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The Constrained Optimisation of Small Linear Arrays of Heaving Point Absorbers. Part I: The Influence of Spacing

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Abstract

This paper describes the optimisation of small arrays of Wave Energy Converters (WECs) of point absorber type. The WECs are spherical in shape and operate in heave alone and a linear array of five devices is considered. Previous work is extended by considering the constrained performance of the array members, where an upper limit on WEC displacements is enforced. Two optimisations are performed. In each case, the objective function is defined as the mean of the averaged interaction factor over the non-dimensional length of the array. The first considers the array layout fixed at a geometry previously identified as optimal in an unconstrained regime and optimises the displacements of the WECs subject to constraints. The second allows both the WEC positions and displacements to vary as optimisation variables. It is shown that the optimal layout of the constrained arrays is different from the unconstrained case. Applying constrained motions results in optimal

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layouts that are more separated, with less grouping of WECs and this will have practical considerations. The effect of the constraints varies depending on the incident wave angle. In some cases, performance is reduced drastically and stability of performance is improved, while in other cases there is a degradation of performance. Thus, a trade-off between performance and stability of performance is seen when displacement constraints are applied.

Keywords: Wave-Power, Arrays, Constrained Optimisation, Interaction, Point Absorber

1. Introduction

The fundamental modelling of arrays of wave power devices of point absorber type was presented independently in [1] and [2]. The point absorber approximation assumes that the ratio of device size to incident wavelength is small enough for the scattered wave field of the device to be neglected. This allows a simplification of the calculations, particularly those relating to WEC arrays. Subsequent papers have applied this theory to assess arrays of differing configurations or array properties, e.g. [3, 4, 5, 6, 7, 8].

In [1],[2] and [3], the devices were assumed to be equally spaced and the concept of positive and negative interference within the array was established. The concept of unequal spacing in a linear array was first considered in [4] and it was shown that unequally spaced arrays performed better in some cases in comparison to equally spaced arrays. However, only a very specific case of unequal spacing was considered. The accuracy of the point absorber approximation is discussed in [5], where it is shown that the approximation gives agreement with the exact multiple scattering method for

17 a non-dimensional device radius of $ka < 0.8$. The extension to arbitrary
 18 array arrangements, without any stipulated geometry or symmetry is con-
 19 sidered in [6], [7] and [8]. In [6] and [7], the point absorber approximation
 20 is applied and the interaction factor is numerically maximised with respect
 21 to WEC positions for both constrained and unconstrained WEC motions. A
 22 full interaction regime is implemented in [8] and the array performance is
 23 maximised using a genetic algorithm for both regular and irregular waves.

24 A major common finding of the of the previous array optimisation studies
 25 (e.g. [3], [6], [7] and [8]) is that the optimal array arrangements were often
 26 found to be only slightly different to those corresponding to very poorly
 27 performing arrays. In many cases, either the best and worst array layouts
 28 were surprisingly close or the optimal array had a sharp peak in performance
 29 surrounded by large troughs. This means that a small change in the non-
 30 dimensional parameters of such arrays, either by a physical mis-alignment or
 31 a change in sea conditions (incident wavelength or wave angle), can have a
 32 potentially disastrous impact on array performance.

33 This issue was addressed in [9] and [10], which considered the optimisation
 34 of linear and circular arrays of five to seven WECs, where the mean of the
 35 interaction factor was maximised, rather than the interaction factor itself. In
 36 these works, the mean was taken over a non-dimensional length/radius mea-
 37 sure, which resulted in arrays that were stable to changes in non-dimensional
 38 separation parameters. However, in some cases, these optimal arrays were
 39 still quite sensitive to changes in incident wave angle. One important issue
 40 is whether high performance or stability (reliability) of performance is more
 41 desirable. Ideally, both would be achieved by an optimal array, however

this may not be possible, particularly with the application of WEC motion constraints.

A main concern when considering array performance is the motions of the individual devices associated with optimal performance. A hydrodynamically optimised array is typically accompanied by large amplitude device motions; this is highlighted in [10]. The large motion of WECs creates engineering difficulties with the control, maintenance and power take-off of the devices. In addition, linear wave theory assumes that all device motions are at most of the same order of the wave amplitude, and violation of this requirement invalidates the underlying assumptions; this is considered in [3], [6], [9] and [10], where the optimal arrays were predicted to exhibit large device motions. Device motion constraints were investigated in [3] and [6], where it was found that, in some cases, these constraints severely limited array performance.

The main aim of this paper is the constrained optimisation of WEC arrays such that the resulting optimal array is stable to changes in array parameters. Having an array that performs well in certain conditions but that is also highly sensitive to changes in wavelength or wave angle is not ideal. Wave conditions in the open ocean can change slightly and ideally a WEC array should maintain optimal or at least near-optimal performance in the case of any such changes. Previous research of the nature is extended by considering constrained performance of the WECs, where the WEC motions are limited to two or three times the incident wave amplitude, as in [3] and [6].

The effect of these constraints are firstly analysed with respect to layouts previously optimised without constraints. The layouts are then re-optimised within the constrained regime and the resulting layouts compared.

67 The work presented herein is conducted within the regime of validity of
68 the point absorber approximation ($ka < 0.8$) as identified in [5], and the
69 non-dimensional radius of the WECs is fixed at $ka = 0.4$. An external model
70 is required in this methodology to determine the device motions and for the
71 chosen device geometry, which is spherical in this case, the motions can be
72 determined using the approach of [11], for a fixed non-dimensional radius of
73 the WECs.

74 This research is motivated by the possibility that unequally spaced lin-
75 ear arrays may perform better than their equally spaced analogs. The work
76 presented in [9] and [10] was similarly motivated, where linear and circular
77 array geometries were enforced and the mean array performance was max-
78 imised with respect to the non-dimensional WEC separations. The mean
79 performance was defined over a range of non-dimensional array length or
80 radius.

81 Section 2 outlines the mathematical theory behind this research, includ-
82 ing the definition of the averaged interaction factor for constrained motions
83 and the optimisation method. The results of the optimisation are presented
84 in section 3. The constrained performance of previously identified uncon-
85 strained optimal array layouts is assessed in section 3.1. In section 3.2, the
86 array layout is not prescribed and an optimisation over both the WEC mo-
87 tions and positions is performed with respect to the mean of the averaged
88 interaction factor. Finally, a discussion of the results is given in section 4
89 along with some conclusions thereof.

2. Mathematical Formulation

2.1. Power Absorption Theory

Consider a linear array of physical length L with N semi-submerged spheres, considered to be point absorbers and which operate in heave alone. It is assumed that linear wave theory is applicable and that regular long-crested waves of amplitude A , frequency ω , wavenumber k and angle β are incident on the array in water of infinite depth, where β is measured in an anticlockwise direction from the positive x -axis.

