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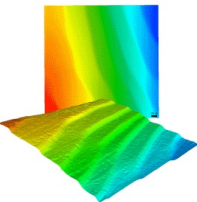
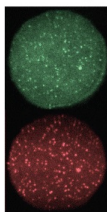


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# Multimode laser beam analyzer instrument using electrically programmable optics

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Presented is a novel design of a multimode laser beam analyzer using a digital micromirror device (DMD) and an electronically controlled variable focus lens (ECVFL) that serve as the digital and analog agile optics, respectively. The proposed analyzer is a broadband laser characterization instrument that uses the agile optics to smartly direct light to the required point photodetectors to enable beam measurements of minimum beam waist size, minimum waist location, divergence, and the beam propagation parameter  $M^2$ . Experimental results successfully demonstrate these measurements for a 500 mW multimode test laser beam with a wavelength of 532 nm. The minimum beam waist, divergence, and  $M^2$  experimental results for the test laser are found to be 257.61  $\mu\text{m}$ , 2.103 mrad, 1.600 and 326.67  $\mu\text{m}$ , 2.682 mrad, 2.587 for the vertical and horizontal directions, respectively. These measurements are compared to a traditional scan method and the results of the beam waist are found to be within error tolerance of the demonstrated instrument. © 2011 American Institute of Physics. [doi:10.1063/1.3669535]

## I. INTRODUCTION

Single mode Gaussian beams are the most commonly encountered laser outputs. They can be completely characterized by the values of minimum beam waist radius and the location of the minimum beam waist.<sup>1</sup> While many lasers are designed to have a single mode Gaussian distribution, in general, lasers with higher transverse order Gaussian modes and/or non-Gaussian distributions exist and the knowledge of the mentioned measurement values to characterize the laser beam is important in many applications. Characterization of these laser beams is done using the  $M^2$  beam propagation parameter.<sup>2-5</sup> For accurate characterization of the  $M^2$  parameter access to both sides of the minimum beam waist is required.<sup>5</sup> When access to both sides of the minimum beam waist is not directly available, a lens is placed in the laser beam path to realize a secondary minimum beam waist such that direct beam waist access from both sides of the auxiliary beam waist is possible.<sup>5</sup> These prior-art methods not only require placement of a lens with a given focal length, but also require physical movement of the laser beam profiler assembly over a span of distances along the beam axis. Additionally, a translating mechanical pinhole within the profiler assembly is used to characterize the second-moment beam radii, adding to the mechanical motion overhead of the overall mechanism for multimode beam characterization.<sup>6,7</sup> This physical movement can lead to errors in system alignment. Hence, the mentioned alignment error can affect the measured beam parameters since alignment of the laser beam with respect to the beam profiler can affect measured beam parameters.<sup>8,9</sup> Hence, these prior-art processes are time-consuming, tedious, and most importantly, prone to poor measurement reliability.

Recently, a beam analyzer using a digital micromirror device (DMD) and an electronically controlled variable focus lens (ECVFL) has been proposed and demonstrated for the characterization of single mode Gaussian laser beams.<sup>10,11</sup> The fundamental design of the deployed beam analyzer is based on the earlier proposed DMD-based optical imager used for irradiance mapping of coherent laser beams<sup>12</sup> and incoherent optical targets.<sup>13</sup> The purpose of this paper is to extend the earlier proposed DMD-ECVFL beam analyzer instrument for the characterization of laser beams using the  $M^2$  beam propagation parameter. Specifically, the paper presents the theory and demonstration of the DMD-ECVFL beam analyzer for measuring the  $M^2$  parameters of a laser beam without requiring the movement of the beam analyzer to different planes along the direction of propagation of the laser beam, thus providing instrument features such as robustness, high repeatability, and improved speed of measurement.

