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The Role of Synchronization in Digital Communications Using Chaos—Part II: Chaotic Modulation and Chaotic Synchronization

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***Abstract*—In a digital communications system, data are transmitted from one location to another by mapping bit sequences to symbols, and symbols to sample functions of analog waveforms. The analog waveform passes through a bandlimited (possibly time-varying) analog channel, where the signal is distorted and noise is added. In a conventional system the analog sample functions sent through the channel are weighted sums of one or more sinusoids; in a chaotic communications system the sample functions are segments of chaotic waveforms. At the receiver, the symbol may be recovered by means of coherent detection, where all possible sample functions are known, or by noncoherent detection, where one or more characteristics of the sample functions are estimated. In a coherent receiver, synchronization is the most commonly used technique for recovering the sample functions from the received waveform. These sample functions are then used as reference signals for a correlator. Synchronization-based coherent receivers have advantages over noncoherent receivers in terms of noise performance, bandwidth efficiency (in narrow-band systems) and/or data rate (in chaotic systems). These advantages are lost if synchronization cannot be maintained, for example, under poor propagation conditions. In these circumstances, communication without synchronization may be preferable. In Part I, the theory and operation of conventional communications systems were surveyed and possible fields of application of chaotic communications were identified. In Part II, the theory of conventional telecommunications is extended to chaotic communications, chaotic modulation techniques and receiver configurations are surveyed, and chaotic synchronization schemes are described. In Part III, examples will be given of chaotic communications schemes with and without synchronization, and the performance of these schemes is evaluated in the context of noisy, bandlimited channels.**

***Index Terms*—Chaos, communication systems, digital communication, digital modulation, spread spectrum communication.**

I. Introduction

RESEARCH into applications of chaos in communications has been motivated by the observation that chaotic systems can be synchronized. The basic idea is that information can be conveyed to a remote receiver by means of a wideband chaotic signal. Since 1992, a number of chaotic synchronization and modulation schemes have been proposed, most of which have been developed using heuristic arguments, without reference to conventional communications measures [1].

The objectives of this work are threefold:

1) to provide a theoretical context in which the performance of modulation schemes based on chaotic synchronization can be evaluated;
2) to develop a unified framework for discussing and comparing conventional and chaotic communication systems;
3) to highlight the special problems that arise when chaotic basis functions are used.

In Part I of this three-part paper [2], we described the major components of a digital communications system, identified the role of synchronization in coherent receivers, and motivated the use of chaotic rather than periodic basis functions.

In Section II of this part, we describe two digital chaotic modulation schemes—chaos shift keying (CSK) and differential chaos shift keying (DCSK)—and identify appropriate coherent, noncoherent, and differentially coherent receiver architectures.

The theoretical performance of chaotic communications receivers with and without synchronization is examined in Section III. We conclude that the synchronization-based recovery of chaotic basis functions from noisy received sample functions offers a potential advantage over noncoherent detection in terms of noise performance and data rate, but only if synchronization can be maintained. Under poor propagation conditions, where synchronization cannot be maintained, the advantages of coherent detection are lost. In such circumstances, a noncoherent receiver offers a more robust and less complex solution.

In Section IV, we consider the state of the art in synchronization of chaotic systems in the context of digital communications and highlight the weaknesses of current chaotic synchronization techniques.

In Part III of the paper [34], the performance of representative coherent and noncoherent chaotic communication schemes is evaluated in the context of a noisy and bandlimited channel.

II. Survey of Chaotic Modulation and Demodulation Techniques

The basic idea of digital communication using a chaotic carrier is that the bits (binary modulation) or symbols (M-ary modulation) are mapped to sample functions of chaotic

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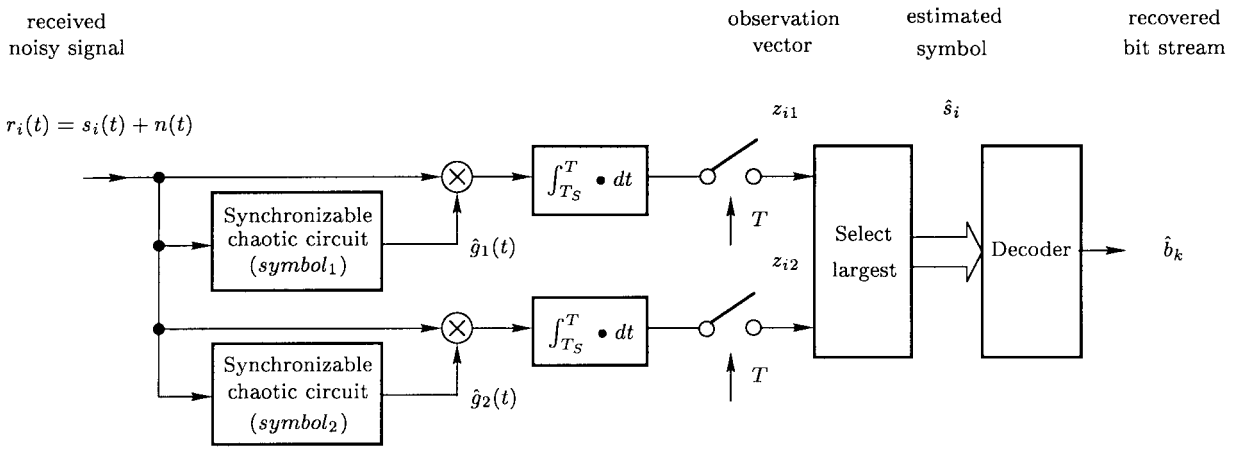

Fig. 1. Block diagram of coherent correlation CSK receiver for $M = 2$.

signals emanating from one or more chaotic attractors. In order to avoid periodicity, the symbols are mapped to the actual nonperiodic outputs of chaotic circuits and not to parameters of certain known sample functions.

The principal difference between a chaotic carrier and a conventional periodic carrier is that the sample function for a given symbol is nonperiodic and is different from one symbol interval to the next. Thus, the transmitted waveform is never periodic, even if the same symbol is transmitted repeatedly.

As in the case of conventional digital communications, we consider four categories of modulation techniques:

1) coherent correlation receiver with chaotic synchronization;
2) coherent matched filter receiver;
3) noncoherent detection techniques;
4) differentially coherent reception.

A. Coherent Correlation Receiver with Chaotic Synchronization

1) Coherent Detection of CSK: Chaos shift keying (CSK) [3], [4] is a digital modulation scheme where each symbol is mapped to a different chaotic attractor. The number of attractors is equal to the size of the signal set in this case. The attractors may be produced by the same dynamical system for different values of a bifurcation parameter or by completely different dynamical systems.

Note that the information to be transmitted is carried not by the shape of the sample function but by the attractor which produces the sample function. The objective of the demodulator is to decide, on the basis of a received noisy and distorted sample function, which attractor is most likely to have produced this waveform.

Using the notation introduced in Part I [2], assume that each attractor produces a basis function $g_j(t)$ and that the elements of the signal set

$$s_i(t) = \sum_{j=1}^{N} s_{ij} g_j(t), \qquad j = 1, 2, \cdots, N$$

are given by $s_i(t) = g_i(t)$ for all i.

In terms of the components of the signal vector, this corresponds to the case $s_{ij} = 0$ if $i \neq j$ and $s_{ij} = 1$ if $i = j$. Furthermore, assume that the autocorrelation of each $g_j(t)$ with itself in each symbol interval T is larger than the cross correlation with any of the other basis functions. In this case, a correlation receiver may be used to identify the attractor which is most likely to have produced the received signal [2].

As in the case of a conventional correlation receiver based on synchronization, a local synchronized copy of each basis function $g_j(t)$ *has to be produced* in the receiver using appropriate synchronization circuitry[1]. In the case of chaotic basis functions, this topic is called *chaotic synchronization*. We will deal with chaotic synchronization in more detail in Section IV, but first let us consider the conceptual process.

