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Examining difficulties in initial algebra: Prerequisite and algebra content areas for Irish post-primary students.

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This research aims to investigate the algebraic under performance of second year post-primary students in Ireland (approximate age 14 years). To this end a diagnostic test for algebra has been developed to profile and identify students who are struggling with algebra. This paper examines the development of the test, which involved the identification of key mathematical content areas that are critical for success in algebra. Both prerequisite, and algebra content areas are key to a students' success in algebra and how each of these areas contribute to a students' progress with algebra is discussed in this theoretical paper. Test items have been selected and adapted from the literature which are aligned with both the key content areas and the Irish mathematics syllabus at junior cycle, the initial three years of post-primary education in Ireland.

Keywords: Algebra, Diagnostic Test, Secondary Education.

Introduction

This paper reports on the development of a diagnostic test for algebra, designed specifically for second year post-primary students in Ireland. It is noted in the literature that few adequate assessments are available to provide formative information on a students' progress with algebra, but they are essential to allow timely and informed instructional decisions for teachers (Ketterlin-Geller, Gifford, & Perry, 2015). An outcome of this research will be a validated diagnostic test for algebra, specifically designed for the Irish syllabus, and relevant for syllabi worldwide. The test can be used by teachers during a normal class period to help inform their instruction of algebra. This paper focuses on identifying key content areas which are required for success in algebra and accordingly developing a bank of questions to support the development of a diagnostic test.

Background and Rationale

The mathematical deficiencies of students entering third level education in Ireland and internationally is widely reported and commonly referred to as the "Maths Problem" (Treacy & Faulkner, 2015). A contributory factor to the problems experienced at third level may be difficulties encountered by students at post-primary level. Over a decade ago it had become apparent that there were issues with the mathematics curriculum at post-primary level in Ireland, and, as a result, radical reform of the syllabi, introduced as "Project Maths" has taken place in the past decade (Treacy & Faulkner, 2015). In Ireland, post-primary education is completed in two cycles; the junior cycle (years 1-3 of secondary level) and the senior cycle (years 5-6 of secondary level). Mathematics is offered at higher, ordinary and foundation levels throughout. The vast majority of students study mathematics throughout their time in post-primary education and the syllabus is delivered in five interwoven strands; 1. Probability and Statistics, 2. Trigonometry and Geometry, 3. Number, 4. Algebra and 5. Functions.

The focus of this study is on algebra given the poor performance of post-primary students in Ireland (Department of Education and Skills, 2015). Despite the changes made to bring the Irish syllabus and teaching methods in line with international standards, it is clear that problems with algebra persist. This is evidenced in the latest chief examiners report which states students need to "gain comfort and accuracy in the basic skills of computation, algebraic manipulation and calculus" (Department of Education and Skills, 2015 p. 29). It is acknowledged that algebra serves as a gateway to higher mathematics and deficiencies in basic algebra result in overall mathematical deficiencies in students (Lawson, 1997; Stephens, Knuth, Blanton, Isler, Gardiner, & Marum, 2013). However, issues with the teaching and learning of algebra remain internationally, and research into progress measuring instruments to inform the area continue (Ketterlin-Geller et al., 2015). Given the poor performance of Irish students in algebra throughout post-primary level, there is a clear requirement to investigate students' ability in algebra at an early stage. To this end a diagnostic test has been developed to give insight into how second year students, age approximately fourteen, perform in the area of algebra. It is noted that many instruments exist to measure performance in algebra, however none that focus on all the prerequisite and algebra content areas have been identified in the literature. This void together with the additional need to have an instrument specifically designed for the Irish context has given rise to the development of this particular diagnostic test.

Theoretical Frame

The teaching and learning of algebra in Ireland aligns closely with Kieran's (2004) model, which outlines three activities that learners of school algebra must participate in: 1. Generational activities, 2. Transformational activities and 3. Global/Meta Level activities. It is noted that two aspects of algebra underlie all others, namely generality and abstraction (Department of Education and Skills, 2016 p. 26). These aspects have led to the definition of three types of algebraic activities that mathematics students in Ireland must engage in - representational, transformational and activities involving generalising and justifying. The test items have been designed to support the implementation of the revised syllabus and in line with the approach to algebra being used in Ireland.