## II. PROPOSED MULTIMODE BEAM ANALYZER

Figure 1 shows the classic prior art method of an  $M^2$  beam propagation parameter beam analyzer operation using a fixed position and focal length lens L to form an auxiliary waist outside of the laser using on-axis translational motion of the classical pinhole profiler along the laser beam. Additionally, an alternate design of the analyzer shown in Fig. 1 involves keeping the profiler location fixed while translating lens L along the beam axis as is done in a commercial instrument.<sup>14</sup> The International Standards Organization (ISO) (Ref. 15) recommends approximately ten measurements on each side of the minimum beam waist, but using the four cuts method,<sup>5</sup> a minimum of four beam profile measurements is sufficient to characterize the beam. Additional measurements

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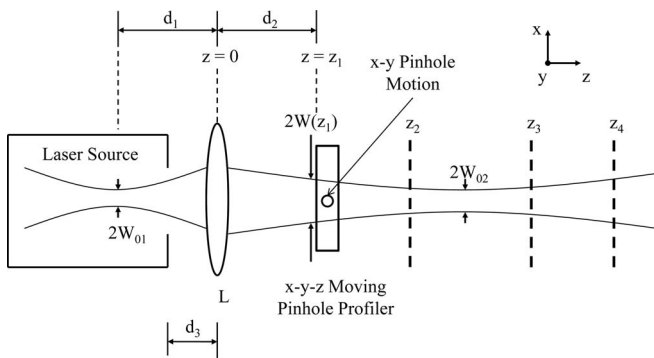
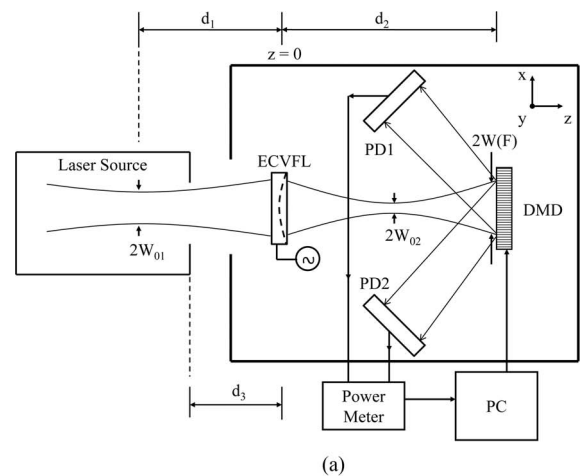


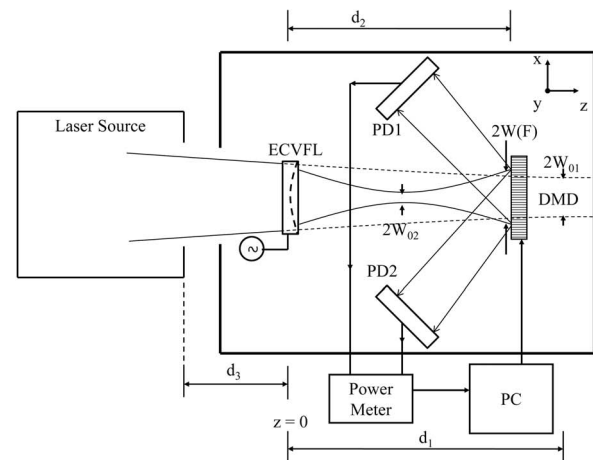
FIG. 1. The traditional moving optics beam analyzer method to determine the  $M^2$  beam parameter. In the shown analyzer operation, the profiler imaging plane is moved along the  $z$  axis and the beam waists are measured at multiple planes like  $z_1, z_2, z_3, z_4$ , etc. The lens  $L$  has a fixed position and focal length  $F$ .

can be taken to reduce the error as is done in commercial devices.<sup>5,14</sup>