A detailed description of the background theory is available in [9] and [10], where array power absorption theory and the point absorber approximation are outlined. In this work, constrained motions are considered and the full power absorption equation is employed without the assumption of optimal motions. As shown in [1], the mean power absorbed by the array is

$$P_{abs} = \frac{1}{8} \mathbf{X}^\dagger \mathbb{B}^{-1} \mathbf{X} - \frac{1}{2} \left(\mathbf{U} - \frac{1}{2} \mathbb{B}^{-1} \mathbf{X} \right)^\dagger \mathbb{B} \left(\mathbf{U} - \frac{1}{2} \mathbb{B}^{-1} \mathbf{X} \right), \quad (1)$$

where $\mathbf{X}$ and $\mathbf{U}$ are complex time-independent column vectors of the exciting forces and velocities of the devices respectively, $\mathbb{B}$ is the radiation damping matrix and † denotes complex conjugate transpose. In this notation, the exciting force and velocity of body m are given by $\text{Re}[X_m e^{-i\omega t}]$ and $\text{Re}[U_m e^{-i\omega t}]$. In order to relate the body displacements to the mean power absorption of the array, the velocities are replaced by

$$\mathbf{U} = -i\omega \mathbf{A} \mathbf{D}, \quad (2)$$

where $\mathbf{D}$ is a complex time-independent column vector containing the body displacements non-dimensionalised with respect to the incident wave amplitude. The expression (1) is not optimal and holds under general conditions.

In order to assess the array performance in the constrained case, the averaged interaction factor is utilised and defined as

$$\bar{q} = \frac{\text{Power absorbed by array subject to constraints}}{N \times \text{Maximum power absorbed by isolated WEC}}. \quad (3)$$

This quantity will usually not achieve a value of unity, unless the constraints applied do not restrict the optimal motions of the WECs. It is also possible for the constrained power absorbed by the array to become negative, in which case the forced displacements cause the array to inject power into the waves, thereby creating waves rather than absorbing them. Assuming the point absorber approximation to be applicable allows the averaged interaction factor to be written, via [11], as

$$\bar{q} = \frac{4\pi(ka)^2}{N} \left(-\text{Re} [(\mathcal{D} + i\mathcal{C})\mathbf{D}^\dagger \boldsymbol{\ell}] - \pi(ka)^2(\mathcal{C}^2 + \mathcal{D}^2)\mathbf{D}^\dagger \mathbb{J} \mathbf{D} \right), \quad (4)$$

where a is the WEC radius, $\mathcal{C}$ and $\mathcal{D}$ are the Havelock coefficients, $\boldsymbol{\ell}$ is a column vector with components $\{\ell_m = e^{ikd_m \cos(\beta - \alpha_m)}; m = 1, \dots, N\}$ and $\mathbb{J}$ is an $N \times N$ matrix with elements $\mathbb{J}_{mn} = J_0(kd_{mn})$, where $J_0(x)$ is the zeroth order Bessel function of first kind.

In this notation, the position of the m^{th} device is given by the cylindrical polar coordinates $(r, \theta, z) = (d_m, \alpha_m, 0)$ and d_{mn} is the distance between the m^{th} and n^{th} devices. One device is fixed at the origin, without loss of generality. As this work concerns linear arrays, all the α_m 's are set to zero. In addition, as consecutive device separations are often employed, the convenient notation $s_m = d_{m(m+1)}$ is introduced.

2.2. Optimisation Process

As in [10], the aim of the hydrodynamic optimisation is to expressly seek array layouts that are stable to changes in non-dimensional parameters as-

sociated with device spacing and incident wavelength. In this paper, the constrained performance of the arrays is examined, where the displacement amplitudes of the WECs are limited to an upper value during the optimisation.

The same re-parameterisation of the separations presented in [10] is utilised here, namely

$$ks_j = n_j kL, \quad (5)$$

where $n_j \in (0, 1)$ is a real parameter that represents the relative separation between devices with respect to the total length. Consistency requires that

$$\sum_{j=1}^{N-1} n_j = 1, \quad (6)$$

which removes one separation variable. Furthermore, relative to [10] there are now an extra N complex displacement variables D_j . In order to formulate the objective function explicitly in terms of real variables, the non-dimensional complex displacements are written as

$$D_j = \delta_j e^{i\psi_j}, \quad (7)$$

where δ_j and ψ_j are the displacement amplitude and phase of the j^{th} WEC respectively.

Using this formulation, the objective function becomes

$$\bar{I}(\mathbf{n}, \boldsymbol{\delta}, \boldsymbol{\psi}; \beta_0) = \frac{1}{kL_u - kL_l} \int_{kL_l}^{kL_u} \bar{q}(\mathbf{n}, \boldsymbol{\delta}, \boldsymbol{\psi}, kL; \beta_0) d[kL], \quad (8)$$

where $\boldsymbol{\delta}$ and $\boldsymbol{\psi}$ are N -component vectors containing the motion amplitudes and phases of each device respectively, $\mathbf{n}$ is an $(N - 2)$ -component vector

151 containing the separation variables, $kL \in [kL_l, kL_u]$ is the integration vari-
 152 able describing the range of non-dimensional length considered and β_0 is a
 153 fixed prescribed incident wave angle. The notation $\bar{I}$ is to indicate that the
 154 mean is defined with respect to $\bar{q}$ and thus considers constrained WEC mo-
 155 tions, which is distinct from the unconstrained optimisation in [10]. The
 156 objective function contains a total of $3N - 2$ and will be maximised using a
 157 similar procedure to that in [10], with appropriate constraints placed on the
 158 variables.

159 The non-dimensional parameter kL can be considered in two ways; for a
 160 fixed wavelength λ it represents a change in physical array length L , while for
 161 a fixed array length it represents a change in incident wavelength. The range
 162 of optimisation over kL is chosen to be $[kL_l, kL_u] = [5, 15]$ in this paper.
 163 These values are chosen arbitrarily but are intended to represent a typical
 164 case. The aim is to represent a target (or mean) value of $kL = 10$, with
 165 the range chosen to allow for variation around this target value. Typical
 166 ocean wavelengths are approximately 200m, in which case the target value
 167 of $kL = 10$ corresponds to an array length of approximately 320m. Consid-
 168 ering a fixed array length of 320m, the lower bound $kL_l = 5$ corresponds to
 169 a wavelength of $\lambda \approx 400\text{m}$, while the upper bound $kL_u = 15$ corresponds to
 170 $\lambda \approx 134\text{m}$. Thus variation over typical ocean wavelengths is accounted for
 171 and the chosen values correspond to reasonable array lengths. The values
 172 are also chosen to allow comparison with previous literature, such as [9, 10],
 173 where similar values are chosen. It should be noted that the method is ap-
 174 plicable for any reasonable values and, if desired, values can be chosen that
 175 correspond to a particular WEC array site if that information is available.

176 Strictly, the displacement amplitude δ_j is required to be positive by def-
 177 inition, so for a maximum displacement constraint of δ_{max} , the range of the
 178 displacement variables would be $0 \leq \delta_j \leq \delta_{max}$ and $0 \leq \psi_j \leq 2\pi$. However,
 179 mathematically this is equivalent to allowing the amplitude to be negative
 180 and restricting the phase to $0 \leq \psi_j \leq \pi$. Since the ψ_j variables are contained
 181 within a complex exponential expression, the variation over this variable
 182 within the optimisation would be more computationally intensive than vari-
 183 ation over δ_j , albeit only slightly. However, given the large number of calls
 184 to the objective function and the large number of runs of the optimisation
 185 necessary, every effort was made to make the calculations more efficient.
 186 Therefore, in the implementation, a new variable χ_j is introduced and the
 187 displacements are written as

$$D_j = \chi_j e^{i\psi_j}. \quad (9)$$

188 If δ_{max} is a given amplitude constraint, then the limits on the displacement
 189 variables are $-\delta_{max} \leq \chi_j \leq \delta_{max}$ and $0 \leq \psi_j \leq \pi$ for $j = 1, \dots, N$.

