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1 **Title: Coast/breakwater-integrated OWC: a theoretical model**

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## 28 **Abstract**

29 Integrating wave energy converters into coastal structures such as breakwaters, seawalls or jetties  
30 not only offers benefits in terms of construction costs but also improves wave energy extraction.  
31 In this paper a novel theoretical model based on linear potential flow theory is developed to study  
32 the performance of an oscillating water column (OWC) integrated into a vertical structure in water  
33 of finite water depth. The model has three fundamental advantages relative to previous works: no  
34 thin-wall restriction (the thickness of the OWC chamber wall is considered), no singularities, and  
35 far fewer truncating terms in the eigen-function expansions. The OWC chamber is a vertical  
36 cylinder semi-embedded in the structure with a submerged, open bottom. As water waves impinge  
37 on the structure, the water column in the chamber oscillates and drives an air turbine installed at  
38 the chamber top to extract wave power. Using linear wave theory, the velocity potential in the  
39 water domain is decomposed into scattering and radiation potentials whose unknown coefficients  
40 are determined by the eigen-function matching method. Upon successful validation, the model is  
41 used to investigate the influence of the thickness of the chamber wall and the radius and  
42 submergence of the chamber on wave power absorption.

43

44 **Keywords:** Oscillating Water Column; Breakwater-integrated OWC; Wave energy; Potential flow;  
45 Excitation volume flow; Hydrodynamic coefficients

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## 47 1. Introduction

48 A large number of wave energy conversion concepts have been proposed since 1970s, which  
49 can be roughly classified as: oscillating water column (OWC) (e.g., [1, 2]), point absorber (e.g.,  
50 [3]), attenuator (e.g., [4]), oscillating wave surge converter (e.g., [5]), overtopping (e.g., [6]), and  
51 others. Despite the large number of concepts proposed and investigated so far, only a few wave  
52 energy converters (WECs) have been tested at a large scale, and even fewer have achieved the  
53 fully commercial stage [7, 8]. The challenges in bringing WECs to the market include: high cost  
54 of construction, installation and maintenance; negative environmental impact; poor reliability; and  
55 low power extraction efficiency [9, 10]. It is not easy to solve all these problems concurrently  
56 since some of them might be in conflict – and therein lie the challenges. For example, the  
57 improvement of the reliability of WECs generally results in an increased cost of construction; to  
58 enhance the power capture efficiency more sophisticated systems are typically necessary, but this  
59 very sophistication is generally detrimental to the overall cost and survivability of the system.

60 The integration of a WEC into a marine structure, e.g., a breakwater, as opposed to its stand-alone  
61 deployment in the open sea is an effective means to overcome a number of these challenges and  
62 significantly increase the attractiveness of wave power exploitation [9, 11]. The integration not  
63 only offers benefits in terms of shared costs of construction, but also improves the robustness of  
64 the WEC and minimizes its environmental impact. Additionally, thanks to the wave power  
65 absorbed by the WEC, wave reflection at the structure is diminished, which is often advantageous  
66 from the points of view of coastal protection and non-interference with shipping. The synergies  
67 between wave energy and marine structures have been investigated in a number of works, e.g.,  
68 integration of an array of WECs with a breakwater [12], integration of an OWC into an offshore  
69 wind turbine [13] or breakwater [14].

70 Among wave energy conversion technologies, OWC systems are especially simple, for the only

71 moving mechanical part is an air turbine/generator located above the water; therefore,  
72 OWC-breakwater integration has received considerable attention [9]. Evans and Porter [15]  
73 developed a theoretical model to simulate a two-dimensional (2D) OWC device composed of a  
74 thin vertical surface-piercing barrier in front of a vertical wall. An integral equation for the  
75 horizontal velocity across the gap under the thin barrier was adopted to deal with the singular  
76 behaviour in the velocity field. Theoretical results showed that increasing the distance between the  
77 barrier and the wall decreased the frequency at which resonance occurred. Later, Morris-Thomas,  
78 Irvin [16] examined effect of the front barrier geometry on the performance of the OWC  
79 experimentally. The hydrodynamic efficiency in short waves was found to decrease with the  
80 increase of the barrier's submergence or thickness. More recently, the impact of the underwater  
81 lips of an offshore OWC device in terms of both thickness and submergence was investigated by  
82 Elhanafi, Fleming [17] with a two-dimensional computational fluid dynamics (CFD) model. By  
83 selecting the optimal combination of the submergence and thickness of the lips, a peak efficiency  
84 exceeding 0.79 was achieved, much larger than the 0.3 for a device with simpler, typical geometry.  
85 Other aspects such as the role of the turbine Power Take-Off (PTO) system and the environmental  
86 conditions in the power extraction of an onshore or bottom-fixed, breakwater-integrated OWC  
87 have also been investigated. Sheng, Alcorn [18] proposed a numerical method based on potential  
88 flow theory to assess the primary energy conversion of two generic OWC WECs (one bottom  
89 fixed and another floating). The hydrodynamics and thermodynamics with consideration of the air  
90 compressibility for different types of the air turbine PTOs (i.e., Wells turbine, impulse turbines and  
91 bi-radial turbines) were coupled in the time-domain, and the numerical results appeared accurate  
92 enough for the OWC power extraction assessment, especially for the bottom-fixed OWC. López  
93 and Iglesias [19] developed a virtual laboratory based on artificial neural networks that can be  
94 employed to obtain the pneumatic efficiency of a given OWC under specific wave condition, tidal  
95 level and turbine damping. Physical model tests of these parameters were carried out as well [20].  
96 In order to achieve an optimal energy transfer, Pereiras, López [21] described a methodology for  
97 matching a nonlinear turbine to the OWC chamber. Elhanafi, Fleming [22] adopted a CFD model  
98 to learn the impacts of both the PTO damping and incoming wave height on the performance of an  
99 onshore OWC. The reflection coefficient and the energy absorption coefficient generally increase  
100 and decrease with wave height. Research has also been directed towards other types of onshore or  
101 bottom-fixed, breakwater-integrated OWCs, e.g., the U-type OWC [23, 24] and the multi-chamber  
102 OWC [25]. Additionally, the integration of OWCs with floating breakwaters was considered by  
103 He, Huang [26], He, Leng [27].

104 The above studies are focused on 2D problems of OWC-breakwater integration. In contrast,  
105 there are few studies on its 3D aspects. For experimental work these require a wave basin rather  
106 than a flume, with a scale model of the breakwater as well as the OWC itself [28]. For numerical  
107 work, if the boundary element method is employed, the surfaces of the OWC device and the  
108 surrounding breakwater or coastline must be divided into elements [29]; if the finite element  
109 method is adopted, a numerical wave basin shall be established with the entire water volume  
110 discretized [30]. Due to the considerable experimental and computational cost, 3D studies of  
111 OWC-structure integration are not common. If the shapes of the structure and OWC are regular,  
112 theoretical models may be used to solve the 3D hydrodynamic problem. Martins-rivas and Mei  
113 [31] proposed a theoretical model based on the 3D wave radiation/diffraction theory and the usual  
114 method of eigen-function expansions to study wave power extraction from an OWC at the tip of a

115 long and thin breakwater. The thin-walled OWC was represented by a hollow cylinder in their  
116 model, in which the method for solving the integral equation of Evans and Porter [15] was used to  
117 treat the singular behaviours in the velocity field beneath the thin wall of the OWC chamber. The  
118 linearized air compressibility in the chamber was taken into account as part of the PTO system.  
119 The effects of the radius and submergence of the OWC chamber, the air compressibility and the  
120 incident wave direction were investigated. It was found that the free surface outside was strongly  
121 dependent on the incident wave direction, whereas the power extracted was roughly insensitive to  
122 the incident direction. Subsequently, Martins-rivas and Mei [32] applied the same theoretical  
123 approach to a thin-walled OWC installed on a straight cliff-like coast. The performance of the  
124 OWC was found to strongly depend on the incident wave direction. Wave reflection at the coast  
125 could lead to up to a doubling in the power absorbed by the OWC. The role of either the radius or  
126 submergence of the OWC was not considered. This theoretical model was later applied by Lovas,  
127 Mei [33] to a vertical OWC at the tip of a general wedge-shaped coast.

128 In this context we propose a novel theoretical model based on linear potential flow theory to  
129 evaluate the hydrodynamic performance and power extraction of the OWC. This novel approach  
130 has three fundamental advantages relative to previous works. First, the thin-wall restriction is  
131 removed, i.e., the thickness of the OWC chamber wall is taken into consideration. Second, there is  
132 no singularity. Finally, far fewer truncating terms of the eigen-function expansions are required to  
133 obtain accurate results. The effects of wall thickness, radius and submergence of the OWC on  
134 power extraction can thus be investigated with the present model – as shown below.

135 The rest of the paper is organized as follows. Section 2 presents the relation between the PTO  
136 system and the hydrodynamic problem. The basic governing equation, the boundary conditions for  
137 wave scattering and radiation problems, and the expressions of the scattering and radiated velocity  
138 potentials in different regions of the water domain are developed in Section 3, alongside the  
139 method for solving the unknown coefficients. The expressions of excitation volume flow and  
140 hydrodynamic coefficients are derived in Section 4. The model validation can be found in Section  
141 5. The influence of the radius, wall thickness and submergence of the OWC chamber on wave  
142 power absorption are investigated in Section 6. Finally, conclusions are drawn in Section 7.

## 143 2. Mathematical model

144 Consider an oscillating water column (OWC) installed on a vertical wall in a water domain of  
145 uniform depth  $h$  (see Fig.1). The OWC chamber is composed of a vertical circular cylinder with a  
146 ring shape cross section and it is half embedded in the wall. The outer and inner radii of the OWC  
147 chamber are denoted as  $R$  and  $R_i$ , respectively. On the seaside, the chamber is open from a finite  
148 submergence, denoted as  $d$ , to the seabed. As water waves propagate in the direction of  $\beta$ , the  
149 water column enclosed by the OWC chamber oscillates and drives a Wells turbine (not plotted in  
150 Fig. 1) installed at the chamber top to extract wave power.

