best 3 dB worse than that of the antipodal modulation scheme described in Section V.

B. Demodulation

1) *Coherent Correlation Receiver*: A coherent correlation receiver, as shown in Fig. 13, may be used to estimate the elements s_{mj} of the signal vector. In the noise-free case, with perfect regeneration of the basis functions, $y_1(t) = g_1(t)$ and $y_2(t) = g_2(t)$, giving

$$\begin{aligned} z_{m1} &= \int_0^T s_m(t)g_1(t) dt \\ &= s_{m1} \int_0^T g_1^2(t) dt + s_{m2} \int_0^T g_1(t)g_2(t) dt \\ &= s_{m1} \int_0^T g_1^2(t) dt \end{aligned} \quad (6)$$

$$\begin{aligned} z_{m2} &= \int_0^T s_m(t)g_2(t) dt \\ &= s_{m1} \int_0^T g_1(t)g_2(t) dt + s_{m2} \int_0^T g_2^2(t) dt \\ &= s_{m2} \int_0^T g_2^2(t) dt. \end{aligned} \quad (7)$$

In the case of chaotic switching with constant E_b , and assuming that the basis functions $g_1(t)$ and $g_2(t)$ are orthonormal in the interval $[0, T]$, the outputs of the correlators become

$$z_{11} = s_{11} \int_0^T g_1^2(t) dt = \sqrt{E_b}$$

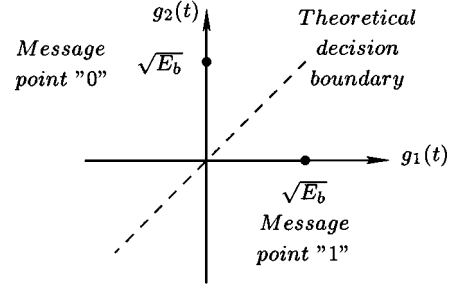


Fig. 12. Signal-space diagram of chaotic switching.

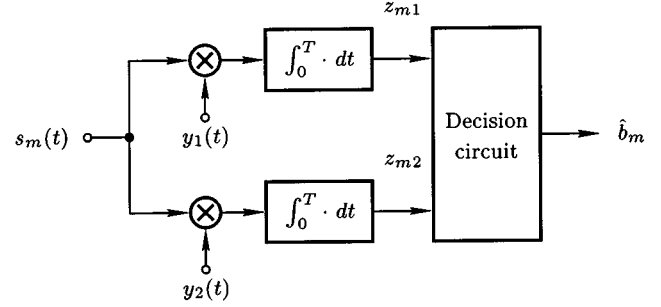


Fig. 13. Idealized coherent correlation receiver for CSK with two basis functions. The effect of the channel filter is neglected.

$$z_{12} = s_{12} \int_0^T g_1(t)g_2(t) dt = 0$$

when symbol “1” is transmitted, and

$$z_{21} = s_{21} \int_0^T g_1(t)g_2(t) dt = 0$$

$$z_{22} = s_{22} \int_0^T g_2^2(t) dt = \sqrt{E_b}$$

when symbol “0” is transmitted.

Thus, the correlation receiver structure may be used to identify which element of the signal set is more likely to have produced the received signal [1]. In particular, if $z_{m1} > z_{m2}$, then the decision circuit decides in favor of symbol “1”; if $z_{m1} < z_{m2}$, then the decision circuit decides in favor of symbol “0”.

Consider now the noise performance of chaotic switching with orthonormal basis functions and a coherent correlation receiver which also includes the channel filter. In this case, the reference signal in each arm is a regenerated basis function $y_j(t)$ which is derived from the filtered noisy received signal $\tilde{s}_m(t) + \tilde{n}(t)$.

The observation signals z_{mj} , $j = 1, 2$, are given by

$$\begin{aligned} z_{mj} &= \int_{T_S}^T [\tilde{s}_m(t) + \tilde{n}(t)]y_j(t) dt \\ &= \int_{T_S}^T [s_{m1}\tilde{g}_1(t) + s_{m2}\tilde{g}_2(t) + \tilde{n}(t)]y_j(t) dt \\ &= s_{m1} \int_{T_S}^T \tilde{g}_1(t)y_j(t) dt + s_{m2} \int_{T_S}^T \tilde{g}_2(t)y_j(t) dt \\ &\quad + \int_{T_S}^T \tilde{n}(t)y_j(t) dt \end{aligned} \quad (8)$$

where we assume that the synchronization transient lasts at most T_S seconds per symbol period.

