Chinese Journal of Mechanical Engineering

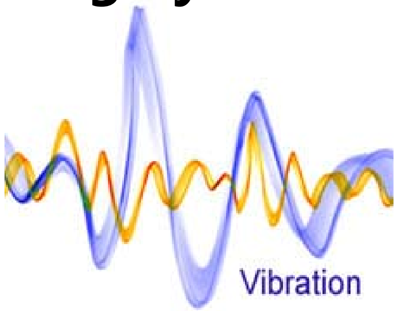
北京航空航天大学
BEIHANG UNIVERSITY

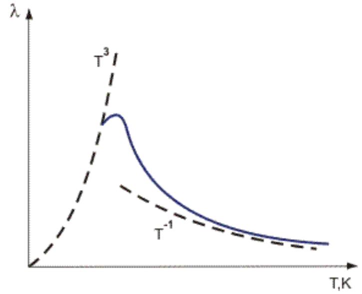
Effect of Degree-of-Symmetry on Kinetostatic Characteristics of Flexure Mechanisms: A Comparative Case Study

HE Xiaobing
2017/11/20

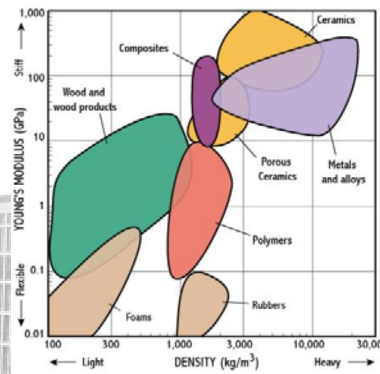
➤ Accuracy of flexure mechanisms is highly sensitive

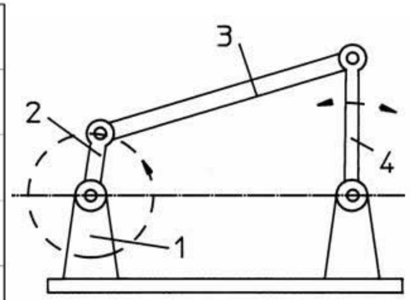
01 External disturbances



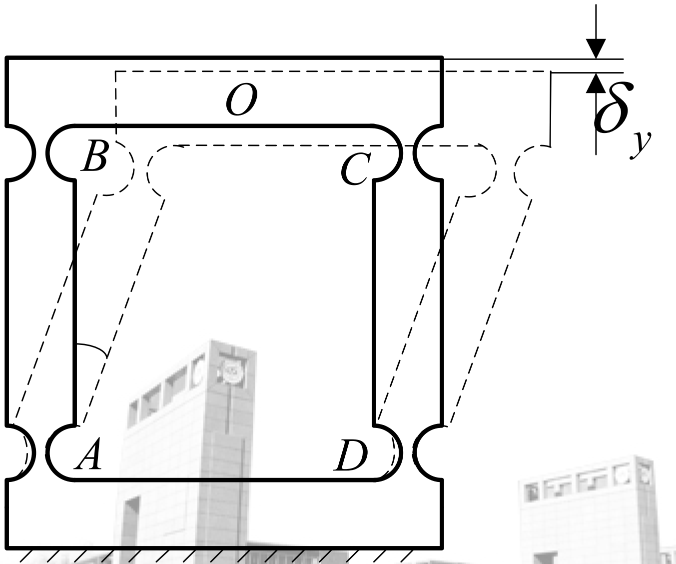


02 Internal factors





Parasitic motion is an index for accuracy



Parasitic motion is defined as any undesirable motion along the constraint directions of a mechanism.

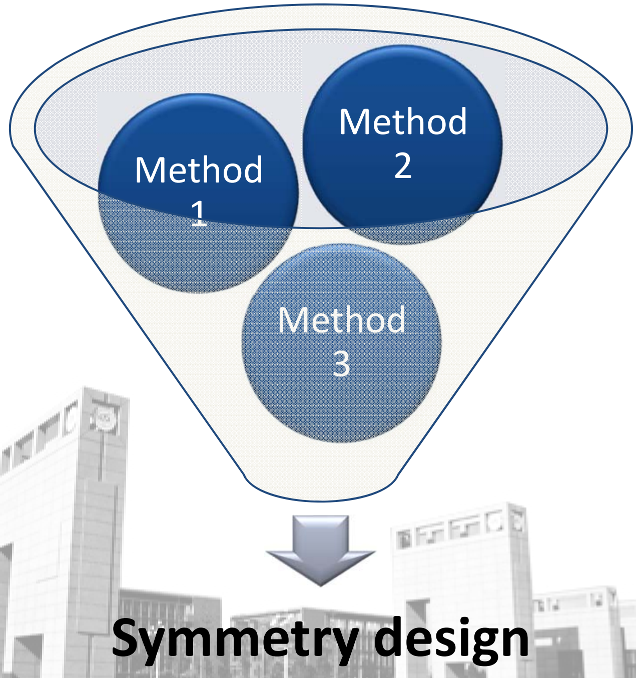
Part1：Question

北京航空航天大学

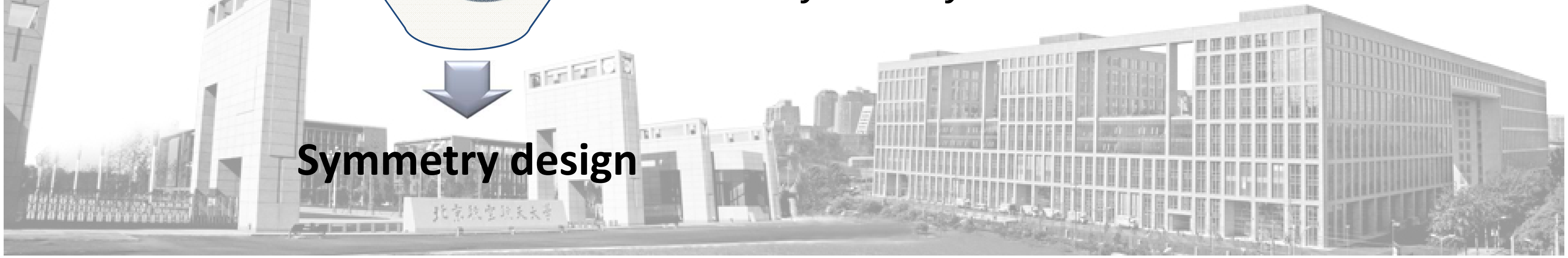
Methods to reduce or eliminate the parasitic motion

