Pakrashi, V., O'Connor, A.J., Basu B., 2008. Importance of Inertial Effects in Damaged Bridge Moving Load Interaction. In: Cannon, E., West, R. (ed.s) *Proceedings BRI08/CRI08 Joint Symposium: Bridge Research in Ireland 2008/Concrete Research in Ireland 2008*. Galway, Ireland 4 - 5 Dec 2008

CRITICAL SPEEDS IN DAMAGED BEAM – MOVING OSCILLATOR INTERACTION: THE IMPORTANCE OF INERTIAL EFFECTS

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Abstract
This paper investigates the inertial effects of the passage of a moving oscillator over a single span simply supported damaged Euler Bernoulli beam for a range of speeds of the moving oscillator and a range of damage conditions. It is observed that the variation of critical speeds due to damage forms a distribution only in the high speed range and such variation is masked by inertial effects of the moving oscillator in the comparatively lower speed range. The damage is modeled as an open crack and a rotational spring analogy is followed whereby it is assumed that the effect of the crack is local on the beam. The moving oscillator is modeled as a two degree of freedom system, where each degree of freedom is characterized by a mass and a spring – dashpot element. The finding is useful for engineers in terms of identification of speed regions of interest in a bridge – vehicle interaction process and for the choice of control systems to suppress such vibrations.

Keywords: Bridge – Vehicle Interaction, Critical Speed, Open Crack, Moving Oscillator, Rotational Spring Model, Damage

1. General

The use of a beam – moving oscillator interaction model is often sufficient when qualitative or phenomenological aspects of bridge – vehicle interaction are to be studied (Green and Cebon, 1994; Fryba, 1996; Billelo et.al, 2004). The absolute maximum values of the responses of a bridge traversed by a vehicle for a range of speed of the vehicle are commonly represented in terms of dynamic amplification factor (DAF) of the most critical point on the bridge (for example, the mid – span for a simply supported bridge in many cases) versus the speed of the traversing vehicle. This representation shows the absolute maximum values of the dynamic displacements and hence the dynamic stresses over a range of vehicle speeds. Frequency matching phenomena can also be isolated by identifying the local maxima values on the DAF versus vehicle velocity curve. The global maximum value selected from the curve is important as it relates to the absolute maximum of the dynamic stresses under expected operating conditions of the bridge. The speeds corresponding to the local maxima of the responses can be defined as critical speeds. The allowable speeds of vehicles over bridges have risen significantly over years and there is a trend of opting for high speed trains and vehicle convoys (Li et.al, 2005).

Velocity and acceleration responses of the bridge and the vehicle have been related to serviceability aspects like riding comfort (Wu et.al, 2003) and the possible stress increase on vehicle suspension (Debrunner and Ta, 2004). The as built condition of the bridge can gradually and significantly change over time. This deterioration may create a shift or a drift of the critical speeds associated with the responses of the bridge or the vehicle. It is important to anticipate the nature and the envelope of the shifts or the spreads of the critical speeds for various damage conditions from numerical experiments. These simulation based experiments, even on simple models, can determine whether the critical speeds do change significantly. The effect of damage depends both on its extent and its position – a larger extent located near the support can often be less influential than a smaller damage near the midspan of a simply supported beam. Once regions of vehicle speeds that are most affected are identified for a real structure using a sufficiently detailed model, they can also be further updated using structural inspection or health monitoring information.

This paper attempts to extract information regarding the variation of critical speeds from a damaged beam – moving oscillator interaction. A simply supported beam with a lumped crack has been considered in this regard. A two degree of freedom (quarter car) moving oscillator is considered to traverse the beam. A wide range of damage locations and extents are considered for a range of vehicle speeds and the maximum responses of the displacement, velocity and the acceleration of the beam and the moving oscillator are obtained. The critical speeds corresponding to each of the responses are identified and their distributions for a range of damage locations and extents are investigated. The findings are valuable to engineers in identifying the envelope of regions of vehicle speeds of interest for the purpose of design and for the development of vibration control strategies.

