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Teaching and learning introductory electrostatics - conformity with nomenclature conventions

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Abstract. The treatment of electric flux in most textbooks on introductory physics, at least in the English language, differs radically from the definitions prescribed by the statutory and intergovernmental organizations (CGPM, BIPM, IUPAP, IUPAC, NIST, etc). The approach also conflicts with the treatment of the topic in many electrical engineering texts. Furthermore, it is at variance with the standard treatment of matter elsewhere in the curriculum (e.g., mechanical or thermal properties). Historical reasons for this discrepancy are investigated and it is suggested that resolving this conflict would be beneficial to the teaching and learning of this and related topics.

1. Introduction

Up until the middle of the 20th century most textbooks and curricula on introductory physics followed a historical sequence, beginning by using macroscopic models and only later proceeding to explain how results so obtained may be derived from microscopic (atomistic) considerations. For example, bulk properties of materials (e.g. pressure of a gas, elasticity of a solid, etc.) were, and usually still are, initially described in terms of continuum mechanics and afterwards interpreted in terms of an atomic/molecular/crystalline picture. Similarly, the basic ideas in thermal physics (temperature, heat capacity, etc.) are introduced as macroscopic concepts to be explained subsequently in terms of molecular (kinetic) theory. The treatment of electromagnetism followed a similar approach, for example, dealing first with fields in dielectric media, Coulomb's law been seen as applying to the force between point charges within an infinite linear, isotropic, homogeneous (l.i.h.) medium.

Towards the middle of the century, however, a number of developments occurred which had important and generally beneficial effect on physics curriculum design and pedagogy. Foremost among these was the movement towards the adoption of an internationally agreed system of units. In 1935, the International Electrotechnical Commission (IEC) had adopted the rationalized MKSA system [1] (originally proposed by Giovanni Georgi in 1901) which was widely embraced in engineering. This subsequently evolved into the Système International d'Unités (SI) which was formally adopted by the 11th Conférence Générale des Poids et Mesures (CGPM) in 1960 [2]. Its implementation, sometimes reluctantly, by the physics community had profound and beneficial effect on physics education.

In parallel with this development were the recommendations for symbols and nomenclature agreed by intergovernmental organizations and national and international standards authorities such as CGPM, Bureau International des Poids et Mesures (BIPM), the International Union of Pure and Applied Physics (IUPAP), the International Union of Pure and Applied Chemistry (IUPAC), the US National Institute of Standards and Technology (NIST) and the International Organization for Standardization (ISO). Such standardization not only enhanced commercial, technical and scientific communication but also led to improvements in curriculum and textbooks used in high school and university science courses. In

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particular, adherence to such internationally agreed standards in units, symbols and nomenclature made it easier for students to read most introductory textbooks and to proceed to intermediate or advanced level courses in areas such as physics, chemistry and engineering.

While almost all these recommendations on symbols and nomenclature have been adopted by the physics pedagogical community, there remains one very obvious exception where agreed nomenclature is not universal. The quantity *electric flux* is defined by IUPAP [3] – the SUNAMCO 'Red Book' – and by IUPAC [4] – the IUPAC 'Green Book' – such that the total electric flux through a surface *S* is given by

$$\Psi_{\mathrm{E}} = \iint_{S} \boldsymbol{D}.d\boldsymbol{S} \tag{1}$$

where $\boldsymbol{D}$ is the electric displacement at each point on the surface. While official documents from BIPM and NIST on nomenclature generally avoid the use of the term 'electric flux', such documents define $\boldsymbol{D}$ as the *electric flux density*, thus indicating conformity with the IUPAP/IUPAC norm. As will be seen below, there are very good reasons for this choice of definition.

This nomenclature is also used in many textbooks on electromagnetism at intermediate or advanced level and in most textbooks on electromagnetics aimed at electrical engineering students. Gauss' law in such texts is normally written as

$$\oint\!\!\oint_{S} \boldsymbol{D}.d\boldsymbol{S} = Q \tag{2}$$

where Q is the net electric charge within the (closed) surface S.

By contrast, the great majority of textbooks aimed at students taking introductory physics courses in university or at the upper level in high school today use a different definition for the quantity 'electric flux', namely

$$\Psi'_{\mathrm{E}} = \iint_{S} \boldsymbol{E}.d\boldsymbol{S} \tag{3}$$

where $\boldsymbol{E}$ is the electric field strength (intensity) at each point on the surface.[1] In this case, Gauss' law is usually presented in the form

$$\oint\!\!\oint_{S} \boldsymbol{E}.d\boldsymbol{S} = \frac{Q}{\varepsilon_0} \tag{4}$$

where ε_0 is the permittivity of vacuum. Note that, from this viewpoint, $\boldsymbol{E}$ is the 'electric flux density', at variance with the IUPAP/IUPAC definition.