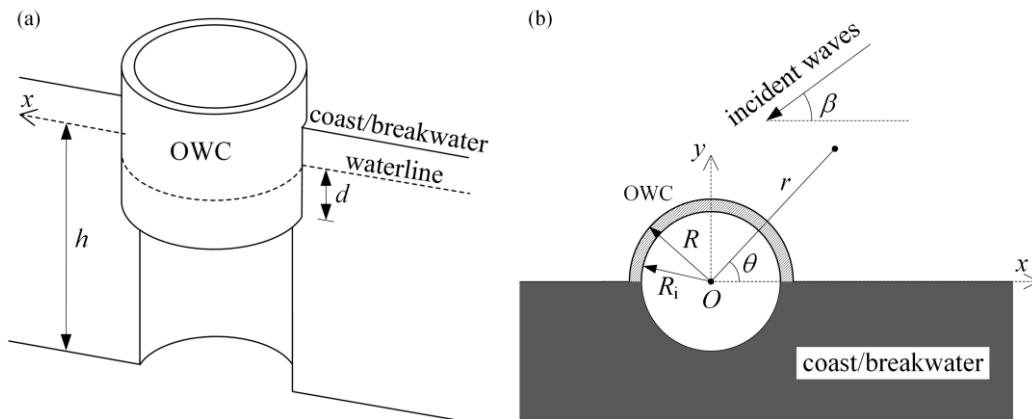


Fig. 1. Oscillating water column integrated into a coast/breakwater: (a) bird view; (b) top view.

As shown in Fig. 1, a general Cartesian coordinate system  $Oxyz$  is adopted with the  $Oz$  axis at the location of the symmetrical vertical axis of the OWC pointing upward and the  $Ox$  axis along the waterline at the coast/breakwater. A polar coordinate  $(Or\theta)$  is defined as given in Fig. 1b.

Subjected to regular waves of angular frequency  $\omega$  with small amplitude, the flow problems may be treated in the linear potential theory regime in the frequency domain based on the assumption that the fluid is isotropic, incompressible inviscid, and the time-harmonic flow is irrotational. The fluid motion can be described by the velocity potential  $\text{Re}[\Phi(r, \theta, z)e^{-i\omega t}]$ , where  $\Phi$  is a complex spatial velocity potential independent of time,  $t$ , and satisfies the Laplace equation,  $i$  represents imaginary unit. In a similar way, the air pressure inside the OWC chamber can be written as  $\text{Re}(pe^{-i\omega t})$ , where  $p$  is the complex air pressure amplitude inside the OWC chamber.

The spatial velocity potential  $\Phi$  can be decomposed into the wave spatial potential,  $\Phi_I$ , which represents the wave field when the vertical wall without OWC (i.e., a flat wall) is subjected to monochromatic incident waves, the diffracted wave spatial potential  $\Phi_D$  induced by the existence of the OWC, and the radiated wave spatial potential as follows

$$\Phi = \Phi_I + \Phi_D + p\Phi_R, \quad (1)$$

where  $\Phi_R$  is the spatial velocity potential due to unit air pressure oscillation inside the OWC chamber.  $\Phi_I$ ,  $\Phi_D$ , and  $\Phi_R$  all satisfy the Laplace equation, the boundary condition at the side wall of the coast/breakwater and the seabed boundary condition. Moreover,  $\Phi_D$  and  $\Phi_R$  must satisfy a radiation condition at infinite distance. Hereinafter, the sum of the incident and diffracted velocity potentials, which is the so-called scattering velocity potential (i.e.,  $\Phi_S = \Phi_I + \Phi_D$ ), is adopted for the sake of simplicity.

Considering the air turbine employed in the OWC is an idealized lossless linear Wells turbine and assuming the mass flux through the Wells turbines is proportional to the chamber air pressure, following Sarmento and Falcão [34] and Martins-rivas and Mei [32], the complex air pressure amplitude,  $p$ , is related to the scattering and radiated velocity potentials by:

$$\left[ -i(a_{\text{PTO}} + a) + (c_{\text{PTO}} + c) \right] p = Q_e, \quad (2)$$

where  $a_{\text{PTO}}$  is used to take into account the effect of air compressibility, and can be expressed as  $a_{\text{PTO}} = \omega V_0 / (v^2 \rho_0)$ , in which  $V_0$  is the air chamber volume,  $v$  denotes the sound velocity in air and  $\rho_0$  represents the static air density;  $c_{\text{PTO}}$  is the damping of the PTO system depending on the

183 rotational speed of turbine blades, the scales of turbine rotor, the design of turbines and  $\rho_0$  as well;  
 184  $Q_e$ , the so-called excitation volume flow, is the rate of upward displacement of the water surface  
 185 inside the column contributed by the scattering potential:

$$186 \quad Q_e = \int_0^{2\pi} \int_0^{R_i} \frac{\partial \Phi_S}{\partial z} \Big|_{z=0} r dr d\theta, \quad (3)$$

187  $c$  and  $a$  are hydrodynamic coefficients that can be derived from the volume flow inside the column  
 188 induced by the radiated potential (i.e.,  $Q_R$ ),

$$189 \quad -(c - ia) = Q_R = \int_0^{2\pi} \int_0^{R_i} \frac{\partial \Phi_R}{\partial z} \Big|_{z=0} r dr d\theta. \quad (4)$$

190 The time-averaged power extraction by the PTO system (i.e., the Wells turbine),  $P$ , can be  
 191 calculated by:

$$192 \quad P = \frac{c_{PTO}}{2} |p|^2 = \frac{c_{PTO}}{2} \frac{|Q_e|^2}{(c + c_{PTO})^2 + (a + a_{PTO})^2}. \quad (5)$$

193 The efficiency of wave power extraction is generally expressed by the relative wave capture  
 194 width

$$195 \quad \eta = \frac{kP}{P_{in}} = \frac{2kP}{\rho g A^2 c_g}, \quad (6)$$

196 where  $P_{in}$  is the incident wave energy per unit width of the wave front;  $c_g$  is the group velocity of  
 197 the incident wave.

198 Note that  $Q_e$ ,  $c$  and  $a$  are fundamental for evaluating the performance of the OWC. In order to  
 199 obtain these parameters, wave scattering and radiation problems, i.e.,  $\Phi_S$  and  $\Phi_R$ , should be solved  
 200 first.

### 201 3 Solution to scattering/radiated potentials

202 The governing equation and the boundary conditions for  $\Phi_\chi$  ( $\chi=S, R$ ) can be written as follows:

$$203 \quad \frac{\partial \Phi_\chi}{\partial z} = 0, \quad z = -h, \quad (7)$$

$$204 \quad \frac{\partial \Phi_\chi}{\partial z} = 0, \quad R_i \leq r \leq R, \quad z = -d, \quad 0 < \theta < \pi, \quad (8)$$

$$205 \quad \frac{\partial \Phi_\chi}{\partial z} - \frac{\omega^2}{g} \Phi_\chi = 0, \quad r \geq R, \quad z = 0, \quad 0 < \theta < \pi, \quad (9)$$

$$206 \quad \frac{\partial \Phi_\chi}{\partial z} - \frac{\omega^2}{g} \Phi_\chi = \begin{cases} 0, & \chi = S \\ \frac{i\omega}{\rho g}, & \chi = R \end{cases}, \quad r \leq R_i, \quad z = 0, \quad 0 \leq \theta \leq 2\pi, \quad (10)$$

$$207 \quad \frac{\partial \Phi_\chi}{\partial \theta} = 0, \quad r > R, \quad -h \leq z \leq 0, \quad \theta = 0, \pi, \quad (11)$$

$$208 \quad \frac{\partial \Phi_\chi}{\partial \theta} = 0, \quad R_1 < r < R, \quad -h \leq z \leq -d, \quad \theta = 0, \pi, \quad (12)$$

$$209 \quad \frac{\partial \Phi_\chi}{\partial r} = 0, \quad r = R, \quad -d \leq z \leq 0, \quad 0 \leq \theta \leq \pi, \quad (13)$$

$$210 \quad \frac{\partial \Phi_\chi}{\partial r} = 0, \quad r = R_1, \quad -d \leq z \leq 0, \quad 0 \leq \theta \leq 2\pi, \quad (14)$$

$$211 \quad \frac{\partial \Phi_\chi}{\partial r} = 0, \quad r = R_1, \quad -h \leq z \leq -d, \quad \pi \leq \theta \leq 2\pi, \quad (15)$$

212 where  $\rho$  is the water density and  $g$  represents the gravity acceleration.

213 The entire fluid domain can be divided into three regions: I, inner region enclosed by the OWC,  
 214 i.e.,  $r \leq R_1, 0 \leq \theta \leq 2\pi, -h \leq z \leq 0$ ; II, ring region beneath the OWC chamber, i.e.,  $R_1 \leq r \leq R, 0 \leq \theta \leq \pi, -h \leq z \leq -d$ ;  
 215 III, outside region, i.e.,  $r \geq R, 0 \leq \theta \leq \pi, -h \leq z \leq 0$ .  $\Phi_\chi$  ( $\chi = S, R$ ) in these three regions are denoted as  
 216  $\Phi_\chi^{\text{in}}$ ,  $\Phi_\chi^{\text{ring}}$  and  $\Phi_\chi^{\text{out}}$ , respectively.