190 The optimisations are implemented in FORTRAN using a similar method
 191 to [10], where Numerical Analysis Group (NAG)¹ routine E04UCF² was
 192 employed to find the maximum of the objective function. This algorithm
 193 searches for the minimum value of the objective function using a sequential
 194 quadratic programming method. This algorithm is essentially identical to
 195 the subroutine NPSOL described in [12]. Appropriate NAG routines were
 196 also employed for the calculation of Bessel functions and quadrature. Motion

¹<http://www.nag.co.uk>

²<https://www.nag.co.uk/numeric/fl/manual/pdf/E04/e04ucf.pdf>

constraints of $\delta_{max} = 2$ and $\delta_{max} = 3$ are examined.

The optimisation routine E04UCF requires a starting point to perform the optimisation. Therefore, in order to ensure a global optimum is found for a given problem, an exhaustive search of the space of starting points must be performed. For an array of five WECs, all possible combinations of $n_l \in \{0.1, 0.2, \dots, 0.7\} \cup \psi_j \in \{0, \frac{\pi}{2}, \pi\} \cup \delta_j \in \{-3, -1, 1, 3\}$ for $l = 1, \dots, 4$ and $j = 1, \dots, 5$ are examined for $\delta \leq 3$. For the lower constraint $\delta \leq 2$, the set of starting points for WEC motion amplitude was taken to be $\delta_j \in \{-2, 0, 2\}$, with starting points for the other variables unchanged. It was found that the optimisation behaved quite well with respect to the starting values of δ_j and ψ_j , as the optimisation converged quickly and repeatedly to the same optimal solution, hence the relatively sparse sampling of starting points of these variables.

3. Constrained Optimisation Results

3.1. Comparison with Unconstrained Optimal Layout

The constrained performance of the optimal formation of an array of five devices in a linear geometry (previously identified in [10]) is now examined. With the optimal spacing denoted by $\mathbf{n}^*$, the array is subject to the direction of the incident waves. As the layout is prescribed prior to constrained optimisation, there are ten variables for $N = 5$ devices, namely the amplitudes δ_j and phases ψ_j of the displacements of each WEC. The objective function is given by (8) with $\mathbf{n} = \mathbf{n}^*$ fixed. This optimisation was performed for $\beta_0 = 0, \frac{\pi}{4}, \frac{\pi}{2}$.

Table 1 lists the optimal layouts $\mathbf{n}^*$ from the unconstrained optimisation

Table 1: Performance of optimal layouts from [10] subject to motion constraints

β_0	n_1^*	n_2^*	n_3^*	n_4^*	$I_{opt}(\delta \leq \infty)$	$\bar{I}_{opt}(\delta \leq 3)$	$\bar{I}_{opt}(\delta \leq 2)$
0	0.0500	0.0500	0.0500	0.8500	1.4802	0.5469	0.4691
$\frac{\pi}{4}$	0.0500	0.8500	0.0500	0.0500	1.1431	0.3070	0.2624
$\frac{\pi}{2}$	0.0500	0.2252	0.3859	0.3359	1.3643	0.9486	0.7693

in [10], along with the performance of these arrays in the unconstrained case (denoted by I_{opt}) and when a WEC motion constraint of $\delta \leq 2$ or $\delta \leq 3$ is enforced (denoted by $\bar{I}_{opt}$). The values of the displacement variables δ_j and ψ_j are listed in table 2. The computation time for each case examined in this section was of the order of ten minutes. This was due to the exhaustive search and optimisation routines scanning over ten variables.

As expected, performance is poorer when constraints are applied, with the lower constraint having a greater impact. For the $\beta_0 = 0, \frac{\pi}{4}$ cases, the application of constraints causes a reduction in performance of at least 63%, with only a relatively small difference between $\delta \leq 2$ and $\delta \leq 3$. This is most likely due to the presence of grouped devices in these layouts and the associated large motions for the unconstrained optimum. Since the optimal motions are predicted to be $\mathcal{O}(100) - \mathcal{O}(1000)$ from [10], it is anticipated that limiting the motions to $\mathcal{O}(1)$ would have a large effect on array performance. This also explains the relatively small difference between the two constraints, as the relative difference between $\delta = 2$ or 3 and $\delta = \mathcal{O}(100) - \mathcal{O}(1000)$ is also small.

The application of constraints seems to have a smaller impact on the $\beta_0 = \frac{\pi}{2}$ array. This is probably due to the larger spacing between most of the

Table 2: Optimal WEC displacement parameters for optimal layouts from [10] subject to constraints

β_0	δ_{max}	δ_1	δ_2	δ_3	δ_4	δ_5	ψ_1	ψ_2	ψ_3	ψ_4	ψ_5
0	2	-2.0000	-2.0000	2.0000	2.0000	-0.5326	0.8933	1.7679	0.0542	1.0235	2.5317
	3	-3.0000	-3.0000	-3.0000	3.0000	-0.5174	0.5977	1.8731	3.1236	1.3434	2.6386
$\frac{\pi}{4}$	2	-2.0000	-2.0000	-1.2500	2.0000	-1.4587	1.1384	2.6780	1.5186	0.8910	0.1221
	3	-3.0000	-3.0000	-1.6884	3.0000	-1.9200	0.8875	2.9290	1.3618	0.8832	0.3028
$\frac{\pi}{2}$	2	-2.0000	-1.1760	-2.0000	-2.0000	-2.0000	1.7266	1.7266	1.7266	1.7266	1.7266
	3	-3.0000	-0.1103	-3.0000	-3.0000	-3.0000	1.7266	1.7266	1.7266	1.7266	1.7266

240 devices in this layout and the smaller associated motions. The application
 241 of the $\delta \leq 3$ and $\delta \leq 2$ constraints results in performance losses of approxi-
 242 mately 31% and 44% respectively. It does not appear to be possible for these
 243 fixed layouts to maintain average constructive interference ($\bar{I} > 1$) after the
 244 application of constraints, although moderate performance of $\bar{I} = 0.94859$,
 245 albeit slightly destructive, is achieved for $\beta_0 = \frac{\pi}{2}$ with $\delta \leq 3$.

246 Table 2 shows that, overall, the majority of the amplitude variables δ_j
 247 converged to the enforced limit of 2 or 3. It should be noted that all op-
 248 timal arrays resulted in one or two of the δ_j values not converging to the
 249 limit but instead to some value in the centre of the allowed range. This
 250 indicates that within the constrained problem, the best solution does not
 251 result from simply setting all device amplitudes to their largest permissible
 252 values. The optimal constrained case appears to be when one or two WECs
 253 oscillate at a smaller amplitude with the appropriate choice of phase. This
 254 could be an artifact of forcing the WECs to be arranged in a layout which

255 was optimised for optimal unconstrained motions. In general, the phases of
 256 each WEC displacements are all different within each optimal solution found,
 257 with the obvious exception of the $\beta_0 = \frac{\pi}{2}$ array. For both constraints applied,
 258 all the WEC phases were equal in the optimal beam seas arrays.

