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Infinity, Infinite Processes and Limit Concepts: Recovering a Neglected Background of Social and Critical Theory

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Abstract
This article seeks to recover a neglected chapter in the historical and theoretical background of social theory in general and Critical Theory in particular with a view to refining the understanding of the presuppositions of a cognitively enhanced critical social science appropriate to our troubled times. For this purpose, it offers a brief reconstruction of the mathematical-philosophical tradition from ancient to modern times by extrapolating that part of it that is marked by the ideas of infinity, infinite processes and limit concepts. It gives suggestive indications not only of how these ideas relate to extant social-theoretical thought, but especially also of the two intertwined cognitive directions – the weak-naturalistic and the sociocultural – in which such thought is currently challenged to go.

Keywords
cognitive sociology, critical theory, culture, Kant, weak naturalism

Introduction

In order to recover a piece of forgotten and neglected historical and theoretical background of social theory, the following paragraphs are devoted to a brief reconstruction of the mathematical-philosophical tradition from ancient to modern times. The account concentrates on that part of this tradition which is marked by the concepts of infinity, infinite processes and limit concept. Along the way, suggestive indications are given of how this tradition's relevant ideas relate to social-theoretical thought.

The account starts with Aristotle's broaching of the problem of infinity and its limits as well as his surprising yet, given ancient Greek presuppositions, his understandably reserved attitude towards infinity and even rejection of a significant part of the problematic. By far the larger part of the account is focused, however, on the early and later modern attitudes toward and treatment of this very problematic. It moves from the initial enthusiastic yet naïve embrace of the idea of infinite processes and the concern with limit concepts, via the sobering late eighteenth-century philosophical summation of previous developments, to the nineteenth-century critical period in mathematics which saw to the refinement and legitimation of the relevant ideas. Anticipated by Galileo, the names of John Wallis, Isaac Newton and Gottfried Leibniz stand for the spirited early modern transformation of the idea, but it was with Immanuel Kant's critical philosophy which creatively synthesized and strictly circumscribed a wide variety of different ideas that the cultural parameters for subsequent developments were made clear. At this stage, the scene was set for the osmotic absorption of the relevant ideas by social theorists. It is at the very time that Hegelian thought, in addition to Kantian ideas, paved the way for the emergence of the differentiated – both social-theoretical and logical – Left-Hegelian response of Karl Marx and Charles Peirce, that Augustin-Louis Cauchy and Niels Henrik Abel inaugurated the critical period in mathematics. From this basis proceeded the significant mathematical development of the late nineteenth century in which Peirce played a seminal role but which is represented especially by Richard Dedekind and Georg Cantor. Against this background, the most basic concepts of social theory come to stand out graphically, available for sharp differentiation and being put in their proper places. The core section of the paper having been covered, the concluding section finally collates the references relevant to social theory and points in the direction of their systematic significance.

Potential infinity, actual infinity and limits

Aristotle's most important contribution to the development of mathematical philosophy and, by extension, of the formal logical principles of the sciences, is his penetrating formulation of the problem of infinity. He was the first to appreciate the need to draw a basic distinction between 'potential infinity' and 'actual infinity' (2015: Book III, Part 6). Although this distinction became quite differently evaluated in the modern period, it has proved to be of great significance not just for mathematical thought but also, more generally, for philosophical thought – both of which served as sources from where it penetrated into social-theoretical thinking. For example, the distinction – albeit in modified form – was central to Kantian philosophy which laid the groundwork for reflection on the whole range of scientific disciplines, including the social and cultural sciences or humanities, but its impact is also visible in theoretical concepts employed by Hegel, Comte and other social theorists.

In accordance with his identification of 'infinity' and 'continuity', Aristotle (2015: Book VII, Part 8, Book III, Part 6) sought to make potential infinity comprehensible by contemplating a sequence that goes on or continues without end or limit. Such a sequence can proceed in one of two directions: either progressing by the addition of another unit, for example, the sequence of natural numbers 1, 2, 3, … which never reaches its end, or by making another subdivision of, for instance, a line between two points and thus engaging in a process which confirms its infinite divisibility. By contrast with the never-ending progression characteristic of potential infinity, either toward the biggest or toward the smallest, Aristotle described actual infinity as 'the infinite in the full sense', as being 'complete (*teleion*)', having an 'end (*telos*)', where 'the end is a limit' (2015: Book III, Part 6). In the philosophical lexicon in his *Metaphysics* (1961: 32), Aristotle included a circumscription of limit: '"Limit" denotes the last point of anything, i.e. the point beyond which it is impossible to find any part of it, but within which all of its parts are found.' Relating this conception to infinity, it means that actual infinity would amount to all the elements of an adding sequence or all the parts of a subdividing sequence being available in their completeness or totality. On contemplating this possibility, however, Aristotle in keeping with the characteristic ancient Greek concern with the concrete summarily rejected the notion of actual infinity as a logical impossibility.[1]

It is against this evaluation and decision of Aristotle's regarding actual infinity that a prominent line of thinkers would subsequently take a diametrically opposed position. Since the Greeks suffered from so-called *horror infiniti*, it had been left to the early modern mathematicians, scientists and philosophers in the context of the revival of learning after a thousand years of Christian stupor to explore the relation of an infinite sequence and its limit. Not only is modern mathematical analysis based on the theory of infinite processes which have for foundation the idea of limit (Dantzig 2007: 133), but modern thought more generally, both philosophical and scientific, including social-theoretical, is pervaded by these very same assumptions.[2]

The theory of infinite series: divergent, convergent and limits

The idea of an infinite sequence was known to ancient Greek mathematicians and philosophers, as Aristotle's critical systematization of the thought of his time shows, but it was only between the 12th and the 17th-18th centuries that the core aspect of the theory of infinity became established (e.g. Kline 1990; Clegg 2003; Dantzig 2007; Elwes 2013; Alexander 2015). This was made possible by the fact that the moderns adopted a positive attitude to infinity and accepted the validity of infinite processes, instead of sequestering the problem of infinity, as did the Greeks, and instead of arresting endless sequences scholastically or theologically by assuming the absolute nature of the unlimited, as did Christianity. Initially having been opened up by the mediaeval businessman and mathematician Leonardo Fibonacci (1170-1240) through contact with Hindu-Arab ideas, the field was explored over the following centuries by mathematicians (e.g. Nicole Oresme, 1323-82; John

Wallis, 1616-1703; Pietro Mengoli, 1626-86; Leonhard Euler, 1707-83), scientists (e.g. Galileo 1564-1642; Johannes Kepler, 1571-1630; Isaac Newton, 1642-1727) and philosophers (e.g. Gottfried Leibniz, 1646-1716; Immanuel Kant, 1724-1804) who prepared the ground for important developments in the nineteenth century. Most basically, it emerged in response to the probing question of what follows from an attempt to add together or find the sum of infinitely many numbers that the historically and theoretically important 'geometrical sequence' can be divided into two distinct series – an increasing or 'divergent' and a diminishing or 'convergent series' (Dantzig 2007: 150; Kline 1990, Vol. 3: 1110).

It should be noted that these two variants of the geometrical sequence are of central importance in the present context since they not only link up with Aristotle's distinction between potential and actual infinity, but also paved the way for the establishment of the basic parametric assumptions of social theory. The conception of these series made possible the consolidation of thinking about two distinct yet nevertheless interrelated dimensions of social reality or the sociocultural form of life and its conditions: corresponding to the convergent series such notions as time, natural history, human history, historical action, practices, sociocultural articulation and elaboration, and transformation and change; and corresponding to the divergent series such notions as differentiation, diversification of sociocultural forms of life, increase in complexity and the evolutionary stabilization of a variety of phylogenetic forms.

