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RESEARCH ARTICLE

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Understanding coupled factors that affect the modelling accuracy of typical planar compliant mechanisms

Abstract In order to accurately model compliant mechanism utilizing plate flexures, qualitative planar stress (Young's modulus) and planar strain (plate modulus) assumptions are not feasible. This paper investigates a quantitative equivalent modulus using nonlinear finite element analysis (FEA) to reflect coupled factors in affecting the modelling accuracy of two typical distributed-compliance mechanisms. It has been shown that all parameters have influences on the equivalent modulus with different degrees; that the presence of large load-stiffening effect makes the equivalent modulus significantly deviate from the planar assumptions in two ideal scenarios; and that a plate modulus assumption is more reasonable for a very large out-of-plane thickness if the beam length is large.

Keywords coupling factors, modelling accuracy, compliant mechanisms, equivalent modulus

1 Introduction

Compliant mechanisms utilize flexibility of material to achieve desired functions associated with motion, load and energy, rather than suppress the flexibility [1–4]. They offer low cost, high performance, and miniaturization for applications in which traditional mechanisms are not satisfactory. Modelling compliant mechanisms accurately enables quick design synthesis and analysis, by providing insightful observation how parameters affect the performances of compliant mechanisms.

This paper discusses static modelling of planar compliant mechanisms. Linear-matrix based linear modelling method is easy to use and straight-forward, which can provide an estimation of the instantaneous motion. For accurate modelling, nonlinearities and other influence factors should be taken into account as discussed below.

Nonlinearity. Nonlinearities in force-displacement characteristics of compliant mechanisms have three sources: Material non-linearity, large geometric non-linearity, and non-linearity of load-equilibrium equations under intermediate deflections [5,6]. The material non-linearity is generally not incorporated in compliant mechanisms due to the fact that plastic deformation is not desired. Because most popular materials such as aluminum alloy used in compliant mechanisms only work within intermediate deformations without yield, the geometry non-linearity can be ignored for simplification [5]. The intermediate deflection is therefore the focus of this paper, referring to that the transverse deflection is limited to be less than 10% of the length of beam but without neglecting non-linearity of load-equilibrium equations. The presence of axial force in the transverse motion equation is the physical embodiment of this type of nonlinearity.

Timoshenko effect. Timoshenko beam theory is usually used for short beams to consider shearing deflection and/or Poisson's ratio effect (cross-section shape change) in modelling beam bending [7–9]. Nevertheless, if the length of a beam is 10 times larger than its bending thickness, the beam is referred to as a slender beam, also Euler-Bernoulli beam, where the shearing deflection and Poisson's ratio effect can be ignored with acceptable modelling accuracy. In this paper, only slender beams are taken into account.

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Modulus assumptions. In compliant mechanisms, Young's modulus E of material is adopted for planar stress assumption where stress occurs only in the bending plane, while plate modulus $E'=E/(1-\nu^2)$ (ν is Poisson's ratio) of material is employed for planar strain assumption where strain perpendicular to the bending plane is negligible [5]. The detailed derivation for plate modulus is demonstrated in Appendix A. For planar stress assumption, beam's out-of-plane thickness is qualitatively assumed to be small enough, and for planar strain assumption, the out-of-plane thickness is qualitatively assumed to be large enough. An ideal planar stress assumption requires a zero out-of-plane thickness, while an ideal planar strain assumption requires an infinitely large out-of-plane thickness. Clearly, a different planar assumption can cause a large error as follows:

$$\frac{E'-E}{E}=\frac{1}{1-\nu^2}-1. \tag{1}$$

There is a need of an accurate equivalent modulus E_e to respond to a different out-of-plane thickness, enabling a quantitative accurate analysis, which is the main objective in this paper. The ratio of equivalent modulus to Young's modulus can be generally represented as follows:

$$\frac{E_e}{E}=f(X,U,D,L,T,...), \tag{2}$$

which is a function of at least beam's out-of-plane thickness (U), beam length (L), displacement (D) that is limited to $0.1L$, and beam's in-plane thickness (T). In the expression, the variable X denotes the type of mechanisms such as the two designs as shown in Fig. 1. If an analytical model for this modulus ratio is hard to derive, an empirical equation based on finite element analysis (FEA) is desired. References [10,11] reported interesting work on equivalent modulus of right circular planar flexure hinges (short beams) using an FEA method, and proposed an empirical equation of the equivalent modulus (or modulus ratio as defined above). It is shown that the equivalent modulus of the right circular planar flexure hinges is larger than the Young's modulus but smaller than the plate modulus, and that it is reasonable to use planar stress (or strain) assumption if beam out-of-plane thickness compared to the in-plane thickness is small enough (or large enough).