### 217 3.1 Expressions of scattering/radiated potentials in different regions

218 In different regions, with the application of the method of separation of variables,  $\Phi_\chi$  ( $\chi = S, R$ )  
 219 can be expressed by orthogonal series as follows [35, 36]:

220 I, inner region

$$221 \quad \Phi_\chi^{\text{in}}(r, \theta, z) = \sum_{m=-\infty}^{\infty} \sum_{l=0}^{\infty} \frac{\tilde{I}_m(k_l r)}{k_l \tilde{I}'_m(k_l R_1)} A_{m,l}^\chi Z_l(z) e^{im\theta} + \Phi_{p,\chi}^{\text{in}}, \quad (16)$$

222 where  $A_{m,l}^\chi$  is the unknown coefficients to be determined;  $\Phi_{p,\chi}^{\text{in}}$  is a particular solution,

$$223 \quad \Phi_{p,\chi}^{\text{in}} = \begin{cases} 0, & \chi = S \\ -\frac{i}{\rho\omega}, & \chi = R \end{cases} \quad (17)$$

$$224 \quad \tilde{I}_m(k_l r) = \begin{cases} J_m(k_l r), & l = 0 \\ I_m(k_l r), & l \neq 0 \end{cases} \quad (18)$$

225 in which  $J_m$  and  $I_m$  denote the Bessel function and the modified Bessel function of the first kind,  
 226 respectively.  $k_0$  is the wave number and  $k_l$  ( $l > 0$ ) is the eigenvalue given by [37, 38],

$$227 \quad \omega^2 = -k_l g \tan(k_l h), \quad l = 1, 2, 3, \dots \quad (19)$$



228  $Z_0(z) = N_0^{-0.5} \cosh[k_0(z+h)]; Z_1(z) = N_1^{-0.5} \cos[k_1(z+h)];$  (20)

229 where

230  $N_0 = \frac{1}{2} \left[ 1 + \frac{\sinh(2k_0h)}{2k_0h} \right], N_1 = \frac{1}{2} \left[ 1 + \frac{\sin(2k_1h)}{2k_1h} \right].$  (21)

231 II, ring region

232  $\Phi_\chi^{\text{ring}}(r, \theta, z) = \sum_{m=0}^{\infty} \left[ F_{m,0}^\chi(r) + \sum_{l=1}^{\infty} \left( C_{m,l}^\chi \frac{I_m(\beta_l r)}{I_m(\beta_l R)} + D_{m,l}^\chi \frac{K_m(\beta_l r)}{K_m(\beta_l R)} \right) \cos[\beta_l(z+h)] \right] \cos m\theta,$

233 (22)

234 where

235  $F_{m,0}^\chi(r) = \begin{cases} C_{m,0}^\chi + D_{m,0}^\chi \left[ 1 + \ln\left(\frac{r}{R}\right) \right], & m=0 \\ C_{m,0}^\chi \left(\frac{r}{R}\right)^{|m|} + D_{m,0}^\chi \left(\frac{r}{R}\right)^{-|m|}, & m \neq 0 \end{cases},$  (23)

236 in which  $C_{m,l}^\chi$  and  $D_{m,l}^\chi$  are the coefficients to be solved;  $K_m$  is the modified Bessel function of  
237 the second kind;  $\beta_l$  is the  $l$ -th eigenvalue which is given by

238  $\beta_l = \frac{l\pi}{h-d}, l=0, 1, 2, 3, \dots$  (24)

239 III, outside region

240  $\Phi_\chi^{\text{out}}(r, \theta, z) = \sum_{m=0}^{\infty} \sum_{l=0}^{\infty} E_{m,l}^\chi \frac{\tilde{K}_m(k_l r)}{\tilde{K}_m(k_l R)} \cos(m\theta) Z_l(z) + \Phi_{p,\chi}^{\text{out}},$  (25)

241 in which  $E_{m,l}^\chi$  is the unknown coefficients to be determined; and  $\Phi_{p,\chi}^{\text{out}}$  is a particular solution,

242  $\Phi_{p,\chi}^{\text{out}} = \begin{cases} \Phi_1, & \chi = \text{S} \\ 0, & \chi = \text{R} \end{cases},$  (26)

243 where, following Zheng and Zhang [37],

244  $\Phi_1(r, \theta, z) = -\frac{2igA}{\omega} \frac{Z_0(z)}{Z_0(0)} \sum_{m=0}^{\infty} \varepsilon_m (-i)^m J_m(k_0 r) \cos m\beta \cos m\theta,$  (27)

245 in which  $\varepsilon_m=1$  for  $m=0$ , whereas  $\varepsilon_m=2$  for  $m \neq 0$ .

246  $\tilde{K}_m(k_l r) = \begin{cases} H_m(k_l r), & l=0 \\ K_m(k_l r), & l \neq 0 \end{cases},$  (28)

247 where  $H_m$  denotes the Hankel function of the first kind.

### 248 3.2 Method of computation for unknown coefficients

249 The scattering and radiated spatial potentials as expressed in Sections 3.1 satisfy all the

250 boundary conditions shown in Eqs. (7) ~ (12) already. Additionally, the boundary conditions at  
 251  $r=R$  and  $r=R_i$ , i.e., Eqs. (13)~(15), together with the pressure and velocity continuity conditions on  
 252 the interfaces of two adjacent regions should be satisfied as well, which can be used to determine  
 253 the unknown coefficients in  $\Phi_\chi$ . These continuity conditions for  $\Phi_\chi$  are given as follows:

254 1) Continuity of normal velocity at the boundary  $r=R_i$ :

$$255 \quad \frac{\partial \Phi_\chi^{\text{in}}}{\partial r} = \begin{cases} 0, & -d < z < 0, r = R_i, 0 \leq \theta \leq \pi; \text{ and } -h < z < 0, r = R_i, \pi \leq \theta \leq 2\pi \\ \frac{\partial \Phi_\chi^{\text{ring}}}{\partial r}, & -h < z < -d, r = R_i, 0 \leq \theta \leq \pi \end{cases} \quad (29)$$

256 2) Continuity of normal velocity at the boundary  $r=R$ :

$$257 \quad \frac{\partial \Phi_\chi^{\text{out}}}{\partial r} = \begin{cases} 0, & -d < z < 0, r = R, 0 \leq \theta \leq \pi \\ \frac{\partial \Phi_\chi^{\text{ring}}}{\partial r}, & -h < z < -d, r = R, 0 \leq \theta \leq \pi \end{cases} \quad (30)$$

258 3) Continuity of pressure at the boundary  $r=R_i$ :

$$259 \quad \Phi_\chi^{\text{ring}} = \Phi_\chi^{\text{in}}, \quad -h < z < -d, r = R_i, 0 \leq \theta \leq \pi \quad (31)$$

260 4) Continuity of pressure at the boundary  $r=R$ :

$$261 \quad \Phi_\chi^{\text{out}} = \Phi_\chi^{\text{ring}}, \quad -h < z < -d, r = R, 0 \leq \theta \leq \pi \quad (32)$$

262 After inserting the expressions of  $\Phi_\chi$  as given in Section 3.1 into these continuity conditions, i.e.,  
 263 Eqs.(29)~(32), and making use of orthogonality of trigonometric functions and eigen-functions,  
 264 the unknown coefficients in  $\Phi_\chi$  can be determined. For convenience, details of the derivations are  
 265 given in Appendix A.

## 266 4 Excitation volume flow and hydrodynamic coefficients

### 267 4.1 Excitation volume flow

268 Once the unknown coefficients are determined, the excitation volume flow  $Q_e$  as given in Eq. (3)  
 269 can be easily calculated by:

$$270 \quad Q_e = \frac{2\pi\omega^2 R_i}{g} \left( -\frac{A_{0,0}^D}{k_0^2} Z_0(z) + \sum_{l=1}^{\infty} \frac{A_{0,l}^D}{k_l^2} Z_l(0) \right). \quad (33)$$

### 271 4.2 Hydrodynamic coefficients

272 In a similar way, the hydrodynamic coefficients as given in Eq. (4) can be rewritten in terms of

273  $A_{m,l}^R$  as:

$$274 \quad -(c - ia) = Q_R = \frac{2\pi\omega^2 R_i}{g} \left( -\frac{A_{0,0}^R}{k_0^2} Z_0(z) + \sum_{l=1}^{\infty} \frac{A_{0,l}^R}{k_l^2} Z_l(0) \right). \quad (34)$$

275 The method, as shown in Eq. (4) or Eq. (34), which is derived from the radiated volume flow  
 276 inside the column is a straightforward way for calculating the hydrodynamic coefficient,  $c$ .  
 277 Actually there are two indirect methods as well for evaluating  $c$ , the one expressed by far-field  
 278 coefficients as

279 
$$c = 2\rho\omega h \sum_{m=0}^{\infty} \frac{1}{\varepsilon_m} \frac{|E_{m,0}^R|^2}{|H_m(k_0 R)|^2}, \quad (35)$$

280 which can be derived from Green's identity [36, 39]; and the other one derived from the excitation  
281 volume flow  $Q_e$  based on Haskind Relation [32, 39]:

282 
$$c = \frac{\omega Z_0(0)^2}{8\pi\rho g^2 h} \int_0^\pi |Q_e(\beta)|^2 d\beta. \quad (36)$$

283 The comparison of the results of  $c$  by using these two indirect methods as given in Eqs. (35) and  
284 (36) with that of the direct method, i.e., Eq. (4) or Eq. (34), can be adopted as an approach to  
285 validate the theoretical model.

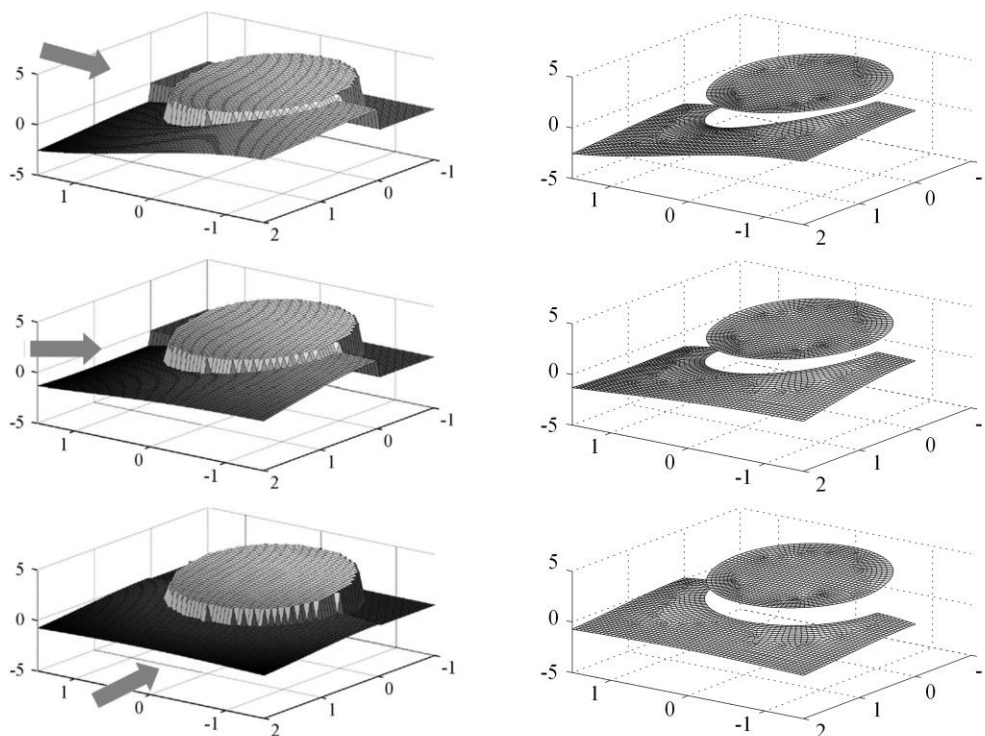
## 286 5 Model validation

287 Martins-rivas and Mei [32] solved the hydrodynamic problems from a thin-walled (i.e.,  $R_i=R$ )  
288 OWC along a straight coast for  $R/h=0.5$ ,  $d/h=0.2$  subjected to regular waves propagating at  
289 different angles  $\beta$  with different values of  $kh$ . The present theoretical model without the thin-wall  
290 restriction is adopted to re-simulate the same case, in which the inner radius is chosen as  $R_i/h=0.49$ ,  
291 i.e.,  $(R-R_i)/R=0.02$ , to represent the thin chamber wall.

