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Theory of improved spectral purity in index patterned Fabry-Pérot lasers

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The spectral purity of a ridge waveguide Fabry-Pérot laser can be improved by patterning the effective refractive index seen by an optical mode propagating in the cavity. Here we present a transmission matrix calculation to first order in the effective index step from which we derive the threshold condition as a function of cavity mode index. This approach enables us to solve the inverse problem relating the index pattern along the cavity to the threshold gain modulation in wavenumber space. Quasiperiodic index patterns are constructed, which lead to improved spectral purity at a predetermined wavelength. © *2005 American Institute of Physics*.
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Approaches to improving spectral purity in edge emitting semiconductor lasers often involve a spatially varying refractive index in the form of a grating. Of these, distributed feedback (DFB) lasers are perhaps the most well established.[1] These devices require complex processing and regrowth steps to manufacture. On the other hand, while edge-emitting ridge waveguide Fabry-Pérot (FP) lasers are among the easiest lasers to manufacture, the plain FP semiconductor laser is limited by a natural tendency to lase in multiple longitudinal modes.

Effective index perturbations within the cavity can introduce a modulation which can improve the spectral purity of the Fabry-Pérot laser.[2,3] One technique utilizing this principle involves the creation of a low density of additional features in the laser ridge waveguide when the ridge itself is formed.[3] The facet reflections in this case are the primary source of feedback necessary for lasing, while the effective index pattern along the cavity leads to improved spectral purity. There has been to date surprisingly little theoretical analysis of such index patterned FP lasers. Previous attempts to design such lasers have therefore relied primarily on experimental approaches or on numerical techniques, including, for example, the use of genetic algorithms to design the effective index pattern and current distribution for a given output spectrum.[4]

We introduce here a straightforward approach to derive the threshold condition as a function of mode index in an index-patterned FP laser. The perturbation we wish to model takes the form of a slot which penetrates into the cladding region of the optical waveguide of the laser. We treat the slot as a separate macroscopic section of the laser cavity where, according to the transverse structure, we assign a different effective index. In the present work, we assume that the effective refractive index step, Δn, is real and that it is small in comparison to the cavity effective index. We then derive the threshold condition from our expression for the complex transmission coefficient. The use of Fourier techniques enables us to solve the inverse problem by specifying a distribution of slots which modify the threshold gain of modes to give improved spectral purity at a predetermined wavelength. While we refer here exclusively to slotted laser structures, our theoretical approach should be applicable wherever a low density of effective index steps can be added to a FP laser cavity.

In the case of a multilayer system in vacuum as shown in Fig. 1, the complex transmission and reflection coefficients can be found by considering a matrix product.[5] We define $\theta_i = k_{iz}\cdot L_i$ where $k_{iz}=n_i k_{0z}$ and L_i and n_i are the length and the effective index of the ith section, respectively. The adjusted optical path length across the cavity is then $\Sigma\theta_i$. Since typically, $\Delta n/n \ll 1$, where n is the cavity effective index, we can treat the influence of the slots by only retaining terms to order $\Delta n/n$. Assuming that the effective index step is the same for all the additional features, the complex transmission coefficient of the cavity is then given by $\tilde{t}=\tilde{t}_0(1+\Sigma_j\Delta_j)^{-1}$, where $\tilde{t}_0$ is the Fabry-Pérot spectrum of a plain cavity with the same adjusted optical path length and

$$\Delta_j = -i\frac{\Delta n}{n}\sin(\theta_j)\frac{r_1\exp(2i\phi_j^-)+r_2\exp(2i\phi_j^+)}{1-r_1 r_2\exp(2i\Sigma\theta_i)}, \qquad (1)$$

the subscript j referring to the slot index. In the above expression, the facet reflectivities are $r_1=|r_1|\exp(i\varphi_1)$ and $r_2=|r_2|\exp(i\varphi_2)$, and, for the case of a real refractive index distribution, the quantities ϕ_j^-, and ϕ_j^+ are the optical path lengths from the center of each slot to the left and right facets, respectively. The factor $\sin(\theta_j)$ expresses the quarter wave condition for maximum slot reflectance. Our first order