259 A more detailed analysis of the constrained performance of these arrays
 260 is given in section 3.2, where the array layout is allowed to vary within a
 261 constrained optimisation. The performance of the array layouts previously
 262 identified as optimal in the unconstrained optimisation are then compared
 263 to the performance of the arrays where the WEC positions are not fixed and
 264 are also fed into the optimisation as variables.

265 3.2. Undetermined Layout

266 The performance of linear arrays is now optimised without a prescribed
 267 layout, so that the array formation and the device displacements are variables
 268 of the optimisation, giving a total of $3N - 2 = 13$ variables for $N = 5$ WECs.
 269 This is performed for two different maximum displacement constraints of
 270 $\delta \leq 2, 3$ and the three values of prescribed incident wave angle $\beta_0 = 0, \frac{\pi}{4}, \frac{\pi}{2}$.
 271 The results of the optimisations are listed in table 3 and the optimal values of
 272 δ_j and ψ_j are listed in table 4. The optimal constrained layouts are denoted
 273 as $\mathbf{n}_{opt}$. In this section, the computation times for each case examined was
 274 of the order of one hour. The increase in computation time was due to the
 275 exhaustive search and optimisation routines scanning over 13 variables, three
 276 more variables than the optimisation in section 3.1.

277 As in the procedure employed in [10], minimum and maximum values
 278 of each separation parameter were enforced within the optimisation so that
 279 $0.05 \leq n_l \leq 0.85$ for $l = 1, \dots, 4$. This ensures that no device will be

Table 3: Optimal linear array layout parameters subject to motion constraints

β_0	δ_{max}	$n_{opt,1}$	$n_{opt,2}$	$n_{opt,3}$	$n_{opt,4}$	$\bar{I}_{opt}$
0	2	0.0978	0.0532	0.1139	0.7351	0.49441
	3	0.1057	0.0504	0.1048	0.7391	0.58438
$\frac{\pi}{4}$	2	0.0940	0.1532	0.2259	0.5269	0.42508
	3	0.1310	0.3066	0.1103	0.4521	0.45507
$\frac{\pi}{2}$	2	0.2679	0.2321	0.2321	0.2679	0.87771
	3	0.2679	0.2321	0.2321	0.2679	1.06779

Table 4: Optimal WEC displacement parameters for constrained optimal layouts in table

3

β_0	δ_{max}	δ_1	δ_2	δ_3	δ_4	δ_5	ψ_1	ψ_2	ψ_3	ψ_4	ψ_5
0	2	-2.0000	-2.0000	2.0000	2.0000	-0.5044	0.9841	2.6244	0.3064	2.0858	2.5309
	3	-3.0000	-3.0000	3.0000	3.0000	-0.4828	0.7737	2.6660	0.3878	2.4455	2.5965
$\frac{\pi}{4}$	2	-2.0000	-2.0000	2.0000	1.5094	0.5083	1.1987	2.6658	0.5846	1.9964	2.0989
	3	-3.0000	3.0000	2.4695	-1.7879	0.5566	1.2333	0.1285	1.0368	0.2747	2.4616
$\frac{\pi}{2}$	2	-2.0000	-2.0000	-2.0000	-2.0000	-2.0000	1.7266	1.7266	1.7266	1.7266	1.7266
	3	-3.0000	-3.0000	-3.0000	-3.0000	-3.0000	1.7266	1.7266	1.7266	1.7266	1.7266

within 5% of the total array length of another device. The upper bound of 0.85 was chosen to allow the possibility that all but one of the separations was exactly the minimum bound. A 5% minimum constraint was chosen as this value also avoided possible difficulties due to numerical inaccuracies and poor behaviour of the objective function caused by small non-dimensional separation arguments. It is also a physically reasonable lower bound on WEC separation distances.

The unconstrained optimal layout $\mathbf{n}^*$ and the constrained optimal layouts $\mathbf{n}_{opt}$ are presented for each case of $\beta_0 = 0, \frac{\pi}{4}$ and $\frac{\pi}{2}$; the performance of the arrays are also analysed for variation in kL and β respectively. There are five curves in each $\bar{q}$ plot for each value of β_0 and these are intended to show the performance of the unconstrained optimal array $q(\mathbf{n}^*)$, the constrained arrays with the unconstrained optimal layout $\bar{q}(\mathbf{n}^*)$ for both $\delta \leq 2$ & $\delta \leq 3$ and the optimal constrained arrays with re-optimised layouts $\bar{q}(\mathbf{n}_{opt})$ for both $\delta \leq 2$ and $\delta \leq 3$.

It is anticipated that each constrained array would perform poorer than the unconstrained equivalent and it is also expected that

$$\bar{I}(\mathbf{n}_{opt}, \delta \leq 3) > \bar{I}(\mathbf{n}^*, \delta \leq 3) > \bar{I}(\mathbf{n}_{opt}, \delta \leq 2) > \bar{I}(\mathbf{n}^*, \delta \leq 2). \quad (10)$$

However, it is unclear how sharp the inequalities will be, i.e how close to equality they can become. It is only by consideration of the individual cases that this information can be obtained.

Similar conclusions to the previous section can be drawn from table 4, where the majority of δ_j values converge to the limit of δ_{max} imposed. In head and intermediate seas, one or two δ_j did not converge to the maximum allowed value and all WECs have different phases ψ_j . However, in beam seas,

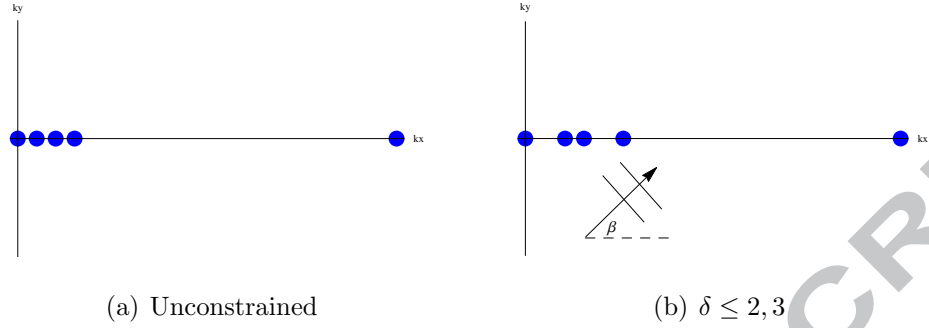


Figure 1: Constrained and unconstrained optimal linear arrays for $\beta_0 = 0$. The optimal layout for the $\delta \leq 3$ case is very similar to the $\delta \leq 2$ case and is omitted for clarity

304 $\delta_j = \delta_{max}$ and the phases are equal for all WECs, as would be expected. This
 305 is unlike the results in table 2, since in this case the WECs were optimised
 306 for constrained motions. Thus for beam seas, where the wave hits all WECs
 307 at the same time, the array is contrived such that the displacement limit is
 308 reached for all WECs, thereby maximising power capture.

309 3.2.1. Head Seas

310 Figure 1 shows the unconstrained and constrained optimal layouts for
 311 $\beta_0 = 0$. The constrained array layouts are quite similar for $\delta \leq 2$ and $\delta \leq 3$
 312 and so the lower value is not shown. The unconstrained and constrained
 313 arrays all have four devices grouped to the left of the array, but, and perhaps
 314 surprisingly, these are more separated for the constrained layouts. Note that
 315 WECs 2 and 3 are still placed very close together, which may still cause some
 316 physical issues such as shadowing and possible collisions.