The theoretical framework guiding this research is that the conceptual errors students make in working with algebra perpetuate the difficulties they encounter, subsequently interfering with their understanding of various algebraic concepts (Russell, O'Dwyer, & Miranda, 2009). The test is designed to identify the conceptual errors students make when working with a particular concept, consequently informing the teacher who can then guide instruction aimed at alleviating these conceptual errors, thereby improving a student's knowledge of an underlying concept and algebra as a whole (ibid.:2009). The following section outlines the content areas identified throughout the literature as pertinent to success in algebra, on which test items have been based.

Prerequisite and Algebra Content Areas

As stated the focus of this paper is the key content areas required for success in algebra. These content areas are outlined in Table 1 (Bush & Karp, 2013; Warren & Cooper, 2008).

Content Areas	Junior Cycle Syllabus
Ratios and proportional relationships	*3.1 Number Systems:* - consolidate their understanding of the relationship between ratio and proportion. 4.4 Examining algebraic relationships: – proportional relationships
Fractions	*3.1 Number Systems*: - Investigate models to think about operation on fractions. - Use the equivalence of fractions, decimals and percentages to compare proportions.
Decimals and Percentages	*3.1 Number Systems*: - Calculate percentages - Use the equivalence of fractions, decimals and percentages to compare proportions.
Integers	*3.1 Number Systems*: - Investigate models, such as the number line, to illustrate the operations on integers
Exponents	3.2 Indices
Order of operations	*3.1 Number Systems*: - Appreciate the order of operations, including use of brackets
Properties of numbers	*3.1 Number Systems*: - Investigate the properties of arithmetic and the relationships between them.
Compare and order numbers	*3.1 Number Systems*: - Use the number line to order natural numbers, integers and rational numbers. - Use the equivalence of fractions, decimals and percentages to compare proportions.
Equality	*3.1 Number Systems:* - Consolidate the idea that equality is a relationship in which two mathematical expressions hold the same value.
Variables	4.6 Expressions: - Using letters to represent quantities that are variable.
Algebraic expressions	4.6 Expressions: - Arithmetic operations on expressions. - Transformational activities
Algebraic equations	4.7 Equations and inequalities: - Selecting and using suitable strategies for finding solutions to equations and inequalities.
Functions	*4.2 Representing situations with table diagrams and graphs:* - use tables, diagrams and graphs as a tool for analysing relations – present and interpret solutions, explaining and justifying methods, inferences and reasoning.
Patterns	*4.1 Generating arithmetic expressions from repeating patterns:* - use tables and diagrams to represent a repeating-pattern situation – generalise and explain patterns and relationships in words and numbers – write arithmetic expressions for particular terms in a sequence.

Table 1: Content Areas for Diagnostic Test and alignment with the Junior Cycle syllabus

One potential criticism of the approach used in developing this test instrument is that it is a test of mathematics globally, given the broad range of content areas included. However, difficulties with algebra lie with both algebra itself and also with other areas of mathematics that students will have encountered, which are seen as prerequisites for the learning and understanding of algebra.

Problems and misconceptions in any one of the content areas can lead to problems in the other thus hindering a student's progress in algebra as a whole.

Related content domains from the Irish syllabus were used as a framework to align the prerequisite and algebra content areas identified in the literature. The Number Strand (3) of the syllabus builds on primary school learning and facilitates the transition from arithmetic to Algebra (Strand 4). The Common Introductory Course (CIC) is the minimum course to be covered by all students at the beginning of the junior cycle, elements from the CIC are in italics within Table 1. The numbering within Table 1 refers to the strand numbers, for instance 3.2 refers to section 2 of strand 3. The diagnostic test has been developed for use with students in second year and therefore it was important to align the test items specifically with the content of the CIC. Once the CIC is complete teachers use their own discretion to introduce their topics (Department of Education and Skills, 2016). There is no prescribed structure for following the syllabus however, it is desirable that students will have completed their basic algebra skills including equation solving by the end of first year (Project Maths, n.d.-a).