The subscript zero in the denominator on the right hand side of Equation (4) is revealing. In this form, Gauss' law and hence Coulomb's law, when first introduced in introductory physics textbooks, are understood to apply *in vacuo* only, but this is not usually stated explicitly. On the other hand, textbooks for which the electric flux is defined by (1), take a macroscopic perspective *ab initio*, specifically developing the laws of electromagnetism in linear, isotropic, homogeneous media and treating vacuum as a special case.

2. Possible origin of the standard approach in physics texts

Why electromagnetism is usually treated differently in introductory physics texts may be traced back to the philosophy underlying the original reforms introduced by the Physical Science Study Committee [5]

[1] Some authors call this the 'electric field flux', presumably to indicate the distinction between the two definitions. Others prefer to discuss 'lines of force' or 'field lines' instead of flux but the same issues arise in this case. It should be pointed out, however, that the situation tends to be more nuanced in textbooks written in languages other than English. In Continental European languages, for example, the tendency is to identify two distinct fluxes: that defined by Eq.(1) above may be translated as 'electric displacement flux' and the flux defined by Eq.(2) as 'electric field strength flux'.

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in the United States in the late 1950s. There can be little doubt that the revolution in physics education which followed the initial PSSC meeting in MIT on 10-12 December 1956 had a huge impact on the improvements in physics curricula and textbooks over the following half century (PSSC, 1960). School and university teaching today owe an enormous debt to those pioneers of physics education and the wonderful textbooks and teaching aids which they spawned.

One feature of the original PSSC approach has left an impact on the teaching of electromagnetism in the US and many other countries. The Committee believed that it would be to the benefit of physics teaching 'if the presentation of the subject matter were focused towards one goal, and that goal ought to be *the atomic picture of the universe*' [6] (italics original). This may go some way to explain why the teaching of electromagnetism so often starts from an *in vacuo* perspective in contradistinction to the way other branches of classical physics are usually presented and developed.[2] Indeed, in the early PSSC approach, the treatment of electromagnetism was integrated with that of Atomic Physics [7].

3. Towards compatibility with IUPAP/IUPAC nomenclature

There is no obvious reason, however, why such an approach needs to be in conflict with IUPAP/IUPAC standards and conventions. All that is required to remove the discrepancy is, for those who wish to introduce electromagnetic concepts by dealing initially with vacuum, is to define electric flux differently, namely as

$$\Psi_{\mathrm{E}} = \iint_S \varepsilon_0 \boldsymbol{E}.d\boldsymbol{S} \tag{5}$$

in which case, invoking the flux model, Gauss' law takes the form

$$\oiint_S \varepsilon_0 \boldsymbol{E}.d\boldsymbol{S} = Q \tag{6}$$

which is essentially the same as Equation (4) in the special case of vacuum.

It would seem to be important, however, that it be made clear to the learner that the treatment, at that stage, is limited to the vacuum case. As in the treatment in most physics textbooks, the quantity $\boldsymbol{D}$ can be introduced later in the context of dielectric materials.

Notwithstanding the above, however, there exists an alternative approach to the development of the formalism of electromagnetism which some students may find easier to master; such an approach is the subject of the remaining sections of this paper.

4. Laws of electromagnetism in linear isotropic media

There is no dispute concerning the definition of *magnetic flux* which all authors and authorities agree is defined as

$$\Psi_{\mathrm{M}} = \iint_S \boldsymbol{B}.d\boldsymbol{S} \tag{7}$$

where $\boldsymbol{B}$ is the magnetic flux density or magnetic induction. The SI unit of Ψ_{M} is weber and that of $\boldsymbol{B}$ is Wb m^{-2}. Gauss' law for magnetism can be written

$$\oiint_S \boldsymbol{B}.d\boldsymbol{S} = 0 \tag{8}$$

in direct analogy with Equation (2) above except for the zero on the right hand side which arises from the fact that there are no magnetic monopoles. If such monopoles existed they would play the same role as sources of magnetic flux as charges do in electrostatics (see Table 1 below). A pole model may be

[2] It was not until towards the end of the century that syllabi constructed throughout on the atomistic nature of matter began to make an appearance. The Matter and Interactions curriculum covered in the text by Chabay and Sherwood (New York: John Wiley and Sons) is the prime example of this approach.

usefully employed, however, to give a simple explanation of everyday magnetic phenomena such as forces between permanent magnets, planetary and stellar magnetism and magnetic compasses.