292 To make a comparison with the published results, the method as adopted in Martins-rivas and  
293 Mei [32] for nondimensionalizing  $Q_e$  and hydrodynamic coefficients,  $c$  and  $a$ , i.e.,  
294  $\bar{Q}_e = \omega Q_e / (ARg)$ ,  $(\bar{c}, \bar{a}) = (c, a)\omega\rho/R$ , is re-employed in the present section.

### 295 5.1 Wave scattering problem

296 The comparison between the present results of the free surface elevation pattern inside and  
297 outside the OWC for  $kh=1.64$  and those of Martins-rivas and Mei [32] is given in Fig. 2.



298

299 Fig. 2. Free surface elevation inside and outside the OWC for  $R/h=0.5$ ,  $d/h=0.2$ ,  $kh=1.64$ ,  $t=\pi/2\omega$ .300 (left) results of Martins-rivas and Mei [32] for thin-walled OWC, i.e.,  $R_i=R$ ; (right) present results

301

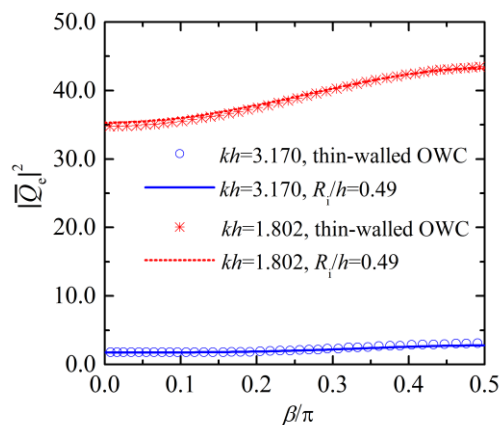
301 with  $R_i/h=0.49$ . The incidence angles  $\beta=0, 0.25\pi, 0.5\pi$ .

302

303 In addition, comparison of the excitation volume flow of the OWC as a function of incidence  $\beta$ 304 for  $R/h=0.5$ ,  $d/h=0.2$ ,  $kh=3.170$  and  $1.802$  by using the present model with that of Martins-rivas

305

305 and Mei [32] is illustrated in Fig. 3.



306

307 Fig. 3. Excitation volume flow of the OWC as a function of incidence  $\beta$  for  $R/h=0.5$ ,  $d/h=0.2$ .308 symbols: results from Martins-rivas and Mei [32] for thin-walled OWC, i.e.,  $R_i=R$ ; lines: present

309

309 results with  $R_i/h=0.49$ .

310

310 The excellent agreement of the present results with those of Martins-rivas and Mei [32], as

311 shown in Figs. 2 and 3, proves that the present theoretical model works pretty well in solving the

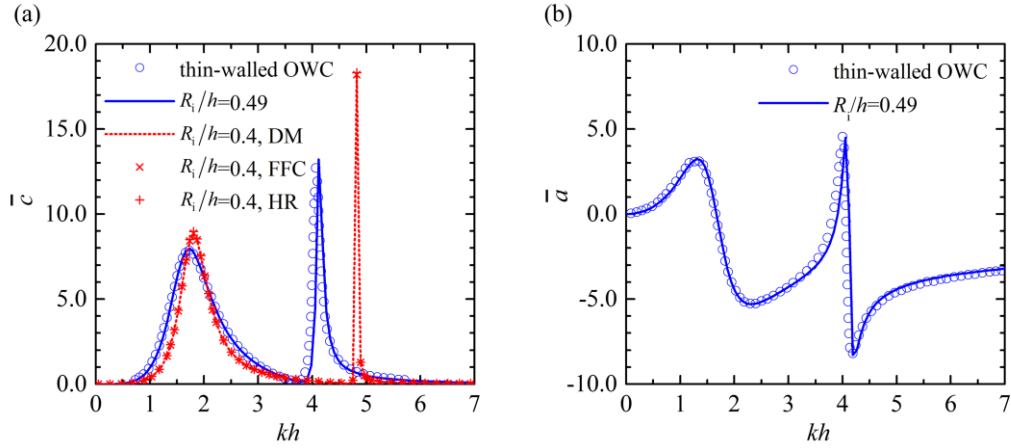
312

312 scattering problem.

313

313 5.2 Wave radiation problem

314 Figure 4 illustrates the frequency response of the hydrodynamic coefficients of the OWC with  
 315  $R/h=0.5$ ,  $d/h=0.2$ . It can be learnt that the present results of  $c$  and  $a$  with  $R_i/h=0.49$ , i.e.,  
 316  $(R-R_i)/R=0.02$ , are in rather good agreement with those under thin-wall restriction [32]. What is  
 317 more, results of  $c$  for the case with  $R_i/h=0.4$  by using direct method (denoted as DM) and by  
 318 adopting the other two indirect methods (denoted as FFC and HR, respectively) agree with each  
 319 other pretty well, meaning the correctness of the present model in solving wave radiation problem.



320  
 321 Fig. 4. Frequency response of radiation damping and added mass of the OWC with  $R/h=0.5$ ,  
 322  $d/h=0.2$ : (a) radiation damping; (b) added mass. Circles: results from Martins-rivas and Mei [32]  
 323 for thin-walled OWC, i.e.,  $R_i=R$ ; lines and crosses: present results.  
 324

325 Note there is an obvious difference of the frequency response of radiation damping by using  
 326  $R_i/h=0.4$  and  $R_i/h=0.49$ , reflecting the significant effect of the OWC chamber's thickness on the  
 327 hydrodynamic characteristics of the OWC along a vertical wall. Influence of the thickness on the  
 328 performance of the OWC deserves more attention and such effect will be discussed in the next  
 329 section.

## 330 6 Results and discussion

331 Hereinafter, following Lovas, Mei [33], the dimensionless coefficients of  $Q_c$  and hydrodynamic  
 332 coefficients,  $c$  and  $a$ , are redefined as follows:

$$333 \quad \bar{Q}_c = \frac{\sqrt{g/h}}{Ahg} Q_c; \quad (\bar{c}, \bar{a}, \bar{c}_{PTO}, \bar{a}_{PTO}) = \frac{\rho\sqrt{g/h}}{h} (c, a, c_{PTO}, a_{PTO}), \quad (37)$$

334 with which, Eq. (6) can be rewritten as

$$335 \quad \eta = \frac{khg}{c_g \sqrt{g/h}} \frac{\bar{c}_{PTO} |\bar{Q}_c|^2}{[(\bar{c} + \bar{c}_{PTO})^2 + (\bar{a} + \bar{a}_{PTO})^2]}. \quad (38)$$

336 Following Martins-rivas and Mei [32] and Lovas, Mei [33],  $a_{PTO}$  is calculated based on  
 337  $\rho/\rho_0=1000$ ,  $v=340$  m/s,  $h=10$  m, and  $V_0=\pi R^2 h$ . The corresponding optimal PTO damping for a  
 338 fixed OWC chamber (fixed  $V_0$ ) is obtained by requiring  $\partial P/\partial c_{PTO}=0$ ,