parameter	compensation	design
Tune the structural parameters and material properties without changing the type of flexure mechanisms.	Mirror two identical flexure modules and arrange them in serial or in parallel to compensate unwanted motion.	Design a practical full-symmetrical compliant mechanism at the phase of conceptual design.

➢Common knowledge of symmetry



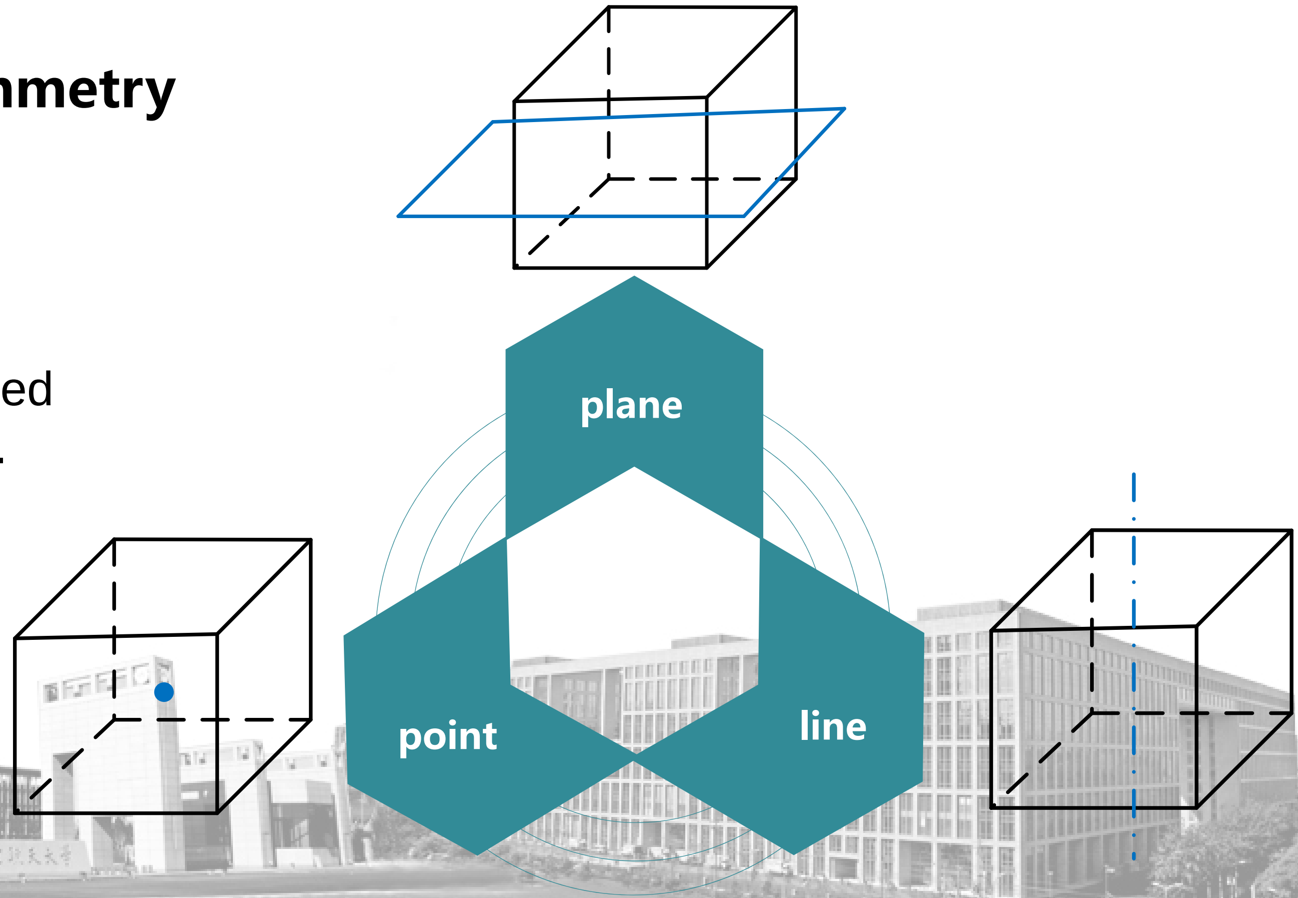
is it necessary to design the flexure mechanisms with symmetrical features as many as possible for better kinetostatic performance, when considering the resulting cost by the symmetry?



Part2：Analysis

➢ Degree of symmetry

Here, the Degree-of-Symmetry (DoS) is specifically constrained with plane symmetry.