317 From figure 2, the overall behaviour of the constrained arrays is similar
 318 to the unconstrained array, in that there is small variation throughout $kL \in$
 319 $[5, 15]$. However, a considerable reduction in performance is caused by the

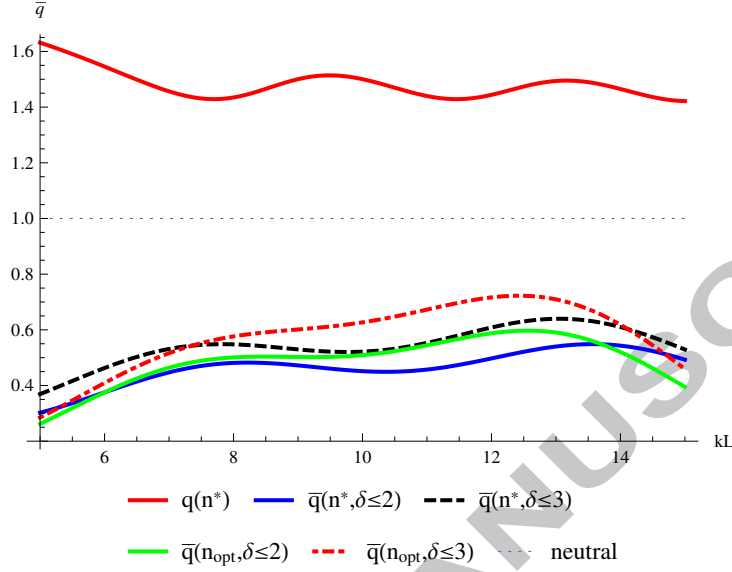


Figure 2: Performance of constrained and unconstrained linear arrays for variation in kL with $\beta = \beta_0 = 0$

application of constraints, as also indicated by utilisation and comparison of tables 1 and 3. Figure 3 shows that the constrained arrays have the advantage of a much broader peak performance in β -variation than the unconstrained array, although the peak is much lower. The unconstrained array has a range where $q > 1$ of approximately $\pm \frac{\pi}{8}$, while the constrained arrays have a larger range of $\pm \frac{\pi}{4}$ where $\bar{q} \approx 0.5$. This coupled with the low variation of $\bar{q}$ with kL suggests a large stability of performance for these constrained arrays in this case, although the performance achieved is rather poor in comparison to the same number of isolated devices. It should also be noted from figure 3 that the $\bar{q}$ values become negative outside a certain range, indicating that the constrained power absorbed by the array is negative in this case and the WECs are injecting power into the waves rather than absorbing power.

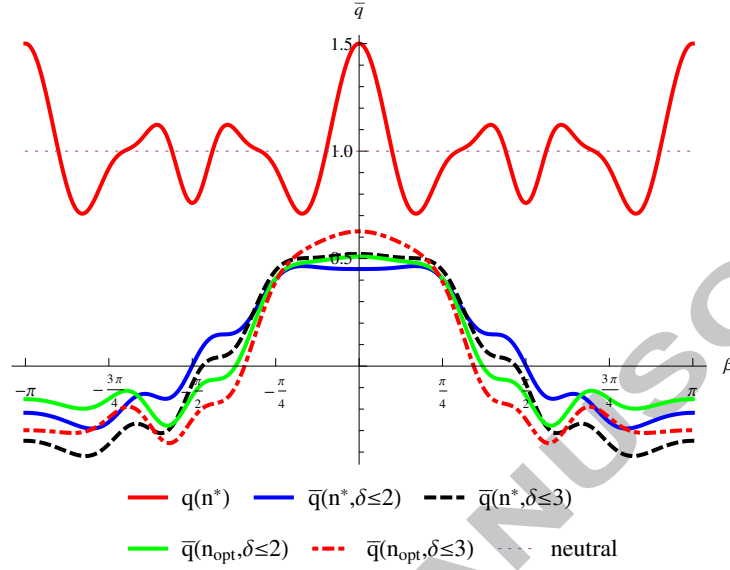


Figure 3: Performance of constrained and unconstrained linear arrays for variation in β with $\beta_0 = 0$ and $kL = 10$.

3.2.2. Intermediate Seas

Figure 4 shows the unconstrained and constrained optimal layouts for $\beta_0 = \frac{\pi}{4}$. As for head seas, the optimal constrained arrays are more separated in comparison to the unconstrained optimal layout. However, in this case, the optimal layouts corresponding to $\delta \leq 2$ and $\delta \leq 3$ differ. In both constrained cases, WEC 5 is relatively isolated at the right of the array. For the $\delta \leq 2$ array, WECs 1-4 have an increasing separation between them, with the smallest separation between WECs 1 and 2 being 9.4% of the total length. In contrast, the $\delta \leq 3$ array has two pairs of devices approximately $0.11kL - 0.13kL$ apart, with the distance between the pairs being approximately $0.3kL$.

Figures 5 and 6 show the performance of the constrained arrays, along

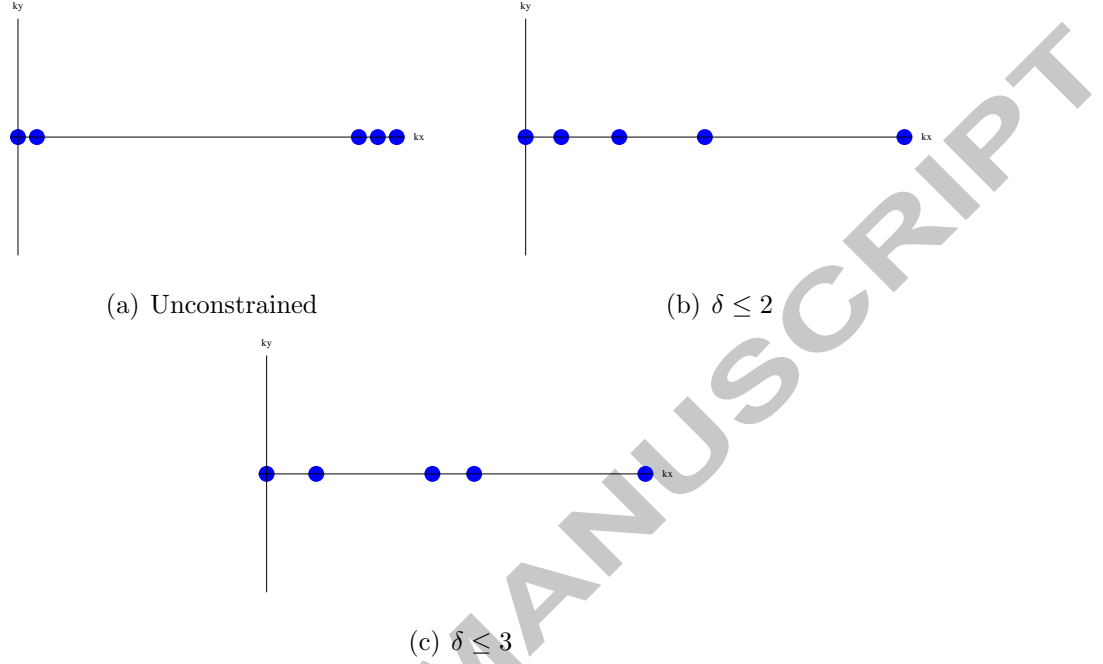


Figure 4: Constrained and Unconstrained Optimal linear arrays for $\beta_0 = \frac{\pi}{4}$.

with the unconstrained case, with kL and β variation respectively for $\beta_0 = \frac{\pi}{4}$. Similar to the head seas case, the application of amplitude constraints has a considerable influence on the array performance, with an overall reduction from $q \in [0.9, 1.3]$ to $\bar{q} \in [0.1, 0.6]$ for the kL variation. This is most likely due to the presence of closely spaced groups of WECs and associated large motions in the optimal unconstrained case.