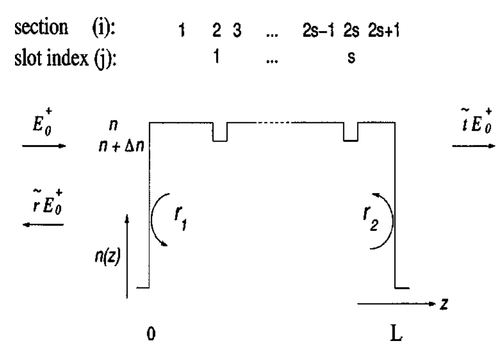

FIG. 1. One-dimensional model of a Fabry-Pérot laser cavity of length L and including s slotted regions. The cavity effective index is n and the slot region has effective index $n+\Delta n$. The cavity is in vacuum with all cavity sections numbered $2s+1 \geq i \geq 1$ beginning on the left. The slotted regions are also numbered with an index j. The complex transmission and reflection coefficients of the cavity are $\tilde{t}$ and $\tilde{r}$, respectively. The complex facet reflectivities are r_1 and r_2 as shown.

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approach describes the coupling of each interface to the cavity mirrors to all orders of scattering while multiple scattering within and between slots is neglected.

Laser oscillation can occur at the complex poles of $\tilde{t}$ as given above. If we set $\phi_j^{\pm} = \phi_j^{\pm'} + i\phi_j^{\pm''}$, $\Sigma\,\theta_i = \Sigma\,\theta_i' + i\Sigma\,\theta_i''$ and, to simplify the analysis, assume that the factor $\Delta n/n \sin\theta_j$ is a real number, then, expanding about the cavity resonance condition: $\Sigma\theta_i' = \phi_j^{-'} + \phi_j^{+'} = m\pi + \delta_m - 1/2(\varphi_1 + \varphi_2)$, we have from the imaginary part of the denominator, again to first order in $\Delta n/n$:

$$\delta_m = \frac{-\Delta n}{n} \frac{\Sigma_j \sin\theta_j (A_j^- + A_j^+)\cos(2\phi_j^{-'} + \varphi_1)}{2|r_1 r_2|\exp(-2\Sigma\theta_i'')}. \tag{2}$$

In the above expression, $A_j^- = |r_1|\exp(-2\phi_j^{-''})$ and $A_j^+ = |r_2|\exp(-2\phi_j^{+''})$. Our equations are valid below lasing threshold, which implies that the quantities $A_j^{\pm}$ are bounded by $1/|r_1 r_2|$. Here we limit the number of slots, s, and the effective index step such that $\delta_m \ll \pi$ and demonstrate that excellent selectivity can be achieved with few additional features. In the language of grating theory we have $\overline{\kappa}\cdot L \ll 1$ where $\overline{\kappa}$ is a suitably defined average coupling coefficient.

If we neglect absorption and scattering losses and assume that the gain is uniformly distributed along the device we have that $\phi_j^{-''} = \eta_j \Sigma\,\theta_i''$, $\phi_j^{+''} = (1-\eta_j)\Sigma\,\theta_i''$, where $0 < \eta_j < 1$ is the position of the slot center as a fraction of the cavity length. The threshold gain, $\gamma_{t(m)}$, of the mth cavity resonance is then given by,

$$\gamma_{t(m)} = \alpha + \frac{1}{L\sqrt{|r_1 r_2|}}$$

$$\times \sum^{j} a_{j(m)}[|r_1|\exp(\epsilon_j L\alpha) - |r_2|\exp(-\epsilon_j L\alpha)], \tag{3}$$

where $a_{j(m)} = \Delta n/n \sin\theta_j \sin(2\phi_j^{-'} + \varphi_1)$, evaluated at $\Sigma\theta_i' = m\pi + \delta_m - 1/2(\varphi_1 + \varphi_2)$, $\alpha = 1/L \ln(1/|r_1 r_2|)$ are the mirror losses of the plain cavity and $\epsilon_j = \eta_j - 1/2$. The change in the threshold gain of modes, $\Delta\gamma_t$, is proportional to the difference in the roundtrip amplitude gains to the left and right of the slot. The amplitude of the modulation of the threshold is then a function of the slot position, and vanishes at the device center in the case where the mirror reflectances are equal.