Part2：Analysis

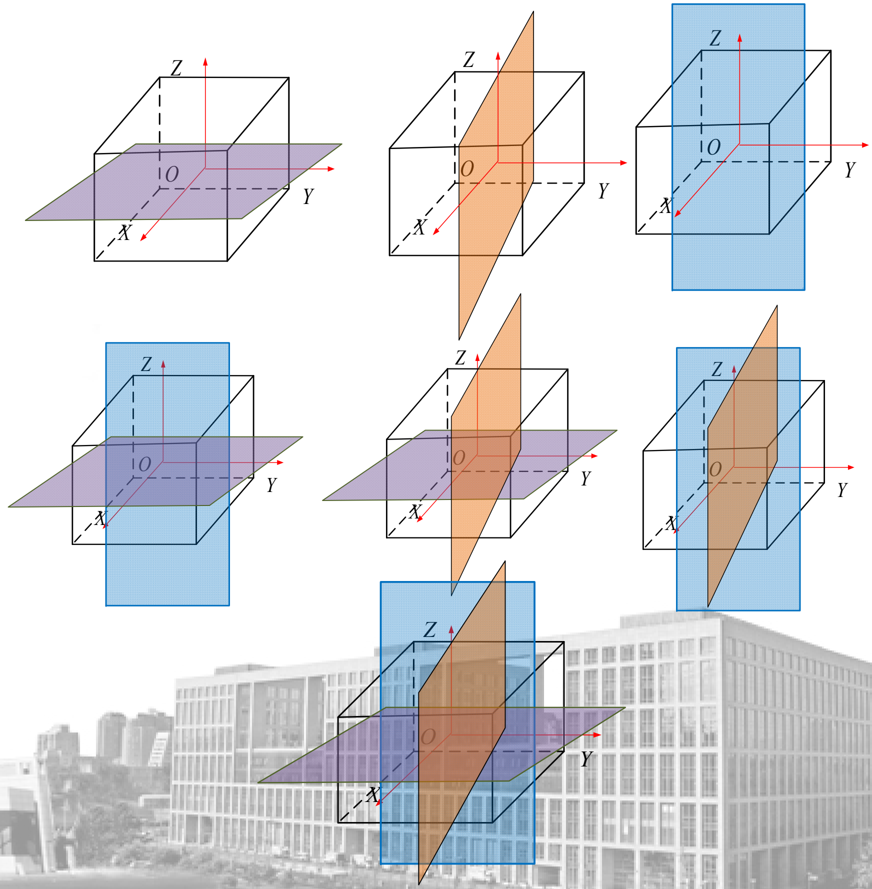
北京航空航天大学

➢ X-DoS

1-DoS

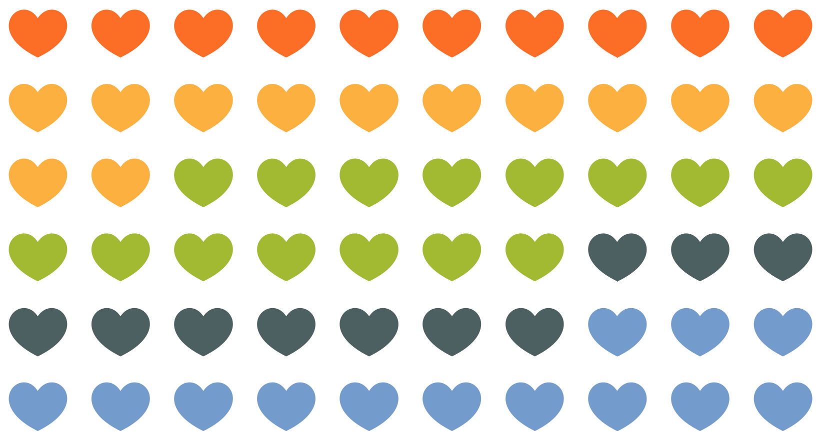
2-DoS

3-DoS



➢ A comparative study

So,
what is the effect of DoS on kinetostatic characteristic of flexure mechanisms?

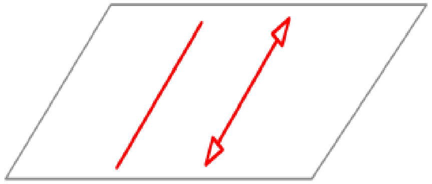


3-DoS > 2-DoS > 1-DoS ? ? ?

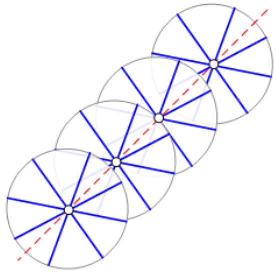
Build the model

1R1T

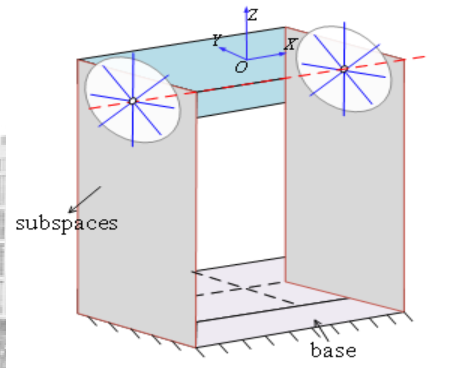
The desired freedom space



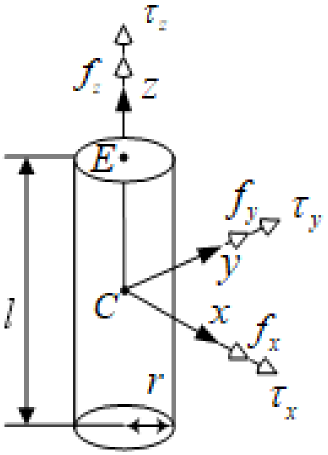
The desired constraint space



One of constraint subspace



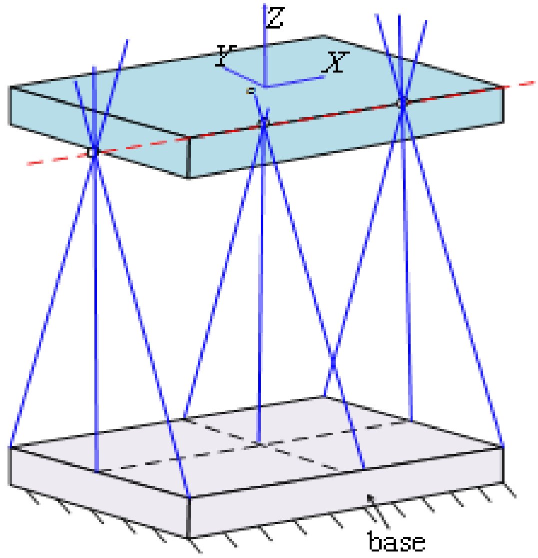
➢ Build the model



$$\frac{c_{c11}}{c_{c33}}=\frac{c_{c22}}{c_{c33}}=\frac{l^3}{12EI_y}\bigg/\frac{l}{EA}=\frac{1}{3}\left(\frac{l}{r}\right)^2=133,$$