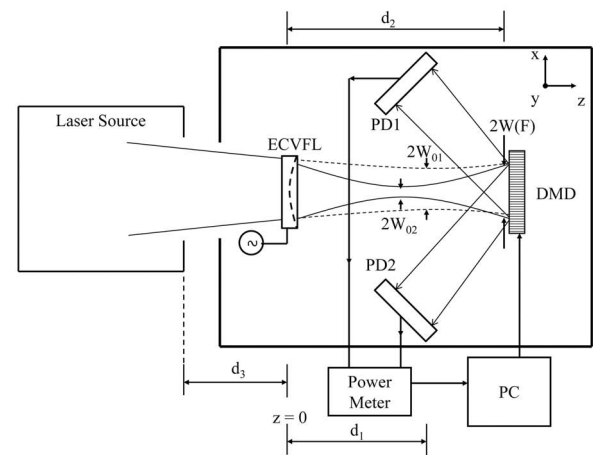
The proposed multimode laser beam characterization instrument is shown in Fig. 2 for three cases of a typical laser minimum beam waist location with respect to the proposed instrument agile optics components. The DMD operates as a digital mode moving pinhole image plane point sampler, while the ECVFL acts as the variable focal length lens operating in analog mode. The distances  $d_1$  and  $d_2$  are the location of the initial minimum beam waist  $W_{01}$  and the distance between the ECVFL and the DMD, respectively. In Fig. 2(a), the input laser beam is located inside the laser assembly and has its minimum beam waist  $2W_{01}$  at location  $d_1$ . The ECVFL is operated with a positive (or convex lens) focal length  $F$ . This causes an auxiliary beam waist  $2W_{02}$  to be formed. The laser beam travels along the optical axis and upon reaching the DMD plane, the beam waist is given as  $W(F, d_2)$  which is dependent upon  $F$ , since  $d_2$  is fixed in this design. DMD sampled light is reflected to the point photodetectors PD1 and PD2 and optical power measurements are normalized to eliminate possible laser power temporal fluctuations during the course of the full 2D beam profile generation. The test beam reflections to PD1 and PD2 are based upon the digital tilt states of the DMD micromirrors. Specifically, the micromirrors have two states, namely, the  $+\theta$  state, which reflects light to PD1, and the  $-\theta$ , which reflects light to PD2. The  $+\theta$  and  $-\theta$  states correspond to the reflections from the moving pinhole micromirrors and the non-pinhole micromirrors, respectively. In contrast to Fig. 2(a), Fig. 2(b) shows the case where  $d_1$  is after the DMD and Fig. 2(c) shows a case where  $d_1$  is between the ECVFL but before the DMD. Note that if the laser minimum beam waist location  $d_1$  is between the ECVFL and DMD, as shown in Fig. 2(b), an ECVFL with negative and positive focal lengths is required to form the necessary auxiliary minimum beam waists, since  $W(F, d_2)$  measurements need to occur on both sides of its minimum value for accurate characterization. In Fig. 2(c), an ECVFL with negative and positive focal lengths might be required to generate the necessary auxiliary minimum beam waists if the initial minimum beam waist  $2W_{01}$  is close (i.e., within a few Rayleigh ranges) to the DMD.



(a)



(b)



(c)

FIG. 2. The proposed multimode beam analyzer when the laser minimum beam waist forms (a) before the ECVFL, (b) after the DMD, and (c) between the ECVFL and DMD.

According to Refs. 2–4, the propagation of a laser beam can be analytically described using the fundamental Gaussian beam mode and the  $M^2$  parameter. In general, multimode

beam radii at the beam waist can be written as

$$W(z) = Mw(z), \quad (1)$$

where  $w(z)$  is the embedded fundamental Gaussian mode second-moment radius definition of the beam irradiance. Now, starting with the analysis for the fundamental Gaussian beam, the multimode beam can be described. The fundamental Gaussian beam optical field can be described according to Ref. 1 as

$$\psi(r, z) \propto \exp\left(\frac{-jkr^2}{2q(z)}\right), \quad (2)$$

where the beam is traveling along in the  $z$  direction,  $r$  is the radial distance from the optic axis,  $k$  is the wave number, and the complex  $q$  parameter is

$$\frac{1}{q(z)} = \frac{1}{R(z)} - j\frac{\lambda}{\pi w^2(z)}, \quad (3)$$

where  $\lambda$  is the wavelength of the light and  $R(z)$  the radius of curvature. At the minimum beam waist, the radius of curvature is infinite and the  $q$  parameter at  $d_1$  reduces to

$$\frac{1}{q_{01}} = -j\frac{\lambda}{\pi w_{01}^2}. \quad (4)$$

The  $q$  parameter at the DMD plane can be found according to Ref. 1 as

$$\frac{1}{q_1} = \frac{Cq_{01} + D}{Aq_{01} + B}, \quad (5)$$

where  $A$ ,  $B$ ,  $C$ , and  $D$  are the ABCD matrix elements describing the transfer of paraxial rays through an optical system. According to Ref. 11, this is given by