$$339 \quad c_{PTO} = \sqrt{c^2 + (a + a_{PTO})^2}. \quad (39)$$

### 340 6.1 Comparison with an isolated offshore OWC and effect of incident wave direction

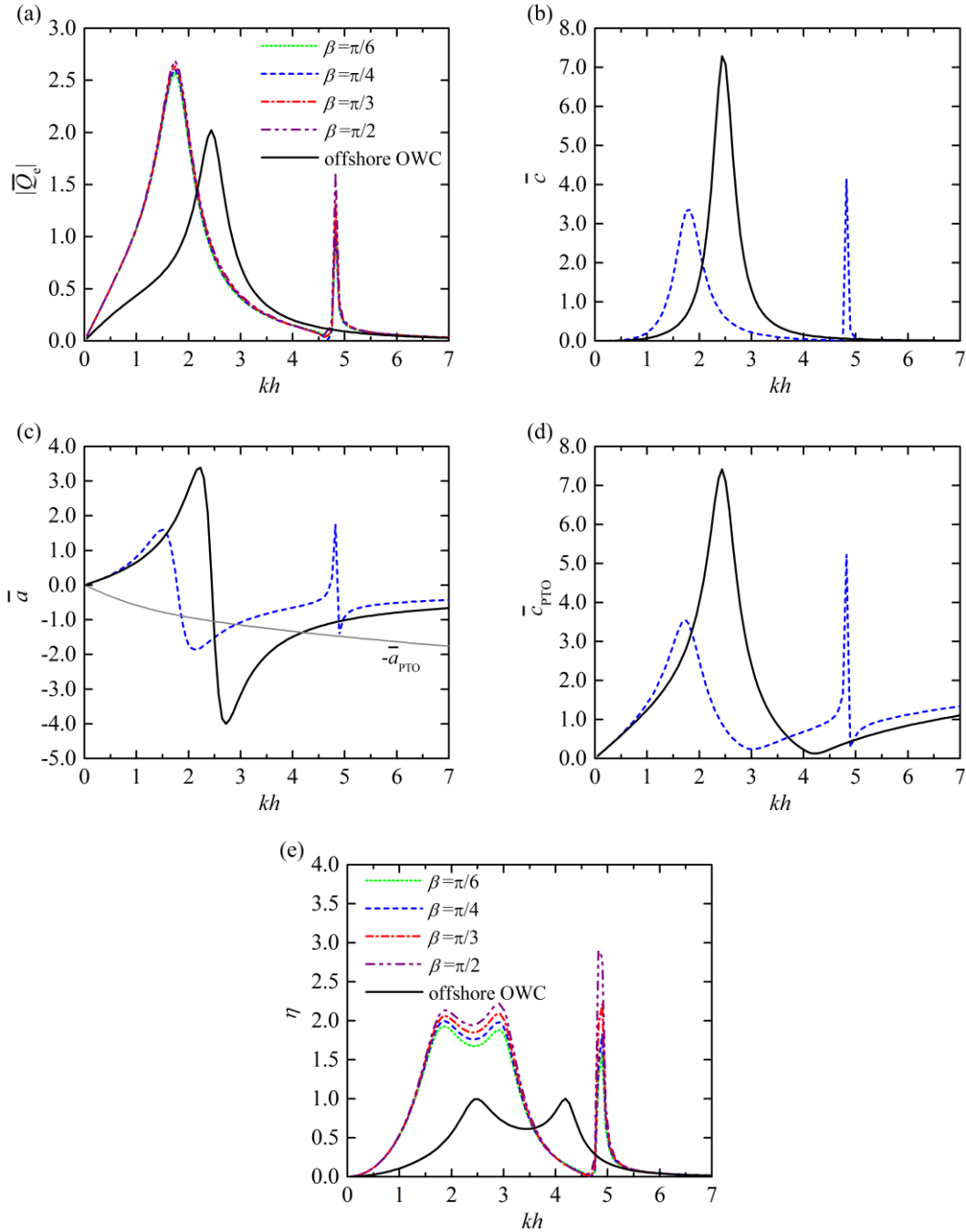
341 Figure 5 presents the frequency responses of wave excitation volume flux, hydrodynamic  
 342 coefficients, turbine parameter and wave power capture factor when the coast/breakwater  
 343 integrated OWC suffers from different incident directions. For reasons of symmetry, only the  
 344 results for  $\beta=\pi/6, \pi/4, \pi/3$  and  $\pi/2$  are presented. For comparison, the results for the same OWC in  
 345 the open sea are also displayed [36]. Only one blue dash curve as plotted in each figure of Figs.  
 346 5b~5d is used to represent the results of  $\bar{c}$ ,  $\bar{a}$  and  $\bar{c}_{\text{PTO}}$  for the coast/breakwater integrated  
 347 OWC, respectively, since they are all independent of  $\beta$ .

348 For wave scattering problem of the isolated offshore OWC, in the computed range of  $kh$ , there  
 349 is only one peak of  $|\bar{Q}_e|-kh$  curve at  $kh\approx 2.44$  (see Fig. 5a). Whereas for the coast/breakwater  
 350 integrated OWC, apart from the main peak of  $|\bar{Q}_e|-kh$  curve at  $kh\approx 1.73$ , a second sharp peak is  
 351 also observed at a higher frequency, i.e.,  $kh\approx 4.82$ . Such a feature can be identified from the view  
 352 of natural modes in a closed cylinder in the radiation problem. For the isolated offshore OWC, the  
 353 only peak of the  $\bar{c}-kh$  curve occurs at  $kh\approx 2.44$  (Fig. 5b), which corresponds to a piston-like  
 354 motion, i.e., the so-called Helmholtz mode of oscillation. In a coast/breakwater integrated OWC,  
 355 the Helmholtz mode cannot exist alone because of the asymmetry of the opening; another mode,  
 356 i.e., the sloshing mode, is excited [32], and dominates the water motion inside the OWC chamber  
 357 at  $kh\approx 4.82$ . As shown in Fig. 5c, the  $\bar{a}-kh$  performs like a N letter shaped and a two-N letter  
 358 shaped curves for an isolated offshore OWC and the integrated case, respectively, and the sign of  
 359  $\bar{a}$  changes rapidly at the  $kh$  where the peak of the  $\bar{c}-kh$  curve occurs. These values of  $kh$  can be  
 360 called the natural frequencies of the OWC in the absence of the PTO. The spiky behaviour of  $\bar{a}$   
 361 around these natural frequencies is connected to the peak of the  $\bar{c}-kh$  curve. Note that the  
 362 chamber coefficient ( $-\bar{a}_{\text{PTO}}$ ) is also plotted in Fig. 5d as a gray solid curve, which intersects the  
 363  $\bar{a}-kh$  curve at  $kh\approx(2.48, 4.18)$  and  $(1.86, 2.90, 4.90)$  for different cases. The  $kh$ -values where  
 364 these intersections occur correspond the resonant frequencies of the OWC with the PTO system.  
 365 For wave conditions corresponding to these resonant frequencies,  $\bar{a}$  and  $\bar{a}_{\text{PTO}}$  cancel each  
 366 other, and it can be readily known from Eq. (2) that the air pressure inside the OWC chamber is in  
 367 phase with the excitation volume flux. The frequency response of the optimal turbine parameter  
 368  $\bar{c}_{\text{PTO}}$  is illustrated in Fig. 5d, in which  $\bar{c}_{\text{PTO}}=\bar{c}$  is satisfied at the resonant frequencies.

369 Compared with the single offshore OWC, the variation of  $\bar{c}_{\text{PTO}}$  for the coast/breakwater  
 370 integrated OWC is less marked and less abrupt (except for  $kh\approx 4.9$ ), which means that it may be  
 371 easier to achieve in practice.

372 Finally,  $\eta$  as a function of  $kh$  for different values of the incident wave angle  $\beta$  is given in Fig. 5e.  
 373 It is apparent that  $\eta$  for the isolated offshore OWC reaches the theoretical maximum value (i.e.,  
 374 1.0) at these two resonant frequencies. Peaks of the  $\eta-kh$  curve for the integrated case are also  
 375 observed at the corresponding resonant frequencies. Thanks to the wave reflection from the  
 376 coast/breakwater, the value of  $\eta$  for the coast/breakwater integrated OWC at  $1.6<kh<3.1$  can be  
 377 around twice as large as the theoretical maximum for the offshore case. Note that there is a

378 frequency between the second and the third resonant frequencies where no power can be  
 379 extracted.



382 Fig. 5. Comparison for different incident wave directions,  $\beta$ . (a) wave excitation volume flux  
 383  $|\bar{Q}_e|$ ; (b) radiation damping  $\bar{c}$ ; (c) added mass  $\bar{a}$  and chamber coefficient  $-\bar{a}_{PTO}$  (gray solid  
 384 line); (d) turbine parameter  $\bar{c}_{PTO}$ ; (e) wave power capture factor  $\eta$ . In every case,  $R/h=0.5$ ,  
 385  $(R-R_i)/h=0.1$ ,  $d/h=0.2$ . Black solid line: isolated offshore OWC.

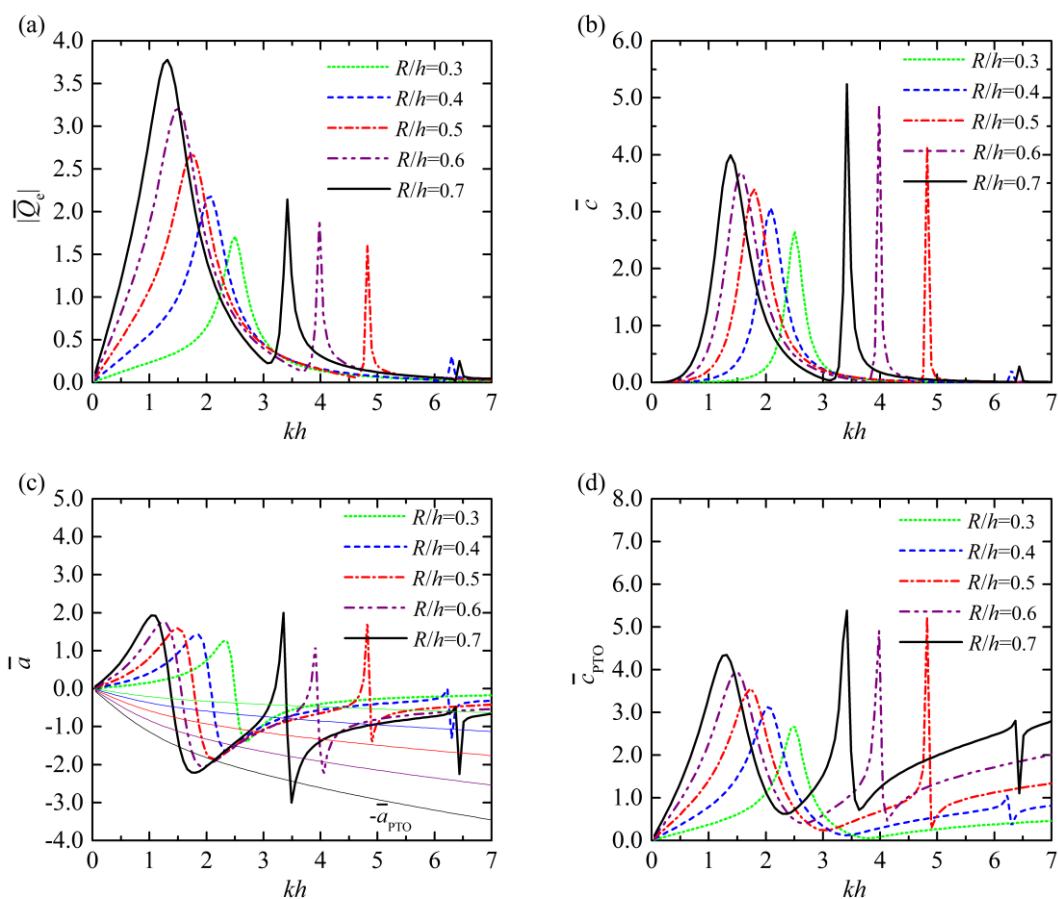
387 The effect of the incident wave direction  $\beta$  on  $|\bar{Q}_e|$  is not obvious (Fig. 5a). As  $\beta$  increases  
 388 from  $\pi/6$  to  $\pi/2$ , this dependence is slightly visible at the natural frequencies. As a comparison, a

389 significant influence of  $\beta$  on  $\eta$  can be observed for  $1.6 < kh < 3.1$  and  $4.9 < kh < 5.0$  (Fig. 5e), where  
 390 the more perpendicular the incident wave direction relative to the coast/breakwater, the more wave  
 391 power that can be captured. Hereinafter, the effects of the other parameters will all be examined  
 392 with  $\beta = \pi/2$ .