2. Damage Model

A lumped crack model is used in this paper as damage. The lumped crack model is a localized damaged model where a rotational spring analogy is used to model the lumped crack. This rotational spring analogy is a good substitute to other more detailed and sophisticated continuous crack models. A simply supported beam of length L with an open crack located at a distance of ‘a’ from the left hand support of the beam is modelled as two uncracked beams connected through a rotational spring at the location of the crack. The crack depth is taken as ‘c’ and the overall depth of the beam is ‘h’. The crack depth ratio (CDR) is defined as c/h and is a measure of the damage extent. The general solution of the modeshape ($\Phi(.)$) from the free vibration equation of the damaged beam can be expressed as

$$\Phi_L = C_{1L}\mathrm{Sin}(\lambda x) + C_{2L}\mathrm{Cos}(\lambda x) + C_{3L}\mathrm{Sinh}(\lambda x) + C_{4L}\mathrm{Cosh}(\lambda x) \qquad 0 \le x < a \tag{1}$$

and

$$\Phi_R = C_{1R}\mathrm{Sin}(\lambda x) + C_{2R}\mathrm{Cos}(\lambda x) + C_{3R}\mathrm{Sinh}(\lambda x) + C_{4R}\mathrm{Cosh}(\lambda x) \qquad a \le x \le L \tag{2}$$

for the sub-beams on the left and the right side of the rotational spring respectively. The distance from the left hand support of the beam is denoted as ‘x’. The terms C(.) are integration constants arising from the solution of the separated fourth order partial differential equation of free vibration of beam in space. The term λ is expressed as

$$\lambda = \left(\frac{\rho A \omega^2}{EI}\right)^{1/4} \tag{3}$$

where the natural frequency of the cracked beam is ω. The symbols ρ, A, E and I refer to the density of the material of the beam, the cross sectional area, the Young's modulus of the material of the beam and the moment of inertia of the beam respectively. Displacements and the moments at the two supports of the beam are zero and continuity in displacement, moment and shear at the location of crack exists. A slope discontinuity is present at the location of the crack location of the crack as

$$\Phi_R'(a) - \Phi_L'(a) = \theta L \Phi_R''(a) \tag{4}$$

where θ is the non-dimensional crack section flexibility dependent on CDR and can be expressed in terms of a polynomial (Narkis, 1994). The boundary conditions, when substituted in the general modeshape equation, give rise to a set of necessary number of linear equations which can be used to determine the natural frequency of the system by setting the determinant of the linear equation system to zero.

3. Damaged Beam – moving Oscillator Interaction Model

A simply supported Euler Bernoulli beam with an open crack is traversed at a speed u_0 by a vehicle modelled as a quarter car consisting of two degrees of freedom. The quarter car comprises of masses corresponding to the lower and the upper degrees of freedom and these are represented as m_w and m_b respectively. A spring – damper assembly consisting of two sets of springs (k_b, k_w) and dampers (c_b, c_w) model the suspension system of vehicle while the angular movements of the vehicle are neglected. Contact loss, bouncing, impact effects and surface roughness are not taken into account to isolate the effects of damage alone. Figure 1 shows the schematic of a damaged beam – moving oscillator interaction.