A divergent series (e.g. Dantzig 2007: 150; Elwes 2013: 97-8) is one in which the addition of more and more numbers leads to a total that outgrows any boundary one might try to impose. For example, in the case of adding 1 + 2 +3 + 4 +5 + …, the total grows exponentially, that is, larger and larger without limit – the total exceeding 100 after 14 steps and surpassing 1000 after 45 steps. In the case of simply counting 1 + 1 + 1 + 1 + 1 + …, the total may be growing much slower yet it does grow without end, the first step reaching 1, the second step 2 and the hundredth step 100.

A convergent series (e.g. Dantzig 2007: 150-51; Elwes 2013: 98) presents an entirely different picture. If instead of whole numbers one adds rational numbers such as decimal fractions, by contrast, the resulting total does not grow bigger and bigger, but rather tends toward a finite limit. For example, adding 0.9 + 0.09 + 0.009 + 0.0009 + … gives rise to a sequence of totals of 0.9, 0.99, 0.999, 0.9999, and so forth that gets ever closer to 1 yet without ever reaching it, not to mention surpassing it. The best known example of a convergent series from mathematics is the transcendental number pi (π). When π is algorithmically derived, a number is obtained which grows ever longer in decimal places and ever closer to π, the growth being infinite and the value of π in principle remaining just out of reach. While for convenience π is accepted as 3.14159, by the late nineteenth century the derivation had yielded more than 700 digits after the point and in our own time the computer has added vastly more, yet still approaching but never reaching the exact value of π. The latter is a finite but ideal limit that cannot be captured perfectly by a number. Decimal places could be added indefinitely, but the calculation of π will never finish.

Another example of a convergent series is offered by non-Euclidean projective geometry which in the fifteenth century formalized the method of focused perspective developed by such artists as Piero della Francesca and Albrecht Dürer (Panofsky 1955) and, subsequently, began to play an increasingly significant role in providing a productive approach to science, as exhibited by Galileo's mathematization of nature through idealisation (e.g. Heidegger 1962). In projective geometry, an ideal point is introduced at infinity on every straight line and then defined as the point at which all lines parallel to the given line converge and intersect; and in every plane an ideal line is introduced which contains all the points at infinity of all the lines in the plane (Kline 1990, Vol. 1: 285-301; Körner 1968).

From Leibniz to Kant

Mathematics served as the source from which Leibniz (1965: 223; 1968: 152) extrapolated what he called the 'law of continuity' (*lex continuitatis*) which he made into the basis of his philosophical and scientific work. In this context, he dealt quite extensively also with the problem of infinity which, in a certain respect, allowed him the claim to have co-founded the modern mathematical branch of calculus together with Newton. Through a series of highly suggestive but also truncated and erroneous ideas, he was instrumental in a number of crucial respects in pointing the way for Kant. One of the most important ideas that Kant appropriated is Leibniz's (1965: 235-6; 1968: 182) distinction between 'truths of fact' and 'truths of reason' which was conditioned by his close study of Galileo (1914) who combatted Aristotle's stance on actual infinity and thus anticipated modern mathematics. Despite this distinction of his and his use of it in his critique of Locke, however, Leibniz nevertheless saw fit to revert to classical Aristotelianism. This is one significant respect in which Kant departed decisively from him.

Kant's critical philosophy is replete not only with the assumption of the basic nature of infinite or continuous series, but also with articulating issues in accordance with it. A crucial instance from the first of his three critiques (Kant 1968) is where he addresses the question of reason as 'the faculty of inferring, i.e. judging mediately' (B386=A330ff). Here he distinguishes two infinite or continuous series: 'a series of inferences that…can be continued…[or]…can be prolonged indefinitely on the side of the conditions (*per prosyllogismos*) or of the conditioned (*per episyllogismos*)' (A331=B387-88). Of these two series, the former '*ascending* series' stands 'in a different relation to the faculty of reason from that of the *descending* series' (A331=B388). It is the case that although 'we can never succeed in comprehending a totality of conditions, the series must none the less contain such a totality, and the entire series must be unconditionally true if the conditioned…is to be counted true' (A332=B389). Thus Kant is able to submit that it can be assumed that 'all the members of the series on the side of the conditions are given (totality in the series of premises); only on this assumption is the judgment before us possible *a priori*: whereas on the side of the conditioned, in respect of consequences, we only think a series *in process of becoming*, not one already presupposed or given *in its completeness*, and therefore an advance that is merely potential' (B388=A331-332).

It is obvious that in the *Critique of Pure Reason* Kant bases himself on the tradition of mathematical-philosophical thinking. What is particularly clear is that he links up with the interpretation of the historically and theoretically important geometrical sequence as classifiable into divergent and convergent series – what he calls 'ascending' and 'descending' series. The interesting question is how he relates to this tradition which had effectively been founded by Aristotle but appropriated in contrary ways by Galileo and Leibniz in the seventeenth century. It is apparent, first, that Kant's distinction between the descending series representing infinity *qua* '*process of becoming*' and the ascending series representing infinity '*in its completeness*' or as 'totality' closely approximates, or is even very similar to, the Aristotelian distinction between 'potential infinity' and 'actual infinity'. Despite this similarity, however, his stance on actual infinity differs considerably from Aristotle's. By contrast to the Greek *horror infiniti* and Aristotle's stricture against actual infinity on logical grounds, Kant not only exhibits the modern positive embrace of infinity, but also regards actual infinity as being logically perfectly possible (e.g. Körner 1968: 30). In fact, he gave it a central place in his critical philosophy by regarding it as in accord with the ideas of reason, thus legitimating it by means of one of his most characteristic concepts.

The designation 'idea of reason' is appropriate to ideas that are unrelated to perception and construction, neither having been derived from sense-experience nor being directly applicable to sense-experience, and in this sense transfinite or 'transcendental'. Despite the fact that such transcendental concepts are beyond the substantive concern of any system or discipline dealing with concrete objects, Kant argued that they could nevertheless amplify such domains as long as they

could be shown to be logically coherent and internally consistent. Indeed, he was convinced that they were not merely useful, but actually necessary for achieving adequate amplification of this kind. Kant not only employed such concepts in his philosophy of mathematics and science, but extended them also to his practical and aesthetic philosophies.

In the *Critique of Judgment* where he compares the mathematical and the aesthetic estimation of magnitude, for instance of natural phenomena, Kant (1972: section 26) elaborates on the relation between the descending or converging and the ascending or divergent series. On the one hand, he depicts the inferential operation of the imagination in its constructive activity as 'go[ing] on without hindrance to infinity' (ibid., 1972: 93). On the other, he stresses that the mind by appealing to reason requires a totality to limit this unbridled progression: 'Reason consequently desires comprehension in *one* intuition, and so the joint *presentation* of all the members of the progressively increasing series. It does not even exempt the infinite…from this requirement; it rather renders it unavoidable to think the infinite…as *entirely given* (according to its totality)' (ibid., 1972: 93). Another aspect of the relation between Kant and Aristotle thus becomes apparent. Like Aristotle, Kant accepts that the concept of limit is a necessary concomitant of infinite processes, but he differs decisively from his Greek predecessor in so far as he forges a link between limit and actual infinity. Central to this link is the concept of 'idea of reason', for in his view this type of idea is necessary to capture actual infinity, complete infinity or infinite totality.