Synchronizable counterparts of the circuits which produce the basis functions $g_j(t)$ in the transmitter are used to recover the basis functions in a coherent correlation receiver, as shown in Fig. 1. Here, the received signal $r_i(t)$ tries simultaneously to synchronize all of the "synchronizable chaotic circuits" in the receiver.

For example, assume that the signal $s_i(t) = g_1(t)$ is transmitted. After a synchronization time T_S, which is analogous to the pull-in time in a phase-locked loop (PLL), the output $\hat{g}_1(t)$ converges to $g_1(t)$. By contrast, $\hat{g}_2(t)$ fails to synchronize with $g_1(t)$. The decision as to which symbol was transmitted is made on the basis of the "goodness" of synchronization. In the ideal case, $\hat{g}_1(t)$ is more strongly correlated with $r_i(t)$ than $\hat{g}_2(t)$ during the interval $[T_S, T]$. Hence, $z_{i1} > z_{i2}$ and the decision circuit decides that symbol 1 was transmitted.

In any realistic situation, the received signal is always corrupted by noise $n(t)$. Even in the case of perfect synchronization, the instantaneous value of the received signal may differ considerably from the recovered chaotic signal. This is why correlators *must* be used for the detection, i.e., to determine the "goodness" of synchronization. Because of the time-averaging involved in correlation, the use of correlators also tolerates loss of synchronization for short periods of time.

In any practical communications system, not only an isolated single symbol, but a sequence of symbols, has to be transmitted.

[1]Recall that the weights s_{ij} are recovered by computing $\hat{s}_{ij} = \int_0^T r_i(t) g_j(t)\, dt$ for $j = 1, 2, \cdots, N$ [2].

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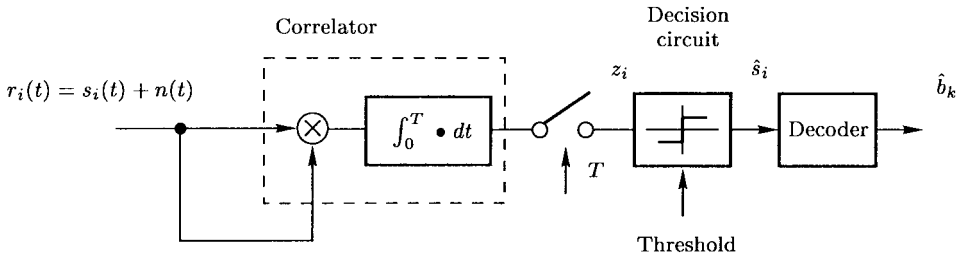

Fig. 2. Block diagram of a noncoherent receiver for COOK or CSK.

Even in a linear channel, interference among the successive symbols may appear, i.e., *intersymbol interference* (ISI) [5] may occur. One source of ISI can be the integrator used in the correlator. If the initial value of the integrator is reset to zero at the beginning of each observation period, by means of an *integrate-and-dump* circuit [5], for example, then this kind of ISI can be avoided.

We assume in the following that the timing information is available in the receiver. The initial value of the integrator(s) is reset to zero at the beginning of every observation period and the observation vector is generated at the end of each symbol interval. In Figs. 1–4, the start and end of each observation period are indicated by the limits of integration and the decision time instants are represented schematically by sampling switches.

2) Data Rate of a Coherent Correlation Receiver with Chaotic Synchronization for CSK: The disadvantage of a coherent correlation CSK receiver is that synchronization is lost and recovered every time the transmitted symbol is changed [4]. The symbol duration is therefore equal to the sum of the synchronization time T_S *plus* the estimation time of the observation vector. The synchronization time puts an upper bound on the symbol rate and thus the data rate.

To maximize the data rate in conventional digital systems, synchronization is always maintained. If the transmitted signal does not contain a signal that can be used as a reference for synchronization (in the case of suppressed carrier modulation schemes, for example) then a nonlinear operation is used to regenerate the reference signal in the receiver [5], [6]. This idea could also be exploited in chaotic communications if a synchronization technique could be found which was sufficiently insensitive to some parameter of the chaotic basis functions. In that case, a selected parameter could be varied according to the modulation and synchronization could be maintained continuously. The symbols to be transmitted would then be mapped to the selected parameter of the chaotic sample function and only one attractor would be necessary; this is analogous to a conventional modulation technique where the attractor is a periodic trajectory whose amplitude, frequency, or phase might be controlled by the modulation.

If synchronization of a chaotic circuit could be maintained in the presence of other chaotic signals, then it would be possible to increase the size of the signal set by generating $s_i(t)$ as a weighted sum of basis functions with more than one nonzero weight, as discussed in Part I of this paper (see [2, Section III-B.1]). To our knowledge, none of the chaotic synchronization techniques which exist in the literature is sufficiently robust to permit augmentation of the signal set in this way [7].

B. Coherent Matched Filter Receiver for CSK

Matched filters can be used only if the waveforms corresponding to each symbol are known in advance and preprogrammed as the impulse responses of filters. In the case of CSK modulation, the symbols are mapped to chaotic attractors and a different sample function is generated each time a symbol is transmitted[2]. Therefore, coherent matched filter receivers simply cannot be used in chaotic communications.

C. Noncoherent Detection

1) Noncoherent Detection of COOK: Chaotic on-off-keying (COOK) offers the simplest solution to chaotic communication. In COOK, the chaotic signal is multiplied directly by the bit sequence to be transmitted, i.e. radiation of a chaotic signal is disabled for bit 0 and enabled for bit 1 [8].

The COOK receiver shown in Fig. 2 estimates the signal energy per bit E_b carried by the transmitted signal and performs the decision by means of a level comparator.

2) Noncoherent Detection of CSK: Signals generated by different chaotic attractors generally have different statistical attributes, such as the mean of the absolute value, variance, and standard deviation. This observation suggests that CSK signals can also be demodulated by noncoherent receivers [9].

Let us consider chaotic sample functions generated by the same attractor but originating from different initial conditions as a *chaotic stochastic process* (for a more precise definition, see Part III of this paper). Let the binary information to be transmitted be mapped to the variances of chaotic stochastic processes. Chaotic stochastic processes with different variances may be obtained by using two different chaotic attractors or by multiplying the sample functions of one attractor by distinct weights.

The block diagram of a noncoherent CSK receiver is shown in Fig. 2. For the sake of simplicity, let us assume that the mean of the received signal is zero. In this case, the receiver can estimate the variance of the received signal using a correlator. The decision is made by a simple level comparator. If a parameter of the chaotic attractor other than its variance is to be evaluated in the demodulation process, the appropriate operation may be substituted for the multiplier in Fig. 2.

Not just statistical attributes but any robust characteristic of a chaotic signal may be exploited in order to implement a noncoherent CSK communication system. For example, in [10], the basis functions are two chaotic signals which have different average frequencies; these may be distinguished at the

[2] Because a chaotic waveform is not periodic, each sample function of length T is different.

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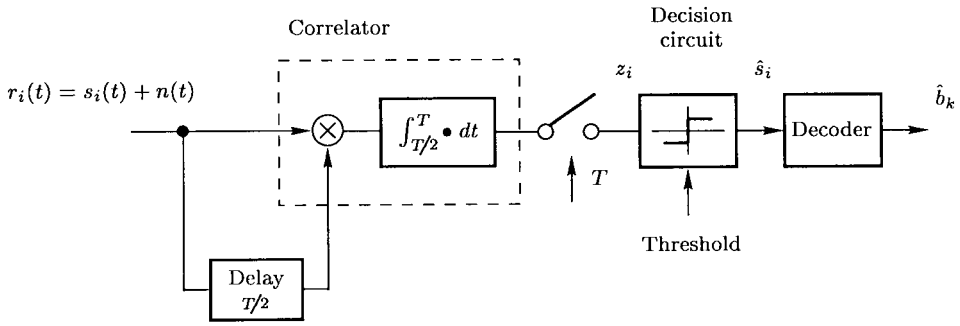

Fig. 3. Block diagram of a DCSK receiver.