The expected trend of $\delta \leq 3$ outperforming $\delta \leq 2$ is not evident in this case, as it is clear from figure 5 that $\bar{q}(\mathbf{n}_{opt}, \delta \leq 2) > \bar{q}(\mathbf{n}^*, \delta \leq 3)$. This is most likely because the optimal array layout is considerably different when constraints are applied. Therefore, applying constraints to the unconstrained

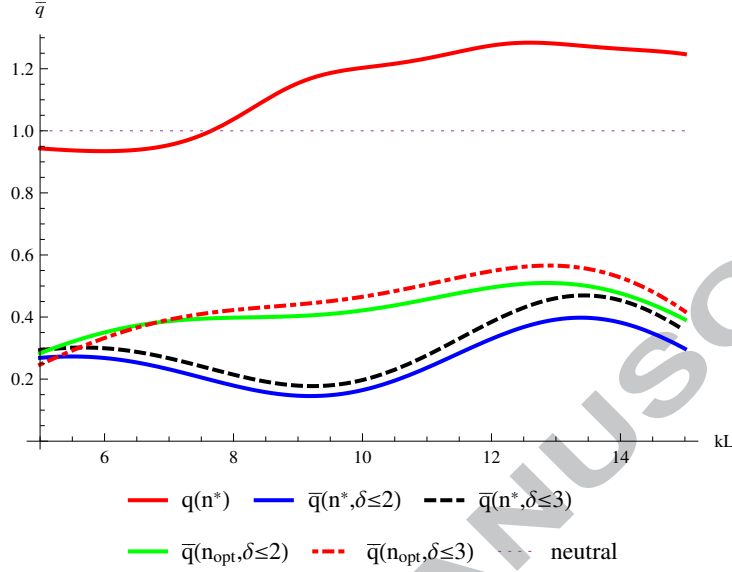


Figure 5: Performance of constrained and unconstrained linear arrays for variation in kL with $\beta = \beta_0 = \frac{\pi}{4}$

354 optimal layout results in very poor performance. This figure also shows that

$$\bar{I}(\mathbf{n}_{opt}, \delta \leq 3) > \bar{I}(\mathbf{n}_{opt}, \delta \leq 2) > \bar{I}(\mathbf{n}^*, \delta \leq 3) > \bar{I}(\mathbf{n}^*, \delta \leq 2), \quad (11)$$

355 in contrast with expectation (10) and the results in head seas.

356 As with head seas, the constrained array performance varies relatively
 357 slowly with kL . This indicates that the performance of the array is relatively
 358 stable to changes in kL , although a large reduction in interaction factor is
 359 again seen when constraints are imposed. Examination of figure 6 shows a
 360 similar behaviour to head seas, where a broader performance with respect
 361 to β is achieved around $\beta = 0$. This is not beneficial in this case, as the
 362 target wave angle is $\beta_0 = \frac{\pi}{4}$, around which are significant variations in $\bar{q}$.
 363 This is particularly evident for $|\beta| > \frac{\pi}{4}$, where the $\mathbf{n}_{opt}$ arrays give $\bar{q} < 0$

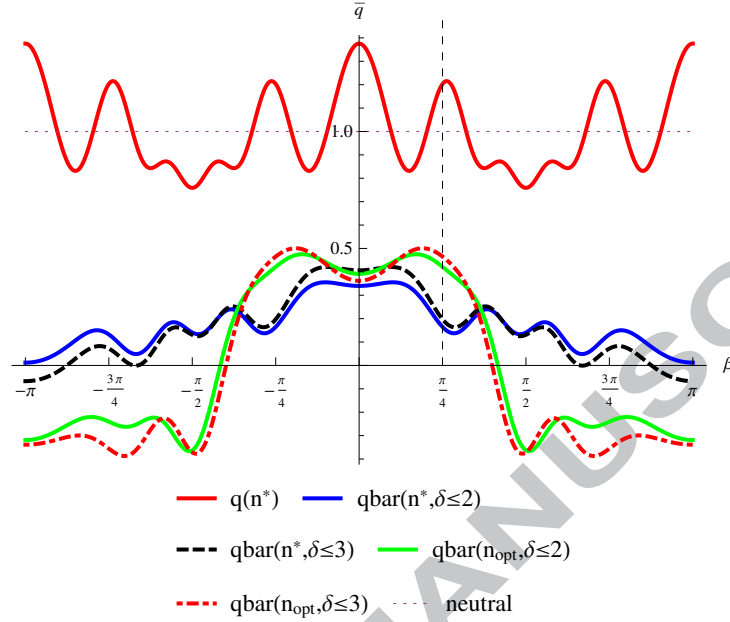
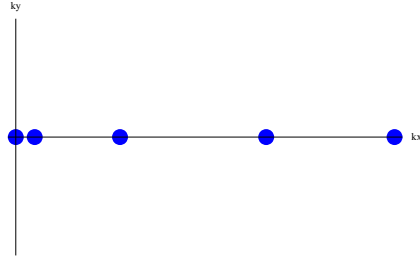


Figure 6: Performance of constrained and unconstrained linear arrays for variation in β with $\beta_0 = \frac{\pi}{4}$ and $kL = 10$. The target wave angle $\beta_0 = \frac{\pi}{4}$ is shown by the vertical dashed line.

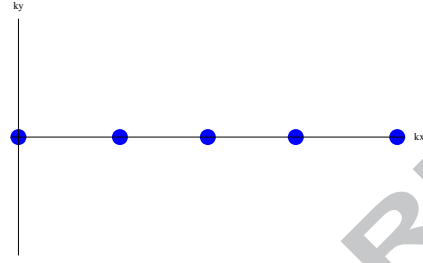
for $|\beta| > \frac{3\pi}{8}$. The $\mathbf{n}^*$ arrays are slightly more stable around the target wave angle, although the performance is not as high as the $\mathbf{n}_{opt}$ arrays.

3.2.3. Beam Seas

Figure 7 shows the optimal constrained and unconstrained array layouts for beam seas. The optimal layout in both constrained cases is very close to a uniform array. This reinforces the intuitive idea that constrained arrays tend to have their optimal layouts more spaced apart, avoiding groups of WECs, with the exception of the head seas. It is also consistent with the idea that greater frontage to the waves gives greater power absorption, since an array with greater frontage to the incident wave has a greater amount of



(a) Unconstrained



(b) $\delta \leq 2, 3$

Figure 7: Constrained and Unconstrained Optimal linear arrays for $\beta_0 = \frac{\pi}{2}$. The optimal layout for $\delta \leq 3$ is identical to the $\delta \leq 2$ case

374 wave-power incident upon it. However, as shown in previous studies, this
 375 does not always translate into increased power absorption of better WEC
 376 interference. The $\bar{I} > 1$ property is achieved for the $\delta \leq 3$ constraint at this
 377 wave angle; this is the only case where average constructive interference is
 378 maintained after the application of constraints.