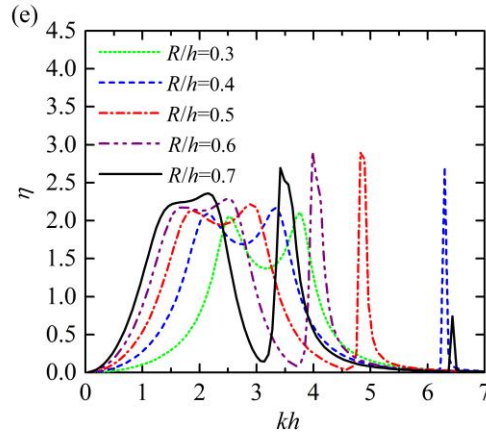
## 393 6.2 Radius of the OWC chamber

394 The coast/breakwater integrated OWCs with  $R/h = 0.3 \sim 0.7$  are selected as five cases to  
 395 investigate the effect of the radius of the chamber on the performance of the OWC (Fig. 6). As  $R/h$   
 396 increases, the highest peak of the  $|\bar{Q}_c|$ - $kh$  curve (Fig. 6a) shifts toward a lower frequency and  
 397 gains intensity. Similar changes affect  $\bar{c}$ ,  $\bar{a}$  and  $\bar{c}_{PTO}$  (Figs. 6b~6d). As  $R/h$  increases, more  
 398 natural frequencies can be observed in the computed range of  $kh$ . For  $R/h = 0.3$ , there is only one  
 399 natural frequency in the range of  $kh$  plotted, whereas for  $R/h = 0.4, 0.5$  and  $0.6$ , there are two. For  
 400  $R/h = 0.7$ , three natural frequencies are readily observable.

401

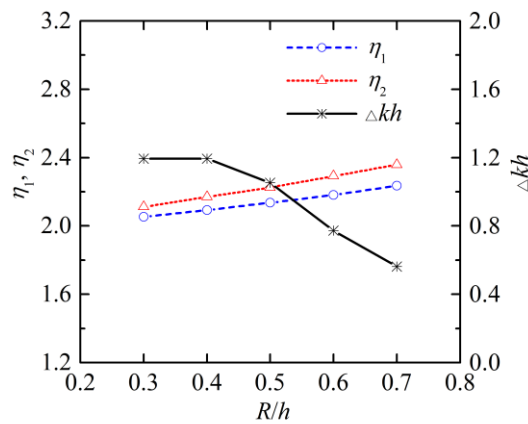






404 Fig. 6. Comparison for different radii of the OWC chamber,  $R/h$ . (a) wave excitation volume  
 405 flux  $|\bar{Q}_e|$ ; (b) radiation damping  $\bar{c}$ ; (c) added mass  $\bar{a}$  and chamber coefficient  $-\bar{a}_{PTO}$  (thin  
 406 solid lines, each of which corresponds to the line of  $\bar{a}$  plotted in the same color); (d) turbine  
 407 parameter  $\bar{c}_{PTO}$ ; (e) wave power capture factor  $\eta$ . In every case,  $(R-R_i)/h=0.1$ ,  $d/h=0.2$ ,  $\beta=\pi/2$ .  
 408

409 Given that the chamber volume  $V_0$  ( $V_0=\pi R^2h$ ) is dependent on  $R$ , there are also five  $(-\bar{a}_{PTO})-kh$   
 410 curves plotted in Fig. 6c corresponding to different values of  $R/h$ . For  $R/h=0.3$ , there are only two  
 411 resonant frequencies in the computed range of  $kh$ . For the other cases, e.g.,  $R/h=0.6$ , there could  
 412 be four resonant frequencies in the same range of  $kh$ . As plotted in Fig. 6d, the larger the  $R/h$ , the  
 413 higher and more abrupt the variation of  $\bar{c}_{PTO}$ . The plot of  $\eta$  (Fig. 6e) shows that when  $kh$  is  
 414 between the first two resonant frequencies, as  $R/h$  increases, the  $\eta-kh$  curve turns higher and flatter,  
 415 and shifts toward lower frequencies. Here,  $\Delta kh$  is adopted to denote the difference between the  
 416 first two resonant frequencies, and  $\eta_1$  and  $\eta_2$  are employed to represent the  $\eta$ -values corresponding  
 417 to the first two resonant frequencies, respectively. Figure 7 presents  $\eta_1$ ,  $\eta_2$  and  $\Delta kh$  as three  
 418 functions of  $R/h$ . It is clear that, as  $R/h$  increases from 0.3 to 0.7, both  $\eta_1$  and  $\eta_2$  increase in a linear  
 419 way approximately, whereas  $\Delta kh$  decreases dramatically from 1.19 to 0.56. Ideally the  $R/h$  ratio  
 420 should be selected to achieve the balance between the peak value of  $\eta$  and its bandwidth, so that  
 421 the OWC can capture the most power for a specified range of wave conditions.



422

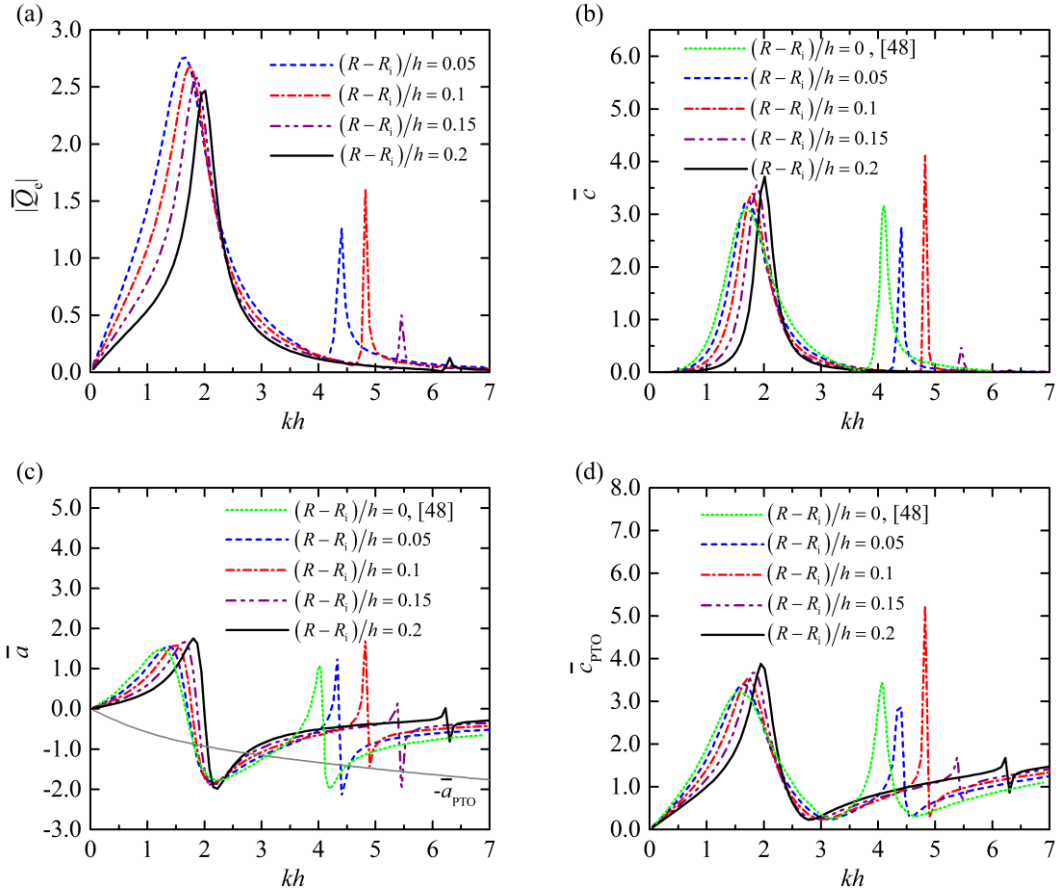
423

Fig. 7. Variation of  $\eta_1$ ,  $\eta_2$  and  $\Delta kh$  with  $R/h$  for  $(R-R_i)/h=0.1$ ,  $d/h=0.2$ ,  $\beta=\pi/2$

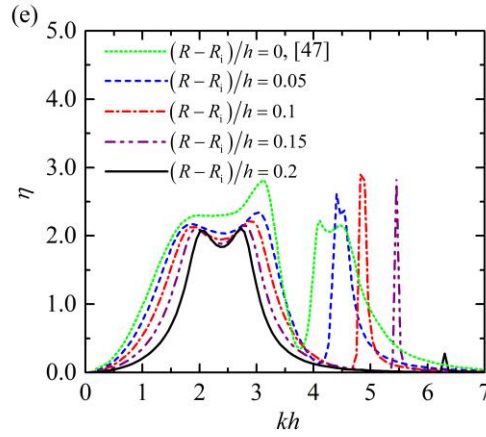
424 6.3 Thickness of the OWC chamber

425 Figure 8 presents the results of the OWC chamber with  $(R-R_i)/h=0.05, 0.1, 0.15$  and  $0.2$ , and the  
 426 other parameters fixed at  $R/h=0.5, d/h=0.2, \beta=\pi/2$ . For comparison, some results of the thin-wall  
 427 case, i.e.,  $(R-R_i)/h=0$ , which were previously displayed by Martins-rivas and Mei [47] and Lovas  
 428 et al. [48], are replotted in Fig. 8 as well. As  $(R-R_i)/h$  increases, the inner radius of the chamber  
 429 decreases, and the highest peak of  $|\bar{Q}_c|$ - $kh$  curve loses intensity and moves toward higher  
 430 frequency as expected (see Fig. 8a). Meanwhile, the main peak of  $\bar{c}$  shifts toward higher  
 431 frequency and turns higher and narrower. Similar changes are also found for  $\bar{a}$  and  $\bar{c}_{PTO}$  as  
 432 given in Figs. 8c and 8d. It should be noted from Fig. 8c that with the increase of  $(R-R_i)/h$ , the first  
 433 two intersection points of  $\bar{a}$  and  $-\bar{a}_{PTO}$  get closer and closer to each other horizontally, hence  
 434 the frequency band of  $\eta$ - $kh$  as plotted in Fig. 8e turns narrower and narrower, whereas the  
 435 frequency position of the middle of the band remains almost the same. In addition, the peaks of  $\eta$   
 436 corresponding to the first two resonant frequencies are lower as  $(R-R_i)/h$  increases. Therefore,  
 437 generally, the thickness of the OWC chamber should be as small as possible, so that larger and  
 438 broader main peaks can be achieved and more wave power absorbed. Needless to say, the  
 439 minimum thickness will be dictated in practice by structural considerations.