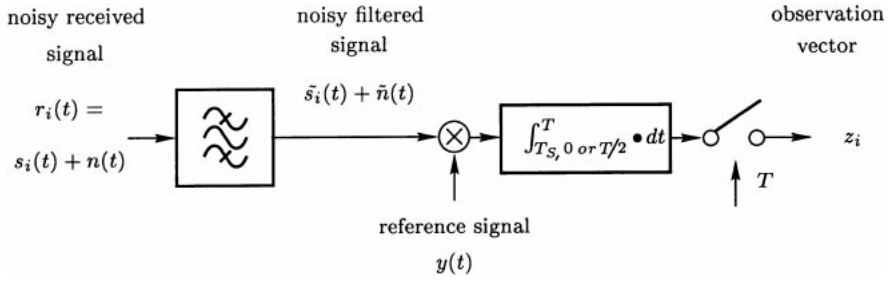

Fig. 4. General block diagram of a CSK receiver.

receiver by measuring the average value of the zero-crossing rate of the received signal.

Other modulation techniques have also been proposed. In [11], the autocorrelation function of a chaotic signal is modified according to the symbols to be transmitted. By measuring the autocorrelation function of the received signal, the transmitted symbol can be identified.

D. Differentially Coherent Reception

One or more chaotic basis functions $g_j(t)$ must be recovered in order to implement a coherent correlation receiver. When propagation conditions are so poor that it is impossible to recover basis functions by chaotic synchronization, a differential modulation scheme (*differential chaos shift keying* (DCSK) [9]) and a differentially coherent correlation receiver may be used [12].

In DCSK, every symbol to be transmitted is represented by two sample functions. The first sample function serves as a *reference* while the second one carries the information. In the case of binary transmission, bit 1 is sent by transmitting a reference signal provided by the chaos generator twice in succession. For bit 0, the reference chaotic signal is transmitted, followed by an inverted copy of the same signal. The two sample functions are correlated in the receiver and the decision is made by a level comparator, as shown in Fig. 3[3].

III. Theoretical Comparison of Chaotic Modulation Techniques

Noise performance is the most important characteristic of a modulation scheme and receiver. Since all of the chaotic modulation techniques discussed in this paper can be considered as variants of CSK modulation, we consider only the noise performance of CSK.

In the previous section, we saw that CSK transmissions can be demodulated in one of three ways:

1) a coherent correlation receiver, where the elements of the signal set are recovered by synchronization;
2) a noncoherent receiver (in this section, we assume that the demodulation is performed by estimating the *variance* of the received chaotic signal or the COOK technique is used); or
3) a differentially coherent receiver.

Note that similar circuitry is used to estimate the observation vector in Figs. 1–3. Each receiver configuration contains one or more correlators; the difference between the schemes is in the manner in which the reference signals are generated. Therefore, we will analyze all three receiver configurations using the common block diagram shown in Fig. 4. Because the channel (selection) filter plays an important role in the DCSK receiver, it is also included explicitly in this figure.

For simplicity, let the elements $s_i(t)$ of signal set be the basis functions $g_j(t)$. We denote by $y(t)$ and $\tilde{s}_i(t) + \tilde{n}(t)$ respectively, the reference signal and the filtered version of the noisy received signal which emerges from the channel filter. The decision is performed based on the observation vector. The probability of wrong decisions, and therefore the BER, depends on the mean value and variance of the observation vector [6].

A. Coherent Correlation Receiver with Chaotic Synchronization

In a coherent correlation receiver, the elements of the signal set are recovered by synchronization from the noisy filtered received signal. The chaotic synchronization techniques which

[3]Note that a DCSK receiver differs fundamentally from a conventional DPSK receiver [6]. Because a reference signal is transmitted in *every* symbol period, the data rate is halved and the required energy per bit is doubled in DCSK compared to DPSK. However, the error propagation problem associated with differential encoding does not arise.

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have been published to date are sensitive to both noise and distortion in the channel (see Section IV). In particular, the signal $s_i(t)$ cannot be recovered exactly when $r_i(t) \neq s_i(t)$.

Therefore, let $\hat{s}_i(t)$ denote the recovered chaotic signal, where $\hat{s}_i(t) \approx \tilde{s}_i(t)$ if $t > T_S$[4]. This corresponds to our reference signal $y(t)$ in Fig. 4.

As explained in Section II-A.1, we assume that synchronization is lost and recovered at the beginning of every new symbol. Since the synchronization transient cannot be used to transmit information, the observation vector must be estimated during the interval $T_S < t \leq T$. Let $s_i(t)$ and $s_j(t)$ denote the elements of the signal set for binary CSK modulation. Then the elements of the observation vector are given by

$$z_{i1} = \int_{T_S}^{T} [\tilde{s}_i(t) + \tilde{n}(t)]\hat{s}_i(t)\, dt = \int_{T_S}^{T} \tilde{s}_i(t)\hat{s}_i(t)\, dt + \int_{T_S}^{T} \tilde{n}(t)\hat{s}_i(t)\, dt \quad (1)$$

$$z_{i2} = \int_{T_S}^{T} \tilde{s}_i(t)\hat{s}_j(t)\, dt + \int_{T_S}^{T} \tilde{n}(t)\hat{s}_j(t)\, dt \quad (2)$$

where $\hat{s}_i(t)$ and $\hat{s}_j(t)$ correspond to $\hat{g}_1(t)$ and $\hat{g}_2(t)$, respectively, in Fig. 1.

Note that z_{i1} and z_{i2} are *random variables*, whose mean value depends on the bit energy of the chaotic signal and the "goodness" of synchronization [see the first term in (1)].

The variance of the estimation is determined by the chaotic signal[5] and the filtered noise. Note that the noise has no *direct* influence on the variance of estimation. As shown by the second terms of (1) and (2), the variance of estimation is influenced only by the cross correlation of noise and the recovered chaotic signal.

For a given chaotic signal and bandwidth of the channel filter, the variance of estimation is inversely proportional to the observation time $(T - T_S)$. The mean value of estimation does not depend on the noise; thus, the receiver is an *unbiased estimator*. In particular, this means that the threshold level required by the level comparator does not depend on the channel noise.

As in the case of a conventional receiver and periodic basis functions, the noise performance of a coherent correlation receiver using chaotic basis functions is theoretically excellent. However, the BER also depends on the "goodness" of synchronization [equivalently, the closeness of the reference signal $y(t)$ to the desired chaotic basis function $g_j(t)$]. Any synchronization error, especially loss of synchronization, results in a large degradation in the noise performance of a correlation receiver.

Loss of synchronization causes the bit error rate (BER) to rise significantly. Recall that a digital communications link is automatically severed at the system level if the BER increases above a predetermined threshold. Therefore, synchronization-based receivers are not suitable for noisy propagation environments.

Let us consider next a noncoherent receiver.

B. Noncoherent Correlation Receiver

Here, we assume that decisions in the noncoherent receiver are made by estimating the variance of the received signal. The reference signal $y(t)$ is equal to the noisy filtered signal $\tilde{s}_i(t) + \tilde{n}(t)$ in this case, and the observation variable can be expressed as

$$z_i = \int_0^T [\tilde{s}_i(t) + \tilde{n}(t)]^2\, dt = \int_0^T \tilde{s}_i^2(t)\, dt + 2\int_0^T \tilde{s}_i(t)\tilde{n}(t)\, dt + \int_0^T \tilde{n}^2(t)\, dt \quad (3)$$

where a new term, which depends only on the filtered noise, appears.

The mean value of estimation depends on both the bit energy of the chaotic signal and the filtered noise (see the first and third terms in (3), respectively). In this case the receiver is a *biased estimator*; the threshold level of the comparator used as a decision circuit now depends on the noise level. In addition, the variance of estimation becomes much greater than in the previous case due to the third term in (3).