A full description of electromagnetic phenomena in vacuum requires only two vector fields. It is usual in such cases to choose the electric field strength ($\boldsymbol{E} = \boldsymbol{D}/\varepsilon_0$) and the magnetic flux density ($\boldsymbol{B}=\mu_0\boldsymbol{H}$). The force on a charge q in an electromagnetic field can be expressed most easily in terms of these field quantities via the Lorentz force $\boldsymbol{F} = q\boldsymbol{E} + q\boldsymbol{v} \times \boldsymbol{B}$.

The treatment of electromagnetism in a material medium, however, requires all four vector fields, together with appropriate constitutive equations (e.g., $\boldsymbol{D} = \varepsilon\boldsymbol{E}$, $\boldsymbol{B}=\varepsilon\boldsymbol{H}$ in a l.i.h.). The fundamental laws of electromagnetism in this case are Maxwell's equations for a medium; in integral form these are

Ampere-Maxwell law:
$$\oint_C \boldsymbol{H.dl} = I + \frac{d}{dt}\iint_S \boldsymbol{D.dS} \tag{9.1}$$

Gauss' law for magnetism:
$$\oiint_S \boldsymbol{B.dS} = 0 \tag{9.2}$$

Gauss' law for electrostatics:
$$\oiint_S \boldsymbol{D.dS} = \sum_i q_i \tag{9.3}$$

Faraday's law:
$$\oint_C \boldsymbol{E.dl} = -\frac{d}{dt}\iint_S \boldsymbol{B.dS} \tag{9.4}$$

Inspection of these equations shows a marked symmetry between $\boldsymbol{D}$ and $\boldsymbol{B}$ and between $\boldsymbol{E}$ and $\boldsymbol{H}$. Furthermore, an obvious implication from the equations is that $\iint_S \boldsymbol{B.dS}$ can be seen as *magnetic flux* and $\iint_S \boldsymbol{D.dS}$ as *electric flux*, consistent with the first approach to the definition of electric flux (1) given in the Introduction above and in keeping with IUPAP, IUPAC, BIPM, NIST conventions and electrical engineering texts.

5. Gravitational flux and Gauss' law for gravity

The approach to the treatment of electric and magnetic flux outlined above has a further advantage in teaching introductory physics. Not only should the greater symmetry between electric and magnetic systems, when flux is defined in this way, make learning easier but the same ideas can also be applied to gravitation.

It is curious how few textbooks avail of a flux model to explain gravitational phenomena. Since Newton's law of gravitation is also an inverse square law relationship, similar considerations apply. The strength of a gravitational source can be given by its mass – unlike electric and magnetic sources, all gravitational sources are *sinks* of flux only. A gravitational flux density (□, say) may be defined similarly to $\boldsymbol{D}$ and $\boldsymbol{B}$ in electromagnetism, in which case Gauss' law for gravitation takes the form

$$\oiint_S \boldsymbol{\Gamma.dS} = \sum_i m_i \tag{10}$$

where the right hand side is the sum of the masses within the closed surface S. This result enables simple derivations of the strength of the gravitational field of symmetric mass distributions (shells, spheres, etc.).

Note that had Newton's law of gravitation been 'rationalized' (that is, the constant G replaced by $\frac{1}{4\pi\gamma}$ where γ is another constant) then the symmetry would be complete.

6. Conclusions

Starting the development of electromagnetism from the viewpoint of ‘electromagnetism in a medium’ has distinct advantages, namely,

- It is consistent with the IUPAP/IUPAC definition of electric flux.
- Introducing the material initially in the context of macroscopic systems is more in line with the usual treatment of other branches of introductory physics with which students are familiar.
- Highlighting the symmetries that arise when the flux model is applied to electric, magnetic and gravitational sources (see Table 1 located at the end of the article) should make life easier for both instructors and learners and should enable students to master the material earlier.
- The approach adopted here leads to a simpler and more and straightforward presentation of electromagnetism than usually encountered in introductory physics courses and textbooks – one possible curriculum sequence is outlined in the Appendix below.

7. Acknowledgments

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9. Appendix

What follows below is an outline curriculum sequence intended to illustrate how the formalism of any inverse force field might be introduced by applying a flux model to sources embedded in l.i.h. media. For clarity, the notations of electrostatics are used but magnetism and/or gravitation can be treated in a similar manner (Table 1).