379 The performance of the arrays for beam seas are shown in figures 8 and 9
 380 for variation in kL and β respectively. Both figures show that the application
 381 of constraints does not have as severe a negative impact on $\bar{q}$ in comparison
 382 with other wave angles. A loss is seen for the $\bar{q}$ values in compared to q , but
 383 constructive interference is still achieved in some cases. As with the $\beta_0 = 0$
 384 case, a constraint of $\delta \leq 2$ has a greater impact on performance than $\delta \leq 3$;
 385 within this pattern, the $\mathbf{n}_{opt}$ arrays perform better than the $\mathbf{n}^*$ layouts, so
 386 that (10) holds true, as expected.

387 As in the previous two configurations, figure 8 shows the slow variation of
 388 $\bar{q}$ with kL , indicating that a small change in kL produces only a small change
 389 in array performance. In general, this figure shows that better performance
 390 is achieved for larger values of kL within the domain examined. Constructive

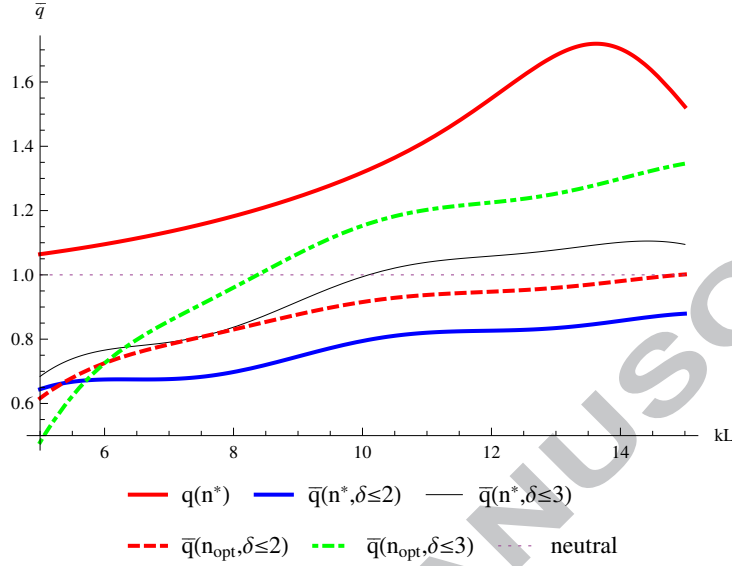


Figure 8: Performance of constrained and unconstrained linear arrays for variation in kL with $\beta = \beta_0 = \frac{\pi}{2}$

interference $\bar{q} > 1$ is achieved for the $\delta \leq 3$ arrays, while the best case for $\delta \leq 2$ is $\bar{q} \approx 1$ at $kL = 15$ for the $\mathbf{n}_{opt}$ layout. Both configurations with $\delta \leq 2$ resulted in $\bar{q} \leq 1$. The fact that $\bar{q}(\mathbf{n}_{opt}, \delta \leq 3) \approx 1.2$ for $kL \in [10, 15]$ is promising, as this indicates that constructive interference is still possible after the imposition of a reasonable constraint. This layout is also almost uniformly-spaced and so avoids the difficulties associated with closely spaced devices.

The β -variation of the array performances are shown in figure 9. Contrary to head and intermediate seas, the imposition of constraints result in a narrower peak performance around $\beta = \beta_0 = \frac{\pi}{2}$ compared to the unconstrained case. A high peak value is achieved with $\max[\bar{q}] \in [0.8, 1.2]$ depending on the constraint and layout but the peak is significantly narrower. This results in

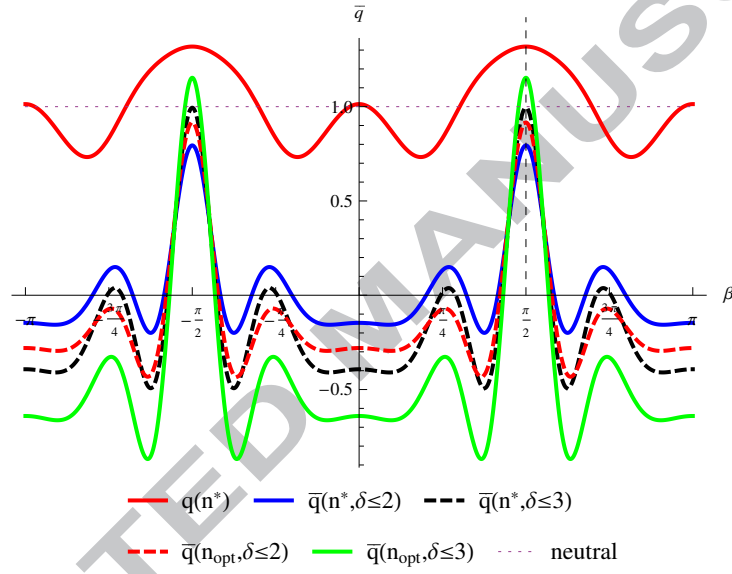


Figure 9: Performance of constrained and unconstrained linear arrays for variation in β with $\beta_0 = \frac{\pi}{2}$ and $kL = 10$. The target incident wave angle $\beta_0 = \frac{\pi}{2}$ is shown by the vertical dashed line.

403 $\bar{q} < 0$ for a relatively small change of $\beta_0 \pm \frac{\pi}{12}$, which may be undesirable and
404 is highly dependent upon the angular variation within the incident wavefield.

405 The results of figure 8 can be compared with the work on constrained
406 motion performance of the uniform array in [3] (figure 6). In both cases,
407 the constrained array examined is almost identical in geometry, since the
408 constrained array presented here ($\mathbf{n}_{opt}$) for beam seas is almost uniform.
409 Note in [3] that the quantity examined is the absorption length scaled by the
410 total WEC covering in the array $\frac{l_{abs}}{10a}$. Note also that this quantity is assessed
411 with respect to variation in the device spacing kd , not the array length kL .
412 Agreement is seen, however, in the overall performance of the array with
413 respect to the application of constraints, i.e. an application of a constraint of
414 three times the wave amplitude still allows for constructive interference, while
415 a constraint of twice the wave amplitude is severely limiting and results in
416 destructive interference dominating.

417 4. Discussion and Conclusion

418 This paper extends the work of [9] and [10] to linear arrays where the
419 WECs are constrained to oscillate at no more than two or three times the in-
420 cident wave amplitude. This is necessary as nearly all unconstrained optimal
421 arrays in these works resulted in predicted optimal displacement amplitudes
422 well in excess of the incident wave amplitude. Such large displacements
423 would not only cause significant physical and engineering difficulties but also
424 violate the underlying linear wave theory, which assumes WEC motions are
425 at most the same order of magnitude as the wave motions and are assumed
426 small in some sense. Therefore, an investigation of placing constraints on

WEC motions is necessary to add validity to the results and conclusions of previous studies in unconstrained regimes.