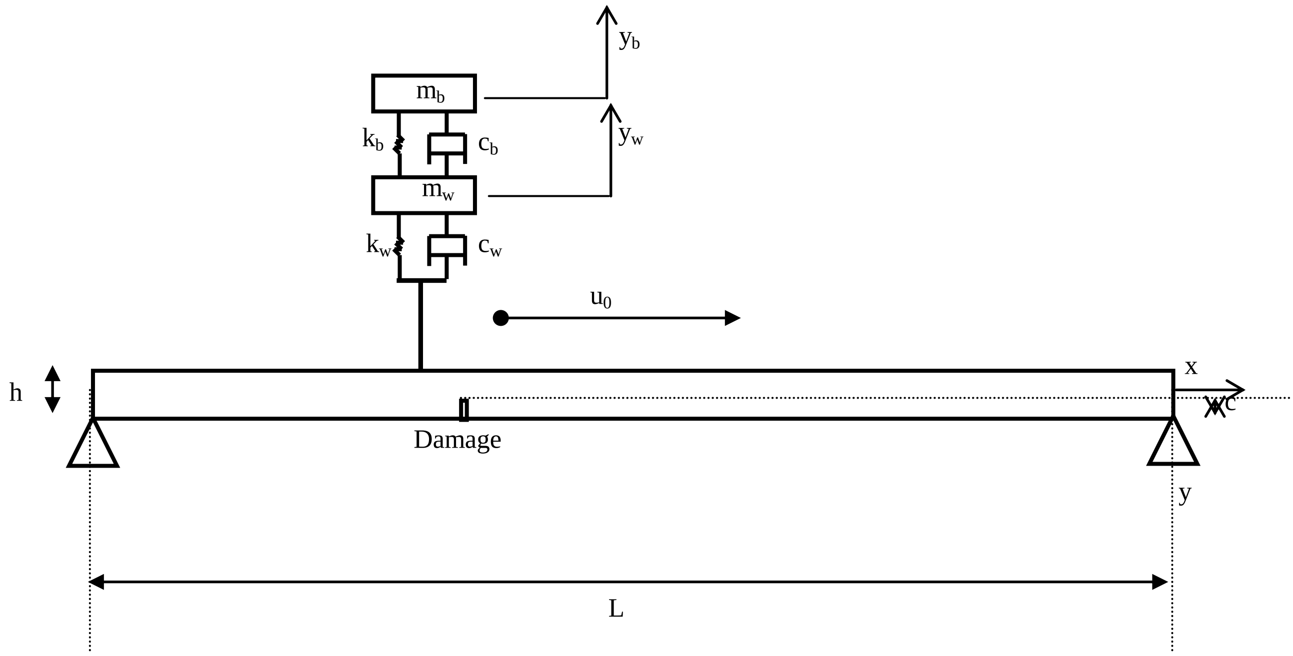

Figure 1 –Damaged Beam – Moving Oscillator Interaction

The dynamic equilibrium conditions for the degrees of freedom along the displacement directions y_b and y_w (representing the upper and the lower degrees of freedom respectively) yield

$$m_b\ddot{y}_b + c_b(\dot{y}_b - \dot{y}_w) + k_b(y_b - y_w) = 0 \tag{5}$$

$$m_b\ddot{y}_b + m_w\ddot{y}_w + c_w(\dot{y}_w + \dot{y}) + k_w(y_w + y) = 0 \tag{6}$$

respectively where y is the dynamic displacement of the beam. The overdots in equations 1, 2 and 3 are the derivatives with respect to time. The partial differential equation for the forced vibration of the beam is

$$EI\frac{\partial^4 y(x,t)}{\partial x^4} + \hat{c}\frac{\partial y(x,t)}{\partial t} + \rho A\frac{\partial^2 y(x,t)}{\partial t^2} = F_p(t) \tag{7}$$

where

$$F_P(t) = (m_b\ddot{y}_b(t) + m_w\ddot{y}_w(t) + (m_b + m_w)g)\hat{\delta}(x - u_0 t) \tag{8}$$

the acceleration due to gravity being g and $\hat{\delta}$ being the Dirac Delta function. The coefficient of damping of the beam is represented as $\hat{c}$. The dynamic displacement of the beam can be resolved into any number of orthogonal modeshapes using the standard technique of separation of variables yielding a second order differential equation in time corresponding to each modeshape as

$$\ddot{q}_j(t) + 2\xi_j\omega_j\dot{q}_j(t) + \omega_j^2 q_j(t) = R_j(t) \tag{9}$$

where ω_j denotes the natural frequency and ξ_j denotes the damping ratio of the beam for the j^{th} mode. The forcing function Rj(t) is

$$R_j(t) = \frac{1}{\rho A\hat{K}}\{(m_w\ddot{y}_w + m_b\ddot{y}_b + (m_w + m_b)g\}\Phi_j(u_0 t)\} \tag{10}$$

where the constant $\hat{K}$ is defined as the integral of the squared modeshape over the length. The vertical acceleration of the vehicle degrees of freedom and the effects of damage both enter into the dynamic loading of the beam. The inertial component of the vehicle in the forcing term in the form of vertical accelerations render the mass matrix of the system of equations so formed, time dependent. This can produce several local maxima in the lower speed regions and it is required to find out how the presence of damage affects the spread if those speeds corresponding to the local maxima. This effect cannot be captured by a closed form response ignoring the inertial effects.

4. Results

The damaged beam – moving oscillator interaction is simulated for a velocity range from 0 to 300 km/hr (0.1 km/hr increment), crack depth ratio range from 0 to 0.35 (0.05 increment) and position (fourteen equally spaced locations from the support to midspan). The details of the values of various parameters involved are provided is Table 1 following Jo et.al. (2001). The beam and the two degrees of the freedom of the oscillator are assumed have zero initial conditions.

Table 1 – Simulation Parameters

Oscillator	$\mathbf{m_b}$=5000 kg $\mathbf{m_w}$=35000 kg	$\mathbf{c_b}$=6x10^4 N-s/m $\mathbf{c_w}$=6x10^4 N-s/m	$\mathbf{k_b}$=5.1x10^6 N/m $\mathbf{k_w}$=9.6x10^6 N/m
Beam	**L**=45m	**EI**=52.5 x10^9 N-m^2	ξ_j = 3%

Figure 2 presents the histograms of the variation of the critical speeds associated with the vertical displacement, velocity and acceleration responses of the beam and the two degrees of freedom of the moving oscillator for a rotational spring model considering the entire damage range. The effect of inertia in the loading term on the critical speeds is apparent. The effect is manifest in the histogram through the presence of closely spaced, dense presence of local maxima values in the lower speed region. The histograms are so closely spaced that a sharp and concentrated region of interest is not present. The lower speed range of the oscillator is observed to be generating a nearly continuous region of interest due to the formation of closely spaced maxima values brought about by the inertial component of the oscillator excitation. Thus, for lower speed regions of the traversing oscillator, the definition of critical speed corresponding to maxima values of bridge or oscillator responses is not helpful. For such range of traversing speeds, it is more important to set a preselected limiting value for the response and then consider the traversing speed range of the oscillator corresponding to all responses exceeding that preselected response value to be of region of interest. The low traversing speed region must be interpreted as a relative value with respect to the speed corresponding to the global maximum of the responses.