Imagine that a histogram of the observed values of z_i is plotted for a large number of transmitted symbols. Because binary modulation is used, the histogram will have two distinct peaks. For a given noise level, channel filter and chaotic signal, the best noise performance can be achieved if the distance between the two peaks is a maximum. The separation of the peaks is determined by the distance between the elements of the signal set. For the case of noncoherent CSK, COOK ensures the maximum distance. In this case, the distance between the elements of the signal set is equal to twice the mean value of the energy per bit.

Is there a way to produce an unbiased estimator with antipodal signals [12] which ensures the maximum distance between the elements of a binary signal set? The answer is yes; DCSK offers a potential solution.

C. Differentially Coherent Reception

In a DCSK receiver, the reference signal $y(t)$ is a delayed version of the filtered noisy signal. Note that different sample functions of filtered noise corrupt the inputs of the correlator. If the time-varying channel varies slowly compared to the symbol rate, then the observable element is

$$z_i = \int_{T/2}^{T} [\tilde{s}_i(t) + \tilde{n}_1(t)][\pm\tilde{s}_i(t) + \tilde{n}(t - T/2)]\, dt = \pm\int_{T/2}^{T} \tilde{s}_i^2(t)\, dt \pm \int_{T/2}^{T} \tilde{s}_i(t)\tilde{n}(t)\, dt + \int_{T/2}^{T} \tilde{s}_i(t)\tilde{n}(t - T/2)\, dt + \int_{T/2}^{T} \tilde{n}(t)\tilde{n}(t - T/2)\, dt \quad (4)$$

[4]Recall that T_S is the synchronization time.

[5]The parameter required for demodulation must be *estimated* from sample functions of finite length. In the case of periodic sample functions, this estimation has zero variance if the observation interval is an integral number of periods. When chaotic basis functions are used, the estimation has a nonzero variance which results from the nonperiodicity of the underlying signals; this increases the overall variance of the observation vector [13]. This problem will be discussed in detail in Part III.

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where the sign of the first and second terms depends on the binary modulation. The signals $\tilde{n}_1(t)$ and $\tilde{n}(t-T/2)$ denote the sample functions of filtered noise that corrupt the reference and information-bearing parts of the received signal, respectively.

By proper design of the channel filter[6], the two sample functions of the noise become uncorrelated and (4) can be simplified as follows

$$z_i = \pm \int_{T/2}^{T} \tilde{s}_i^2(t)\,dt \pm \int_{T/2}^{T} \tilde{s}_i(t)\tilde{n}(t)\,dt + \int_{T/2}^{T} \tilde{s}_i(t)\tilde{n}(t-T/2)\,dt. \quad (5)$$

Note that the receiver is an *unbiased estimator* in this case. This means that the threshold level of the decision circuit is zero and is independent of the noise level.

The mean value of the estimation depends on the bit energy of the chaotic signal; the variance of estimation is determined by the chaotic signal and the filtered noise. The application of antipodal signals ensures the maximum distance between the elements of the signal set for DCSK [see the first term in (5)]. As in the case of a coherent correlation receiver with synchronization, the noise has no direct influence on the variance of estimation. However, in contrast with (1), two cross-correlation terms appear in (5). For correct operation, we need to ensure that the first term, corresponding to the bit energy, dominates the two cross-correlation terms contributed by the noise.

D. Coherent CSK versus DCSK

The main advantage of DCSK over CSK is that both the reference and information-bearing components of the transmitted signal pass through the *same* channel so they undergo the same transformation. This transformation does not change the correlation that carries the information, provided that the time-varying channel remains almost constant for the symbol duration.

Because there is no need for synchronization, the DCSK technique can be used even under poor propagation conditions. However, the symbol rate is halved compared with a synchronization-based receiver in which synchronization is maintained.

Recall, however, that the synchronization time T_S of a coherent receiver is wasted—no information can be carried during this interval. If each symbol must be synchronized independently and the synchronization time T_S is comparable to the correlation time $(T-T_S)$, then a DCSK system can in principle operate at the *same symbol rate* as a synchronization-based coherent receiver, with the added advantage of superior performance under poor propagation conditions.

Thus, synchronization-based recovery of chaotic basis functions from a noisy received signal offers superior performance to DCSK in terms of data rate only if synchronization can be maintained. This advantage is lost if the modulation technique requires the loss and recovery of synchronization at the beginning of every new symbol or if poor propagation conditions make it impossible to maintain synchronization.

[6]Let the channel filter be an ideal bandpass filter with bandwidth BW_{RF}. The cross-correlation of $\tilde{n}_1(t)$ and $\tilde{n}(t-T/2)$ becomes zero if $BW_{RF}T = n, n = 1, 2, 3, \cdots$ [6].

IV. Synchronization in Chaotic Communications Systems

As explained in Part I of this paper, the primary use of synchronization in digital communications systems is for recovering basis function(s) in coherent correlation receivers, as illustrated for binary CSK in Fig. 1. In a real system, the signal which is received differs from that which was transmitted. At the very minimum, the signal is corrupted by additive noise as it passes through the channel; usually, it is also bandpass filtered. The transformation may be more severe if the channel is nonlinear, time-varying, or suffers from multipath effects.

Thus, the objective of the synchronization process is to recover basis functions from the noisy received signal in order to maximize the probability of correctly identifying the transmitted symbols. In this section, we examine chaotic synchronization techniques from this perspective.

A. Chaotic Synchronization Schemes

Chaotic steady-state solutions are characterized by sensitive dependence on initial conditions: trajectories of two identical autonomous continuous-time dynamical systems started from slightly different initial conditions quickly become uncorrelated [14]. Surprisingly perhaps, it is nevertheless possible to synchronize these systems in the sense that a trajectory of one asymptotically approaches that of the other.

Several notions of synchronization have been proposed for chaotic systems, the strongest and most widely-used of which is *identical synchronization*, where the state of the receiver system converges asymptotically to that of the transmitter [14]. More recently, two weaker notions of synchronization, called *generalized synchronization* [15], [16] and *phase synchronization* [17], [18] have been introduced.

1) Identical Synchronization: Two continuous-time dynamical systems

$$\dot{\boldsymbol{x}} = \boldsymbol{f}(\boldsymbol{x}) \quad (6)$$

and

$$\dot{\boldsymbol{x}}' = \boldsymbol{f}'(\boldsymbol{x}') \quad (7)$$

are said to *synchronize identically* if

$$\lim_{t\to\infty} \|\boldsymbol{x}'(t) - \boldsymbol{x}(t)\| = 0$$

for any combination of initial states $\boldsymbol{x}(0)$ and $\boldsymbol{x}'(0)$.

From a communications perspective, we may think of system (6) as the transmitter and (7) as the receiver. The signal $s_i(t)$ which is transmitted is a linear combination of basis functions $g_j(t)$. For simplicity, we consider the case where only one basis function $g(t)$ is used and we assume that $s_i(t) \equiv g(t)$. At the receiver, we must recover the scalar basis function $g(t) = h(\boldsymbol{x}(t))$ which has been derived from the state of the transmitter (6). By synchronizing the state of the receiver

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identically with that of the transmitter, and applying the same readout function $h(\cdot)$, the basis function can be recovered. In particular, if $\boldsymbol{x}'(t)$ can be made to converge to $\boldsymbol{x}(t)$ then the estimate $\hat{g}(t) = h(\boldsymbol{x}'(t))$ will converge to $g(t)$.

2) Generalized Synchronization: Systems (6) and (7) are said to exhibit *generalized synchronization* if there exists a transformation $\boldsymbol{M}$ such that

$$\lim_{t\to\infty} \|\boldsymbol{x}'(t) - \boldsymbol{M}(\boldsymbol{x}(t))\| = 0$$

where the properties of the transformation are independent of the initial conditions $\boldsymbol{x}(0)$ and $\boldsymbol{x}'(0)$.

Generalized synchronization occurs, for example, in unidirectionally-coupled chaotic systems where the driven system (the "synchronizable chaotic circuit" block in Fig. 1) is asymptotically stable [21].