Prerequisite Content Areas

Difficulties with algebra lie with both algebra itself and also with other areas of mathematics that students will have encountered, which are seen as prerequisites for the learning and understanding of algebra. Proportional reasoning is a key aspect of numeracy and it leads to relational thinking which is important in the development of algebraic skills, it is highly conceptual and a skill that develops gradually. Equally, fractions are an integral part of algebra and can be found as coefficients, constants and solutions to equations, the slope of a line, and, in general, proportions are written in fraction form in algebra (Bush & Karp, 2013). Knowledge of decimals, their value and placement on a number line, computation with decimals, and the ability to convert between decimals, fractions and percentages is also important for success in algebra (Bush & Karp, 2013). Studies have been conducted to identify what element of decimal and fraction understanding best indicate a students' performance in algebra. It has been found that the relational understanding of the bipartite format of a fraction and unidimensional magnitude, measured with the placement of decimals on a number line are the best predictors (DeWolf, Bassok, & Holyoak, 2015).

In learning about fractions, decimals and percentages, with the use of number lines and graphs, students at junior cycle are expected be able to compare and order numbers. This provides students with the skills and knowledge to apply the rules correctly when working with variables. It also enables a student to assess if a solution to an equation or inequality is reasonable. However, if students do not understand a fraction, decimal or percent, they are unable to extend their understanding to which is greater than or less than or equivalent (Bush & Karp, 2013).

Furthermore, a solid understanding and procedural fluency with integers is required for success in algebra. Misconceptions about negative integers can impede progression, where, for example, a student may fail to accept a negative number as a solution to an equation. Research has suggested that the number line and graphs of functions can be used to help correct students' misunderstandings and conceptual errors with integers (Bush & Karp, 2013). Equally, an understanding of exponents is required for both the transformational skills, in dealing with expressions, and the generational and global/meta level skills, where knowledge of the shape of functions are required (Bush & Karp, 2013). Moreover, to succeed in the transformational rules of algebra it is essential to understand the order of operations. Some students believe that order of operations do not matter, that the same answer will result regardless. Others believe that the context of the problem determines the order of operations and in the absence of context operations should be performed from left to right. Research suggests that students should learn the hierarchy of operations more naturally by attending to more complicated operations first (Bush & Karp, 2013).

Finding equivalent expressions is frequently required in algebra, and this manipulation requires an underlying sense of the properties of numbers. Allowing students to investigate the properties of numbers will assist in learning, retaining knowledge and developing relational understanding, which in turn will create a strong foundation for algebra (Bush & Karp, 2013). Numerous studies have focused on development of the concept of the equal sign in the early stages of learning algebra (Bush & Karp, 2013). Students often misinterpret the meaning of the sign viewing it as an operational sign. Those who interpret the equal sign correctly and see it as a relational symbol have more flexibility when working with equations.

Algebra Content Areas

Kieran (1992) asserts that many misconceptions and common errors in algebra are generally rooted in the meaning of symbol or the letters used in algebra. Much research has been conducted into students' difficulties in working with algebraic variables and the misconceptions student's hold. These misconceptions include viewing variables as labels, the belief that the value of a variable has something to do with its position in the alphabet, and the belief that a variable is just a missing value rather than something which has varying values. These difficulties are then compounded when a student attempts to create and manipulate an algebraic expression (Bush & Karp, 2013).

The underlying misconceptions and difficulties students hold in relation to variables, expressions and indeed all the prerequisite content areas can then lead to difficulties in solving algebraic equations. The ability to solve equations is reliant on both procedural and conceptual understanding. Conceptual understanding is strongly related to student's equation solving performance, as without it students learn by rote a series of transformational rules for dealing with equations. A solid understanding of how to use variables to write algebraic expressions, form subsequent equations and solve when necessary is the essence of success in algebra at junior cycle level (Bush & Karp, 2013).

A function is defined as a correspondence between two sets (Kieran, 1992), and there are two general approaches to teaching and learning functional relationships mentioned in the literature; a correspondence approach and a covariation approach (Ayalon, Watson, & Lerman, 2015). The correspondence approach deals with an input-output model, whereby an output value y is calculated for a given input value x, often listed in a table of values or as couples. This approach allows for determining the rule which generates the y-value from the x-value and is in line with the approach to teaching functions in Ireland. The concept of a function is not simple when you consider that at least three representations are used to convey the notion of a function; a table, a graph and an equation. True procedural fluency and competency in working with functions is obtained when one can move between the different representations of a function with ease and this aligns with the multi-representations approach advocated in the Irish syllabus (Bush & Karp, 2013; Project Maths, n.d.-a)

Finally, algebra can be seen as the language used to describe patterns and relationships for the ultimate goal of problem solving and as a systematic way of expressing generality (Project Maths, n.d.-a). Students at junior cycle learn to identify the relationship which lies between the pattern and its position is a functional relationship meaning an expression or formula must be created using variables. In doing this a context for the use of variables is set for Irish students, assisting their understanding of a variable as a varying quantity rather than a specific unknown, laying down the foundation for understanding expressions and solving equations in what is known overall as a functions based approach to algebra (Project Maths, n.d.-a).