Note that z_{mj} is a *random variable*, whose mean value depends on the energy per bit of the chaotic signal and the “goodness” with which the basis functions have been recovered [see the first two terms in (8)].

At best, $y(t) \approx \tilde{g}_j(t) \approx g_j(t)$, and synchronization of $y(t)$ with $g_j(t)$ is maintained throughout the transmission, i.e., $T_S = 0$. In this case,

$$z_{mj} = s_{m1} \int_0^T g_1(t)g_j(t) dt + s_{m2} \int_0^T g_2(t)g_j(t) dt + \int_0^T \tilde{n}(t)g_j(t) dt. \quad (9)$$

We have seen that the variance of $\int_0^T g_j^2(t) dt$ can be reduced to zero by choosing appropriate basis functions. The cross-correlations $\int_0^T g_k(t)g_j(t) dt$, $k \neq j$, can be zeroed by selecting orthogonal basis functions.

By choosing orthonormal basis functions, such as the DCSK functions described in Section III-B-1,

$$z_{mj} = s_{mj} + \int_0^T \tilde{n}(t)g_j(t) dt$$

provides an unbiased estimate of s_{mj} .

Fig. 14 shows the simulated upper bound on the noise performance of chaotic switching with two basis functions. The noise performance curve of coherent FSK is shown for comparison.

2) *Conclusion*: The noise performance of chaotic switching can be maximized by choosing orthonormal basis functions. In the limit, chaotic switching can match the performance of coherent FSK.

3) *Differentially Coherent Correlation Receiver for DCSK*: Although chaotic switching can, in principle, achieve the noise performance of coherent FSK, this level of performance can be achieved only if the two basis functions $g_1(t)$ and $g_2(t)$ can be regenerated exactly at the receiver and if they are orthonormal. We have already noted in Section V-B-2 that regeneration of chaotic basis functions is difficult.

However, the structure of the DCSK basis functions—each consists of a piece of chaotic waveform followed by a noninverted or inverted copy of itself—makes it possible to perform the demodulation by evaluating the correlation between the reference and information-bearing chips.

In a binary differentially coherent DCSK receiver, the reference signal $y(t)$ is the filtered noisy received signal, delayed by half a bit period. Note that different sample functions of filtered noise corrupt the inputs of the correlator.

The observation signal is defined by

$$z_m = \int_{T/2}^T [\tilde{s}_m(t) + \tilde{n}(t)] [\tilde{s}_m(t - T/2) + \tilde{n}(t - T/2)] dt. \quad (10)$$

If the time-varying channel varies slowly compared to the symbol rate, then the filtered DCSK signal is given by

$$\tilde{s}_m(t) = \begin{cases} \tilde{c}(t), & 0 \leq t < T/2, \\ (-1)^{m+1} \tilde{c}(t - T/2), & T/2 \leq t < T \end{cases} \quad (11)$$

where $\tilde{c}(\cdot)$ is the filtered version of $c(\cdot)$.

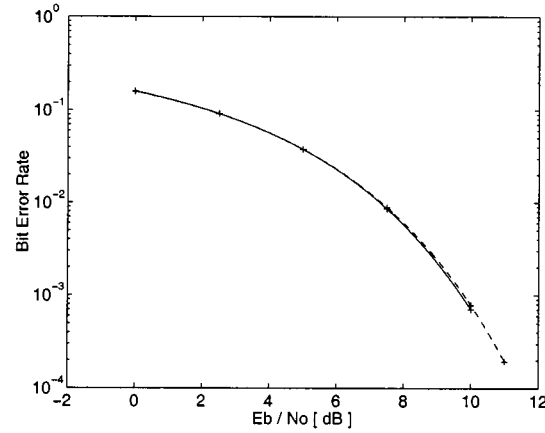


Fig. 14. Simulated optimum noise performance of chaotic switching with two orthonormal basis functions (solid curve). The noise performance of coherent FSK (dashed) is shown for comparison.