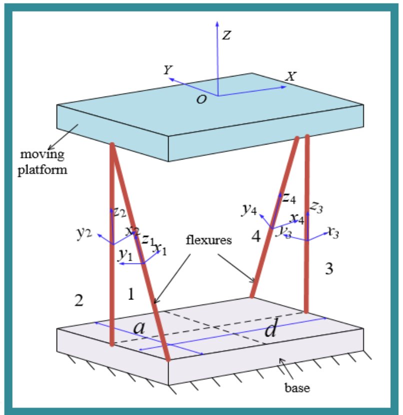
$$\frac{c_{c44}l^2}{c_{c33}}=\frac{c_{c55}l^2}{c_{c33}}=1600,\quad \frac{c_{c66}l^2}{c_{c33}}=\frac{2l^2}{3(1+\mu)r^2}\geq 1000$$

Each mechanism is a cylindrical joint that is composed of several identical beams distributed in two planes orthogonal to the direction of motion axes.

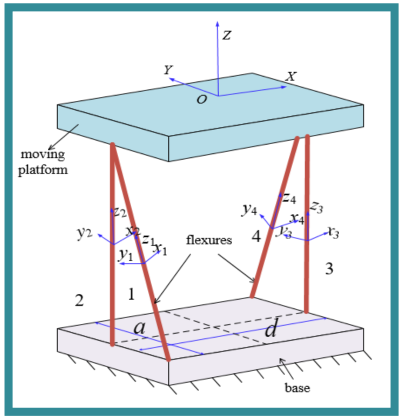


Part2：Analysis

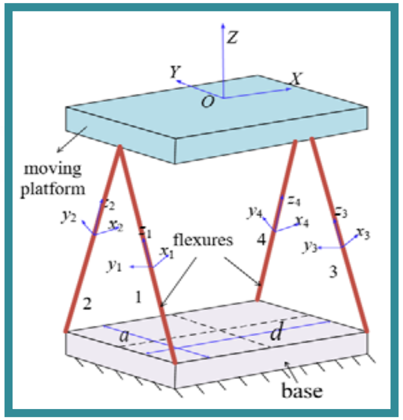
➢ Build the model



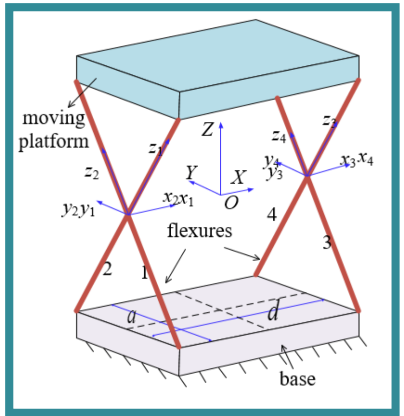
0-DoS



1-DoS



2-DoS



3-DoS

Overall compliance matrix

According to the screw theory and the theory of linear elasticity, the deformation denoted by the twist

$$\xi=(\boldsymbol{\theta};\boldsymbol{\delta})=(\theta_x,\theta_y,\theta_z;\delta_x,\delta_y,\delta_z)$$

and the load wrench

$$\boldsymbol{F}=(\boldsymbol{\tau};\boldsymbol{f})=(\tau_x,\tau_y,\tau_z;f_x,f_y,f_z)$$

are connected by the generic 6×6 compliance matrix.

$$\begin{pmatrix}\theta_x\\ \theta_y\\ \theta_z\\ \delta_x\\ \delta_y\\ \delta_z\end{pmatrix}=\begin{pmatrix}c_{11}&c_{12}&c_{13}&c_{14}&c_{15}&c_{16}\\ c_{21}&c_{22}&c_{23}&c_{24}&c_{25}&c_{26}\\ c_{31}&c_{32}&c_{33}&c_{34}&c_{35}&c_{36}\\ c_{41}&c_{42}&c_{43}&c_{44}&c_{45}&c_{46}\\ c_{51}&c_{52}&c_{53}&c_{54}&c_{55}&c_{56}\\ c_{61}&c_{62}&c_{63}&c_{64}&c_{65}&c_{66}\end{pmatrix}\begin{pmatrix}\tau_x\\ \tau_y\\ \tau_z\\ f_x\\ f_y\\ f_z\end{pmatrix}$$

