Supplementary Material: Nonstationary Signal Decomposition using Quadratic Time–Frequency Distributions

J.M. O'Toole and N.J. Stevenson

Table S1

Parameters for the cross time–frequency distribution (cross-TFD) and TV (time-varying) filtering method for different test signals, expressed in terms of signal length N. Kernel parameters of the $N \times N$ TFD include the length of the lag and Doppler kernels. The Doppler kernel is a Hamming window and lag kernel is a Chebyshev window with 100 dB sidelobe attenuation. The cross-TFD method oversamples the TFD in the frequency direction for the two-tone (two sinusoids) signal, as $N_{\text{freq}} > N$; for the time-varying (TV) filtering method $N_{\text{freq}} = N$ for all signals. Also included are 2 parameters associated with setting a lower-limit of a valid IF component for both methods: minimum duration (L_{min}) and minimum rate of change (discrete-frequency difference for 1 discrete-time sample, R_{min}).

		cross-TFD			TV filter		IF tracks	
signal	N	N_{freq}	lag	Doppler	lag	Doppler	L_{min}	R_{min}
two sinusoids [a]	1024	$8N$	$2\sqrt{N}$	$16\sqrt{N}$	$2\sqrt{N}$	$\sqrt{N}$	$4\sqrt{N}$	$\sqrt{N}/2$
bat signal	400	N	$2\sqrt{N}$	$4\sqrt{N}$	$2\sqrt{N}$	$4\sqrt{N}$	$4\sqrt{N}$	$\sqrt{N}/2$
multi-component nonlinear signal	256	N	$6\sqrt{N}$	$4\sqrt{N}$	$6\sqrt{N}$	$4\sqrt{N}$	$4\sqrt{N}$	$\sqrt{N}/2$
Gaussian noise[b]	128	N	$3\sqrt{N}$	$6\sqrt{N}$	$3\sqrt{N}$	$6\sqrt{N}$	$\sqrt{2}\sqrt{N}$	$\sqrt{N}/2$

[a] for both the time-disjoint and time-overlapping versions
[b] for both white and fractional Gaussian noise signals

Table S2

Estimation of signal components for multiple methods with three test signals: a bat echolocation signal (bat), a multicomponent nonlinear signal (nonlinear) with additive noise, and fractional Gaussian noise. Correlation and relative-error are averaged over the 3 components for the nonlinear test signal.

	correlation			relative error		
	bat	nonlinear	noise	bat	nonlinear	noise
TV filtering (ours)	**1.000**	0.869	**1.000**	**0.000**	0.579	**0.000**
cross-TFD (ours)	0.997	**0.968**	0.890	0.099	**0.248**	0.460
EFD	0.946	0.003	0.853	0.395	1.000	0.893
TV-EMD	**1.000**	0.271	**1.000**	**0.000**	0.975	**0.000**
SSFT	0.944	0.896	0.966	0.487	0.517	0.437
VMD	0.999	0.677	0.982	0.032	1.180	0.191
VN-CMD	0.719	0.630	0.751	0.696	0.679	0.660

Key: TV, time varying; EFD, empirical Fourier transform; EMD, empirical mode decomposition; TFD, time–frequency distribution; VMD, variational mode decomposition; SSFT, synchrosqueeze short-time Fourier transform; VN-CMD, variational nonlinear chirp mode decomposition