It should be noted that all models of the type implemented within this work inherently overestimate the the actual power absorption of a WEC. This model considers the hydrodynamic power absorbed by the device, so a PTO is not directly implemented within this work. If a PTO term was included in the equation of motion and the power absorbed calculated from this term alone, then this term would absorb a fraction of the total hydrodynamic power. However, this would result in more intensive calculations and impede a numerical optimisation of the type performed in this preliminary work.

The imposition of constraints has been shown to have a significant impact on array performance, particularly when optimal performance was accompanied by by very large device motions. In previous studies, the impression of good performance was given by the large values of optimal interaction factor q achieved. However, these were accompanied in most cases by unacceptably large device motions. Therefore, the application of constraints was expected to have a large negative impact on array performance. This was particularly true in those cases with groups of closely spaced devices, which were associated with the largest predicted optimal motions.

This effect is most clearly seen by comparing the results of head seas and beam seas in figures 1 and 7. The $\beta_0 = 0$ unconstrained optimal layout from [10] contained a group of four devices and predicted very unrealistic motions of the order of 1000 times the wave amplitude. When constraints are applied, the array performance is reduced by approximately 60% and

452 resulted in the domination of destructive interference ($\bar{q} < 1$). In contrast,
 453 the $\beta_0 = \frac{\pi}{2}$ unconstrained optimal array was more spaced, although still
 454 contained a closely spaced pair of WECs. The application of constraints
 455 here resulted in a smaller performance reduction of approximately 30% (for
 456 $\delta \leq 3$) and allowed the possibility of constructive interference ($\bar{q} > 1$).

457 When the array layout parameters were added as optimisation variables,
 458 noticeably different layouts were obtained in comparison to the unconstrained
 459 optimisation (i.e. $\mathbf{n}^* \neq \mathbf{n}_{opt}$). This resulted in a more separated layout in
 460 each case, which reduced the number closely-spaced WECs within the array
 461 or eliminated these groups of WECs altogether. For $\beta_0 = 0$, the constrained
 462 optimal layout separated the group of four devices slightly but still retained
 463 a closely spaced pair. This was very similar for both $\delta \leq 2$ and $\delta \leq 3$. In
 464 the intermediate case of $\beta_0 = \frac{\pi}{4}$, no closely spaced devices remained in the
 465 constrained optimal layouts. Most notably, the different constraints resulted
 466 in significantly different optimal layouts for this wave angle. A symmetric
 467 and almost uniform layout was found to be optimal when the constraints
 468 were applied in the beam seas case, with the same layout found for both
 469 $\delta \leq 2$ and $\delta \leq 3$. This optimisation eliminated the pair of closely spaced
 470 devices on the left of the unconstrained optimal layout for this wave angle.
 471 This was also the best performing constrained array with the largest $\bar{I}$ for
 472 both constraints, with mean constructive interference ($\bar{I} > 1$) maintained for
 473 the $\delta \leq 3$ constraint. The fact that the optimal array layout changes with
 474 the constraint imposed agrees with the result of [13], which shows that the
 475 control problem is related to the array layout problem.

476 Although both constraints considered were within the $\mathcal{O}(1)$ regime nec-

477 essary, the $\delta \leq 2$ constraint had a more severe impact on array performance;
 478 this was not unexpected. In general, the arrays with the $\delta \leq 3$ constraint
 479 applied performed better than the $\delta \leq 2$ arrays, with varying differences
 480 between these depending on the wave angle and layout considered. Previous
 481 studies, such as [3] and [6], have discussed how the imposition of a constraint
 482 of three times the wave amplitude still allows for constructive interference in
 483 some cases, while a constraint of two times the wave amplitude is severely re-
 484 strictive. This idea is echoed here, where $\delta \leq 2$ had a greater negative impact
 485 on all arrays considered, while constructive interference was still possible in
 486 some cases for $\delta \leq 3$.

487 It would be reasonable to argue that the best linear array presented herein
 488 was the almost uniform layout found for $\beta_0 = \frac{\pi}{2}$. This array had the greatest
 489 overall performance with constraints imposed, by a considerable margin. The
 490 array was widely spaced and symmetric and thus avoided issues of closely
 491 spaced WECs. Most importantly, mean constructive interference was possi-
 492 ble for the larger constraint and stable performance with respect to changes
 493 in kL was also observed. However, the array was very sensitive to changes
 494 in incident wave angle. Moving away from the target wave angle by $\pm \frac{\pi}{12}$
 495 resulted not only in destructive interference, but also $\bar{q} < 0$, indicating that
 496 the array is adding power to the waves rather than extracting it.

497 It is often envisaged that WECs should be placed in large arrays or small
 498 arrays. In principle, the method presented in this paper can be applied to
 499 larger arrays of more than five WECs. However, the main issue with this
 500 is the increase in optimisation variables and the associated increase in com-
 501 putation time; this phenomenon is sometimes called "parameter explosion".

502 The optimisation must scan over the possible starting point space of all vari-
 503 ables, and this scan must be fine enough to ensure that the global optimum
 504 is reliably and repeatedly found. The computation times become prohibitive
 505 for arrays of larger numbers of WECs. For example, an array of ten WECs
 506 would have twenty displacement variables and eight position variables, giving
 507 a total of 28 variables for a constrained layout optimisation. It is estimated
 508 that in order to conduct a sufficient scan of the starting point space in this
 509 case, the optimisation would take of the order of 50-100 hours on one stan-
 510 dard machine. Thus the present study is limited to arrays of five WECs, as
 511 this also allows comparison with previous research such as [3, 6, 8, 9, 10].

512 The results presented here show that a trade-off is made either in overall
 513 performance of the array or in the sensitivity of the optimal array. When
 514 examining the β plot for the head seas case in figure 3, it is clear that the
 515 imposition of constraints widens the peak performance of the $\bar{q}$ vs β curve
 516 compared to the unconstrained case, although the overall performance is
 517 severely reduced. However, the opposite is seen for beam seas in figure 9,
 518 where decent performance is maintained under the imposition of constraints
 519 but the peak performance is significantly narrowed, thus severely increasing
 520 the sensitivity of the array to changes in the incident wave angle. Within
 521 the current analysis, it does not seem to be possible to have an array un-
 522 der motion constraints that both performs well and is stable to parameter
 523 changes.

524 Future work should include a more detailed investigation of this trade-off.
 525 One possible method to combat this issue would be to consider the objective
 526 function as the mean performance over the incident wave angle, rather than

a non-dimensional length. This is motivated by the greater effect that β has on the optimal array formation in comparison to changes in kL . This formation of the objective function would also allow for a generalised 2-D array layout optimisation, since no array geometry need be imposed. This will be considered in part 2 of this paper.

Acknowledgments

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- The constrained optimisation of linear arrays of heaving point absorber WECs is considered.
- Previous research is extended by constraining the WEC motion amplitudes to two or three times the incident wave amplitude.
- The objective function of the optimisation is taken to be the mean performance of the array, with respect to isolated devices, over a range of non-dimensional array length. This is defined using the averaged interaction factor.
- The results of the constrained optimisation are compared with previous results in an unconstrained regime.
- It found that the optimal constrained layouts are more separated than the unconstrained cases. Most notably, an almost uniform layout is found to be optimal for beam seas.