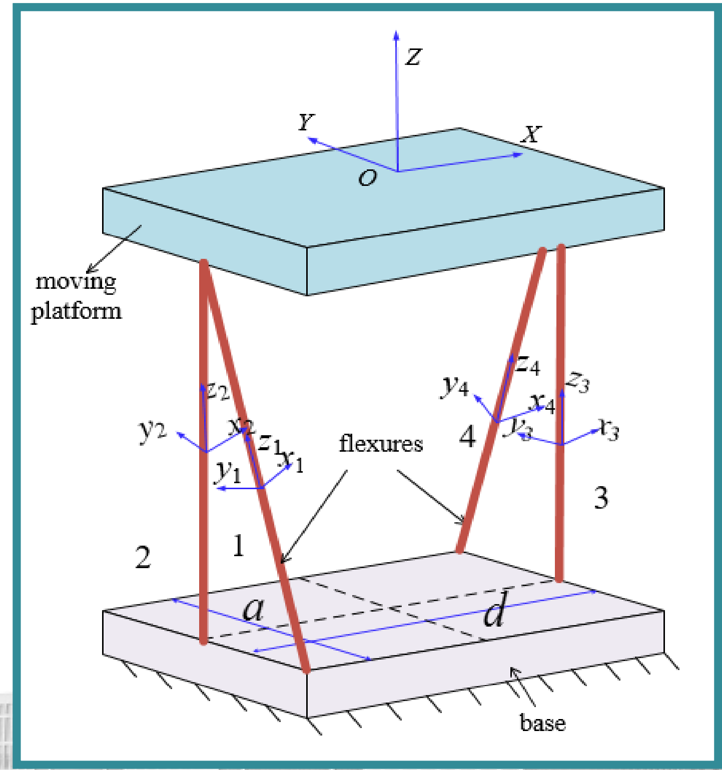
➢ Overall compliance matrix

Only a force f_x is imposed on the mobile platform.

$$\begin{pmatrix} \theta_x \\ \theta_y \\ \theta_z \\ \delta_x \\ \delta_y \\ \delta_z \end{pmatrix} = \begin{pmatrix} c_{11} & c_{12} & c_{13} & c_{14} & c_{15} & c_{16} \\ c_{21} & c_{22} & c_{23} & c_{24} & c_{25} & c_{26} \\ c_{31} & c_{32} & c_{33} & c_{34} & c_{35} & c_{36} \\ c_{41} & c_{42} & c_{43} & c_{44} & c_{45} & c_{46} \\ c_{51} & c_{52} & c_{53} & c_{54} & c_{55} & c_{56} \\ c_{61} & c_{62} & c_{63} & c_{64} & c_{65} & c_{66} \end{pmatrix} \begin{pmatrix} \tau_x \\ \tau_y \\ \tau_z \\ f_x \\ f_y \\ f_z \end{pmatrix}$$

➢ Parasitic motion error analysis

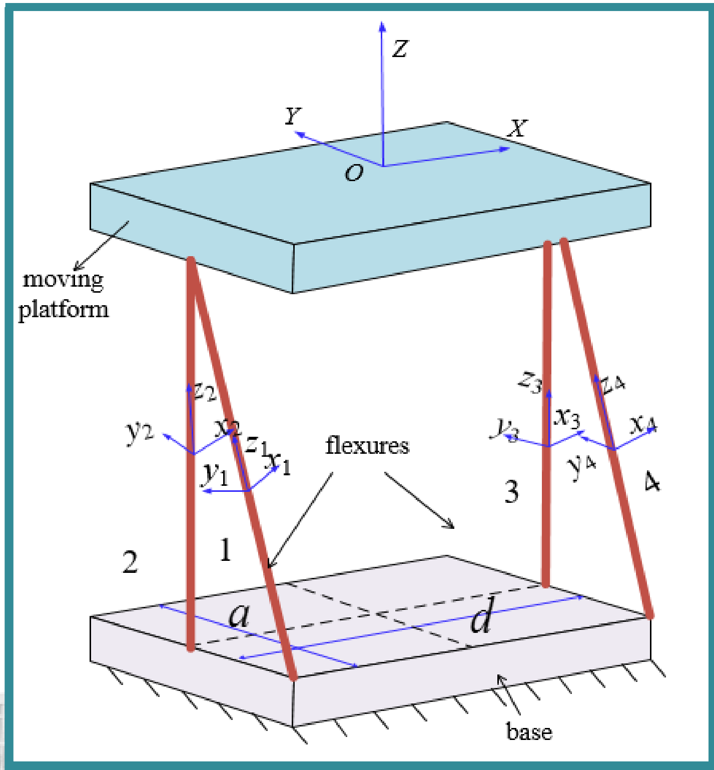
$$
\boldsymbol{C}_{0-\mathrm{DOS}} = \begin{pmatrix} c_{11} & c_{12} & 0 & c_{14} & c_{15} & 0 \\ c_{21} & c_{22} & 0 & c_{24} & c_{25} & 0 \\ 0 & 0 & c_{33} & 0 & 0 & c_{36} \\ c_{41} & c_{42} & 0 & c_{44} & c_{45} & 0 \\ c_{51} & 0 & c_{53} & c_{54} & c_{55} & 0 \\ 0 & 0 & c_{63} & 0 & 0 & c_{66} \end{pmatrix}
$$



$$
\xi_0 = \theta_x + \theta_y + \delta_x + \delta_y = c_{14} f_x + c_{24} f_x + c_{44} f_x + c_{54} f_x
$$

➢ Parasitic motion error analysis

$$
\boldsymbol{C}_{1-\mathrm{DOS}} = \begin{pmatrix} c_{11} & 0 & 0 & 0 & c_{15} & c_{16} \\ 0 & c_{22} & c_{23} & c_{24} & 0 & 0 \\ 0 & c_{32} & c_{33} & c_{34} & 0 & 0 \\ 0 & c_{42} & c_{43} & c_{44} & 0 & 0 \\ c_{51} & 0 & 0 & 0 & c_{55} & c_{56} \\ c_{61} & 0 & 0 & 0 & c_{65} & c_{66} \end{pmatrix}
$$