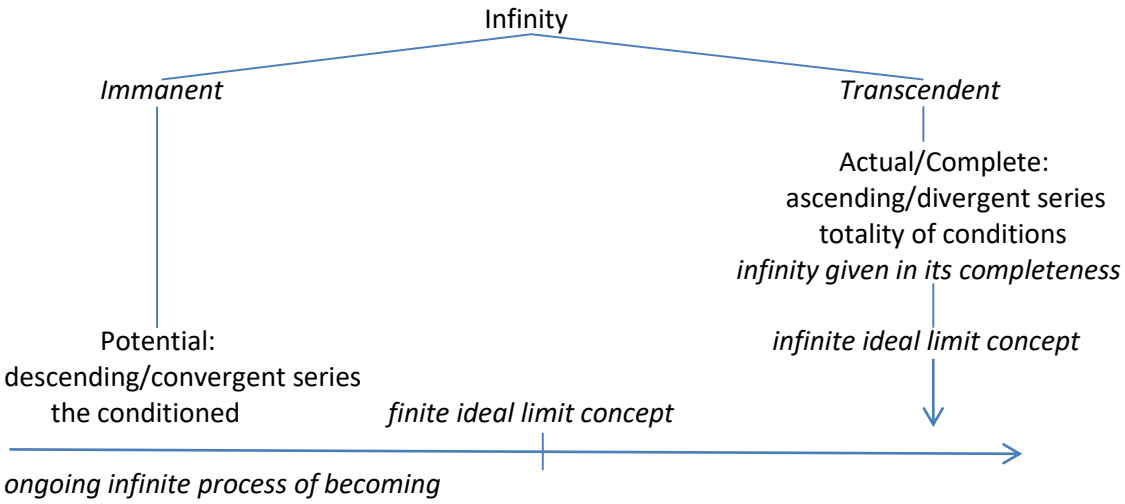
What Kant in an earlier quotation calls 'the conditioned' includes the relevant appearances of the object of inference, while what he conceives as the 'totality of conditions', embracing actual infinity, complete infinity or infinite totality, is the very core of the ideas of reason. We know, too, that he associates the former with the descending or convergent series and the latter in turn with the ascending or divergent series. Now, this means that as the core of the ideas of reason, the totality of conditions represented by the ascending or divergent series provides the infinite ideal limit of any descending or convergent series. Ideas of reason serve as ideal limit concepts, and as such they render the descending series *qua* processes of becoming intelligible and manageable. Through them, reason or the power of the mind is brought to bear on the process of becoming and rendered into a suitable object of knowledge and action. Considering all three of Kant's critiques (1956, 1968, 1972), the most important ideas of reason he identifies include the following: 'world' (the totality of objects of possible experience), 'immortal soul' or 'freedom' and 'autonomy' (the subject of experience, knowledge and action), 'beauty' (the cognitive mode of imagination and reflection) and finally 'validity' ('God' or later 'the faculty of judgement'). These ideas all designate totalities that operate as infinite ideal limit concepts, some as forms of thought and others as forms of elementary practical concepts. However, whereas Kant tended rather strongly toward dualistic thinking, in the wake of a variety of post-Kantian developments in philosophy and mathematics, it became clear that ideas of reason as ideal limit concepts can be regarded as involving idealisation in the sense of a mode of procedure that occupies a position between Platonic idealism and Aristotelian empiricism, being as they are neither Forms independent of concrete reality nor simply parts of empirical objects. Of them it could now be said that they are idealizing abstractions that emerge through mediating reflexivity from human activities of all sorts and become stabilized over long periods of time as the humanly most significant conceptual (linguistic) and quasi-conceptual (logical and mathematical) foundations of the sociocultural form of life.[3]

While the ideas of reason *qua* infinite ideal limit concepts provide the overall framework of intelligibility and practical orientation relevant to ongoing processes of becoming, this still leaves the finite yet also ideal limit concepts associated with the descending or convergent series – that is, the finite yet ideal limits like the value of pi (π) toward which such processes tend but are never able to reach. For Kant is concerned not just with the transcendental nature of ideas of reason corresponding to the ascending series, but also with their '*immanent*…employment' (1968:

B383=A327) by regulating the theoretical and practical concepts corresponding to the descending series which give effect to them at the lower level. In this respect, he submits that reason involves not exclusively ideas, but also 'ideals', 'archetypes', 'models' or 'examples' (1968: A569-571=B597-599). Ideas have the task of ordering and unifying concepts 'with a view to obtaining totality in various series', whereas ideals depend on concepts of the understanding and moral concepts which in turn unify the elements of objects in such a way that 'such *series* of conditions *come into being*' (1968: A643=B672). As in the case of the mathematical convergent series which at best only tends toward a limit value, however, Kant on many an occasion stresses that any attempt to realize the ideal contained in a model or example, whether by knowledge production or action, is 'impracticable' (1968: A570=B598) – and that in the sense that while 'a great goal…is set before it…it can never of itself reach' that ideal (1956: 152). Knowledge is always incomplete and perfectly emulating the completeness of the ideal of, say, 'the wise man' is an 'illusion'.

From the above, the following preliminary consolidation emerges.

Figure 1: Infinity, infinite processes and limit concepts in Kant



Intermediate social-theoretical reflections

Reflection on this diagrammatic icon from the perspective of social theory leads to the identification of historical-theoretical precedents of familiar social-theoretical constructs as well as contact points for the proposal of basic concepts for a newly conceived cognitive social theory.

Familiar or standard constructs which have become settled as fundamental theoretical assumptions in social theory have been inspired by the distinction between the descending or convergent and the ascending or divergent series that was developed and stabilized in the course of the unfolding of mathematical-philosophical tradition. These constructs are the basically assumed parameters of social reality expressed by the conceptual pair of dynamics and statics or, differently, by the notions of the process of the creation, construction or production of society and of the structure, structuration or reproduction of society.

This takes care of the two series but still leaves the question of the types of limit concepts, the finite ideal and the infinite ideal limit concepts, which are necessary concomitants of the two infinite series from a mathematical-philosophical viewpoint. On this matter, contributions have indeed been made by classical and contemporary social theorists in so far as they stress generality, yet without thinking the problem of generality through and without recognition of the status of their contributions as being explicable in terms of limit concepts. On the whole, moreover, there is a great

lack of clarity and much equivocation in social theory – indeed, to a degree that could be judged as having been damaging to it. A major desideratum under contemporary conditions demanding a reconfiguration of the relation to self, to social relations and of society to nature is the elimination of this debilitating state of affairs. My proposal is that this can be achieved by conceptualizing the two distinct types of limit concepts in cognitive theoretical terms – that is, the infinite ideal limit concepts as context-transcendent or transcendental cognitive order principles, and the finite ideal limit concepts as context-immanent, cognitively structured cultural models of different levels and scales. I take up this issue in the final section of the paper.

The post-Kantian situation

When interest in the problem of infinity was rekindled in the Renaissance as one of the first steps toward the revival of learning under the impact of Arab culture, the moderns embraced it enthusiastically yet quite naively, certainly without the critical rigour characteristic of the ancient Greeks. On all historical accounts (e.g. Dantzig 2007: 135; Glegg, 2003: 102; Alexander 2015: 262), a veritable orgy of the reckless use of confusing terminology, rough-and-ready methods and wild and woolly calculations prevailed for some two centuries, implicating figures from Kepler, through Wallis, Leibniz and Newton, to d'Alembert. This exuberance was brought to a halt by the cultural change marked by Kant's critical philosophy which was followed by the rigour of the nineteenth-century critical period in mathematical thought. It was embodied, for example, by mathematicians like Karl Weierstrass, Richard Dedekind, Ernst Schröder and Georg Cantor as well as the outstanding philosopher, logician and mathematician Charles Peirce, the founder of pragmatism, who is also of great social-scientific significance (Apel 1995; Strydom 2011a). This critical disciplining of argumentation involved rendering the ideas of infinite processes and limit concepts more precise in order to bring the endless progression of such processes under control. A philosophical example of such disciplining was earlier offered by Kant whose critical philosophy was aimed at curtailing the speculative or dogmatic employment of the cognitive capacities and reason, but the mathematicians pursued it in their own terms – terms that centrally implicated the convergent and divergent series and their limits.