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 1 - d_2/F & d_1 + d_2 - d_1d_2/F \\ -1/F & 1 - d_1/F \end{bmatrix}. \quad (6)$$

Now, according to Ref. 11 using Eqs. (4)–(6), one can find that

$$w^2(F, d_2) = w_{01}^2 \left[ (1 - d_2/F)^2 + \left( \frac{\lambda(d_1 + d_2 - d_1d_2/F)}{\pi w_{01}^2} \right)^2 \right]. \quad (7)$$

Substitution of Eq. (1) into Eq. (7) and solving for the beam waist at the DMD plane gives

$$W^2(F, d_2) = W_{01}^2 \left[ (1 - d_2/F)^2 + \left( \frac{M^2\lambda(d_1 + d_2 - d_1d_2/F)}{\pi W_{01}^2} \right)^2 \right]. \quad (8)$$

Now, the full angle divergence and Rayleigh range of the multimode beam before the system can be written as<sup>5</sup>

$$\Theta_{01} = \frac{2\lambda M_{01}^2}{\pi W_{01}}, \quad (9)$$

$$z_{R1} = \frac{\pi W_{01}^2}{\lambda} = \frac{\pi W_{01}^2}{M_{01}^2 \lambda}. \quad (10)$$

All beam waist measurements are defined using second-order moments since this is what the  $M^2$  beam parameter is based

upon.<sup>15</sup> According to Refs. 16–20, this definition is valid for any arbitrary real laser beam. The ISO also recommends using this second-order moment in laser beam measurements.<sup>15</sup> Now, using a pinhole for measurements, this beam radius can be written according to Refs. 3 and 11 as

$$W_H(F, d_2) = 2\sqrt{\frac{\sum_x \sum_y I(x, y)(x - x_0)^2}{\sum_x \sum_y I(x, y)}}, \quad (11)$$

for the horizontal beam waist at the DMD plane. Here,  $I(x, y)$  is the irradiance falling on the DMD pinhole located at the point  $(x, y)$  and  $x_0$  is the centroid of the beam in the  $x$  direction given as

$$x_0 = \frac{\sum_x \sum_y I(x, y)x}{\sum_x \sum_y I(x, y)}. \quad (12)$$

Similarly, for the vertical beam waist radius,

$$W_V(F, d_2) = 2\sqrt{\frac{\sum_x \sum_y I(x, y)(y - y_0)^2}{\sum_x \sum_y I(x, y)}}, \quad (13)$$

where  $y_0$  is the centroid of the beam in the  $y$  direction given as

$$y_0 = \frac{\sum_x \sum_y I(x, y)y}{\sum_x \sum_y I(x, y)}. \quad (14)$$

As shown in Fig. 2 to measure the beam waist, a moving pinhole is created using a DMD-based profiler.<sup>10–13</sup> The DMD-based profiler creates a digital moving pinhole which is highly repeatable, accurate, and has a spatial mapping resolution equal to one pixel pitch of the DMD. When compared to prior macro-motion mechanical profiling techniques, the DMD-based profiler offers 100% accurate digital spatial mapping repeatability. Using software control, the DMD realized pinhole scans across the test beam at the DMD plane and the instrument calculates the beam waists  $W_H(F, d_2)$  and  $W_V(F, d_2)$  for the horizontal and vertical directions using Eqs. (11)–(14). Note that  $d_2$  is a chosen fixed value for the measurement. Once the beam waists have been measured at a given  $F$  value of the ECVFL,  $F$  is changed while keeping  $d_2$  fixed and the beam at the DMD plane is profiled again. This process is continued until several (e.g., four or more) beam waist measurements for different  $F$  values have been taken to provide a good data fit to Eq. (8). The curve fitting to Eq. (8) gives values for  $W_{01}$ ,  $d_1$ , and  $M^2$ . Next, using Eqs. (9) and (10) with the experimentally determined curve fit values, the divergence and Rayleigh range of the multimode beam can be found. Hence, using the proposed instrument and the steps described for operation of the instrument in Fig. 2, the multimode test beam is completely characterized.