440



442



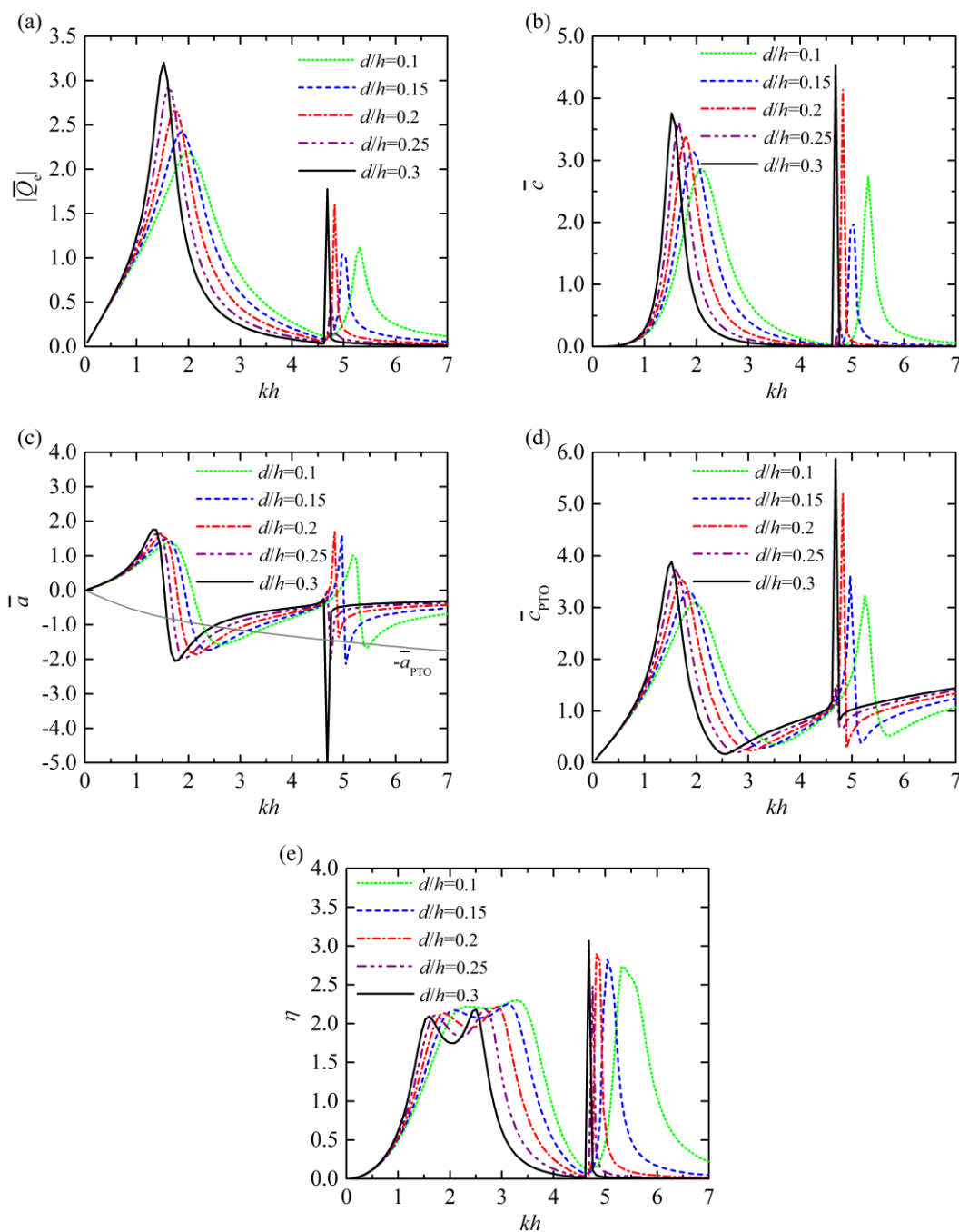
443  
 444 Fig. 8. Comparison for different thicknesses of the OWC chamber wall,  $(R-R_i)/h$ . (a) wave  
 445 excitation volume flux  $|\bar{Q}_e|$ ; (b) radiation damping  $\bar{c}$ ; (c) added mass  $\bar{a}$  and chamber  
 446 coefficient  $-\bar{a}_{PTO}$  (gray solid line); (d) turbine parameter  $\bar{c}_{PTO}$ ; (e) wave power capture factor  $\eta$ .

447 In every case,  $R/h=0.5$ ,  $d/h=0.2$ ,  $\beta=\pi/2$ .

#### 448 6.4 Submergence of the OWC chamber

449 Figure 9 compares the results for the coast/breakwater integrated OWC with different  
 450 submergence of the chamber, i.e.,  $d/h = 0.1, 0.15, 0.2, 0.25$  and  $0.3$ , with  $R/h=0.5$ ,  $(R-R_i)/h=0.1$ ,  
 451  $\beta=\pi/2$ . The results of  $|\bar{Q}_e|$  (Fig. 9a) show that with the increase of  $d/h$ , the highest peak of  $|\bar{Q}_e|$   
 452 turns higher and narrower, and shifts toward lower frequencies. Similar changes apply to  $\bar{c}$  and  
 453  $\bar{a}$  (Figs. 9b and 9c). This is reasonable, for the radiation loss becomes weaker. All the resonant  
 454 frequencies in the computed range of  $kh$  reduce as  $d/h$  increases. The larger the  $d/h$  ratio, the  
 455 higher and more abrupt the variation of the corresponding  $\bar{c}_{PTO}$  with  $kh$  (Fig. 9d), which may be  
 456 more difficult to achieve in practice. The plot of  $\eta$  in Fig. 9e shows that, due to the change in the  
 457 resonant frequencies, the peaks of  $\eta$  are shifted toward lower frequencies as well with the increase  
 458 in  $d/h$ . Meanwhile, both  $\eta_1$  and  $\eta_2$  are found to decrease slightly, and the main bandwidth in terms  
 459 of  $\Delta kh$  also decreases. The  $\eta$  corresponding to the  $kh$  between the first two resonant frequencies  
 460 decreased more dramatically than  $\eta_1$  and  $\eta_2$ . It might be concluded that a better result can be  
 461 obtained by using a smaller value of  $d/h$ .

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Fig. 9. Comparison for different submergence of the OWC chamber,  $d/h$ . (a) wave excitation volume flux  $|\bar{Q}_e|$ ; (b) radiation damping  $\bar{c}$ ; (c) added mass  $\bar{a}$  and chamber coefficient  $-\bar{a}_{PTO}$

(gray solid line); (d) turbine parameter  $\bar{c}_{PTO}$ ; (e) wave power capture factor  $\eta$ . In every case,

$$R/h=0.5, (R-R_i)/h=0.1, \beta=\pi/2.$$

Note that  $d/h$  cannot be too small in practice, otherwise the opening might not be continuously submerged, especially when the OWC is subjected to incident waves with a large amplitude, to a large tidal range, or both.

## 473 7 Conclusions

474 In this paper a theoretical model based on linear potential flow theory is proposed to study the  
 475 performance of an OWC along a vertical coast/breakwater without the thin-wall restriction of  
 476 previous works. The water domain is divided into three regions, i.e., the interior region enclosed  
 477 by the OWC chamber, the half-ring shaped region beneath the OWC chamber and the exterior  
 478 region in front of the coast/breakwater extending towards infinite distance horizontally. Subjected  
 479 to small amplitude incident regular waves, wave-structure interaction is decomposed into wave  
 480 scattering and wave radiation problems. In order to determine the unknown coefficients of the  
 481 scattering and radiated potentials in these three regions, the eigen-function matching method is  
 482 employed. The wave power extraction of the OWC with linear PTO system is then evaluated in  
 483 the frequency domain.

484 The influence of the vertical coast/breakwater is briefly discussed by comparing the  
 485 performance of the integrated OWC with that of a similar isolated OWC deployed in the open sea.  
 486 Finally, the effects of the radius, thickness and submergence of the chamber on the performance of  
 487 the OWC along a coast/breakwater are investigated by means of the theoretical model. The  
 488 following conclusions may be drawn.

489 The value of  $\eta$  for the coast/breakwater integrated OWC at specified ranges of  $kh$  can be around  
 490 twice as large as the theoretical maximum of  $\eta$  for the offshore case due to the wave reflection  
 491 from the coast/breakwater. The more perpendicular the incident wave direction relative to the  
 492 coast/breakwater, the more wave power that can be captured by the OWC.

493 As the  $R/h$  ratio increases, more natural and resonant frequencies can be observed in the  
 494 computed range of  $kh$ . The main peaks of  $\eta$  shift toward lower frequencies and the peak values  
 495 increase nearly linearly with  $R/h$ , whereas the bandwidth reduces drastically.