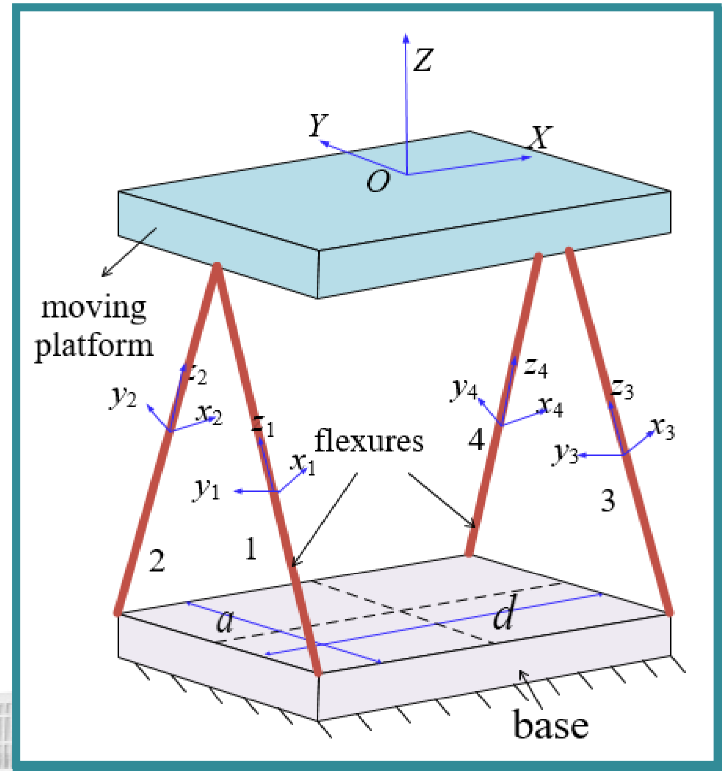


$$
\xi_1 = \theta_y + \theta_z + \delta_x = c_{24} f_x + c_{34} f_x + c_{44} f_x
$$

➢ Parasitic motion error analysis

$$\boldsymbol{C}_{2-\mathrm{DOS}} = \begin{pmatrix} c_{11} & 0 & 0 & 0 & c_{15} & 0 \\ 0 & c_{22} & 0 & c_{24} & 0 & 0 \\ 0 & 0 & c_{33} & 0 & 0 & 0 \\ 0 & c_{42} & 0 & c_{44} & 0 & 0 \\ c_{51} & 0 & 0 & 0 & c_{55} & 0 \\ 0 & 0 & 0 & 0 & 0 & c_{66} \end{pmatrix}$$

$$\xi_2 = \theta_y + \delta_x = c_{24} f_x + c_{44} f_x$$

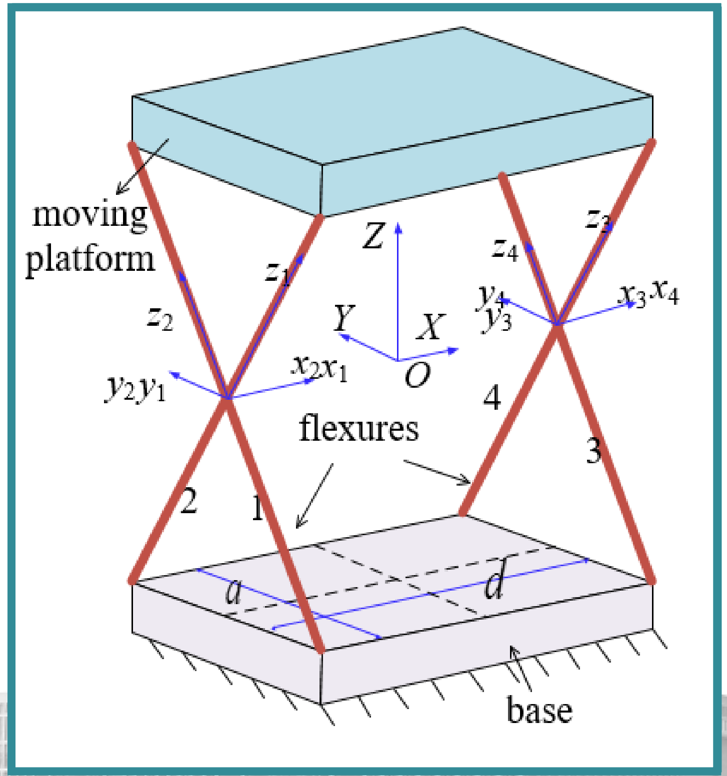


Part2：Analysis

➢ Parasitic motion error analysis

$$\boldsymbol{C}_{3\text{-DOS}} = \begin{pmatrix} c_{11} & 0 & 0 & 0 & 0 & 0 \\ 0 & c_{22} & 0 & 0 & 0 & 0 \\ 0 & 0 & c_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & c_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & c_{55} & 0 \\ 0 & 0 & 0 & 0 & 0 & c_{66} \end{pmatrix}$$