Kant's immediate successors

Kant's successors Salomon Maimon and especially Johann Gottlieb Fichte completed the pursuit of his thought into a full-blown idealism. Maimon (2010) regarded a reality outside of consciousness, Kant's 'thing-in-itself', as a contradiction, indeed as an impossible concept. He treated it as the limit concept of the infinitely decreasing series (comparable to π) from complete consciousness to the irrational infinitesimal quantity, the thing-in-itself as the merely given, which means that complete knowledge of the given can never be attained. Fichte (1982) executed the full idealistic disintegration of the concept of the thing-in-itself. Since being is comprehensible only as a product of reason, an object exists only for a subject. On the basis of this idealistic principle, he regarded the real series of objects as being perceived in the ideal series of mental representations. These are clearly quite striking designations for the convergent and divergent series respectively, despite the questionable idealistic tapering.

Hegel, Marx and Peirce

In the subsequent period, however, Kant's transcendental idealism and its one-sided extrapolation by his successors were drastically reformulated. Although Hegel, following Fichte, developed idealism in the extreme, his elaboration of Kant's concept of mediation opened the way for such left-Hegelians as the two contemporaries Marx and Peirce to recuperate the previously idealistically evaporated dimension of praxis or action as well as nature-in-itself or the objective world of objects of possible experience and reference. Peirce did so in terms of a mathematically grounded, logically rigorous, triadic semiotic theory of signs providing the basis for his pragmaticism; and Marx

responded with a materialistic and system transforming emphasis within an overall framework to a large extent comparable to Peirce's (Strydom 2011a).

From Kant and Fichte, Hegel turned to the historical self-creative process of spirit, mind or the idea, whether the history of subjective mind, objective mind or absolute mind. The focus for him was on the 'logic' (Hegel 1969) of the unfolding of the self-creative process at each of these levels through interlinked infinitesimal shifts of thesis, antithesis and synthesis which represent stages marking the tendency toward the realization of the whole. While the finite limit concepts of each of these convergent series would have to be specified in historically concrete terms, Hegel effectively identified first individual self-consciousness (Hegel 1966), second right and morality (Hegel 1967), and finally the absolute in art, religion and philosophy as the infinite ideal limit concepts determining their respective parameters.

For his part, Marx (1969a) inherited both Hegel's transformation of philosophy into social theory and his historical perspective, but corrected and extended it in two respects. Where Hegel's historical emphasis left him in the lurch in natural philosophy, Marx introduced the view of the human species as forming part of nature; and where Hegel as conservative philosopher of the Restoration fudged a crucial moment in his *Science of Logic*, he insisted on being explicit about the potentiality of the continuous progression of the infinite process of self-creation and its revolutionary significance.[4] For Marx (1969b: 252), furthermore, 'communism…[is]…the next stage of historical development' and emphatically 'not the aim of human development or the final form of human society' – which means that it is at best only the ideal yet finite limit concept toward which the infinite convergent process of human becoming tended at that point in history. It is for this reason that he employed the notion of 'association' (1969b: 251, 253) to designate the total set of conditions of 'a community of free individuals' located on the ascending or divergent axis that serves as the overall or infinite ideal limit concept. The parameters it suggests possess the sense of proper relations not only among individuals, but also between humans and nature of which humans themselves form an integral part.

Peirce grew up in a mathematical environment and from early developed also an extensive knowledge of philosophy. This foundation allowed him to carve out his own unique position by drawing from Kant, Leibniz and Berkeley, all of whose thought he traced back to Scholastic realism. It is from Kant that he drew inspiration for his 'pragmaticism' or Kantian pragmatism which focuses on the relation between action and its 'ends' as contained in 'the ideas of human life' (1998: 360, 457, 197), but not without first replacing Kant's (1968: A255=B311, A29, B66=A49, A228) most controversial 'limit concept' (*Grenzbegriff*), the 'unknowable…thing-in-itself' or 'noumenon' which is visible in its appearances only, by an in principle pragmatically knowable objective world. The names of Leibniz and Berkeley indicate that he learned from the early modern mathematical debate about infinity, especially the problem of infinitesimals, in which also Newton was embroiled. Not only did he give infinity or continuity a central place in his thinking, but by his fortieth year he was also able to make a seminal logical contribution to the development of modern mathematics. Peirce acknowledged Aristotle's distinction between potential infinity which is carried by activity (ένέργεία), what he himself called *esse in futuro* (1998: 180), and actual infinity or complete reality (έντελέχεια). And he distinguished his own position from the predominant modern nominalist trend in philosophy by insisting on the logical cogency and philosophical importance of the latter mode of infinity, what he most characteristically called 'Thirdness'. While he regarded the most basic ideas of human life, namely 'Truth, Right, and Beauty', as the prime examples of the latter, he included also reality, writing for example: 'Now Reality is an affair of Thirdness as Thirdness, that is, in its mediation between Secondness and Firstness' (1998: 197).

Peirce offered accounts of both truth and reality from different viewpoints, including in terms of both convergent and divergent series as exemplified by intellectual and scientific investigation.

According to the former (convergence) viewpoint, '[r]eality can be regarded as the limit of the endless series of symbols' (1998: 323), whereas '[t]he opinion which is fated to be ultimately agreed to by all who investigate, is what we mean by truth' (1992: 139).[5] The relation between truth and reality at this level of infinite processes is that 'the object represented in this opinion is the real' (1992: 139). As regards divergence, truth is one of the '*Ends*' (1998: 197) of inquiry and as such a 'real general' or mighty force influencing human conduct, which unmistakeably indicates that he understands it as actual or complete infinity. Insofar as such an end is real, it should be obvious that he conceives reality in accordance with the divergent series. All in all, then, this means that Peirce not only proceeds from the well-established distinction between convergent and divergent series, but that he simultaneously also identifies two distinct types of limit concept. For example, truth is on the one hand the 'finite' ideal 'value' or 'definite limit' that the ratio of frequency of investigation has 'in the long run' and toward which the ongoing process 'indefinitely converge[s]' (1998: 100); on the other, truth as the end of investigation and the entelechy of reality is the infinite ideal limit that lays down the parameters of the former.[6]

Reflecting on the post-Kantian developments with the concepts of the convergent and divergent infinite processes and their limit concepts in mind, it strikes one that the left-Hegelian principle assumed by both Marx and Peirce and informing Critical Theory to this day – that is, the principle of the historical accumulation of rational potentials that then incursively and recursively work back in a structuring way on the actions, practices and processes which drove the accumulation in the first place[7] – is unthinkable without this seminal conceptual background.