496 The smaller the  $(R-R_i)/h$  ratio, the larger and broader the main peaks of  $\eta$ , i.e., more wave power  
 497 absorbed, and the frequency position of the middle of the band remains almost the same. Needless  
 498 to say, an appropriate thickness, rather than zero thickness of the OWC chamber, will be dictated  
 499 in practice by overall considerations, including not only wave power extraction but also structural  
 500 survivability.

501 With the increase of  $d/h$ , the peaks of  $\eta$  are shifted toward lower frequencies. Meanwhile, both  
 502  $\eta_1$  and  $\eta_2$  are found to decrease slightly, and the main bandwidth in terms of  $\Delta kh$  is reduced.

503 To capture wave power on a large scale, it is expected that multiple OWCs along a  
 504 coast/breakwater will be required. It is possible to extend the present theoretical model to multiple  
 505 OWCs, as will be reported elsewhere.

506

## 507 Acknowledgements

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## 511 Appendix A. Integral equations of the scattering and radiation problems

512 After inserting Eqs. (16) and (22) into Eq.(29), multiplying both sides by  $Z_\zeta(z)e^{-i\tau\theta}$  and  
 513 integrating for  $z \in [-h, 0]$  and  $\theta \in [0, 2\pi]$ , for any pair of integer  $(\tau, \zeta)$ , it can be obtained that

$$514 \quad 2\pi h A_{\tau,\zeta}^{\chi} = \sum_{l=0}^{\infty} \left[ \frac{\pi}{\varepsilon_{|\tau|}} \left( X_{|\tau|,l}^{(1)} C_{|\tau|,l}^{\chi} + Y_{|\tau|,l}^{(1)} D_{|\tau|,l}^{\chi} \right) + i \sum_{\substack{m=0 \\ m \neq |\tau|}}^{\infty} \frac{\tau \left[ (-1)^{\tau-m} - 1 \right]}{\tau^2 - m^2} \left( X_{m,l}^{(1)} C_{m,l}^{\chi} + Y_{m,l}^{(1)} D_{m,l}^{\chi} \right) \right] L_{l,\zeta},$$

$$515 \quad (A.1)$$

516 where

$$517 \quad X_{\tau,\zeta}^{(1)} = \begin{cases} \frac{\tau}{R} \left( \frac{R_1}{R} \right)^{\tau-1}, & \zeta = 0 \\ \frac{\beta_{\zeta} I'_{\tau}(\beta_{\zeta} R_1)}{I_{\tau}(\beta_{\zeta} R)}, & \zeta \neq 0 \end{cases}; \quad Y_{\tau,\zeta}^{(1)} = \begin{cases} \frac{1}{R_1}, & \zeta = 0, \tau = 0 \\ -\frac{\tau}{R} \left( \frac{R}{R_1} \right)^{\tau+1}, & \zeta = 0, \tau \neq 0. \\ \frac{\beta_{\zeta} K'_{\tau}(\beta_{\zeta} R_1)}{K_{\tau}(\beta_{\zeta} R)}, & \zeta \neq 0 \end{cases} \quad (A.2)$$

$$518 \quad L_{l,\zeta} = \int_{-h}^{-d} \cos[\beta_l(z+h)] Z_{\zeta}(z) dz = \begin{cases} \frac{(-1)^l (h-d)^2 k_0 Z_0(0) \sinh[k_0(h-d)]}{[(h-d)^2 k_0^2 + l^2 \pi^2] \cosh(k_0 h)}, & \zeta = 0 \\ \frac{(-1)^l (h-d)^2 k_{\zeta} Z_{\zeta}(0) \sin[k_{\zeta}(h-d)]}{[(h-d)^2 k_{\zeta}^2 - l^2 \pi^2] \cos(k_{\zeta} h)}, & \zeta \neq 0 \end{cases} \quad (A.3)$$

519 After inserting Eqs. (22) and (25) into Eq.(30), multiplying both sides by  $Z_{\zeta}(z)\cos(\tau\theta)$  and  
520 integrating for  $z \in [-h,0]$  and  $\theta \in [0,\pi]$ , for any pair of integer  $(\tau, \zeta)$ , we have

$$521 \quad \sum_{l=0}^{\infty} \left( X_{\tau,l}^{(2)} C_{\tau,l}^{\chi} + Y_{\tau,l}^{(2)} D_{\tau,l}^{\chi} \right) L_{l,\zeta} - h Z_{\tau,\zeta}^{(2)} E_{\tau,\zeta}^{\chi} = f_2^{\chi}, \quad (A.4)$$

522 in which

$$523 \quad f_2^{\chi} = \begin{cases} -\frac{2\delta_{\zeta,0} \text{ig} A k_0 h}{\omega Z_0(0)} \varepsilon_{\tau} (-i)^{\tau} J'_{\tau}(k_0 R) \cos(\tau\beta), & \chi = S \\ 0, & \chi = R \end{cases}, \quad (A.5)$$

$$524 \quad X_{\tau,\zeta}^{(2)} = \begin{cases} \frac{\tau}{R}, & \zeta = 0 \\ \frac{\beta_{\zeta} I'_{\tau}(\beta_{\zeta} R)}{I_{\tau}(\beta_{\zeta} R)}, & \zeta \neq 0 \end{cases}; \quad Y_{\tau,\zeta}^{(2)} = \begin{cases} \frac{1}{R}, & \zeta = 0, \tau = 0 \\ -\frac{\tau}{R}, & \zeta = 0, \tau \neq 0, \\ \frac{\beta_{\zeta} K'_{\tau}(\beta_{\zeta} R)}{K_{\tau}(\beta_{\zeta} R)}, & \zeta \neq 0 \end{cases} \quad (A.6)$$

$$525 \quad Z_{\tau,\zeta}^{(2)} = \begin{cases} \frac{k_0 H'_\tau(k_0 R)}{H_\tau(k_0 R)}, & \zeta = 0 \\ \frac{k_\zeta K'_\tau(k_\zeta R)}{K_\tau(k_\zeta R)}, & \zeta = 1, 2, 3, \dots \end{cases} \quad (A.7)$$

526 After inserting Eqs. (16) and (22) into Eq.(31), multiplying both sides by  $\cos[\beta_\zeta(z+h)]\cos(\tau\theta)$   
 527 and integrating for  $z \in [-h, -d]$  and  $\theta \in [0, \pi]$ , for any pair of integer  $(\tau, \zeta)$ , it can be obtained that

$$528 \quad \sum_{l=0}^{\infty} \left( \frac{\pi}{2} \left( \frac{\tilde{I}_\tau(k_l R_i)}{k_l \tilde{I}'_\tau(k_l R_i)} A_{\tau,l}^\chi + \frac{\tilde{I}_{-\tau}(k_l R_i)}{k_l \tilde{I}'_{-\tau}(k_l R_i)} A_{-\tau,l}^\chi \right) - i \sum_{\substack{m=-\infty \\ m \neq \pm \tau}}^{\infty} \frac{m \left[ (-1)^{m-\tau} - 1 \right]}{m^2 - \tau^2} \frac{\tilde{I}_m(k_l R_i)}{k_l \tilde{I}'_m(k_l R_i)} A_{m,l}^\chi \right) L_{\zeta,l}^\chi, \\ = \frac{\pi(h-d)}{\varepsilon_\tau \varepsilon_\zeta} \left( X_{\tau,\zeta}^{(3)} C_{\tau,\zeta}^\chi + Y_{\tau,\zeta}^{(3)} D_{\tau,\zeta}^\chi \right) + f_3^\chi$$

$$529 \quad (A.8)$$

530 where

$$531 \quad f_3^\chi = \begin{cases} 0, & \chi = S \\ \frac{\delta_{\tau,0} \delta_{\zeta,0} i \pi (h-d)}{\rho \omega}, & \chi = R \end{cases}, \quad (A.9)$$

$$532 \quad X_{m,l}^{(3)} = \begin{cases} \left( \frac{R_i}{R} \right)^m, & l = 0 \\ \frac{I_m(\beta_l R_i)}{I_m(\beta_l R)}, & l \neq 0 \end{cases}; \quad Y_{m,l}^{(3)} = \begin{cases} 1 + \ln \left( \frac{R_i}{R} \right), & l = 0, m = 0 \\ \left( \frac{R}{R_i} \right)^m, & l = 0, m \neq 0 \\ \frac{K_m(\beta_l R_i)}{K_m(\beta_l R)}, & l \neq 0 \end{cases} \quad (A.10)$$

533 After inserting Eqs. (22) and (25) into Eq.(32), multiplying both sides by  $\cos[\beta_\zeta(z+h)]\cos(\tau\theta)$   
 534 and integrating for  $z \in [-h, -d]$  and  $\theta \in [0, \pi]$ , for any pair of integer  $(\tau, \zeta)$ , we have

$$535 \quad \frac{h-d}{\varepsilon_\zeta} \left( C_{\tau,\zeta}^\chi + D_{\tau,\zeta}^\chi \right) - \sum_{l=0}^{\infty} E_{\tau,l}^\chi L_{\zeta,l}^\chi = f_4^\chi, \quad (A.11)$$

536 where

$$537 \quad f_4^\chi = \begin{cases} -\frac{2\varepsilon_\tau \text{ig} A L_{\zeta,0}}{\omega Z_0(0)} (-i)^\tau J_\tau(k_0 R) \cos(\tau\beta), & \chi = S \\ 0, & \chi = R \end{cases} \quad (A.12)$$

538 Eqs.(A.1), (A.4), (A.8) and (A.11) form a linear algebraic system, which can be used to solve

539  $A_{m,l}^\chi$ ,  $C_{m,l}^\chi$ ,  $D_{m,l}^\chi$  and  $E_{m,l}^\chi$  numerically after truncation. In the present model, the infinite

540 terms of  $e^{-im\theta}/\cos(m\theta)$ , and  $Z_l(z)/\cos[\beta_l(z+h)]$  are truncated at  $m=M$  and  $l=L$ , respectively. Accurate

541 results can be obtained by choosing  $M=12$ ,  $L=20$ .

542

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