### III. EXPERIMENTAL RESULTS

To characterize the multimode laser beam shown in Fig. 1, prior art method was implemented using a 500 mW Nd:YAG  $\lambda = 532$  nm frequency doubled laser source with a lens  $L$  of focal length 15 cm, and a DMD-based profiler. This visible band TI DMD has  $\theta = 12^\circ$ , a pixel pitch of 13.68  $\mu\text{m}$ , and 1024 by 768 micromirrors. The distance between the laser aperture and  $L$  was  $d_3 = 16.7$  cm and for eye safety, the laser

TABLE I. Second-moment beam radii as a function of  $d_2$  with  $F = 15$  cm.

$d_2$ (cm)	$W_V$ ( $\mu\text{m}$ )	$W_H$ ( $\mu\text{m}$ )
7.96	165.701	223.305
8.46	151.984	202.412
8.96	121.902	165.093
9.46	114.136	149.015
9.96	93.199	126.767
10.46	76.555	105.068
11.46	56.863	74.962
12.46	62.95	74.114
13.46	85.65	100.283
14.46	121.24	145.05
15.46	155.899	192.288
16.46	198.144	252.744
17.46	242.807	307.636
19.96	328.184	433.399

beam was attenuated using a neutral density filter with a 1% transmission. The light reflected off the DMD was spatially filtered so that only the first-order reflections were captured by PD1 and PD2. These detectors are 1 cm<sup>2</sup> active area 918D-UV detectors from Newport connected to a 2931C Newport Power Meter. The required beam irradiance  $I(x, y)$  measurements were taken at multiple  $d_2$  values along the z axis. Using the DMD profiler with this measured  $d_2$ -dependent  $I(x, y)$  data and using Eqs. (11)–(14), the experimentally deduced values of  $W_V(F, d_2)$  and  $W_H(F, d_2)$  are computed and displayed in Table I. Next, as shown in Fig. 3, curve fitting of the data from Table I is applied to Eq. (8) and the parameters  $W_{01}$ ,  $d_1$ , and  $M^2$  are found to be equal to 241.47  $\mu\text{m}$ ,  $-44.39$  cm, 1.337 and 302.93  $\mu\text{m}$ ,  $-48.48$  cm, 2.095 for the horizontal and vertical directions, respectively. The negative sign on distances means that the beam waist occurs after the ECVFL. Next, using Eqs. (9) and (10) with the experimentally determined curve fit values of  $W_{01}$ ,  $d_1$ , and  $M^2$ , the divergence and Rayleigh range of the multimode Gaussian beam are found to be 1.875 mrad, 25.76 and 2.342 mrad, 25.87 cm for the vertical and horizontal directions, respectively. Note that all  $I(x, y)$  have a spatial error/tolerance of one pinhole size, which, in this case, is one micromirror pitch (i.e., 13.68  $\mu\text{m}$ ) of the DMD.

Next, the proposed multimode beam analyzer in Fig. 2(a) was set up using the same DMD, laser source, detectors, and power meter shown in Fig. 1. The lens L in the setup shown in Fig. 1 is replaced by the ECVFL in the setup shown in Fig. 2. The distances  $d_1$  and  $d_3$  are the same as the experiment shown in Fig. 1. The distance between the ECVFL and DMD was fixed to  $d_2 = 32$  cm. The ECVFL used is a Varioptic Artic France Model 320 liquid lens, which has a 3.0 mm clear aperture and can operate as a concave or convex lens.<sup>21</sup> The Varioptic voltage controller with a drive signal was used to adjust  $F$  from 8 to 36 cm. When aligning the system, it is important to note that according to Refs. 8 and 9, any changes in the alignment of the transverse beam axes with respect to the beam profiler axes and off-axis propagation can affect instrument-measured beam parameters. One can right away see that such a problem can be very severe using classic prior