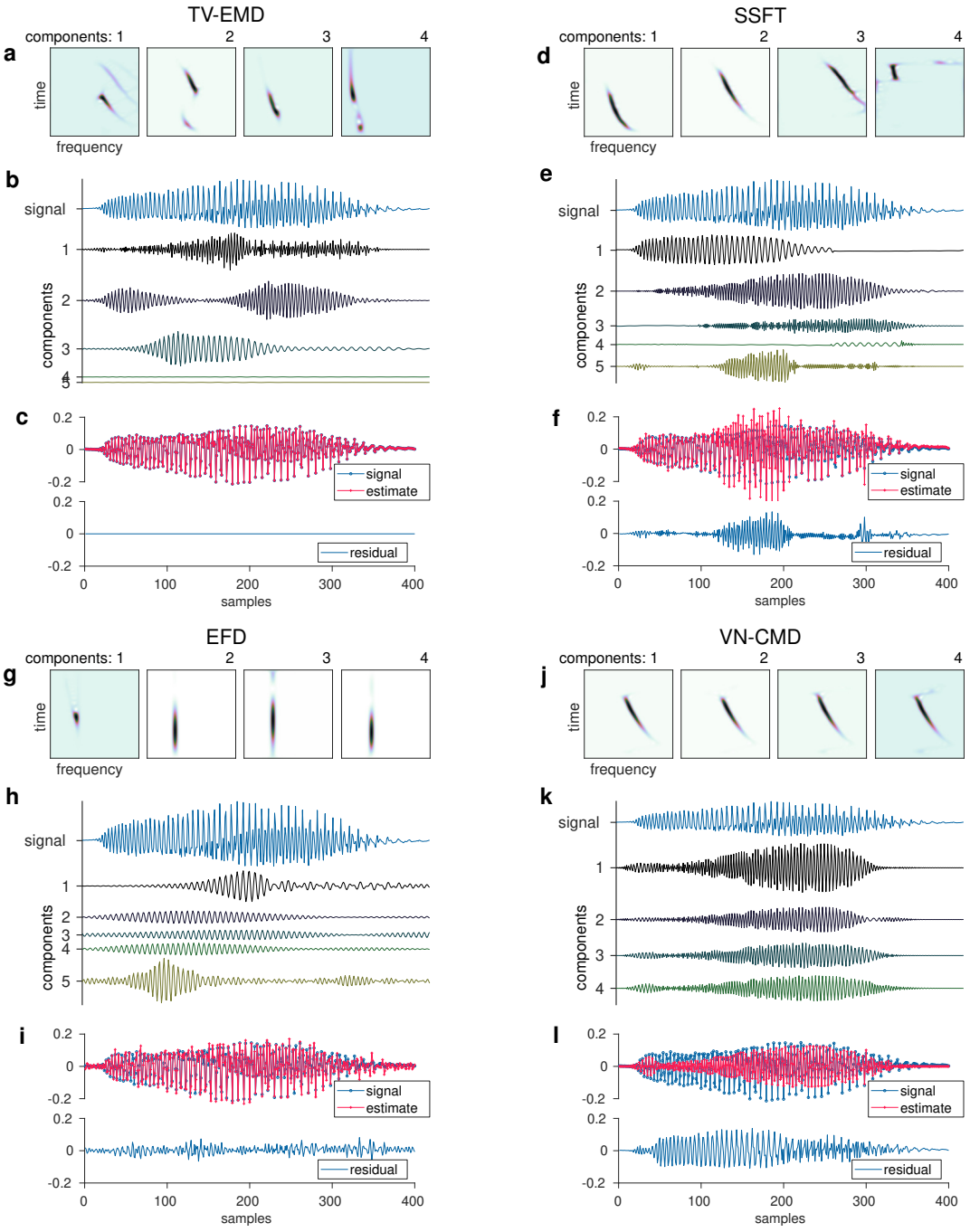

Figure S1: Signal decomposition with bat signal. Methods: top left, time-varying empirical mode decomposition (TV-EMD) in (**a**–**c**); top right: sychrosqueeze short-time Fourier transform (SSFT) in (**d**–**f**); bottom left, empirical Fourier decomposition (EFD) in (**g**–**i**); and bottom right, variational nonlinear chirp mode decomposition (VN-CMD) in (**j**–**l**). Time–frequency representation of the first 4 components in (**a**), (**d**), (**g**), and (**j**). Amplitude of components are plotted relative to signal amplitude for (**a**), (**c**), and (**e**). Difference (error) between sum of all components and signal in (**c**), (**f**), (**i**), and (**l**).

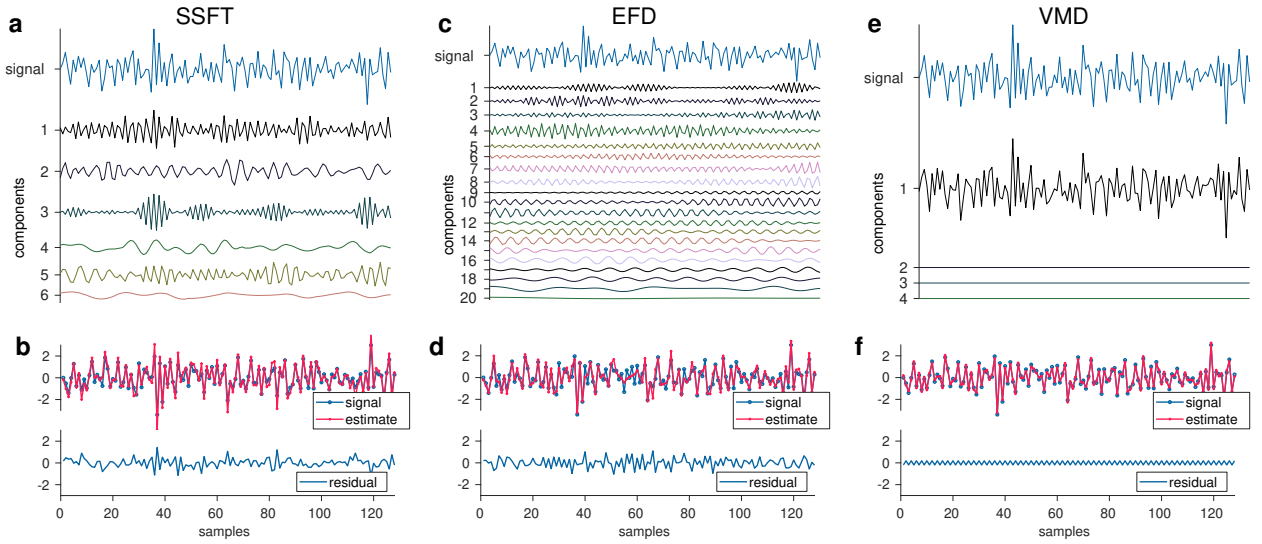

Figure S2: Signal decomposition with fractional Gaussian noise. Methods: left: synchrosqueeze short-time Fourier transform (SSFT) in (**a**) and (**c**); middle: empirical Fourier decomposition (EFD) in (**c**) and (**d**); and right: variational mode decomposition (VMD) in (**e**) and (**f**). Signal with components in (**a**), (**c**), and (**e**). Difference (error) between sum of components and signal in (**b**), (**d**), and (**f**). Amplitude of components are plotted relative to signal amplitude for (**a**), (**c**), and (**e**).