This paper intends to investigate how coupled parameters affect the equivalent modulus of two commonly-used distributed-compliance mechanisms, with its remainder organized as follows. Section 2 describes the study objectives followed by nonlinear FEA simulations and analysis in Section 3. Finally, conclusions are drawn in Section 4.

2 Study description

The study objects in this paper are a compound basic parallelogram mechanism (CBPM) (Fig. 1(a)) and a compound double parallelogram mechanism (CDPM) (Fig. 1(b)). The two mechanisms are both 1-DOF (degree of freedom) and symmetrical translational compliant mechanisms, which are composed of identical slender beams. Figure 1(a) is the one without under-constraints and free of buckling, which can induce load-stiffening effect by the inherent internal axial force caused by the primary translation (along the Y-axis) only. Figure 1(b) is the one with under-constrained stages (secondary stages), but without load-stiffening effect under the primary translation. This paper only studies the equivalent modulus under the primary motion. In Fig. 1(a), each beam can be simplified as a fixed-clamped beam, while in Fig. 1(b) each beam can be simplified as a fixed-guided beam.

The closed-form primary motion equations for the two designs (Fig. 1) are shown below [12,13]. In the two equations, we normalize the geometrical dimension by the beam length (L), the force by E_eI/L^2, and moment by E_eI/L (I: Moment of inertia of cross-section areas) with all

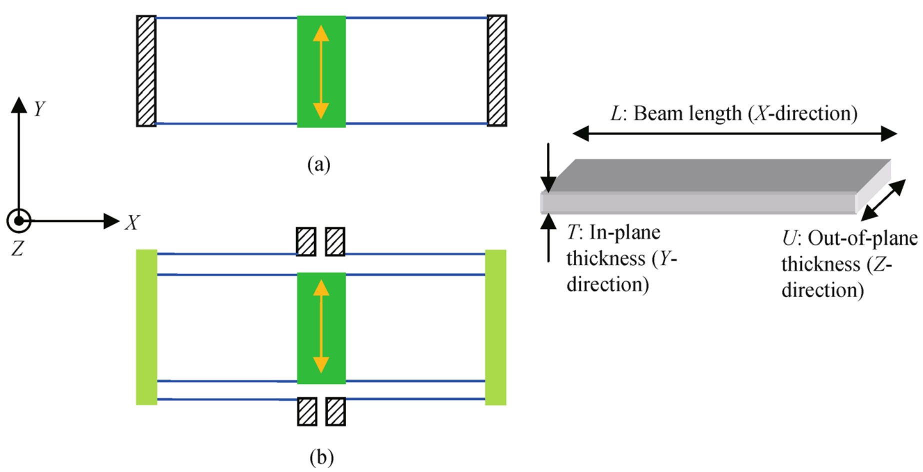

Fig. 1 Two types of commonly-used translational compliant mechanisms

normalized parameters denoted by corresponding lower-case symbols [14].

$$y_s \approx \frac{f_y}{48 + \dfrac{2.88 y_s^2}{1/d + y_s^2/700}}, \tag{3}$$

$$y_s \approx \frac{f_y}{24}, \tag{4}$$

where $d = 12/(T/L)^2$ is a normalized parameter in which T is the beam in-plane thickness.

Since the required actuation force is proportional to the used modulus under the same primary motion [10,11], the following two equations can be obtained:

$$\frac{F_{3D}}{F_{SS}} = \frac{E_e}{E}, \tag{5}$$

$$\frac{F_{SN}}{F_{SS}} = \frac{E'}{E}, \tag{6}$$

where F_{3D}, F_{SS} and F_{SN} are the actual required actuation forces for the same primary motion (D) under no any planar assumption, planar stress assumption, and planar strain assumption, respectively, which will be simulated by FEA in the next section.

3 FEA simulation results and analysis

Commercial software, Comsol, was used to conduct FEA to obtain the required actuation forces for the same primary motion. Here, the nonlinear simulation function is activated, and finest free meshing for beams is chosen for two cases with planar stress and planar stress assumptions, respectively, and a case without any planar assumption. Simulation results are shown in Figs. 2 to 5. For convenience, beam thickness is fixed to 1 mm and other parameters vary case by case.