Substituting (11) into (10), the observation signal becomes

$$z_m = (-1)^{m+1} \int_{T/2}^T \tilde{c}^2(t - T/2) dt + \int_{T/2}^T \tilde{n}(t) \tilde{c}(t - T/2) dt + (-1)^{m+1} \int_{T/2}^T \tilde{c}(t - T/2) \tilde{n}(t - T/2) dt + \int_{T/2}^T \tilde{n}(t) \tilde{n}(t - T/2) dt \quad (12)$$

where $\tilde{n}(t - T/2)$ and $\tilde{n}(t)$ denote the sample functions of filtered noise that corrupt the reference and information-bearing parts of the received signal, respectively.

At best, $\tilde{c}(t) = c(t)$, giving

$$z_m = (-1)^{m+1} \int_{T/2}^T c^2(t - T/2) dt + \int_{T/2}^T \tilde{n}(t) c(t - T/2) dt + (-1)^{m+1} \int_{T/2}^T c(t - T/2) \tilde{n}(t - T/2) dt + \int_{T/2}^T \tilde{n}(t) \tilde{n}(t - T/2) dt. \quad (13)$$

Since $\int_{T/2}^T c^2(t - T/2) dt = E_b/2$, the first term in (13) is equal to $\pm E_b/2$. The second, third, and fourth terms, which represent the contributions of the filtered channel noise, are zero-mean random processes. Therefore, the receiver is an *unbiased estimator* in this case; the threshold level of the decision circuit is zero and is independent of the SNR.

Although the fourth term in (13) has zero mean, it has a non-Gaussian distribution. Its variance increases with the bit duration T and the bandwidth of the channel filter $2B$. Consequently, the noise performance of chaotic switching with two DCSK basis functions and a differentially coherent receiver decreases with either increasing bit duration or filter bandwidth; this is illustrated in Fig. 15. If $BT = 1$, then the noise performance of this modulation scheme is as good as that of noncoherent binary FSK [20]. Of

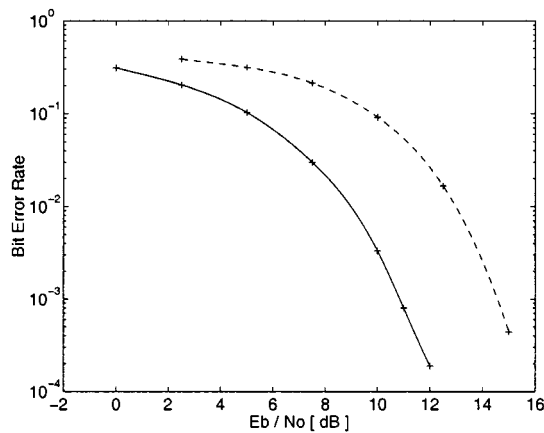


Fig. 15. Simulated noise performance of binary chaotic switching with DCSK basis functions and a differentially coherent receiver with short ($BT = 1$, solid curve) and long ($BT = 17$, dashed curve) bit durations.

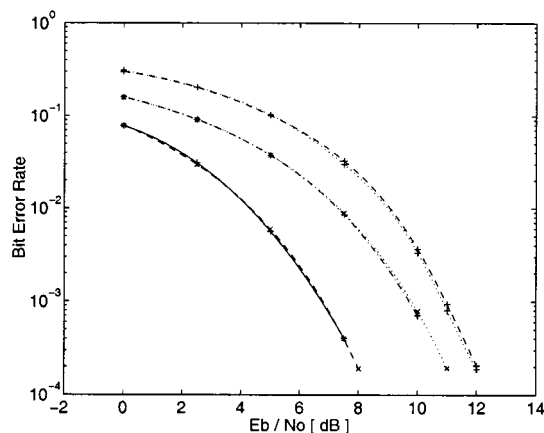


Fig. 16. Simulated optimum noise performance of antipodal CSK modulation with coherent demodulation (solid curve with “+” marks [left]), COOK with noncoherent demodulation (dashed curve with “+” marks [right]), chaotic switching with orthonormal basis functions and coherent demodulation (dash-dot curve with “+” marks [center]), and chaotic switching with DCSK basis functions and a differentially coherent receiver (dotted curve with “+” marks [right]). The noise performance curves for BPSK (dashed curve with “x” marks [left]) and coherent FSK (dotted curve with “x” marks [center]) are also shown, for comparison.

course, in this case the DCSK signal becomes a narrow-band signal and the superior multipath performance of DCSK [26] cannot be exploited. An explicit expression for the noise performance of this system has been derived in [20].