For single frequency index patterned FP lasers, the approach we follow is to select a particular cavity resonance, $m = m_0$, and introduce the reflective features along the cavity in order to preferentially lower the threshold gain of that mode. It is instructive to first consider the threshold modulation in wave number space where the modes are defined for positive integers m by the cavity resonance condition given above. An ideal functional form for $\Delta\gamma_t(m_0 + \Delta m)$ would have a maximum at $\Delta m = 0$ and would equal zero at all other integer values of Δm. Such a function is $\mathrm{sinc}(\Delta m) = \sin\pi\Delta m/\pi\Delta m$, which can be written as the Fourier transform of the unit rectangle or top-hat function $\Pi(\epsilon)$. That is, $\mathrm{sinc}(\Delta m) = \int_{-1/2}^{1/2}\cos[2\pi\epsilon\Delta m]d\epsilon$.[6]

In practice, cavity resonances over a finite band of frequency are of interest. We therefore define a periodic istribution of sinc functions with spacing a cavity modes and take the product with a Gaussian envelope function to make our object spectrum equal to the

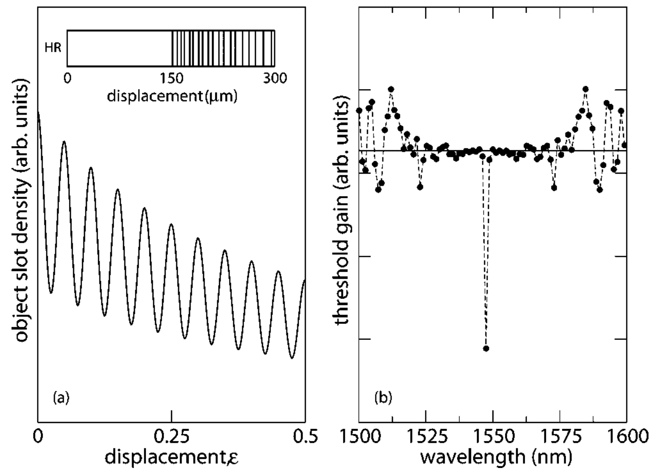

FIG. 2. (a) Object slot density function which we approximate. Inset: Laser cavity schematic indicating the locations of the slots; (b) threshold gain of modes for the laser cavity schematically pictured in the inset of (a). The horizontal line is at the value of the mirror losses of the plain cavity.

function $(g\cdot p)(\Delta m) = g(\Delta m)\cdot\Sigma_{n=-\infty}^{\infty}\mathrm{sinc}(\Delta m - na)$, where $g(\Delta m) = \exp[-\pi\tau^2(\Delta m)^2]$. The factor τ determines the decay of the envelope and thus the magnitude of the threshold gain modulation at a distance a cavity modes from the selected mode. The Fourier transform of our object function now consists of a series of Gaussians functions of the form $\Gamma(\epsilon) = \exp[-\pi\epsilon^2/\tau^2]$, centered at the origin, and with equal spacing a^{-1} inside the window $-1/2 \leqslant \epsilon \leqslant 1/2$.

We first apply the method to the symmetric, as-cleaved case which is particularly intuitive. We have $\varphi_1 = \varphi_2 = 0$, and $|r_1| = |r_2|$ and to proceed we at first neglect the optical path length corrections which result from the introduction of the slots, taking $\eta_j \propto \phi_j^{-'}$. A given cavity mode m_0 is chosen and we expand in terms of $m = m_0 + \Delta m$. We then find that the threshold modulation can be expressed in terms of its even and odd components with respect to Δm as follows:

$$\sin(2\phi_j^{-'} + \varphi_1) \simeq \cos m_0\pi \cos\Delta m\pi$$

$$\times[\sin 2\pi\epsilon_j m_0 \cos 2\pi\epsilon_j\Delta m + \cos 2\pi\epsilon_j m_0 \sin 2\pi\epsilon_j\Delta m]. \tag{4}$$

Because the function, which determines the amplitude of the threshold modulation, $f(\epsilon) = |r_1|[\exp(\epsilon L\alpha) - \exp(-\epsilon L\alpha)] \sim \sinh\epsilon L\alpha$, is odd with respect to ϵ in this case, a $\pi/2$ phase shift must be added to the index pattern at the device center. Each slot is positioned such that $\sin 2\pi\epsilon_j m_0 = \pm 1$ according to whether the slot is placed to the left or right of the device center. One can show that the resultant index pattern is related to that of the phase-shifted DFB laser with a half wavelength subcavity at the device center and all other subcavities quarter wave.