Figure 2 shows the force ratio (F_{3D}/F_{SS}) significantly changes with primary motion with the highest value at the home position for the CBPM. It also suggests that larger out-of-plane thickness makes the force ratio (F_{3D}/F_{SS}) larger as imagined. However, it seems that force ratio (F_{SN}/F_{SS}) is not sensitive to the primary motion and the out-of-plane thickness as suggested by Eq. (6). Figure 3 shows that primary motion increases the force ratio (F_{3D}/F_{SS}) slightly but has no effect on the force ratio (F_{3D}/F_{SS}) for the CDPM. The increase of the out-of-plane thickness significantly increases the force ratio (F_{3D}/F_{SS}) but has little influence on force ratio (F_{SN}/F_{SS}), as predicted by Eq. (6).

In order to reflect the effect of beam length along with other parameters on the force ratio, the average force ratio over primary motion is adopted as shown in Figs. 4 and 5 for both CBPM and CDPM. Similar to the results as illustrated in Figs. 3 and 4, the increase of out-of-plane thickness enlarges the average force ratio (F_{3D}/F_{SS}), but has an insignificant influence on the average force ratio (F_{SN}/F_{SS}). In addition, the average force ratio (F_{SN}/F_{SS}) is almost independent of the beam length, which is nearly same as E'/E as shown in Eq. (6).

There is an exciting finding that a shorter beam can increase F_{3D}/F_{SS} in a larger degree, and F_{3D}/F_{SS} can even