If the transformation $\boldsymbol{M}$ is invertible, then

$$\hat{g}(t) = h(\boldsymbol{M}^{-1}(\boldsymbol{x}'(t)))$$

approaches $g(t)$. However, the transformation $\boldsymbol{M}$ is not necessarily invertible, so recovering the state $\boldsymbol{x}'(t)$ in a coherent receiver does not necessarily permit one to recover the required basis function.

3) Phase Synchronization: Phase synchronization of two coupled systems occurs if the difference $|\phi'(t) - \phi(t)|$ between the "phases" of the two systems is bounded by a constant [19], where the "phase" $\phi(t)$ is some suitably chosen monotonically increasing function of time. For example, in the case of a spiral Chua attractor [22], the angle of rotation about the unstable equilibrium point in a two-dimensional projection of the attractor would be a suitable choice.

In this work, we are concerned with recovering basis functions *exactly*, so we focus exclusively on *identical synchronization*. Since we are dealing with one-way communication between a transmitter (the drive system) and a receiver (the response system), we consider only unidirectional coupling between two systems. This is called a "drive-response" or "master-slave" configuration. In the following subsections, we present two classical approaches to identical drive-response synchronization of unidirectionally-coupled systems: Pecora-Carroll synchronization and error-feedback synchronization. In the language of control theory, Pecora-Carroll synchronization corresponds to open-loop state estimation, and error-feedback synchronization corresponds to asymptotic state estimation.

B. Pecora-Carroll Synchronization

In the drive-response synchronization scheme proposed by Pecora and Carroll [23], a chaotic dynamical system

$$\dot{\boldsymbol{x}} = \boldsymbol{f}(\boldsymbol{x}) \tag{8}$$

with scalar output $g(t) = h(\boldsymbol{x})$, as shown in Fig. 5, is decomposed into two subsystems with states $\boldsymbol{x}_1$ and $\boldsymbol{x}_2$, respectively:

$$\dot{\boldsymbol{x}}_1 = \boldsymbol{f}_1(\boldsymbol{x}_1, \boldsymbol{x}_2) \tag{9}$$

$$\dot{\boldsymbol{x}}_2 = \boldsymbol{f}_2(\boldsymbol{x}_2, g(t)) \tag{10}$$

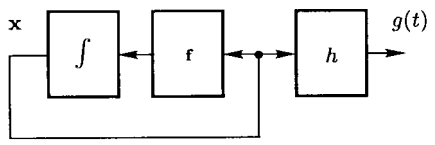

Fig. 5. Block diagram of drive system described by (8).

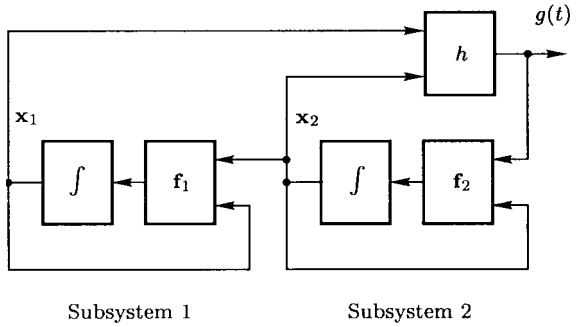

Fig. 6. Pecora-Carroll decomposition of (8) into two subsystems described by (9) and (10).

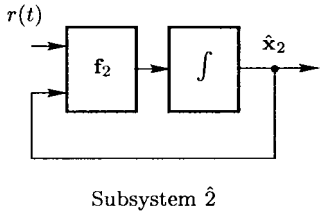

Fig. 7. Pecora-Carroll drive-response synchronization. The response system is a copy of the second subsystem in the drive system shown in Fig. 6.

where $\boldsymbol{x} = (\boldsymbol{x}_1, \boldsymbol{x}_2)$, and

$$g(t) = h(\boldsymbol{x}_1(t), \boldsymbol{x}_2(t))$$

is the scalar output signal, as before; this is illustrated in Fig. 6.

The system is partitioned in such a way that the conditional Lyapunov exponents[7] (CLE's) [23] of the second subsystem (10) are negative.

Qualitatively, the CLE's characterize the stability of the second subsystem (10) when driven by $g(t)$. If all CLE's are negative, the trajectory $\boldsymbol{x}_2(t)$ is asymptotically stable [23]. This means that the states of two or more copies of the second subsystem will synchronize identically when driven by the *same input* $g(t)$. This is the basis of Pecora-Carroll drive-response synchronization.

In particular, consider "Subsystem $\hat{2}$" in Fig. 7. This system is described by

$$\dot{\hat{\boldsymbol{x}}}_2 = \boldsymbol{f}_2(\hat{\boldsymbol{x}}_2, r(t)). \tag{11}$$

If the CLE's of this subsystem (called the "response system") are all negative and $\hat{\boldsymbol{x}}_2(0)$ is sufficiently close to $\boldsymbol{x}_2(0)$,

[7] Lyapunov exponents (LE's) [24] quantify the average exponential rate of separation of trajectories in a dynamical system under steady-state conditions. If one or more LE's is positive, then there is a "direction" in the system along which trajectories are stretched apart exponentially. In this case, the system is said to be chaotic. Conditional Lyapunov exponents (CLE's) are a measure of the average local exponential rate of separation of trajectories in a dynamical system along a reference trajectory which is defined by a prescribed input.

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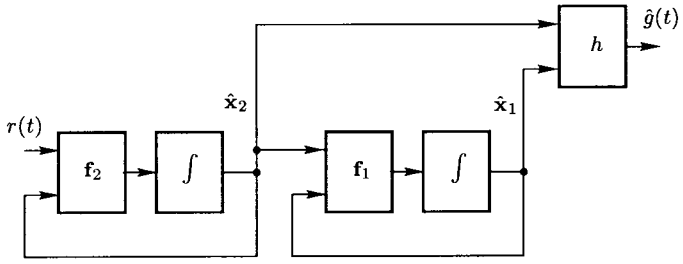

Fig. 8. Basis function recovery using cascaded Pecora-Carroll drive-response synchronization.

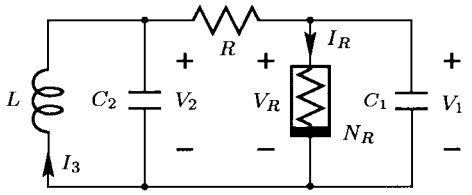

Fig. 9. Chua's circuit consists of a linear inductor L, two linear capacitors (C_2, C_1), a linear resistor R, and a voltage-controlled nonlinear resistor N_R.

and $r(t) = g(t)$, then the state $\hat{\boldsymbol{x}}_2$ of the response system converges asymptotically to $\boldsymbol{x}_2$, i.e.

$$\lim_{t\to\infty} \|\hat{\boldsymbol{x}}_2(t) - \boldsymbol{x}_2(t)\| = 0.$$

In terms of a communications system, the drive system (8) produces a chaotic basis function $g(t)$ which we assume, for simplicity, is transmitted directly through the channel and received, noisy and distorted, as $r(t)$.

Recall that the objective of synchronization in a coherent receiver is to estimate $g(t)$, given $r(t) \neq g(t)$. Therefore the response system must play the role of basis function recovery. It is not sufficient to recover $\boldsymbol{x}_2$; we need to recover both $\boldsymbol{x}_1$ and $\boldsymbol{x}_2$. This can be accomplished using cascaded drive-response synchronization. A second subsystem is added which is driven by the first, as shown in Fig. 8. Here,

$$\dot{\hat{\boldsymbol{x}}}_2 = \boldsymbol{f}_2(\hat{\boldsymbol{x}}_2, r(t)) \tag{12}$$

$$\dot{\hat{\boldsymbol{x}}}_1 = \boldsymbol{f}_1(\hat{\boldsymbol{x}}_1, \hat{\boldsymbol{x}}_2). \tag{13}$$

If the CLE's of (12) are all negative, and $\hat{\boldsymbol{x}}_2(0)$ is sufficiently close to $\boldsymbol{x}_2(0)$, and $r(t) = g(t)$, then $\hat{\boldsymbol{x}}_2(t)$ converges asymptotically to $\boldsymbol{x}_2(t)$. If, in addition, the CLE's of (13) are negative, then $\hat{\boldsymbol{x}}_1$ converges asymptotically to $\boldsymbol{x}_1$, i.e.

$$\lim_{t\to\infty} \|\hat{\boldsymbol{x}}_1(t) - \boldsymbol{x}_1(t)\| = 0,$$

and the output $\hat{g}(t) = h(\hat{\boldsymbol{x}}_1(t), \hat{\boldsymbol{x}}_2(t))$ converges asymptotically to $g(t)$. In this way, a basis function $g(t)$ may in principle be recovered from the received signal $r(t)$.