$$\xi_3 = \delta_x = c_{44} f_x$$



The best design

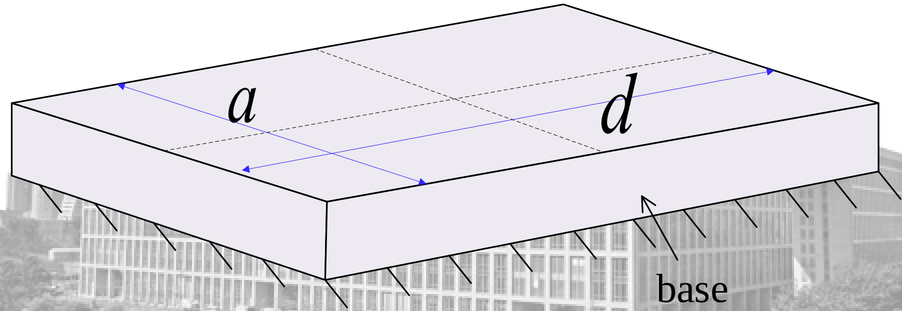
3-DoS type

Because of all symmetric planes

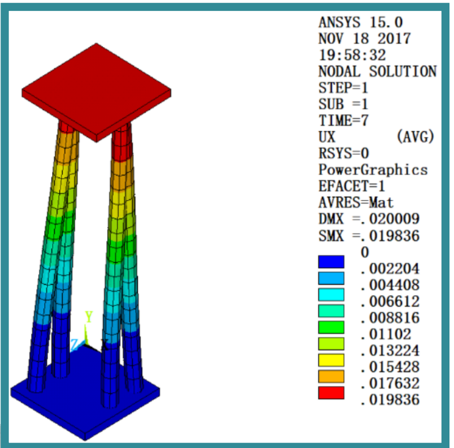
Part3：Verification

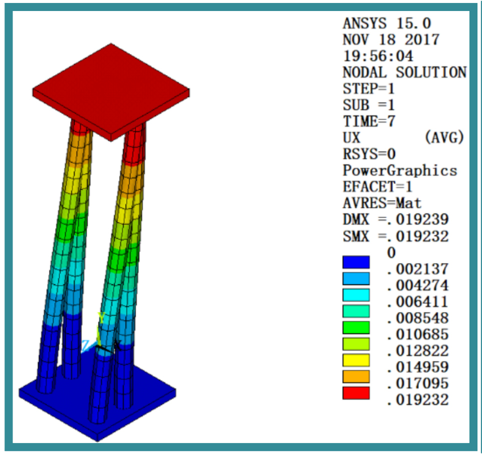
➢ Finite element analysis

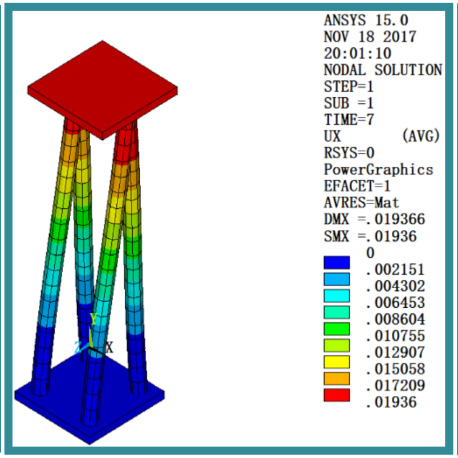
ANSYS 15.0	Rigid platform: SOLID 186
	Flexure beam: BEAM 189
	E=70 GPa, μ=0.34
	Cross section radius: 5 mm
	Height between platforms: 200 mm
	a=b= 60 mm