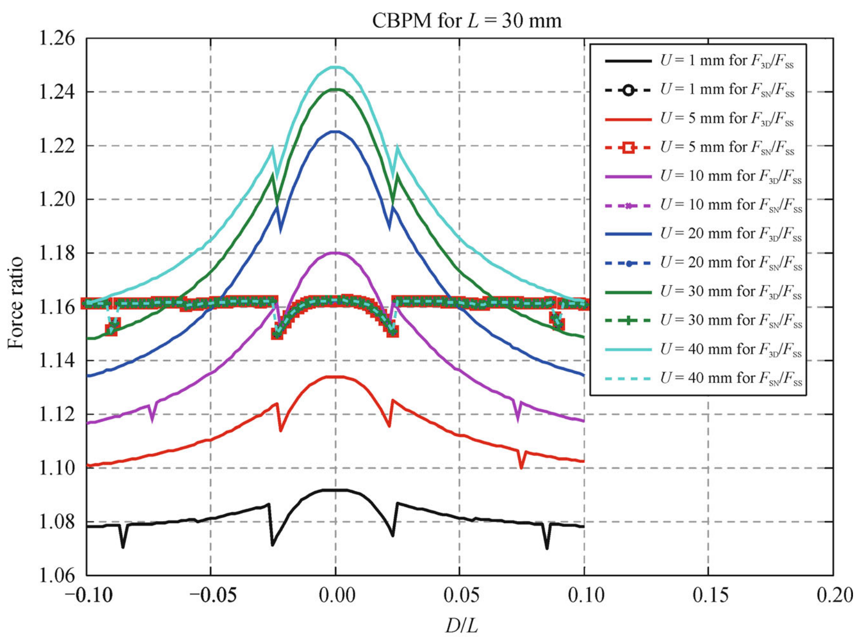

Fig. 2 CBPM force ratio results for fixed beam length

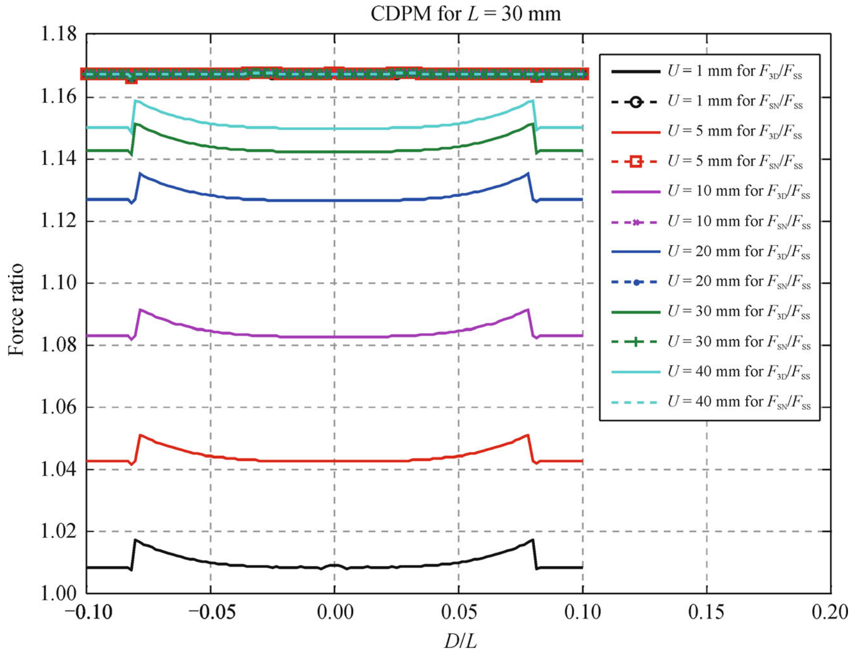

Fig. 3 CDPM force ratio results under fixed beam length

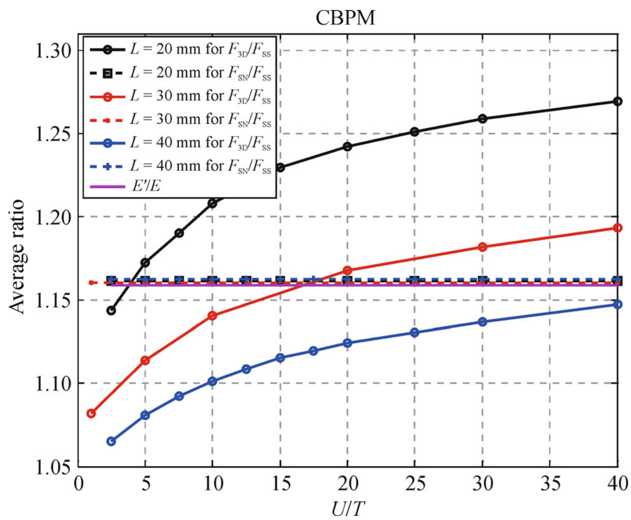

Fig. 4 Average force ratio for CBPM

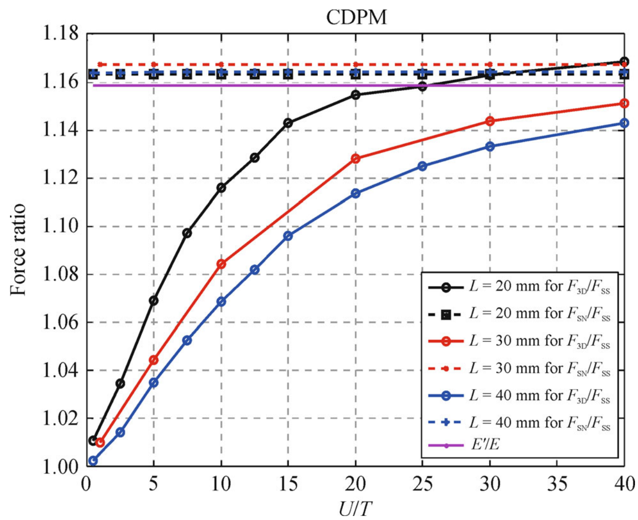

Fig. 5 Average force ratio for CDPM

be much larger than F_{SN}/F_{SS} under certain combination of L and U. This finding is different from the result in Refs. [10,11] and betrays our expectation. The influence from beam length of the CBPM is stronger than that of the CDPM, due to the fact that the CBPM exhibits load-stiffening behavior. The presence of significant load-stiffening effect results in large internal axial force, which may amplify the Timoshenko effect to make equivalent modulus deviate from the planar (stress and strain) assumptions in two ideal scenarios. Figures 4 and 5 both reveal that a plate modulus assumption for both CBPM and CDPM is still valid for large out-of-plane thickness if the beam length is more than 40 times of beam in-plane thickness. This means that Euler-Bernoulli beam works well for plate modulus assumption under large out-of-plane thickness. Furthermore, a Young's modulus assumption for CDPM is largely acceptable for small out-of-plane thickness in spite of different beam lengths. Therefore, based on the recommendations in Figs. 4 and 5, an equivalent modulus can be selected to produce more accurate analytical model using Eqs. (3) and (4).

The cubic (3th degree polynomial) fitting curve equation for the CBPM for a given beam length (Fig. 4) is shown below

$$y_1 = p_1x^3 + p_2x^2 + p_3x + p_4, \tag{7}$$

where $x = U/T$, and y_1 denotes the average of $F_{3D}/F_{SS} = E_e/E$.