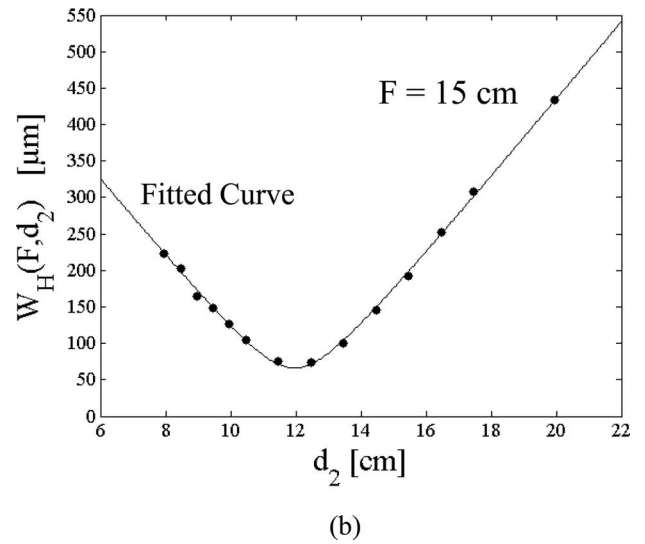
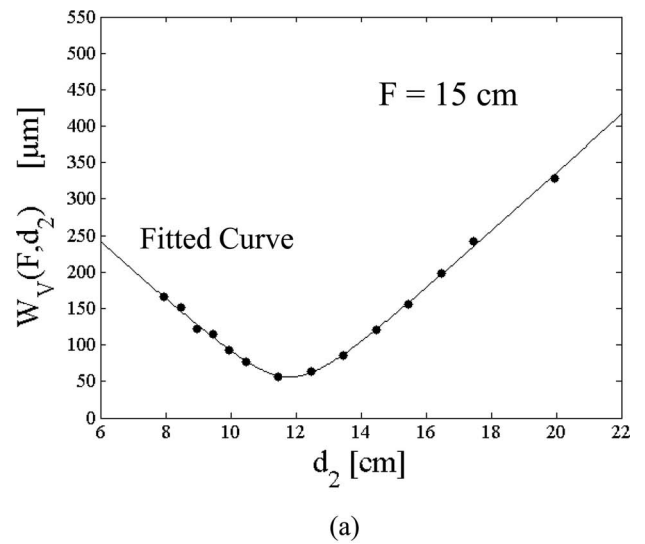


FIG. 3. Characterization of the multimode laser beam using the classic prior-art method with physical motion of the analyzer (i.e., the DMD) to measure the second-moment irradiance beam radii  $W_V(F, d_2)$  and  $W_H(F, d_2)$  at different  $d_2$  positions. Data plots are for (a) the vertical (or y direction) beam waist and (b) the horizontal (or x direction) beam waist.

art methods as shown in Fig. 1 where many moving optics are deployed.

Using the DMD profiler with this  $F$ -dependent measured  $I(x, y)$  data and using Eqs. (11)–(14), the experimentally deduced values of  $W_V(F, d_2)$  and  $W_H(F, d_2)$  are computed. Next curve fitting the data in Table II to Eq. (8), as shown in Fig. 4, the parameters  $W_{01}$ ,  $d_1$ , and  $M^2$  are found to be equal to 257.61  $\mu\text{m}$ ,  $-46.53$  cm, 1.600 and 326.67  $\mu\text{m}$ ,  $-48.99$  cm, 2.587 for the vertical and horizontal directions, respectively. Next, using Eqs. (9) and (10) with the experimentally determined curve fit values, the divergence and Rayleigh range of the laser beam are found to be 2.103 mrad and 24.50 cm for the vertical direction and 2.682 mrad and 24.36 cm for the horizontal direction. The proposed design is compared with the traditional method in Table III. Since the traditional scan method using the DMD profiler has a maximum error of one micromirror and the proposed DMD-ECVFL-based method has a maximum error of one micromirror for each beam waist

TABLE II. Second-moment beam radii as a function of  $F$  with  $d_2 = 32$  cm.