The critical period in mathematics

That the concern with infinite processes and their limits was of immense intellectual significance was confirmed by a number of key achievements in the course of the critical period in mathematics. Augustin-Louis Cauchy (1789-1857) inaugurated this period in mathematics in the early nineteenth century by showing in precise mathematical terms that the geometrical sequence can indeed be classified into a divergent and a convergent series (Kline 1990, Vol. 3: 1110; Dantzig 2007: 150). The foundations of calculus deriving from Leibniz and Newton's struggle with infinity remained ambiguous, however, which inspired Karl Weierstrass (1815-1897) not only to resolve the problem of the lack of soundness of calculus in the early 1840s, but also to considerably refine it in subsequent years. By the 1880s, the question of infinity and limit came to a head in the intricate set of relations involving Peirce, Ernst Schröder, Richard Dedekind and Georg Cantor. Towards the close of the critical period, these men conducted a searching analysis of the whole problem of infinite processes and their limits with a view to eliminating the vagueness and ambiguity still marring these concepts. While Peirce's (1992, 1998) strict logical analyses contributed, this important development can be briefly stated with reference to Cantor and Dedekind whose apparently conflicting contributions nevertheless productively complemented each other (Dantzig 2007: 181-82). For the purposes of his astounding transfinite mathematics, Cantor adopted a dynamic theory according to which the limit value is generated by the motion of a point on a continuous or infinite line, while Dedekind adopted a static theory according to which the power of the mind by means of a *Schnitt* or cut imposes a special classificatory scheme on an infinite process to generate its limit value.

In this debate, both the infinite convergent and divergent processes and their ideal limit concepts moved to the foreground. But most significant is that it was unequivocally established that the concepts of infinity, infinite processes and limits endow mathematics with generality, with the implication that reality can by no means be restricted to the immediate experience of the human senses. There is a transfinite or context-transcendent dimension beyond the immanent that Kant earlier sought to pinpoint philosophically under the title of the transcendental – albeit without establishing an adequate version of the necessary relation of mediation between the two dimensions. Such mediation is what Peirce (1998), in contact with the leading mathematicians and

abreast of the cutting-edge mathematics of his time, sought to accomplish by means of his triadic semiotics which, besides its general significance, is of great social-scientific importance.

Social-theoretical relevance of infinite processes and limit concepts

When one contemplates the interrelated developments reviewed in the above from a social-theoretical perspective, the question arises as to precisely what the significance of this submerged mathematical-philosophical background is for the development of social theory – not just for extant social theory, but especially for future social theory and its potential contribution to facing the current challenges of having to reconfigure the relation of humans to nature, to others and to themselves. For there can be no doubt about the fact that it has significance – the only question is how much and in what sense. In the intermediate reflections above, to recall, it was suggested that the neglected mathematical-philosophical background of social theory contains historical-theoretical precedents of what have become standard social-theoretical parametric constructs as well as fecund anchor points for the formulation of certain key concepts for a newly conceived cognitive social theory which is compatible with a weak naturalistic ontology. Here these two matters – that is, parametric constructs (infinite processes) and fecund anchor points (limit concepts) – need elaboration beyond the bare indications offered earlier.

The first set of considerations turns on the notion of infinite processes, particularly the differentiation between the descending or convergent and the ascending or divergent series that originally goes back to Aristotle's distinction between potential and actual infinity, was transformed by early modern thinkers like Galileo, Wallis, Leibniz and Newton, then philosophically articulated by Kant on the basis of the mathematical-philosophical tradition and, finally, unequivocally established in mathematics by Cauchy and elaborated by Dedekind and Cantor. Significance can be ascribed to this distinction in so far as it represents a thoroughly thought-through conceptual achievement attained and legitimated over an extended period of time that laid the foundations for the conceptualization of a variety of intellectual domains, including the social sciences. In the case of social theory, this distinction allowed basic conceptual constructs that became transposed into standard fundamental theoretical assumptions operating as parameters of social reality. Among the most basically assumed parameters is the social-theoretical conceptual pair of dynamics and statics which corresponds directly to the two distinct series. Roundabout 1840, Auguste Comte (1974) was able to formulate this social dynamics–social statics distinction for the new domain of sociology on the basis of his knowledge as a mathematician, undoubtedly in the wake of Cauchy's contribution, and to correlate it with progress and order. Noteworthy is that he regarded statics as the conditions and pre-conditions of social order and, further, that all of these were knowable, apparently informed by his understanding of the divergent series and its infinite ideal limit concept. This was the first authoritative step toward the general acceptance in social theory up to the present of the closely allied concepts of process and structure or, more fully, of the process of the creation, construction or production of society and the structure, structuration or reproduction of society.[8]

It would be relatively easy to provide a broad survey of social theory in which the adoption and operation of this fundamental parametric assumption is demonstrated in the writings of the major social theorists past and present, but I am less interested here in such a boring task than in making a proposal that could benefit the development of social theory at the crossroads and, hence, the point of decision where it stands today. Were one to embark on such a survey, one would for example have to explicate the following schemes: Comte's social dynamics–social statics, Marx's praxis–general concepts, Durkheim's division of labour–obligatory categories, Weber's rationalization–value spheres, Mead's evolution/social interaction–generalized other/universal society, Piaget's developing structures– completed structures, Habermas' evolution/communication–formal pragmatic world concepts, Luhmann's autopoiesis–world, and so forth. It is of course of the utmost importance to see and appreciate the mode of thinking regarding infinite processes exhibited by

these and other social theorists which derives from the historically and theoretically related distinctions between potential and actual infinity and between convergent and divergent series. Leaving the suggested exercise to one side, however, I propose instead to turn to the second set of considerations relating to the problem of limit concepts and to gesture toward what a more independent systematic elaboration of it could mean for social theory.

The fundamental insight from which a social-theoretical consideration of the problem of limit concepts applicable to infinite processes has to proceed is the one regarding reality going beyond immediate sense-experience that Leibniz suggested, Kant cast in transcendental terms and eventually was unequivocally established in the form of complete infinity by the late nineteenth-century mathematicians – the insight, namely, that the concepts of infinity and especially limits endow thought, whether mathematics, philosophy or science, including social science and social theory, with generality. The central problem here, however, is to think through the problem of generality in a philosophically defensible manner relevant to social theory. This is something that social theorists have not done properly, which accounts for the conspicuous lack of clarity and much equivocation on the matter in social theory. The key to the solution of this problem is the vital distinction between generality and universality.[9]

In terms of what has thus far been discussed, the distinction between generality and universality can be aligned with the distinction between convergent and divergent series and, therefore, can be made intelligible with reference to the associated limit concepts. Accordingly, it can be submitted that generality is captured by the finite ideal limit concepts punctuating convergent series, while universality is captured by infinite ideal limit concepts punctuating divergent series. Given this mathematical-philosophical basis, it becomes possible in a second step to give abstract theoretical content to the still more abstract concepts of generality and universality. To begin with, let it be noted that both types of limit concepts, as their designation as 'ideal' indicates, necessarily involve idealization – a characteristic stressed by both mathematical and philosophical thought (e.g. Dantzig 2007; Körner 1968; Habermas 1984). This means that both generality and universality are modes of idealization, yet as such they differ in a theoretically decisive sense. The former is context-immanent and contingent, while the latter is context-transcendent and necessary in the sense of making the human sociocultural form of life possible. My proposal for the purposes of a cognitive enhancement of social theory is accordingly as follows: to conceive, first, of the context-immanent, contingent, finite ideal limit concepts associated with convergent series as taking the form of cultural models of different levels and scales;[10] and, second, to conceive of the context-transcendent, necessary, infinite ideal limit concepts associated with divergent series as transcendental cognitive order principles. The key cognitive social-theoretical concepts are cultural model and the cognitive order of society, and treating them in this way allows them, by contrast with the existing confused and opaque state of affairs in social theory, to be unambiguously located at their distinct levels. Before providing a social-theoretical overview of both types of limit concept and the infinite processes they punctuate, more should be said towards characterizing them and clarifying their status, while some preliminary examples also need mentioning.