The Diagnostic Test

Test items were taken from previous relevant studies pertinent to measuring ability in the core content areas required for algebra outlined in Table 1. The diagnostic test currently contains twenty one questions summarized in Table 2 where the source of each test question is detailed.

Content Areas	**Test Question Number and Source**
Ratio and proportion	1. Number Line/Decimal Magnitude from DeWolf et al. (2015) 7. Proportional reasoning from Hilton, Hilton, Dole, and Goos (2013)
Fractions	2. 4. 5. Fraction Knowledge from DeWolf et al. (2015) 3. Fraction Knowledge multiplication (Bush & Karp, 2013)
Decimals and Percentages	1. Number Line/Decimal Magnitude from DeWolf et al. (2015) 11. Comparing and Ordering Numbers, Project Maths (n.d.-b)
Integers	15. Integers and equations adapted from Vlassis (2008)
Exponents	6. and 8. adapted from discussion in Mok (2010)
Order of operations	9. Order of operations adapted from Linchevski and Livneh (1999)
Properties of numbers	10. Distributive property, adapted from discussion in Mok (2010)
Comparing and ordering numbers	11. Comparing and Ordering Numbers, Project Maths (n.d.-b)
Equality	12. 13. adapted from Stephens et al. (2013)
Variables	14. Variable as label adapted from Küchemann (1981)
Algebraic expressions	16. adapted from Hodgen, Kuchemann, Brown, and Coe (2009) 17. Simplifying expressions based on errors discussed in Kieran (1992)
Algebraic equations	20. adapted from Clement, Lochhead, and Monk (1981) 18. 19. Next step of solution adapted from Chung and Delacruz (2014) 21.3 Forming equations adapted from Ayalon et al. (2015)
Patterns	21 Interpreting from a geometric pattern from Ayalon et al. (2015)

Table 2: Summary of content and source of items on the diagnostic test

An example of a test item, which assesses relational fraction knowledge, together with understanding of a variable and algebraic expression is taken and adapted from (DeWolf et al., 2015) as follows;

n is a whole number greater than 0. If n continues to get bigger in value, please circle one of the following options A, B or C in the answer box for what happens to $\frac{1}{n}$. Hint: Think about the following sequence of numbers $\frac{1}{10}, \frac{1}{20}, \frac{1}{30}, \ldots$

Figure 1: Question 5 on the diagnostic test based on relational fraction knowledge

Students have space for workings and are then asked to circle the correct answer from the following options; A. $\frac{1}{n}$ gets very close to 1, B. $\frac{1}{n}$ gets very close to 0, or C. $\frac{1}{n}$ increases in value too. Adaptations from the original question include changing the word "integer" to "a whole number" and offering the "Hint", to ensure the question is more in line with the age profile of those being tested as informed by the pilot of this test and feedback from teachers. The above question was included as relational understanding of a fraction was an element identified in the DeWolf et al. (2015) study to predict performance in algebra. All test items have been developed with such a theoretical underpinning that is using multiple choice responses based on possible conceptual errors. In addition language was adjusted where necessary to make the test items more accessible for fourteen year old students.

Conclusion

There are fresh concerns in relation to student attainment in mathematics in Ireland, specifically algebra and for progression to third level education (Treacy & Faulkner, 2015). There is a clear need to intervene early in the effort to address the issues students are facing with learning and understanding algebra. The overall aim in using this test is to identify conceptual errors that students make in both algebra and the prerequisite content areas required for success in algebra, therefore assisting to identify possible root causes of the students' errors, and as a result, through appropriate intervention improve students' knowledge of algebra and therefore general mathematical ability. Ultimately, this will be a tool for teachers to use in the classroom allowing them to make informed decisions and to plan appropriate interventions (Russell et al., 2009).

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