4) *Conclusion:* Given two basis functions and a noncoherent correlation receiver, the best noise performance can be achieved by chaotic switching with orthonormal DCSK basis functions and a differentially coherent receiver.

VII. SUMMARY

In this paper, we have considered the noise performance of CSK with one and two basis functions.

In the case of a single basis function, if one can recover this basis function *exactly* at the receiver, then the noise performance of antipodal CSK can theoretically reach that of BPSK. This is

the best possible noise performance which can be achieved by any digital modulation scheme over an AWGN channel.

In order for this level of performance to be achieved, the chaotic basis function must be recovered independently of the modulation; we are not aware of any receiver structure in the literature that can do this in a sufficiently robust manner.

If the basis function cannot be recovered independently of the modulation, then COOK offers the best noise performance for the single basis function case. The disadvantages of COOK are that the dynamic range of the transmitted power level varies between zero and twice the average transmitted power level and that the decision threshold at the receiver depends on the SNR.

Chaotic switching offers a two-basis function modulation scheme, where the average power level of the transmitted signal can be kept constant and the decision threshold at the receiver is independent of the SNR. The noise performance of chaotic switching with coherent detection can reach that of coherent FSK provided that orthonormal basis functions are used. In particular, chaotic switching with DCSK basis functions can reach the performance of coherent FSK if the basis functions can be regenerated at the receiver.

If the basis functions cannot be recovered, a DCSK transmission can be demodulated using a differentially coherent receiver. The noise performance of this chaotic communications system is 3 dB worse than that of DPSK with autocorrelation demodulation.

The best possible noise performance curves for antipodal CSK modulation with coherent demodulation, COOK with noncoherent demodulation, chaotic switching with orthonormal basis functions and coherent demodulation, and chaotic switching with DCSK basis functions and a differentially coherent receiver are summarized in Fig. 16. The noise performance curves for BPSK and coherent FSK are also shown, for comparison.

VIII. CONCLUSIONS

If the propagation conditions are so good that the basis function(s) can be regenerated at the receiver, then digital modulation schemes using conventional orthonormal (typically periodic³) basis functions, and orthonormal chaotic basis functions can achieve similar levels of noise performance [20]. The main question from an implementation perspective is the ease with which the basis functions can be regenerated.

We believe that it is fundamentally easier to regenerate a periodic basis function than a chaotic one. We conjecture, therefore, that the noise performance of digital chaotic modulation with coherent correlation receivers will always lag behind that of equivalent modulation schemes using periodic basis functions.

If the propagation conditions are such that coherent detection is impossible, then chaotic switching with DCSK basis functions and a differentially coherent receiver (DCSK, for short), offers the best possible performance for a chaotic digital modulation scheme. In the limit, the noise performance of DCSK lags

³The application of noise as a carrier for digital communications system was proposed in [27]. A system configuration for the qualitatively similar FM-DCSK system was described in [28]. The novelty of the latter solution over the former is that the estimation problem has been recognized and solved and the chaos generator provides a more robust and simpler source of nonperiodic basis functions.

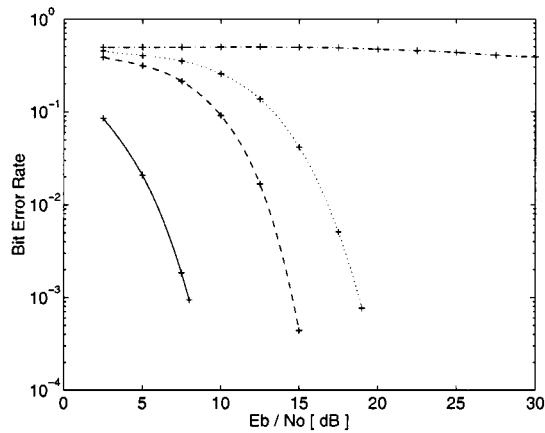


Fig. 17. Simulated noise performance curves for DPSK and wideband DCSK in a single-ray channel (solid and dashed, respectively) and a multipath channel where coherent detection is impossible (dash-dot and dotted, respectively). While DCSK disimproves by about 4 dB, DPSK fails completely.

only 3 dB behind that of DPSK with autocorrelation demodulation [20].

In this case, the choice of periodic or chaotic basis functions is determined by the propagation conditions. In particular, the multipath performance of a DCSK system can be improved by increasing the transmission bandwidth.