In so far as finite and infinite limit concepts are idealizations, they are dependent on language or, more specifically, on the rule structure of language. Idealization in the medium of ordinary everyday language takes the form of meaning which implies that, in the first instance, idealization is a semantic matter, more specifically, a manifestation of semantic generality.[11] Both types of limit concept thus possess meaning or semantic generality. Examples of finite ideal limit concepts in the sense of context-immanent cognitive structures or forms exemplifying generality of this kind would include, among many others of different levels and scales, such cultural models as democracy, Irishness, French republicanism, Catholicism, communism and so forth. They are all structures or forms that generalize certain values, norms, symbols and orientations so that all those involved

share a particular culture which gives direction to and guide their engagements and practices, even in cases where the engagements and practices diverge. In terms of the previous analysis of convergent series, then, cultural models can be understood as limit concepts that represent finite ideals – context-immanent ideals toward which practices tend but can never be reached or fully realized, like pi (π) as an example of the limit value of potential infinity.

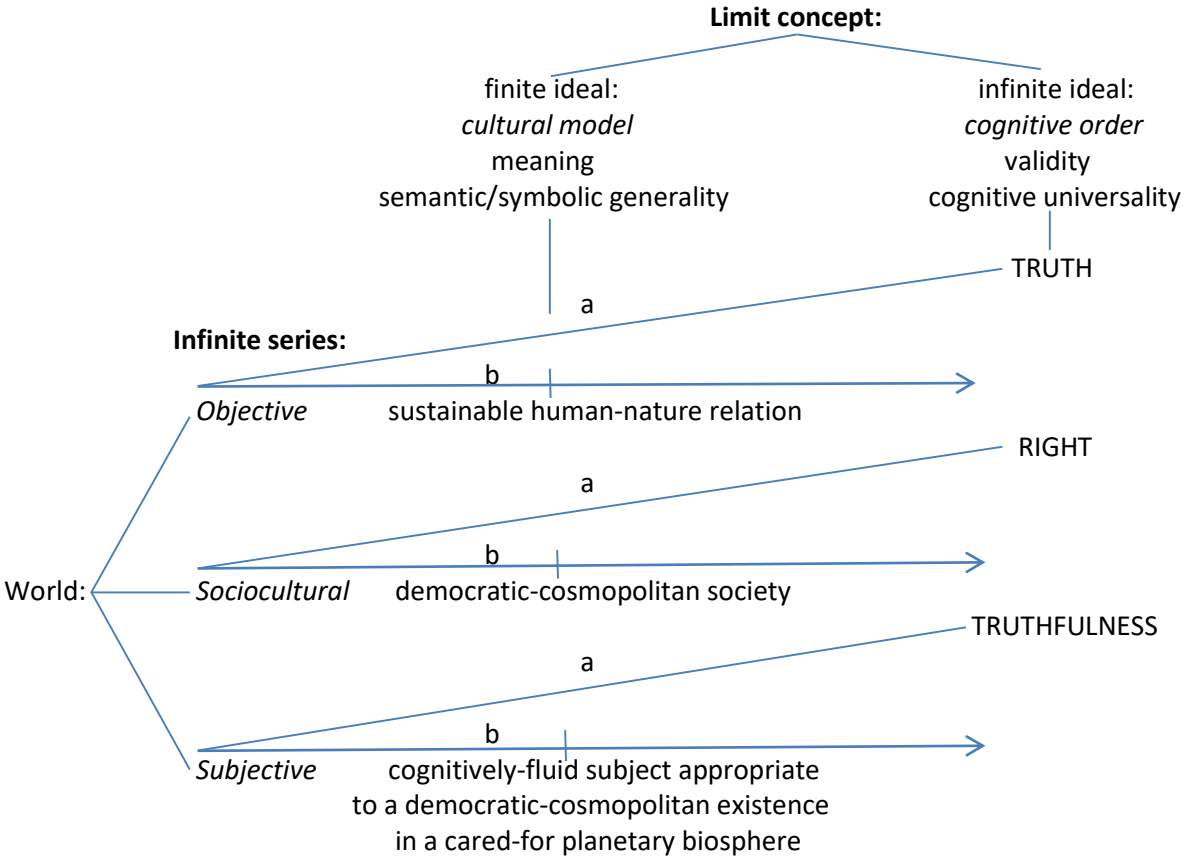
As already indicated, infinite ideal limit concepts in the sense of context-transcendent cognitive order principles such as truth, right, justice, equality, truthfulness, authenticity and so forth indeed also exemplify semantic generality in so far as they are concepts meaningfully articulated in language. But since these limit concepts above all possess universality, they cannot be conceived exclusively in terms of semantics or symbolically packaged meaning. Most characteristically, in fact, they are seats of validity, or what philosophically is considered under the description of 'unconditionality' (e.g. Peirce 1998: 457; Habermas 2003: 99). Beyond meaning, they have validity. While meaning is tied to the rules of language, validity is anchored cognitively in the brain-mind – that is, in the organic endowment and the culturally articulated phylogenetically acquired mind of the human species.[12] By contrast with meaning as an instance of semantic generality, validity is one of cognitive universality. It is therefore not just a matter of appreciating the difference between generality and universality and between meaning and validity, but also the distinction between the semantic and the cognitive. Besides language or concepts, the cognitively rooted universality of infinite ideal limit concepts depends on what one might perhaps call quasi-conceptual media represented, for instance, by logic and mathematics – that is, cognitive rule systems given with the human organic endowment other than language that nevertheless provide meaning with a structure and underpin the correct use of language. Given their semantic and basically cognitive nature, the components of the cognitive order *qua* infinite ideal limit concepts can be regarded as the conceptual and quasi-conceptual foundations of the human sociocultural form of life.[13] Rather than simply complexes of culturally specific values, norms, symbols and orientations that are generally shared by a particular group in distinction to other comparable groups, like cultural models, cognitive order principles are commonly presupposed by human beings *qua* human beings who have grown up and live in the characteristic human sociocultural form of life. In terms of the previous analysis of divergent series that end in actual or complete infinity, then, cognitive order principles can be regarded as limit concepts that represent infinite ideals providing classification and ordering schemes which, in turn, serve as a basis for cultural models, lay down their parameters, structure them from on high and make possible the regulation of their embodiment and realization.

Systematizing the preceding discussion, Figure 2 below starts from the reconstructed mathematical-philosophical conceptualization and proposes a social-theoretical interpretation of the world for us humans as consisting of three sets of infinite processes, each of which consists of an ongoing convergent process of becoming and a structurally accumulating divergent process. The three sets of infinite processes are the objective, the sociocultural and the subjective.

First, the objective set covers the ongoing natural-historical and evolutionary process of which the sociocultural form of life is a continuation and extension in a specifically human form. It should be stressed that the conceptualization of this dimension is intentionally geared toward admitting a weak-naturalistic ontology[14] as a necessary complement to the proposed cognitive twist of social theory. The objective world of all possible objects of experience and reference embraces neither just sociocultural reality nor just nature as seen by the natural sciences, but both of these as well as nature-in-itself or *natura naturans* in the sense of the infinite process that spontaneously brings forth a plethora of diverse forms, including humans and their form of life. Second, the sociocultural set houses the ongoing process of the creation, construction, production and formation as well as the organization of the sociocultural form of life internally and in relation to nature. Finally, the subjective set consists of the ongoing process of subject-formation and self-transformation, whether

under objective or sociocultural pressure or, alternatively, initiating change. In turn, these three ongoing processes are overdetermined by the concomitant complementary processes of the historically long-term accumulation of rational potentials that take on an incursive and recursive structuring role in respect of the ongoing processes – which leads to a consideration of limit concepts in the form of cultural models and cognitive order principles.