1) Example: Pecora-Carroll Synchronization in Chua's Circuit: We illustrate basis function recovery using Pecora-Carroll cascaded drive response synchronization in Chua's circuit [25]. This widely-studied circuit, shown in Fig. 9, consists of a linear inductor, a linear resistor, two linear capacitors, and a single nonlinear resistor N_R.

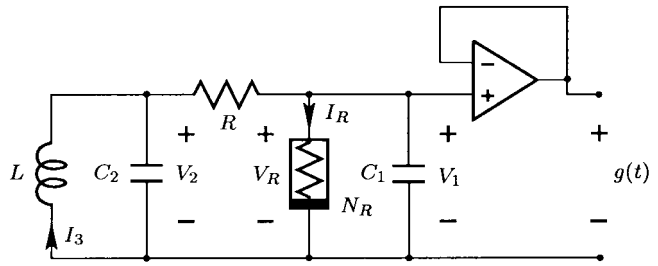

Fig. 10. Drive system using Chua's circuit to produce chaotic basis function $g(t)$.

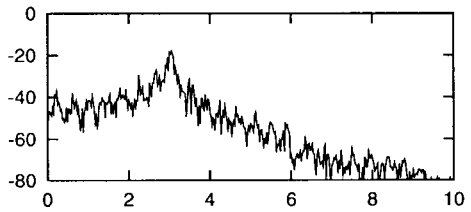

Fig. 11. Power spectrum of chaotic basis function $g(t)$ from Chua's circuit. Horizontal axis: f (kHz); vertical axis $P(f)$ (dB).

The dynamical behavior of the circuit is described by three ordinary differential equations:

$$\dot{V}_1 = \frac{G}{C_1}(V_2 - V_1) - \frac{1}{C_1} f(V_1), \tag{14}$$

$$\dot{V}_2 = \frac{G}{C_2}(V_1 - V_2) + \frac{1}{C_2} I_3, \tag{15}$$

$$\dot{I}_3 = -\frac{1}{L} V_2 \tag{16}$$

where $G = 1/R$ and $f(V_R) = G_b V_R + \frac{1}{2}(G_a - G_b)(|V_R + E| - |V_R - E|)$ is the piecewise-linear driving-point characteristic of the nonlinear resistor N_R.

This system may be partitioned into two subsystems in a number of different ways. Since the subcircuit consisting of R, L, and C_2 [described by (15) and (16)] is passive, and therefore has negative CLE's, we choose this as "Subsystem 2." "Subsystem 1" is described by (14) and the drive signal $g(t)$ is $V_1(t)$.

When $L = 18$ mH, $C_1 = 10$ nF, $C_2 = 100$ nF, $R \approx 1800\ \Omega, G_a = -50/66$ mS, $G_b = -9/22$ mS, and $E = 1$ V, the circuit shown in Fig. 10 produces a chaotic basis function $g(t)$ whose power spectrum is shown in Fig. 11; this is our drive system.

The response system contains a cascaded drive-response configuration. The first section, denoted "Subsystem $\hat{2}$," and described by

$$\dot{\hat{V}}_2 = \frac{G}{C_2}(r(t) - \hat{V}_2) + \frac{1}{C_2}\hat{I}_3 \tag{17}$$

$$\dot{\hat{I}}_3 = -\frac{1}{L}\hat{V}_2 \tag{18}$$

is a copy of Subsystem 2. As shown in Fig. 12, this circuit is followed by a copy of subsystem 1, which we label "Subsystem $\hat{1}$:"

$$\dot{\hat{V}}_1 = \frac{G}{C_1}(\hat{V}_2 - \hat{V}_1) - \frac{1}{C_1} f(\hat{V}_1) \tag{19}$$

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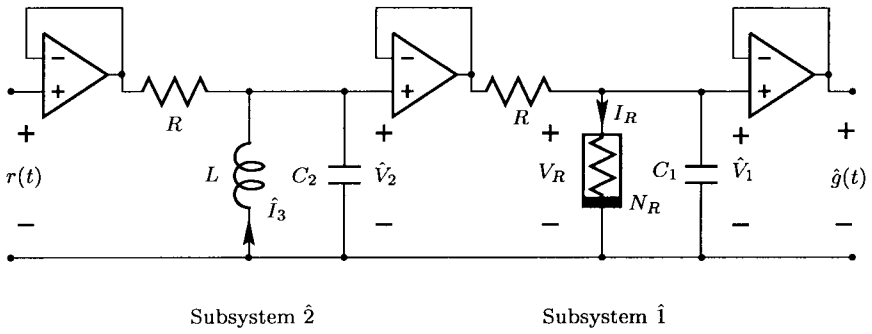

Fig. 12. Recovery of $g(t)$ from $r(t)$ using Chua's circuit in a Pecora-Carroll cascaded drive-response configuration.

If $r(t) \approx g(t)$, then $\hat{V}_2$ approaches V_2 asymptotically. If $\hat{V}_2 \approx V_2$ and, in addition, $\hat{V}_1(0)$ is sufficiently close to $V_1(0)$ and the CLE's of "Subsystem $\hat{1}$" are negative, then $\hat{V}_1$ approaches V_1 asymptotically and $\hat{g}(t) \approx g(t)$.

In this way, a basis function can be recovered from the received signal if $r(t) \approx g(t)$ and the parameters of the drive and response system are matched.

C. Robustness of Pecora-Carroll Synchronization

In the discussion above, we have assumed that the parameters of the drive and response systems are identical, that a single basis function $g(t)$ is transmitted, that $r(t) \approx g(t)$, that the CLE's of the response subsystems are negative, and that the initial conditions of the systems are close; this rather extensive set of assumptions allows us to recover $g(t)$.

However, we pointed out in Part I of this paper that the minimum channel nonidealities which must be considered in a practical communications system are additive white Gaussian noise and linear bandpass filtering. Therefore, we must consider carefully the robustness of the modulation/demodulation scheme. How well can we recover $g(t)$ using synchronization from the noisy distorted version $r(t)$?

In every practical implementation of a telecommunications system, the transmitter and receiver circuits operate under different conditions, so it is necessary to consider the case of a mismatch between the parameters of the transmitter and receiver. The parameter mismatch also depends on temperature, aging, etc. The effect of parameter mismatch on the recovery of $g(t)$ has not been widely studied; further research is required in this area.

Pecora-Carroll drive-response synchronization is fundamentally an open-loop state estimation technique, the objective of which is to reconstruct the state $\boldsymbol{x}$ of the transmitter, given a noisy observation $r(t)$ of a basis function $g(t)$. Open-loop state estimators are sensitive to noise and parameter mismatch. Consequently, identical synchronization using the Pecora-Carroll technique is not robust, as we shall demonstrate by example in Part III of the paper.

CLE's are a local concept which characterize the behavior of a system close to a prescribed reference trajectory. If a driven system is nonlinear, then different inputs may drive the system through different regions of its state space and produce different CLE's. If a synchronization scheme relies on negative CLE's to recover $g(t)$, then the CLE's should be negative for all possible inputs under expected operating conditions.