We stress that, although we have referenced the performance bounds for chaotic modulation schemes to the limits for conventional narrow-band modulation techniques, the comparison is not fair in the sense that chaotic modulation is intended for use as an inherently wide-band communications system. The advantage of DCSK is that the fall-off in its performance in a wide-band multipath channel is more gradual than that of an equivalent narrow-band modulation scheme.

Fig. 17 shows the performance degradation in narrow-band DPSK ($BT = 1$, classical DPSK with optimum receiver configuration [21]) and wide-band DCSK ($BT = 17$) systems operating in single-ray and multipath channels. The bit duration was set to $2 \mu\text{s}$ in both cases. Although the single-ray performance of DCSK is worse than that of DPSK, its multipath performance is significantly better. Therefore, DCSK offers a performance advantage over DPSK in multipath environments when the propagation conditions are so poor that coherent detection is not possible; this issue is addressed in detail in [26].

ACKNOWLEDGMENT

The figures were prepared by G. Kis and Z. Jákó using an ultrafast chaotic communications system simulator [29] developed in the framework of the INSPECT Project; their input is greatly appreciated.

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Géza Kolumbán (M'92–SM'98) received the M.S. and Ph.D. degrees from the Technical University of Budapest and the C.Sc. degree from the Hungarian Academy of Sciences in 1976, 1990, and 1990, respectively. After his graduation, he was employed as a Research Engineer by the Fine Mechanical Enterprise, Hungary, where he developed local generators, microwave transistor power amplifiers and VCO circuits for high-capacity microwave analog radio relay systems. He joined the Research Institute for Telecommunications, Hungary, in

1980, where he was involved in many system engineering projects such as SCPC-type satellite telecommunications system, microwave satellite up-and down-converters, low-capacity microwave digital radio system, etc. He led a group of engineers developing frequency synthesizers and local generators for frequency hopping spread spectrum and satellite systems. He spent one year (1991–1992) with Bilkent University in Turkey and another year (1992–1993) with the Eastern Mediterranean University in Cyprus. He returned to the Budapest University of Technology and Economics (called Technical University of Budapest before 2000) in 1993, where he is employed as an Associate Professor at the Department of Measurement and Information Systems. He has been a Visiting Researcher to the Electronics Research Laboratory, UC Berkeley, University College Dublin, EPFL, Switzerland, and TU Dresden, Germany. His current research and professional interests include nonlinear dynamics of different-type phase-locked loops, frequency synthesis by sampling phase-locked loop, mixed signal processing, computer simulation of complex systems, chaotic communications, and applications of chaotic signals in measurement engineering.



Michael Peter Kennedy (S'84–M'91–SM'95–F'98) received the B.E. (electronics) degree from the National University of Ireland in 1984, and the M.S. and Ph.D. degrees from the University of California at Berkeley in 1987 and 1991, respectively, for his contributions to the study of neural networks and nonlinear dynamics.

He worked as a Design Engineer with Philips Electronics, a Postdoctoral Research Engineer at the Electronics Research Laboratory, UC Berkeley, and as a Professeur Invité at the EPFL, Switzerland.

From 1992 to 1999, he was on the faculty of the Department of Electronic and Electrical Engineering at University College Dublin (UCD), where he taught electronic circuits and computer-aided circuit analysis, and directed the undergraduate Electronics Laboratory. He was appointed Professor and Head of the Department of Microelectronic Engineering at University College Cork in 1999. He has published over 180 articles in the area of nonlinear circuits, holds two patents, and has taught courses on nonlinear dynamics and chaos in England, Switzerland, Italy, and Hungary. His research interests are in the simulation, design, and analysis of nonlinear dynamical systems for applications in communications and signal processing.

Dr. Kennedy received the 1991 Best Paper Award from the International Journal of Circuit Theory and Applications, was the IEE Kelvin Lecturer in 1997, was made a Fellow of the IEEE in 1998, received the Best Paper Award at the European Conference on Circuit Theory and Design in 1999, and was awarded the IEEE Circuits and Systems Society Golden Jubilee Medal and the IEEE Third Millennium Medal in 2000. He served as Associate Editor of the IEEE TRANSACTIONS ON CIRCUITS AND SYSTEMS PART I from 1993 to 1995 and PART II since 1999.