1) **Field strength** is defined as force/unit source strength, that is, $\boldsymbol{E} = \frac{F}{q}\hat{\boldsymbol{t}}$ where $\hat{\boldsymbol{t}}$ is unit vector tangential to the local field line.
2) **Flux density** is defined as $\boldsymbol{D} = \lim_{\Delta S \to 0} \frac{\Delta \Psi_E}{\Delta S}\hat{\boldsymbol{t}}$ (see Figure 1)
3) Since in a l.i.h. medium $\boldsymbol{E} \,||\, \boldsymbol{D} \;\rightarrow\; \boldsymbol{D} = \varepsilon \boldsymbol{E}$, where ε is a constant characteristic of the medium.
4) The value of the flux density $\boldsymbol{D}$ at any point in an infinite l.i.h. medium due to some arbitrary distribution of charge is the vector sum of the contribution of each point source or, in the case of a continuous distribution, from each infinitesimal source element. For systems with a high degree of symmetry, Gauss’ law enables simple derivations.

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5) The field strength ***E*** at any point does depend on the medium and is determined by dividing ***D*** by the value of ε for the medium.
6) Flux density is independent of the medium and flux is continuous across interfaces between media. The latter condition requires that the component of ***D*** perpendicular to an interface is continuous across the interface.
7) Since the electric field is conservative ($\oint \boldsymbol{E}.d\boldsymbol{l} = 0$), it follows that the component of ***E*** parallel to the interface must be continuous across the interface.

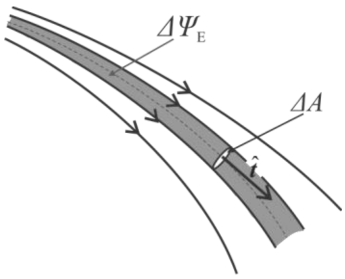

Figure 1. Definition of electric flux density

At this point most standard electrostatic problems may be addressed, such as field strength due to specific charge distributions (e.g., dipoles), capacitors, boundary value problems, method of images, etc. Phenomena arising from the microscopic properties of dielectric media such as polarization, susceptibility, etc. may then be treated and the macroscopic results, e.g. ***D*** **=** □***E***, interpreted and explained. Only in this context do issues such as ‘free’ and ‘bound’ charges need to be introduced.

Table 1: Summary of the symmetries apparent when electric, magnetic and gravitational fields in a medium are described in terms of flux models

	electric quantities	magnetic quantities*	gravitational quantities
source	q	p	m
field	$\boldsymbol{E}$	$\boldsymbol{H}$	$\boldsymbol{g}$
force on source	$\boldsymbol{F} = q\boldsymbol{E}$	$\boldsymbol{F} = p\boldsymbol{H}$	$\boldsymbol{F} = m\boldsymbol{g}$
flux density	$\boldsymbol{D}$	$\boldsymbol{B}$	$\boldsymbol{\Gamma}$
flux	$\Psi_{\mathrm{E}} = \iint_S \boldsymbol{D}.d\boldsymbol{S} = \varepsilon \iint_S \boldsymbol{E}.d\boldsymbol{S}$	$\Psi_{\mathrm{M}} = \iint_S \boldsymbol{B}.d\boldsymbol{S} = \mu \iint_S \boldsymbol{H}.d\boldsymbol{S}$	$\Psi_{\mathrm{G}} = \iint_S \boldsymbol{\Gamma}.d\boldsymbol{S} = \frac{1}{4\pi G} \iint_S \boldsymbol{g}.d\boldsymbol{S}$
Gauss’ law	$\oiint_S \boldsymbol{D}.d\boldsymbol{S} = \sum_i q_i$	$\oiint_S \boldsymbol{B}.d\boldsymbol{S} = \sum_i p_i = 0$	$\oiint_S \boldsymbol{\Gamma}.d\boldsymbol{S} = \sum_i m_i$
flux density due to point source	$\boldsymbol{D} = \frac{q}{4\pi\varepsilon r^2}\hat{\boldsymbol{r}}$	$\boldsymbol{B} = \frac{p}{4\pi\mu r^2}\hat{\boldsymbol{r}}$	$\boldsymbol{\Gamma} = G\frac{m}{r^2}\hat{\boldsymbol{r}}$
Force between point sources	$\boldsymbol{F} = \frac{q_1 q_2}{4\pi\varepsilon r^2}\hat{\boldsymbol{r}}$	$\boldsymbol{F} = \frac{p_1 p_2}{4\pi\mu r^2}\hat{\boldsymbol{r}}$	$\boldsymbol{F} = G\frac{m_1 m_2}{r^2}\hat{\boldsymbol{r}}$

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* Note that magnetic monopoles do not exist; relationships involving the symbol p for magnetic pole strength in this table are useful in the study of permanently magnetized specimens which can be modelled as a system of point poles, for example a magnetic needle.