The critical speed range corresponding to the absolute global maximum responses is clustered separate from those corresponding to low traversing speeds of the oscillator. They are observed to be varying significantly. These clusters are narrow and well defined. Consequently, the definition of critical speeds related with the identification of a maximum of the response is valid. The location of these clusters shift according to the type of response. The choice for a vibration control system for the suppression of the global response should be directed towards suppressing those clustered critical speed ranges. Since the spread of the critical speeds are significant potential robust control systems should cater for the anticipated spread.

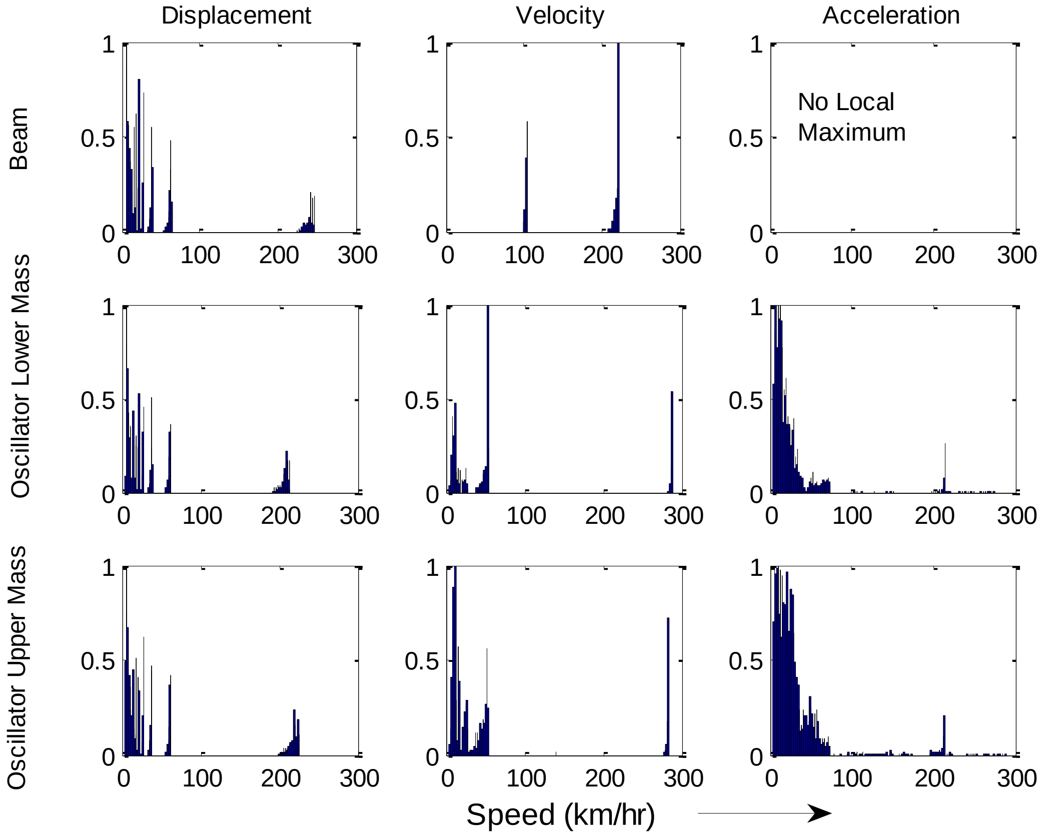

Figure 2 - Normalized Responses of the Beam for a Range of Damage Conditions versus the Speed of the Traversing Oscillator

5. Conclusions

The variation of critical vehicle speeds for a damaged beam – moving oscillator interaction is presented. It is observed, that at lower speed ranges the inertial effects of the oscillator governs and thus the standard definition of critical speed based on formation of a local maximum of a response is not useful. The exceedence of a preselected value of a response for a traversing speed range is more critical here. The critical speeds corresponding to the absolute global maxima of responses form well defined clusters of their own in the high traversing speed range and undergo significant variation due to the presence of damage. This duality of definition is important since many bridge structures are traversed by vehicles below a certain maximum allowable speed.

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