For $L = 20$ mm, the coefficients are $p_1 = 4.3494\times10^{-6}$, $p_2 = -0.0003865$, $p_3 = 0.012315$, and $p_4 = 1.1174$; for $L = 30$ mm, the coefficients are $p_1 = 3.1976\times10^{-6}$, $p_2 = -0.00028066$, $p_3 = 0.0091061$, and $p_4 = 1.0737$; and for $L = 40$ mm, the coefficients are $p_1 = 2.2863\times10^{-6}$, $p_2 = -0.00020164$, $p_3 = 0.0068409$, and $p_4 = 1.0502$.

Similarly, the cubic fitting curve equation for the CDPM for a given beam length (Fig. 5) is shown below

$$y_2 = p_1x^3 + p_2x^2 + p_3x + p_4, \tag{8}$$

where $x = U/T$, and y_2 denotes the average of $F_{3D}/F_{SS} = E_e/E$.

For $L = 20$mm, the coefficients are $p_1 = 6.3392\times10^{-6}$, $p_2 = -0.0005568$, $p_3 = 0.016328$, and $p_4 = 1.0009$; for $L = 30$ mm, the coefficients are $p_1 = 2.5216\times10^{-6}$, $p_2 = -0.00028307$, $p_3 = 0.011118$, $p_4 = 0.99798$; and for $L = 40$ mm, the coefficients are $p_1 = 1.3393\times10^{-6}$, $p_2 = -0.00018976$, $p_3 = 0.0091325$, and $p_4 = 0.99502$.

For a beam length in either CBPM or CDPM that is not equal to 20, 30 or 40 mm as shown in Figs. 4 and 5, a *linear interpolation method* can be applied to obtain the average E_e/E corresponding to this beam length. We can observe from the above cubic fitting equations that they have the similar coefficient characteristic where only p_2 is negative and the other coefficients are positive. Moreover, the increase of beam length can cause the decrease of p_1, $|p_2|$, p_3 or p_4.

4 Conclusions

This paper discusses the effects of coupled factors on the equivalent modulus for the modelling accuracy in planar compliant mechanisms. Two typical parallelogram mechanisms are simulated by nonlinear FEA to demonstrate such effects. The cubic fitting curve equation for the average of E_e/E corresponding to a different beam length and a different beam out-of-plane thickness has been obtained. It has been found:

① That the beam's out-of-plane thickness (U), beam length (L), and displacement (D) all have influences on the force ratio (F_{3D}/F_{SS}) with different extents;

② That the presence of load-stiffening effect makes the equivalent modulus largely deviate from the planar assumptions in two ideal scenarios; and

③ That a plate modulus assumption is still valid for a very large out-of-plane thickness if the beam length is more than 40 times of beam in-plane thickness.

A closed-form model of the equivalent modulus to accommodate all parameter effects is greatly desired in the future. Investigating the equivalent modulus for more typical compliant mechanisms is also the further work.

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Appendix A: Plate modulus

Figure A1 shows the normal stresses of an infinitesimal element along three orthogonal directions. If a planar strain assumption in the XY plane is made, we have the normal strain equation in the Z-direction based on the generalized Hook law:

$$\varepsilon_{zz} = \frac{\sigma_{zz}}{E} - \frac{\nu}{E}\left(\sigma_{xx} + \sigma_{yy}\right) = 0, \tag{A1}$$

where $\sigma_{yy} = 0$ since there is no applied stress in this direction, causing planar stress in the XZ plane.

Equation (A1) can be simplified as

$$\sigma_{zz} = \nu\sigma_{xx}. \tag{A2}$$

Thus the normal strain equation in the X-direction can be obtained as:

$$\varepsilon_{xx} = \frac{\sigma_{xx}}{E} - \frac{\nu}{E}\left(\sigma_{yy} + \sigma_{zz}\right). \tag{A3}$$

The substitution of Eq. (A2) into Eq. (A3) yield

$$\varepsilon_{xx} = \frac{\sigma_{xx}}{E/(1-\nu^2)}. \tag{A4}$$

Therefore, the Plate modulus is derived for the bending in the *XY* plane as

$$E' = \frac{E}{1-\nu^2}.$$

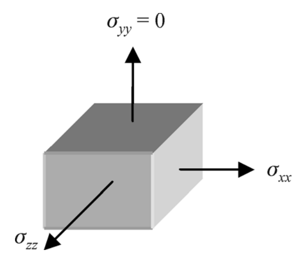

Fig. A1 Normal stresses of an infinitesimal element along three orthogonal directions