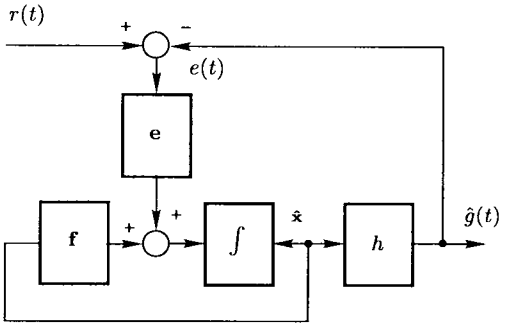

Fig. 13. Error-feedback synchronization.

One way to guarantee that the CLE's of the response subsystem are negative is to make that subsystem passive, as we did in our Chua's circuit example above. In this case, generalized synchronization always occurs, even if the parameters of the drive and response subsystems are mismatched. In this sense, generalized synchronization is a robust phenomenon. However, *identical synchronization*, which is required for recovering chaotic basis functions, occurs only if the parameters are matched.

Here, we have discussed a continuous-time state estimator. Variations on this theme which estimate the state by means of Takens-type delay reconstructions [26] are also possible.

The performance of the receiver in a drive-response configuration may be improved significantly by adding feedback in the state estimator. This technique, which we discuss in the next section, is called error-feedback synchronization.

D. Error-Feedback Synchronization

The goal of the "synchronizable chaotic circuit" block in a receiver (see Fig. 1) is to estimate the basis function $g(t)$, given a noisy observation $r(t)$ of the signal. Error-feedback synchronization is a technique whereby the instantaneous difference between the estimate $\hat{g}(t)$ and the received signal $r(t)$ produces a scalar error signal $e(t)$ which modifies the state of the receiver so as to minimize the error.

Assuming that the basis function $g(t)$ has been generated by the system shown in Fig. 5, then the corresponding error-feedback synchronizable system has the structure given in Fig. 13.

Here,

$$\dot{\hat{\boldsymbol{x}}} = \boldsymbol{f}(\hat{\boldsymbol{x}}) + \boldsymbol{e}(e(t)), \tag{20}$$

where $e(t) = r(t) - \hat{g}(t)$ and $\hat{g}(t) = h(\hat{\boldsymbol{x}})$, as before.

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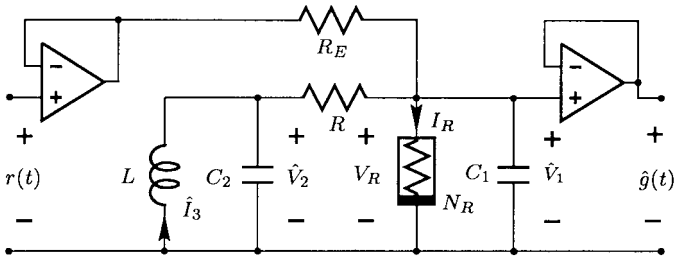

Fig. 14. Error-feedback synchronization in Chua's circuit.

With appropriate choices for $h(\cdot)$ and $\boldsymbol{e}(\cdot)$,

$$\lim_{t\to\infty} \|\hat{\boldsymbol{x}}(t) - \boldsymbol{x}(t)\| = 0.$$

If $\hat{\boldsymbol{x}}$, converges to $\boldsymbol{x}$, then $\hat{g}(t)$ converges to $g(t)$.

1) Example: Error-Feedback Synchronization in Chua's Circuit: Consider again the drive system formed by a Chua's circuit shown in Fig. 10 which produces a chaotic basis function $g(t)$. Error-feedback synchronization using linear feedback may be implemented by the circuit configuration given in Fig. 14.

Here,

$$\dot{\hat{V}}_1 = \frac{G}{C_1}(\hat{V}_2 - \hat{V}_1) - \frac{1}{C_1}F(\hat{V}_1) + \frac{G_E}{C_1}e \tag{21}$$

$$\dot{\hat{V}}_2 = \frac{G}{C_2}(r(t) - \hat{V}_2) + \frac{1}{C_2}\hat{I}_3 \tag{22}$$

$$\dot{\hat{I}}_3 = -\frac{1}{L}\hat{V}_2 \tag{23}$$

where $G_E = 1/R_E$ and $e(t) = r(t) - \hat{V}_1(t)$.

For a sufficiently small value of the coupling resistor R_E, $\hat{V}_1$ synchronizes with V_1 and $\hat{g}(t) \approx g(t)$.

E. Proof of Synchronization

Synchronization in Pecora-Carroll drive-response systems may be established by numerically calculating the CLE's of the system. This approach is unsatisfactory in two respects: extensive simulation is required to calculate the CLE's. Moreover, the synchronization theorem is valid only for trajectories in the receiver which come sufficiently close to the reference trajectory in the transmitter.

Although one may justifiably argue that because the basis function $g(t)$ belongs to an attractor and that ergodicity on this attractor ensures that the trajectory $\hat{\boldsymbol{x}}_2(t)$ will eventually come close to $\boldsymbol{x}_2$ and "pull-in," there is no *a priori* upper bound on the pull-in time [4]. For a practical communications system, this is unacceptable. Worse still, *local* stability of the reference trajectory $\boldsymbol{x}_2(t)$ is not sufficient to guarantee that $\hat{\boldsymbol{x}}_2(t)$ will remain close to $\boldsymbol{x}_2(t)$ when random perturbations are added to the drive signal, as happens in a realistic channel model.

Recent work has highlighted the qualitative difference between "weak" and "strong" forms of chaotic synchronization in systems which have identical transversal Lyapunov exponents [27]. In the case of strong synchronization, additive noise produces a small synchronization error which is related to the noise level. Weak synchronization is characterized by intermittent desynchronization bursts of large amplitude when noise or parameter mismatches are present. Consequently, weak synchronization is not suitable for chaotic communications.

By contrast, several examples exist in the literature where *global* stability can be proven for the case $r(t) = g(t)$ when strong error-feedback synchronization is used [28]–[30]. This can be accomplished by using Lyapunov's direct method [31] to prove that the error system $(\hat{\boldsymbol{x}}(t) - \boldsymbol{x}(t))$ has a globally asymptotically stable equilibrium point at the origin.

In the case of error-feedback synchronization, design of the feedback and analysis of the stability of the error system reduces to a nonlinear observer design problem [32]. Synchronization performance in the presence of noise can also be improved by filtering the error signal $e(t)$ before applying it to the summing node in Fig. 13 [33].

V. Concluding Remarks

Much of the recent research in chaotic communications has focused on synchronization. Our objectives in this work have been:

1) To provide a theoretical context in which the performance of modulation schemes based on chaotic synchronization can be evaluated,
2) To develop a unified framework for discussing and comparing conventional and chaotic communications systems, and
3) To highlight the special problems that arise when chaotic basis functions are used.

In Part I, we surveyed the theory and operation of the conventional digital communications systems and identified the minimum requirement for a realistic channel model. The use of chaotic carrier signals was motivated by highlighting the limitations of narrowband communications.

In Section II of Part II, we described the CSK, COOK, and DCSK modulation techniques.

We compared the theoretical performance of a coherent correlation receiver with synchronization, a noncoherent correlation receiver, and a DCSK correlation receiver in Section III.

We concluded that synchronization-based recovery of chaotic basis functions from noisy received sample functions offers potential advantages in terms of data rate and noise performance, but only if synchronization can be maintained. If synchronization cannot be maintained, then noncoherent detection represents a better choice.

In Section IV we surveyed the state-of-the-art in synchronization of chaotic systems in the context of digital communications.

In Part III, performance targets for chaotic communications techniques are summarized and examples (CSK with synchronization, noncoherent CSK and COOK, and DCSK correlation receiver) are given. We evaluate the performance of these systems in the context of a noisy and bandlimited channel. We also highlight a fundamental problem in chaotic communications of minimizing the variance of the estimation in the correlator.