$F$ (cm)	$W_V$ ( $\mu\text{m}$ )	$W_H$ ( $\mu\text{m}$ )
9.0	272.612	333.124
11.5	153.630	216.444
13.0	114.228	161.864
15.5	76.470	114.233
17.0	62.985	79.437
20.2	88.700	97.943
22.2	159.705	207.131
34.6	286.000	354.599

measurement, the maximum possible error between the measurements from the two instruments should be two micromirrors (i.e.,  $27.36 \mu\text{m}$ ). Thus, it is seen from Table III that the two-instrument experimental difference is within this two micromirror tolerance for  $W_0$  for which the difference is 16.14

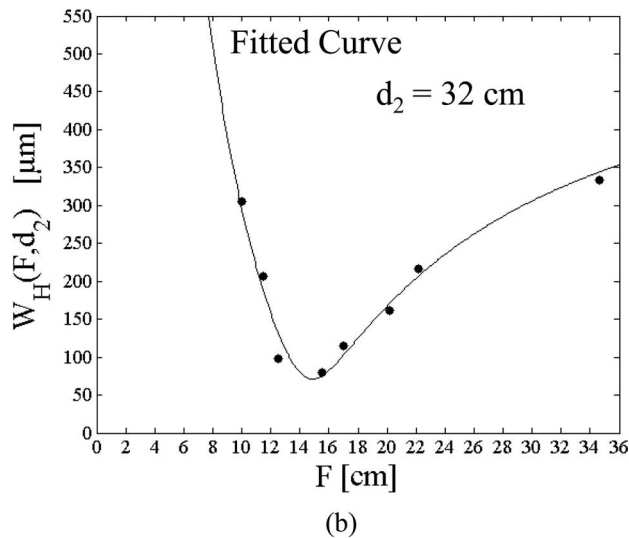
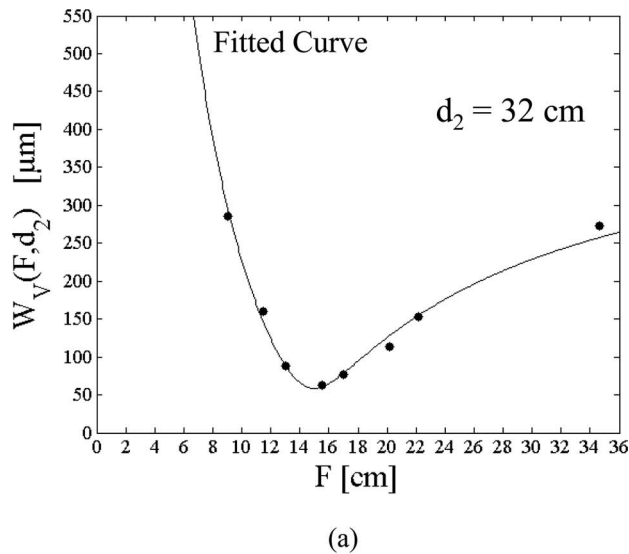


FIG. 4. Characterization of a multimode laser beam using the proposed DMD-ECVFL beam analyzer. Data shown are the second-moment irradiance beam waist radius as a function of the ECVFL focal length  $F$  for (a) the vertical (or  $y$  direction) and (b) the horizontal (or  $x$  direction) beams.

TABLE III. Multimode beam propagation parameters via the experimentally demonstrated traditional and proposed multimode beam analyzers.

	Horizontal		Vertical	
	Traditional	Proposed	Traditional	Proposed
$W_0$ ( $\mu\text{m}$ )	302.93	326.67	241.47	257.61
$\Theta$ (mrad)	2.342	2.682	1.875	2.103
$d_I$ (cm)	-48.48	-48.99	-44.39	-46.53
$z_{R1}$ (cm)	25.87	24.36	25.76	24.50
$M^2$	2.095	2.587	1.337	1.600

and  $23.74 \mu\text{m}$  for the vertical and horizontal directions, respectively. It is important to remember that measurements be taken on both sides the minimum  $W_V(F, d_2)$  and  $W_H(F, d_2)$ , as seen in Fig. 4. Also, increasing the number of measurements near the minimum increases the accuracy of the minimum beam waist size.