Figure 2: Infinite Processes and Limit Concepts in Social Theory



Key:
a = increasing or divergent series: historically long-term accumulation of rational potentials
b = decreasing or convergent series: indefinitely tending toward a finite limit value

In all three cases, Figure 2 identifies the limit concepts relevant to each of the infinite processes. For the three ongoing convergent processes, a selection is made from among the vanishingly large number of possible cultural models of finite ideal limit concepts of ones that have only very recently emerged as, perhaps, the most urgently required directing and guiding cultural models under the currently rather challenging conditions. This particular choice is made since the aim of this paper is not to present an analysis or typology of typical cultural models, but rather to make a contribution to the renewed development of social theory with a view to being relevant to the challenges humankind is confronted with at present on the brink of the emerging world society and the new age of 'the Anthropocene' (Crutzen 2002; Strydom 2017).

In view of the sobering realization in the age of the Anthropocene that society forms part of nature and that humans have become a geophysical force contributing to global warming and climate change, the limit value or cultural ideal toward which the ongoing objective process since the 1970s tends is that of a sustainable human-nature relation. In the wake of globalization, mondialization

and the emergence of world society, second, the ongoing sociocultural process has in the 1990s begun to tend toward the limit value or cultural model of a democratic-cosmopolitan society. As regards the ongoing subject-formation process, finally, the pressures emanating from both objective and sociocultural processes reinforce and further structure the changes in the formative process which were inaugurated in the 1960s, so that it is beginning to become clear today that the fitting limit value or cultural model in this case is that of a cognitively fluid subject appropriate to a democratic-cosmopolitan existence in a cared-for planetary biosphere (Strydom 2011b; Strydom 2015b).

In respect of the divergent processes in the sense of the historically long-term accumulation of rational potentials, infinite ideal limit concepts are listed in the diagram in the objective, sociocultural and subjective cases that reflect those accumulated potentials which take the form of the most basic cognitive order principles of the human sociocultural form of life. They cover not just the main principle directly related to the internal organization of that form of life, namely right, but also the main principles respectively related to its subjective component, truthfulness, and to its objective nature and conditions, that is truth. While the principles of truth, right and truthfulness tower over the three sectors of the cognitive order, the latter of course contains a vanishingly extensive number of principles,[15] any one of which can function as an infinite ideal limit concept if the situation so requires. In any concrete case, the identification of these principles and the incursive and recursive roles they play immanently is of the essence of social-scientific analysis led by clear cognitive social-theoretical thinking.

Conclusion

From a social-theoretical perspective, the brief reconstruction of the mathematical-philosophical tradition in this paper opened a way for seeing more clearly and thus understanding better the direction that social theory and, more generally, the social sciences are compelled to take and, to be sure, have already begun to take under contemporary conditions – that is, an interrelated cognitive and weak naturalistic direction. Not only did it allow the two most basic conceptual parameters of social reality corresponding to the convergent and divergent infinite processes to be backed up by historical and theoretical evidence deriving from centuries of hard research and design work done by philosophers, mathematicians and scientists. At the same time, it made possible also the foregrounding of the important notion of limit concept, indeed, two types of limit concept which provide anchor points for much needed conceptual clarification and development. While the notion of a limit concept due to its dependence on the power of the mind has a direct bearing on the cognitive upgrading of social theory, one of the three sets of infinite processes, the objective process which generates the world of all possible objects of experience and reference, brings the need for the adoption of a weak naturalistic ontology within the cognitive social-theoretical purview.

The conceptual achievement involving the introduction of the new concept of the cognitive order of society and its unambiguous differentiation from the concept of cultural model feeds, as indicated, into the cognitive enhancement of social theory. This it does by enabling the employment of cognitive structures and structuration emanating from both the cognitive order and cultural models for the purposes of hitherto either haphazard or lacking conceptual and substantive analyses. That such a cognitive direction is unavoidable for social theory today is given with the fact that long-standing taken-for-granted assumptions and ideas regarding the relations of humans to nature, to others and to themselves have to be brought to the level of conscious reflection and, further, that central cherished ideas have to be subjected to penetrating evaluation, reconsideration, critique, transformation and even replacement where necessary. In all these cases, we have to do with historically accumulated rational potentials such as schemata, forms and formats stemming from the power of the mind, some context-immanent and others context-transcendent, that require

cognitively inspired theorizing and analyses, both with reference to their structural properties and dynamics.[16]

The acknowledgement of the ongoing infinite objective process opens the possibility, as indicated, of building a weak naturalistic ontological assumption into social theory as a complement to and support for its articulation in cognitive terms. This is the case since the process' continuous and multidimensional nature draws attention to the fact that social theory, as far as is relevant, has to take into account its full range. The objective process is multidimensional in so far as it embraces all possible objects of experience and reference which cover the ongoing spontaneous natural-historical and evolutionary process in its diversity; and it is continuous in so far as it is an infinite process of which the sociocultural form of life is a continuation and extension in a specifically human form. Far from such a weak naturalism entailing a naturalistic reduction of the sociocultural world, however, it distinguished itself from strong naturalism precisely by making space for the integrity of a relatively independent sociocultural world. Especially important is that it dovetails perfectly with a sociocultural cognitive approach by virtue of the fact that nature-in-itself or *natura naturans* spontaneously gives rise to natural cognitive forms. They come in the elementary or primitive social forms exhibited by our primate ancestors as well as many animal species, like kinship, group membership, dominance, cooperation, competition, sharing, playing, fighting, reconciling and so forth – all forms that humans presuppose yet then socially and culturally elaborate into a variety of domains directed and guided by cognitive order principles and cultural models and further articulate by corresponding social arrangements and practices.

The conclusion that follows, finally, is that a rethinking of social theory by recourse to a recovery of its historically and theoretically important mathematical-philosophical background, particularly as captured by the notions of infinity, infinite processes and limit concepts, enables its channelling in an interrelated cognitive and weak naturalistic direction. And it is exactly such a departure that is urgently called for by our currently fraught situation in which we are compelled to begin to transform ourselves so as to be able to think in new terms and to relate to both others and nature in an appropriate way.

Notes

[1] An exception was Archimedes, but his thought made no impact at the time, his life having been brutally cut short by the sword of an invading Roman soldier.
[2] Since modern thought is by no means monolithic, the developments covered here are not reducible to the long-predominant nominalist or positivist trend as they deny that reality can be restricted to immediate sense-experience, while recognizing the unconditional, open horizons and uncertainty.
[3] Such idealizing abstraction forms the basis of the procedure of reconstruction that is methodologically characteristic of Critical Theory (Strydom 2011a).
[4] See Badiou (2013) for an exemplary analysis of Hegel's relapse and Marx's alternative approach.
[5] In a carefully qualified extraction of Peirce's understanding of the meaning of truth, Legg (2014) presents an analysis of his 'limit concept of truth' expressly in terms of the convergent series, not unlike the interpretation of Apel (1995) which Habermas (2003) revised only in 1999. However, this leaves open the question of the relevance of the divergent series for Peirce's conception of truth.
[6] The debate generated by Albrecht Wellmer's (e.g. 2003) persistent criticism since the late 1980s of Apel and Habermas' not entirely justifiable talk of anticipating the realization of the ideal communication community in the real one, which eventually led Habermas (2003) to revise his position on truth, has not produced adequate clarity on the issue. I am convinced that it can be critically deconstructed by means of the distinction between different infinite processes and the

corresponding limit concepts, matters of which the participants in this debate take no notice at all. See Strydom (in press).

[7] Note that this principle coincides with what may be called the cognitive meta-problematic: that is, that something belonging to the world is able nevertheless to take distance from the world, distinguish itself from the world, develop a perspective on the world, establish any of a number of relations to the world and to act upon it (Strydom 2011a).