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REFERENCES

[1] M. Hasler, "Engineering chaos for secure communication systems," *Phil. Trans. R. Soc. London A*, no. 1701, p. 353, Oct. 1995.
[2] G. Kolumbán, M. P. Kennedy, and L. O. Chua, "The role of synchronization in digital communication using chaos—Part I: Fundamentals of digital communications," *IEEE Trans. Circuits Syst. I, (Special Issue on Chaos Synchronization and Control: Theory and Applications)*, vol. 44, pp. 927–936, Oct. 1997.
[3] M. P. Kennedy and H. Dedieu, "Experimental demonstration of binary chaos shift keying using self-synchronizing Chua's circuits," in *Proc. IEEE Int. Specialist Workshop on Nonlinear Dynamics of Electronic Systems*, Dresden, germany, July 23–24, 1993, pp. 67–72.
[4] H. Dedieu, M. P. Kennedy, and M. Hasler, "Chaos shift keying: Modulation and demodulation of a chaotic carrier using self-synchronizing Chua's circuits," *IEEE Trans. Circuits Syst. II, (Special Issue on Chaos in Nonlinear Electronic Circuits—Part C: Applications)*, vol. 40, pp. 634–642, Oct. 1993.
[5] I. Frigyes, Z. Szabó, and P. Ványai, *Digital Microwave Transmission*. Amsterdam, The Netherlands: Elsevier Science, 1989.
[6] S. S. Haykin, *Communication Systems*, 3rd ed. New York: Wiley, 1994.
[7] G. Kolumbán, H. Dedieu, J. Schweizer, J. Ennitis, and B. Vizvári, "Performance evaluation and comparison of chaos communication systems," in *Proc. 4th Int. Workshop on Nonlinear Dynamics of Electronic Systems*, Sevilla, Spain, June 1996, pp. 105–110.
[8] G. Kolumbán, M. P. Kennedy, and G. Kis, "Performance improvement of chaotic communications systems," in *Proc. European Conf. on Circuit Theory and Design*, Budapest, Hungary, Aug. 30–Sept. 3, 1997, pp. 284–289.
[9] G. Kolumbán, B. Vizvári, W. Schwarz, and A. Abel, "Differential chaos shift keying: A robust coding for chaotic communication," in *Proc. 4th Int. Workshop on Nonlinear Dynamics of Electronic Systems*, Sevilla, Spain, June 27–28, 1996, pp. 87–92.
[10] T. Yang, "Recovery of digital signals from chaotic switching," *Int. J. Circuit Theory Appl.*, vol. 23, pp. 611–615, 1995.
[11] J. Schweizer, "A stochastic approach to spread spectrum communication using chaos," in *Proc. SPIE Chaotic Circuits for Communication*, Philadelphia, PA, Oct. 1995, pp. 115–125.
[12] A. J. Viterbi, *Principles of Coherent Communication*. New York: McGraw-Hill, 1966.
[13] G. Kolumbán, M. P. Kennedy, and G. Kis, "Determination of symbol duration in chaos-based communications," in *Proc. 5th Int. Workshop on Nonlinear Dynamics of Electronic Systems*, Moscow, June 26, 1997, pp. 217–222.
[14] M. P. Kennedy, "Three steps to chaos part I: Evolution," *IEEE Trans. Circuits Syst. I, (Special Issue on Chaos in Nonlinear Electronic Circuits—Part A: Tutorial and Reviews)*, vol. 40, pp. 640–656, Oct. 1993.
[15] H. Fujisaka and T. Yamada, "Stability theory of synchronized motion in coupled oscillator systems," *Prog. Theor. Phys.*, vol. 69, pp. 32–47, 1983.
[16] L. M. Pecora and T. L. Carroll, "Synchronization in chaotic systems," *Phys. Rev. Lett.*, vol. 64, no. 8, pp. 821–824, 1990.
[17] N. F. Rulkov, M. M. Sushchik, L. S. Tsimring, and H. D. Abarbanel, "Generalized synchronization of chaos in directionally coupled chaotic systems," *Phys. Rev. E*, vol. 51, no. 2, pp. 980–994, Feb. 1995.
[18] L. Kocarev, U. Parlitz, and T. Stojanovski, "Generalized synchronization of chaotic signals," in *Proc. NOLTA'95*, Las Vegas, NV, Dec. 10–14, 1995, pp. 953–956.
[19] M. G. Rosenblum, A. S. Pikovsky, and J. Kurths, "Phase synchronization of chaotic oscillators," *Phys. Rev. Lett.*, vol. 76, no. 11, pp. 1804–1807, Mar. 11, 1996.
[20] U. Parlitz, L. Junge, W. Lauterborn, and L. Kocarev, "Experimental observation of phase synchronization," *Phys. Rev. E*, vol. 54, no. 2, pp. 2115–2117, Aug. 1996.
[21] L. Kocarev and U. Parlitz, "Generalized synchronization, predictability, and equivalence of unidirectionaly coupled dynamical systems," *Phys. Rev. Lett.*, vol. 76, no. 11, pp. 1816–1819, Mar. 1996.
[22] M. P. Kennedy, "Three steps to chaos part II: A Chua's circuit primer," *IEEE Trans. Circuits Syst. I, (Special Issue on Chaos in Nonlinear Electronic Circuits—Part A: Tutorial and Reviews)*, vol. 40, pp. 657–674, Oct. 1993.
[23] L. M. Pecora and T. L. Carroll, "Driving systems with chaotic signals," *Phys. Rev. A*, vol. 44, no. 4, pp. 2374–2383, 1991.
[24] M. P. Kennedy, "Bifurcation and chaos," in W. K. Chen, Ed., *The Circuits and Filters Handbook*. Boca Raton, FL: CRC Press, 1995, pp. 1089–1164.
[25] R. N. Madan, Ed., *Chua's Circuit: A Paradigm for Chaos*. Singapore: World Scientific, 1993.
[26] T. S. Parker and L. O. Chua, *Practical Numerical Algorithms for Chaotic Systems*. New York: Springer-Verlag, 1989.
[27] M. Hasler, "Strong and weak forms of synchronization of chaotic systems," in *Proc. 5th Int. Workshop on Nonlinear Dynamics of Electronic Systems*, Moscow, Russia, June 26, 1997, pp. 2–7.
[28] R. He and P. G. Vaidya, "Analysis and synthesis of synchronous periodic and chaotic systems," *Phys. Rev. A*, vol. 46, no. 12, pp. 7387–7392, Dec. 1992.
[29] C. W. Wu and L. O. Chua, "A unified framework for synchronization and control of dynamical systems," *Int. J. Bifurc. Chaos*, vol. 4, no. 4, pp. 979–998, 1994.
[30] J. Schweizer, M. P. Kennedy, M. Hasler, and H. Dedieu, "Synchronization theorem for a chaotic system," *Int. J. Bifurc. Chaos*, vol. 5, no. 1, pp. 297–302, Feb. 1995.
[31] M. Vidyasagar, *Nonlinear Systems Analysis*. Englewood Cliffs, NJ: Prentice-Hall, 1978.
[32] H. Nijmeijer and I. Mareels, "An observer looks at synchronization," *IEEE Trans. Circuits and Syst. I, (Special Issue on Chaos Synchronization and Control: Theory and Applications)*, vol. 44, pp. 882–890, Oct. 1997.
[33] T. L. Carroll and L. M. Pecora, "Synchronizing nonautonomous chaotic circuits," *IEEE Trans. Circuits Syst. II, (Special Issue on Chaos in Nonlinear Electronic Circuits—Part C: Applications)*, vol. 40, pp. 646–650, Oct. 1993.
[34] G. Kolumbán, M. P. Kennedy, and L. O. Chua, "The role of synchronization in digital communication using chaos—Part III: Performance evaluation," *IEEE Trans. Circuits Syst. I*, submitted for publication.

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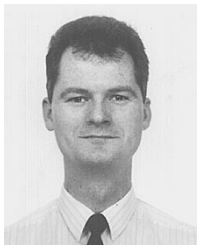

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