The index pattern we now construct represents a discrete approximation to the continuous Fourier transform of our object function, corrected to allow for the variation of the amplitude of the threshold modulation with position along the device. To generate the approximate slot positions, ϵ_j, we integrate the product of the Fourier transform of our object spectrum with the inverse of the modulation amplitude function. The approximate feature positions are then generated using

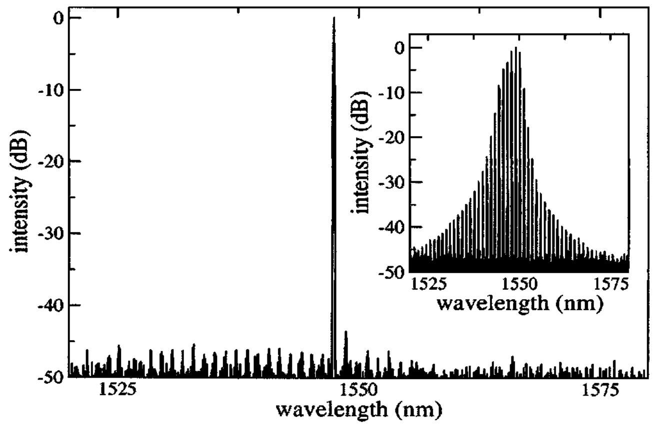

FIG. 3. Laser spectrum of the index patterned device at twice threshold. Inset: Spectrum at twice threshold of an equivalent Fabry-Pérot laser.

$$A\sum_n \int_{\epsilon_{\min}}^{\epsilon_j} [f(x)]^{-1}\Gamma(x-n/a)dx = j-1/2, \tag{5}$$

where $j=1,2,\ldots,s$ and A is normalized to the number of slots to be introduced. Once the approximate ϵ_j are known, the correct positions correspond to the nearest fractions, $\alpha_j = \phi_j^-/\Sigma\theta_i$, of the total optical path length which satisfy the appropriate phase requirement, being $\sin(2\pi\alpha_j m_0)=\pm 1$ in this case. These positions are found taking into account the optical path length corrections resulting from the introduction of the slots themselves. Fractions of the cavity length, η_j, corresponding to fractions of the optical path length α_j are related by $\alpha_j=(\eta_j+s_j^-\Delta n/n\beta)/(1+s\Delta n/n\beta)$. Here s_j^- is the number of slots to the left of slot j, and β is the slot length as a fraction of the cavity length. The corrected sequence η_j then determines a quasiperiodic index pattern with respect to the wavelength of the mode m_0 within the cavity.

To illustrate the application of the method, we have designed a single mode laser cavity of length 300 μm (free space wavelength 1547.5 nm, $m_0=1236$) using $s=19$ slots distributed over one half of the device beginning at the center. Parameters used were $a=20$ modes and $\tau\simeq 0.036$, corresponding to an 80% suppression of the threshold gain modulation amplitude at $\Delta m=\pm a$ modes. Together with these values, the facet coatings determine the object slot density function shown in Fig. 2(a) while a schematic picture of the device, high-reflection (HR) coated as indicated, is shown in the inset. The resultant threshold gain spectrum of the cavity, found by iteration using Eqs: (2) and (3) is plotted in Fig. 2(b). Although we estimate the change in the threshold gain of the selected mode to be less than 10% of the plain cavity mirror losses, the side mode suppression ratio at twice threshold exceeds 40 dB, as shown in Fig. 3. For comparison, an equivalent spectrum of a Fabry-Pérot laser fabricated on the same bar is shown in the inset.

Uncertainty in the device length and position of the index pattern with respect to the facets will mean that single mode lasers in the intended wavelength interval can be expected on a statistical basis using this approach. This mirrors the sensitivity of the performance of DFB lasers to the grating phase in the presence of a facet reflection.[7] Predetermination will be easier to achieve with longer wavelength ridge waveguide and plasmon enhanced quantum cascade laser sources.[8] Other applications of the technique include the possibility of tailoring multimode FP spectra for multisection discretely tunable sources as well as device applications requiring mode locked laser operation.[9]

In conclusion, we have derived an expression for the threshold condition as a function of mode index for a Fabry-Pérot laser incorporating a low density of weak effective refractive index steps along the laser cavity. We demonstrated that Fourier techniques then allow for the construction of quasiperiodic effective index patterns, which provide improved spectral purity at a predetermined wavelength.

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