#### IV. CONCLUSION

Presented for the first time, to the best of the authors' knowledge, is a multimode beam analyzer using an ECVFL and a programmable digital spatial light modulator. The analyzer can be used in determining the beam waist size, the minimum beam waist location, the beam divergence, and the  $M^2$  parameter of laser beams.<sup>22</sup> Because the analyzer is electronically controlled, it does not require precision motion mechanics, increasing its reliability, speed, and repeatability. Specifically, having no moving macro-optics gives an advantage over traditional multimode beam analysis methods as misalignment of the beam propagation axis with the optical system as the lens or profiler is moved can affect the measured multimode beam propagation parameters. Experiments conducted with a 532 nm laser beam show that the multimode beam parameters using the proposed analyzer closely match the characterization results obtained using a traditional mechanically moving scan method.

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<sup>1</sup>H. Kogelnik and T. Li, *Appl. Opt.* **5**, 1550 (1966).

<sup>2</sup>D. R. Hall, P. E. Jackson, and M. W. Sasnett, *Physics and Technology of Laser Resonators* (Taylor & Francis Group, New York, 1989), pp. 132–142.

<sup>3</sup>A. E. Siegman, *OSA Trends in Optics and Photonics Series* (OSA, Washington, DC, 1998), Vol. 17, p. 184.

<sup>4</sup>P. A. Bélanger, *Opt. Lett.* **16**, 196 (1991).

<sup>5</sup>T. F. Johnston, *Appl. Opt.* **37**, 4840 (1998).

<sup>6</sup>M. W. Sasnett and T. F. Johnston, Jr., U.S. patent 5,214,485 (25 May, 1993).

<sup>7</sup>A. E. Siegman and S. W. Townsend, *IEEE J. Quantum Electron.* **29**, 1212 (1993).

<sup>8</sup>W. Li, G. Feng, Y. Huang, G. Deng, Y. Huang, J. Chen, and Sh. Zhou, *Laser Phys.* **19**, 455 (2009).

<sup>9</sup>O. Mendoza-Yero, *Proc. SPIE* **5622**, 1094 (2004).

<sup>10</sup>M. Sheikh, P. J. Marraccini, and N. A. Riza, *Proc. SPIE* **7675**, 767508 (2010).

- <sup>11</sup>M. Sheikh and N. A. Riza, *Appl. Opt.* **49**, D6 (2010).
- <sup>12</sup>S. Sumriddetchkajorn and N. A. Riza, *Appl. Opt.* **41**, 3506 (2002).
- <sup>13</sup>N. A. Riza, S. A. Reza, and P. J. Marraccini, *Opt. Commun.* **284**, 103 (2011).
- <sup>14</sup>ModeMaster PC M2 Beam Propagation Analyzer, Coherent, Auburn, CA Datasheet, 2011.
- <sup>15</sup>“Test methods for laser beam parameters: beam widths, divergence angle, and beam propagation factor,” International Standards Organization, Pforzheim, Germany, ISO/TC 172/SC9/WG1, ISO/DIS 11146.
- <sup>16</sup>A. E. Siegman, *Tutorial presented at the OSA Annual Meeting*, Long Beach, California, 1997.
- <sup>17</sup>T. Fang, X. Ye, J. Niu, and J. Xu, *Chin. Opt. Lett.* **4**, 586 (2006).
- <sup>18</sup>Q. Cao and X. Deng, *Opt. Commun.* **142**, 135 (1997).
- <sup>19</sup>P. M. Mejis and R. Martinez-Herroer, *Opt. Lett.* **20**, 660 (1995).
- <sup>20</sup>C Paré and P. A. Bélanger, *Opt. Commun.* **123**, 679 (1996).
- <sup>21</sup>Model Arctic 320 Liquid Lens Technical Datasheet: Optical and Optomechanical Data, Varioptic SA, Lyon, France, Datasheet 2006.
- <sup>22</sup>P. J. Marraccini and N. A. Riza, *Proc. SPIE* **8026**, 8026E (2011).