[8] For an interpretation of this construct, see for example the diagram titled 'Immanent Transcendence' in Strydom (2011a: 98) which is central to the programme for a new cognitive sociology.

[9] The issue is dealt with in a critical treatment of Habermas n Strydom (2015a).

[10] For the sake of clarity, I focus for the moment on cognitive cultural models alone and leave out of account any related sociocultural and social organizational schemes such as social models and social systems.

[11] Typically, semantic generality is symbolically packaged in order to be rendered communicable.

[12] Philosophers treat validity in deontological terms, but social-scientifically it can and must be regarded also as a meta-cultural phenomenon which, nevertheless, has roots in the human brain-mind. The proposal is to conceive of this meta-cultural and weak naturalistic complex in terms of the cognitive properties pervading it at all levels.

[13] Jackendoff (1999: 74-5) speaks of the 'conceptual foundations of social organization [that provide] a skeleton of issues around which cultures are built'. Habermas (2003) acknowledges rule systems other than the strictly conceptual, but he consistently focuses only on the conceptual rule system of language; see my criticism in Strydom (2015a).

[14] See Habermas (2007) for his latest statement on weak naturalism.

[15] A social-theoretically meaningful, research oriented but by no means exhaustive breakdown of the cognitive order is proposed in Strydom (2015a: 278).

[16] See, for example, Strydom (2012) for such an analyses, as well as Delanty (2013) and O'Mahony (2013) for applications.

References

Alexander, Amir (2015) *Infinitesimal: How a Dangerous Mathematical Theory Shaped the Modern World*. London: Oneworld.

Apel, Karl-Otto (1995) *Charles S. Peirce: From Pragmatism to Pragmaticism*. Atlantic Heights, NJ: Humanities Press.

Aristotle (1961 [350 BC]) *Metaphysics*. London: Dent and New York: Dutton.

Aristotle (2015 [350 BC]) *Physics*. Adelaide: University of Adelaide (E-book).

Badiou, Alain (2013) *Theory of the Subject*. London: Bloomsbury.

Comte, Auguste (1974 [1855]) *Positive Philosophy*. New York: AMS.

Crutzen, Paul J. (2002) 'Geology of Mankind: the Anthropocene', *Nature* 415: 23.

Dantzig, Tobias (2007 [1930]) *Number: The Language of Science*. London: Plume.

Delanty, Gerard (2013) *Formations of European Modernity*. Basingstoke: Palgrave Macmillan.

Elwes, Richard (2013) *Maths in 100 Key Breakthroughs*. London: Quercus.

Fichte, Johann Gottlieb (1982 [1794]) *The Science of Knowledge*. Cambridge: Cambridge University Press.

Galilei, Galileo (1914 [1638]) *Dialogues Concerning Two New Sciences*. New York: Macmillan. <http://oll.libertyfund.org/titles/753> accessed 31 October 2015.

Habermas, Jürgen (1984) *The Theory of Communicative Action*, Vol. 1. London: Heinemann.

Habermas, Jürgen (2003) *Truth and Justification*. Cambridge: Polity.

Habermas, Jürgen (2007) 'The Language Game of Responsible Agency and the Problem of Free Will', *Philosophical Explorations* 10(1): 13-50.

Hegel, Georg Wilhelm Friedrich (1966 [1807]) *The Phenomenology of Mind*. London: Allen & Unwin.

Hegel, Georg Wilhelm Friedrich (1967 [1821]) *Philosophy of Right*. London: Oxford University Press.
Hegel, Georg Wilhelm Friedrich (1969 [1812]) *Science of Logic*. London: Allen & Unwin.
Heidegger, Martin (1962) *Die Frage nach dem Ding*. Tubingen: Niemeyer.
Jackendoff, Ray (1999) *Languages of the Mind*. Cambridge, MA: MIT Press.
Kant, Immanuel (1956 [1788]) *Critique of Practical Reason*. Indianapolis: Bobbs-Merrill.
Kant, Immanuel (1968 [1781]) *Critique of Pure Reason*. London: Macmillan.
Kant, Immanuel (1972 [1790]) *Critique of Judgement*. New York: Hafner.
Kline, Morris (1990) *Mathematical Thought from Ancient to Modern*, Vol. 1-3. New York and Oxford: Oxford University Press.
Körner, Stephan (1968) *The Philosophy of Mathematics*. London: Hutchinson.
Legg, Catherine (2014) 'Charles Peirce's Limit Concept of Truth', *Philosophical Compass* 9(3): 204-13. DOI: 10.1111/phc3.12114.
Leibniz, Gottfried Wilhelm (1965 [1714]) *The Monadology and Other Philosophical Writings*. London: Oxford University Press.
Leibniz, Gottfried Wilhelm (1968) *Philosophical Writings*. London: Dent.
Maimon, Salomon (2010 [1790]) *Essay on Transcendental Philosophy*. London and New York: Continuum.
Marx, Karl (1969a) *The Economic and Philosophic Manuscripts of 1844*. New York: International Publishers.
Marx, Karl (1969b) *Selected Writings in Sociology and Social Philosophy*. Harmondsworth: Penguin.
O'Mahony, Patrick (2013) *The Contemporary Theory of the Public Sphere*. Oxford: Peter Lang.
Panofsky, Erwin (1955) *Meaning in the Visual Arts*. Garden City, NY: Doubleday.
Peirce, Charles S. (1992) *The Essential Peirce*, Vol. 1 (1867-93). Bloomington: Indiana University Press.
Peirce, Charles S. (1998) *The Essential Peirce*, Vol. 2 (1893-1913). Bloomington: Indiana University Press.
Strydom, Piet (2011a) *Contemporary Critical Theory and Methodology*. London: Routledge.
Strydom, Piet (2011b) 'The Communicative Turn and Cosmopolitan Ecological Consciousness in the Global Risk Society: An Assessment of the Discourse', in P. O'Mahony and K. Keohane (eds) *Irish Environmental Politics: The Communicative Turn*. Manchester: Manchester University, pp. 12-30.
Strydom, Piet (2012) 'Toward a Global Cosmopolis? On the Formation of a Cosmopolitan Cultural Model', *Irish Journal of Sociology* 20(2): 28-50. DOI: 10.7227/IJS.20.2.3.
Strydom, Piet (2015a) 'The Latent Cognitive Sociology in Habermas: Extrapolated from *Between Facts and Norms*', *Philosophy & Social Criticism* 41(3): 273-91. DOI: 10.1177/0191453714563877.
Strydom, Piet (2015b) 'Cognitive Fluidity and Climate Change: A Critical Social-Theoretical Approach to the Current Challenge', Special Issue on Climate Change, *European Journal of Social Theory* 18(3): 236-56. DOI: 10.1177/1368431015579961.
Strydom, Piet (2017) 'The Sociocultural Self-Creation of a Natural Category: Social-theoretical Reflections on Human Agency under the Temporal Conditions of the Anthropocene', special 20th-century anniversary issue 'The Anthropocene and Social Theory', *European Journal of Social Theory* 20(1): 61-79. DOI: 10.1177/1368431016643330.
Strydom, Piet (in press) 'The Problem of Limit Concepts in Habermas: Toward a Cognitive Approach to the Cultural Embodiment of Reason', *Philosophical Inquiry*.
Wellmer, Albrecht (2003) 'Der Streit um die Wahrheit: Pragmatismus ohne regulative Ideen', in D. Böhler, M. Kettner and G. Skirbekk (eds) *Reflexion und Verantwortung*. Frankfurt: Suhrkamp, pp. 143-70.