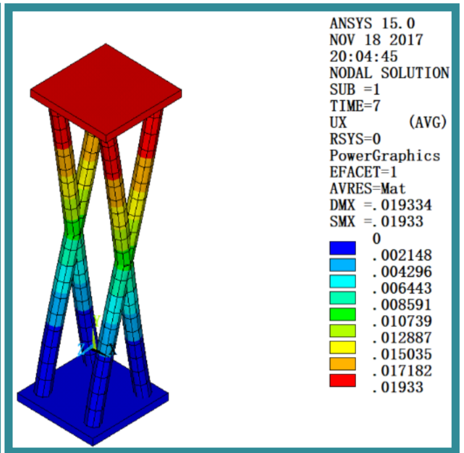


➢FEA simulations of each mechanism









0-DoS
DMX=0.020009
SMX=0.019836

1-DoS
DMX=0.019239
SMX=0.019232

2-DoS
DMX=0.019366
SMX=0.01936

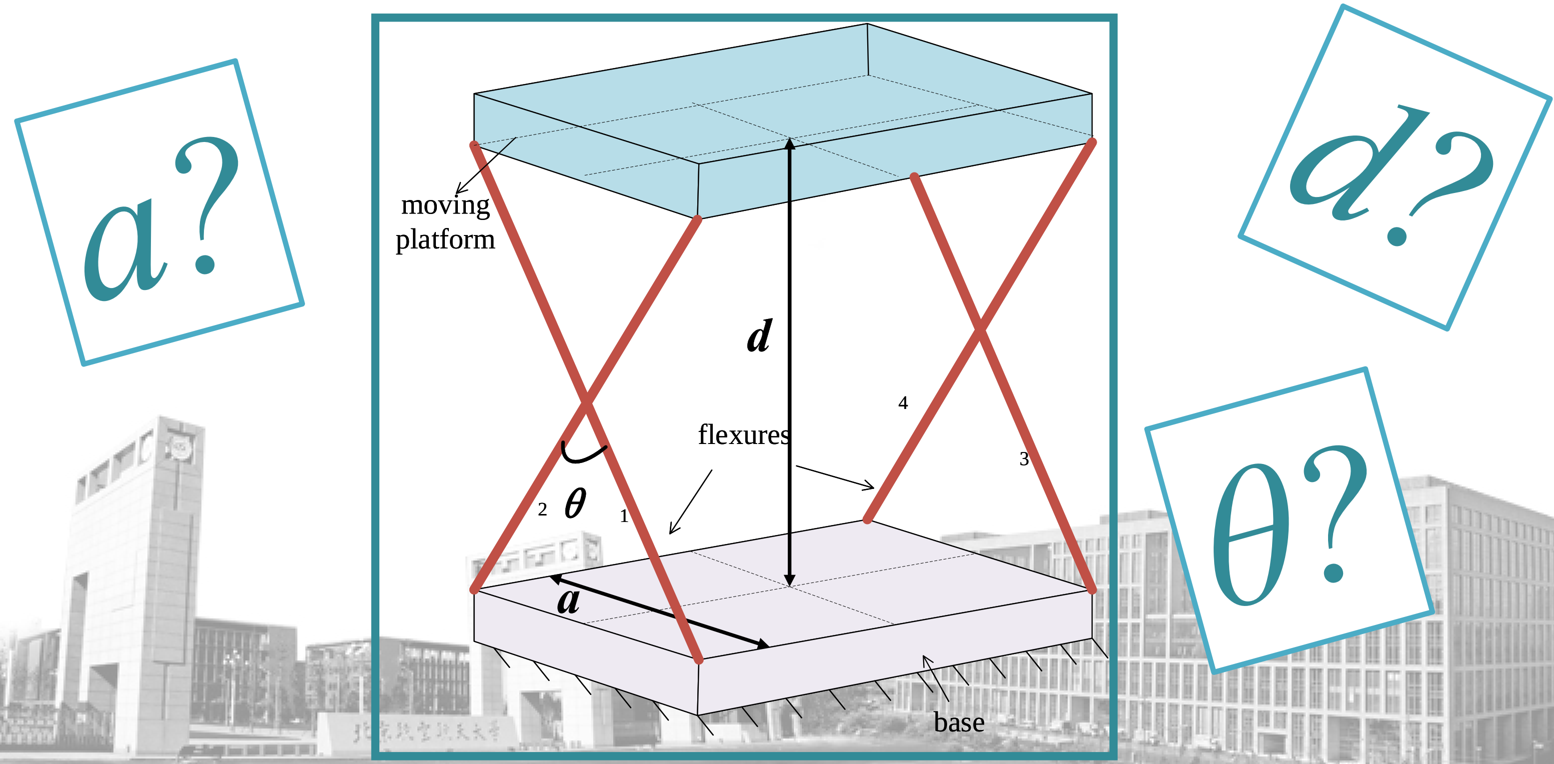
3-DoS
DMX=0.019334
SMX=0.01933

Optimize the 3-DoS flexure mechanism

The entry c_{11} and c_{44} should be the dominant ones for ensuring a rotation along the x-axis and a translational about x-axis.

$$
\begin{cases}
c_{11} = \dfrac{a}{E\pi r^4 \sin\theta} \\
c_{22} = \dfrac{a^3}{\pi r^2 \sin\theta \left(2Ga^2r^2\sin^2\theta + 12Ed^2r^2\sin^4\theta + Ea^2r^2\cos^2\theta + 4Ea^2d^2\cos^2\theta\right)} \\
c_{33} = \dfrac{a^3}{\pi r^2 \sin\theta \left(2Ga^2r^2\cos^2\theta + Ea^2r^2\sin^2\theta + 12Ed^2r^2\cos^2\theta\sin^2\theta + 4Ea^2d^2\sin^2\theta\right)} \\
c_{44} = \dfrac{a^3}{12E\pi r^4 \sin^3\theta} \\
c_{55} = \dfrac{a^3}{4E\pi r^2 \sin^3\theta \left(3r^2\cos^2\theta + a^2\right)} \\
c_{66} = \dfrac{a^3}{4E\pi r^2 \sin\theta \left(3r^2\sin^4\theta + a^2\cos^2\theta\right)}
\end{cases}
$$

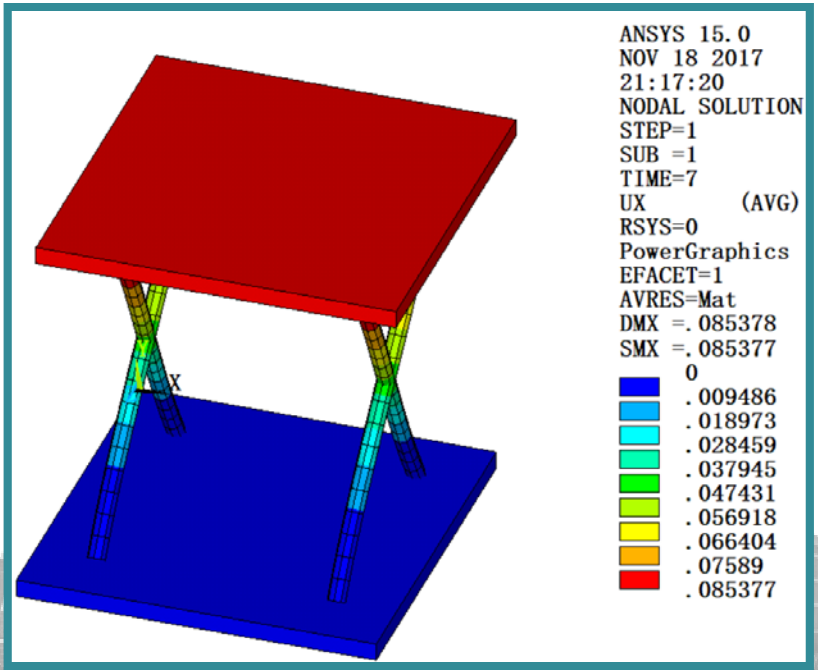
➢Optimize the 3-DoS flexure mechanism



Optimize the 3-DoS flexure mechanism

Optimal parameters for 3-DoS flexure model

Beam orientation (*θ*)	***θ*=π/4**
Two end points distance (*a*)	**200 mm**
Interval distance (*d*)	**200 mm**
Radius of beam (*r*)	**5 mm**



➢ Larger DoS, better result.

In the design process, it is indeed better to generate as many DoS as possible from the very beginning. With the different DoS, people can further adopt certain methods to alleviate the unwanted parasitic motion.

➢ Application of flexure mechanisms with cylindrical motion

The joint for realizing a snack-link robot's three dimensions' gaits.

For the in-pipe inspection robots with fast and continuous rate together with required accuracy.

Part5 : Conclusion

01 Design

Using FACT method to design flexure mechanisms with different number of symmetric planes.

02 Analysis

Using overall compliance matrix to analyze the effect of symmetrical geometry on the kinetostatic characteristics.

03 Verification

Using FEA simulations to verify the effect of different DoS on the parasitic motion and obtain the optimization of 3-DoS model.

Acknowledge

北京航空航天大学

Thank for the grant support of NSFC with No. 51575017.

北京航空航天大學
BEIHANG UNIVERSITY

THANK YOU